

### ΤΥΠΟΛΟΓΙΟ

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \quad \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{\nabla P}{\rho} + \frac{\mathbf{f}}{\rho} + \frac{\eta + 3\eta_\tau}{3\rho} \nabla(\nabla \cdot \mathbf{u}), \quad \frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla \cdot \boldsymbol{\Pi} = \mathbf{f}, \quad \frac{\mathbf{f}}{\rho} = -\nabla \Phi_g - 2\boldsymbol{\Omega}_\oplus \times \mathbf{u},$$

$$\Pi_{ij} = \rho u_i u_j + P \delta_{ij} - \sigma'_{ij}, \quad \sigma'_{ij} = \eta \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \nabla \cdot \mathbf{u} \delta_{ij} \right) + \eta_\tau \nabla \cdot \mathbf{u} \delta_{ij}, \quad (\mathbf{u} \cdot \nabla) \mathbf{u} = \nabla \left( \frac{u^2}{2} \right) + \boldsymbol{\zeta} \times \mathbf{u} \dot{\mathbf{n}} u \frac{\partial u}{\partial \ell} \hat{\varepsilon} + \frac{u^2}{\mathcal{R}} \hat{\mathbf{n}}.$$

$$\text{Καρτεσιανές: } d\mathbf{r} = dx\hat{x} + dy\hat{y} + dz\hat{z}, \quad \nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}, \quad \nabla \cdot \mathbf{u} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z},$$

$$\nabla \times \mathbf{u} = \left( \frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z} \right) \hat{x} + \left( \frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x} \right) \hat{y} + \left( \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} \right) \hat{z}, \quad \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}.$$

$$\text{Κυλινδρικές } (x = \varpi \cos \phi, \quad y = \varpi \sin \phi, \quad \hat{x} = \hat{\varpi} \cos \phi - \hat{\phi} \sin \phi, \quad \hat{y} = \hat{\varpi} \sin \phi + \hat{\phi} \cos \phi): \quad d\mathbf{r} = d\varpi \hat{\varpi} + \varpi d\phi \hat{\phi} + dz \hat{z},$$

$$\nabla f = \frac{\partial f}{\partial \varpi} \hat{\varpi} + \frac{1}{\varpi} \frac{\partial f}{\partial \phi} \hat{\phi} + \frac{\partial f}{\partial z} \hat{z}, \quad \nabla \cdot \mathbf{u} = \frac{1}{\varpi} \frac{\partial(\varpi u_\varpi)}{\partial \varpi} + \frac{1}{\varpi} \frac{\partial u_\phi}{\partial \phi} + \frac{\partial u_z}{\partial z}, \quad \nabla \times \mathbf{u} = \left( \frac{1}{\varpi} \frac{\partial u_z}{\partial \phi} - \frac{\partial u_\phi}{\partial z} \right) \hat{\varpi} + \left( \frac{\partial u_\varpi}{\partial z} - \frac{\partial u_z}{\partial \varpi} \right) \hat{\phi} +$$

$$\frac{1}{\varpi} \left[ \frac{\partial(\varpi u_\phi)}{\partial \varpi} - \frac{\partial u_\varpi}{\partial \phi} \right] \hat{z}, \quad \nabla^2 f = \frac{1}{\varpi} \frac{\partial}{\partial \varpi} \left( \varpi \frac{\partial f}{\partial \varpi} \right) + \frac{1}{\varpi^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}, \quad \frac{\partial u_\varpi}{\partial t} + \mathbf{u} \cdot \nabla u_\varpi - \frac{u_\varpi^2}{\varpi} = \frac{f_\varpi}{\rho} - \frac{1}{\rho \varpi} \frac{\partial P}{\partial \phi} + \frac{\eta}{\rho} \left( \nabla^2 u_\varpi + \frac{2}{\varpi^2} \frac{\partial u_\varpi}{\partial \phi} - \frac{u_\varphi^2}{\varpi^2} \right), \quad \frac{\partial u_z}{\partial t} + \mathbf{u} \cdot \nabla u_z = \frac{f_z}{\rho} - \frac{1}{\rho} \frac{\partial P}{\partial z} + \frac{\eta}{\rho} \nabla^2 u_z,$$

$$\sigma'_{\varpi \varpi} = 2\eta \frac{\partial u_\varpi}{\partial \varpi}, \quad \sigma'_{\varpi \phi} = \sigma'_{\phi \varpi} = \frac{\eta}{\varpi} \left[ \frac{\partial u_\varpi}{\partial \phi} + \varpi^2 \frac{\partial}{\partial \varpi} \left( \frac{u_\phi}{\varpi} \right) \right], \quad \sigma'_{\varpi z} = \sigma'_{z \varpi} = \eta \left( \frac{\partial u_\varpi}{\partial z} + \frac{\partial u_z}{\partial \varpi} \right),$$

$$\sigma'_{\phi \phi} = \frac{2\eta}{\varpi} \left( u_\varpi + \frac{\partial u_\phi}{\partial \phi} \right), \quad \sigma'_{\phi z} = \sigma'_{z \phi} = \eta \left( \frac{\partial u_\phi}{\partial z} + \frac{1}{\varpi} \frac{\partial u_z}{\partial \phi} \right), \quad \sigma'_{z z} = 2\eta \frac{\partial u_z}{\partial z}.$$

$$\text{Σφαιρικές } (x = r \sin \theta \cos \phi, \quad y = r \sin \theta \sin \phi, \quad z = r \cos \theta, \quad \hat{z} = \hat{r} \cos \theta - \hat{\theta} \sin \theta): \quad d\mathbf{r} = dr \hat{r} + rd\theta \hat{\theta} + r \sin \theta d\phi \hat{\phi},$$

$$\nabla f = \frac{\partial f}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \hat{\phi}, \quad \nabla \cdot \mathbf{u} = \frac{1}{r^2} \frac{\partial(r^2 u_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(u_\theta \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial u_\phi}{\partial \phi}, \quad \nabla \times \mathbf{u} = \frac{1}{r \sin \theta} \left[ \frac{\partial(u_\phi \sin \theta)}{\partial \theta} - \frac{\partial u_\theta}{\partial \phi} \right] \hat{r} +$$

$$\frac{1}{r} \left[ \frac{1}{\sin \theta} \frac{\partial u_r}{\partial \phi} - \frac{\partial(r u_\phi)}{\partial r} \right] \hat{\theta} + \frac{1}{r} \left[ \frac{\partial(r u_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta} \right] \hat{\phi}, \quad \nabla^2 f = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2},$$

$$\frac{\partial u_r}{\partial t} + \mathbf{u} \cdot \nabla u_r - \frac{u_\theta^2 + u_\phi^2}{r} = \frac{f_r}{\rho} - \frac{1}{\rho} \frac{\partial P}{\partial r} + \frac{\eta}{\rho} \left[ \nabla^2 u_r - 2 \frac{u_r}{r^2} - \frac{2}{r^2 \sin \theta} \frac{\partial(u_\theta \sin \theta)}{\partial \theta} - \frac{2}{r^2 \sin \theta} \frac{\partial u_\phi}{\partial \phi} \right],$$

$$\frac{\partial u_\theta}{\partial t} + \mathbf{u} \cdot \nabla u_\theta + \frac{u_r u_\theta}{r} - \frac{u_\phi^2 \cos \theta}{r \sin \theta} = \frac{f_\theta}{\rho} - \frac{1}{\rho r} \frac{\partial P}{\partial \theta} + \frac{\eta}{\rho} \left( \nabla^2 u_\theta + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r^2 \sin^2 \theta} - \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial u_\phi}{\partial \phi} \right),$$

$$\frac{\partial u_\phi}{\partial t} + \mathbf{u} \cdot \nabla u_\phi + \frac{u_r u_\phi}{r} + \frac{u_\theta u_\phi \cos \theta}{r \sin \theta} = \frac{f_\phi}{\rho} - \frac{1}{\rho r \sin \theta} \frac{\partial P}{\partial \phi} + \frac{\eta}{\rho} \left( \nabla^2 u_\phi + \frac{2}{r^2 \sin \theta} \frac{\partial u_r}{\partial \phi} + \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial u_\theta}{\partial \phi} - \frac{u_\phi}{r^2 \sin^2 \theta} \right),$$

$$\sigma'_{rr} = 2\eta \frac{\partial u_r}{\partial r}, \quad \sigma'_{r\theta} = \sigma'_{\theta r} = \frac{\eta}{r} \left[ \frac{\partial u_r}{\partial \theta} + r^2 \frac{\partial}{\partial r} \left( \frac{u_\theta}{r} \right) \right], \quad \sigma'_{r\phi} = \sigma'_{\phi r} = \frac{\eta}{r \sin \theta} \left[ \frac{\partial u_r}{\partial \phi} + r^2 \sin \theta \frac{\partial}{\partial r} \left( \frac{u_\phi}{r} \right) \right],$$

$$\sigma'_{\theta\theta} = \frac{2\eta}{r} \left( u_r + \frac{\partial u_\theta}{\partial \theta} \right), \quad \sigma'_{\theta\phi} = \sigma'_{\phi\theta} = \frac{\eta}{r \sin \theta} \left[ \frac{\partial u_\theta}{\partial \phi} + \sin^2 \theta \frac{\partial}{\partial \theta} \left( \frac{u_\phi}{\sin \theta} \right) \right], \quad \sigma'_{\phi\phi} = \frac{2\eta}{r \sin \theta} \left( u_r \sin \theta + u_\theta \cos \theta + \frac{\partial u_\phi}{\partial \phi} \right).$$

$$\frac{\partial}{\partial t} \left( \frac{\rho \mathbf{u}^2}{2} + \rho e \right) + \nabla \cdot \left( \frac{\rho \mathbf{u}^2}{2} \mathbf{u} + \rho h \mathbf{u} - \boldsymbol{\sigma}' \cdot \mathbf{u} \right) = \mathbf{f} \cdot \mathbf{u} + \rho q, \quad \frac{de}{dt} + P \frac{d}{dt} \left( \frac{1}{\rho} \right) = \frac{\rho q + \boldsymbol{\sigma}' : \boldsymbol{S}}{\rho}, \quad e = C_V T \dot{\mathbf{n}} \frac{1}{\gamma - 1} \frac{P}{\rho}, \quad h = e + \frac{P}{\rho},$$

$$\frac{\mathbf{u}^2}{2} + h + \Phi_g = \mathcal{E}, \quad -\frac{\partial \Phi}{\partial t} + \frac{|\nabla \Phi|^2}{2} + h + \Phi_g = f(t),$$

$$\mathbf{u} = -\nabla \Phi, \quad \mathbf{u} = \nabla \Psi(x, y) \times \hat{z}, \quad \mathbf{u} = \nabla \Psi(\varpi, \phi) \times \hat{z}, \quad \mathbf{u} = \frac{\nabla \Psi(\varpi, z) \times \hat{\phi}}{\varpi}, \quad \mathbf{u} = \frac{\nabla \Psi(r, \theta) \times \hat{\phi}}{r \sin \theta},$$

$$\nabla^2 \left[ a_0 + b_0 \ln \varpi + c_0 \phi + \sum_{m=1}^{\infty} \left( a_m \varpi^m + \frac{b_m}{\varpi^m} \right) \cos(m\phi) + \sum_{m=1}^{\infty} \left( c_m \varpi^m + \frac{d_m}{\varpi^m} \right) \sin(m\phi) \right] = 0,$$

$$\nabla^2 \left[ \sum_{\ell=0}^{\infty} \left( a_\ell r^\ell + \frac{b_\ell}{r^{\ell+1}} \right) P_\ell(\cos \theta) \right] = 0, \quad P_0(\cos \theta) = 1, \quad P_1(\cos \theta) = \cos \theta, \quad P_2(\cos \theta) = \frac{3 \cos^2 \theta - 1}{2},$$

$$\omega = ck, \quad c = \sqrt{\frac{\gamma P}{\rho}}, \quad [\rho u_n] = 0, \quad [\rho u_n^2 + P] = 0, \quad [\rho u_n \mathbf{u}_t] = 0, \quad \left[ \frac{\rho u^2}{2} u_n + \frac{\gamma}{\gamma - 1} P u_n \right] = 0, \quad \omega = \omega_{co} + \mathbf{U} \cdot \mathbf{k},$$

$$\omega = \sqrt{gk \tanh(kH)} \approx \begin{cases} \sqrt{gk} & , \\ k \sqrt{gH} & , \end{cases} \quad \omega = \frac{\rho \mathbf{k} \cdot \mathbf{U}}{\rho + \rho'} \pm \sqrt{\frac{\rho - \rho'}{\rho + \rho'} gk - \frac{\rho \rho' (\mathbf{k} \cdot \mathbf{U})^2}{(\rho + \rho')^2}}, \quad \omega = \sqrt{f^2 + gHk^2}, \quad \omega = \frac{-\beta_0 R^2 k_x}{k^2 R^2 + 1}, \quad \omega = \pm N \frac{k_x}{k},$$

$$R = \frac{\sqrt{gH}}{f}, \quad R' = \frac{\sqrt{g'H}}{f}, \quad g' = \frac{\Delta \rho}{\rho_0} g, \quad N = \sqrt{-\frac{g}{\rho_0} \frac{d\rho}{dz}}, \quad f \approx f_0 + \beta_0 y, \quad f_0 = 2\Omega_\oplus \sin \varphi_0, \quad \beta_0 = \frac{2\Omega_\oplus \cos \varphi_0}{R_\oplus},$$

$$\mathbf{u}_H = \mathbf{u}_g \left( 1 - e^{-z/d} \cos \frac{z}{d} \right) + \hat{z} \times \mathbf{u}_g e^{-z/d} \sin \frac{z}{d} = -\frac{\nabla_H P \times \hat{z}}{f \rho_0} \left( 1 - e^{-z/d} \cos \frac{z}{d} \right) - \frac{\nabla_H P}{f \rho_0} e^{-z/d} \sin \frac{z}{d}, \quad d = \sqrt{\frac{2\mathcal{A}_V}{f}}, \quad \mathcal{A}_V = \frac{\eta}{\rho_0},$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -g \frac{\partial \xi}{\partial x} + fv, \quad \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -g \frac{\partial \xi}{\partial y} - fu, \quad \frac{\partial \xi}{\partial t} + \frac{\partial [(H + \xi - b)u]}{\partial x} + \frac{\partial [(H + \xi - b)v]}{\partial y} = 0,$$

$$w = u \frac{\partial b}{\partial x} + v \frac{\partial b}{\partial y} - \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) (H + z - b), \quad P = P_a - \rho_0 g z + \rho_0 g \xi, \quad h = H + \xi - b, \quad q = \frac{f + \zeta_z}{h},$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -g' \frac{\partial h}{\partial x} + fv, \quad \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -g' \frac{\partial h}{\partial y} - fu, \quad \frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = 0,$$

$$g = 9.8 \text{ m s}^{-2}, \quad R_\oplus = 6371 \text{ km}, \quad \Omega_\oplus = 7.3 \times 10^{-5} \text{ rad s}^{-1}, \quad \rho_{atm} = 1.2 \text{ kg m}^{-3}, \quad \rho_{H_2O} \approx 1000 \text{ kg m}^{-3}, \quad P_{atm} \approx 10^5 \text{ N m}^{-2},$$

$$m_p = 1.67 \times 10^{-27} \text{ kg}, \quad k_B = 1.38 \times 10^{-23} \text{ J/K}.$$