Δευτέρα 11-3 Διάλεξη 3

έναρξη στις 13:15

- ένα θεώρημα
- αρχή στάσιμης δράσης ϗ κβαντομηχανική

Έστω $\vec{f}(t), \vec{n}(t)$ συναρτήσεις για τις οποίες ισχύει

$$\int_{t_1}^{t_2} \vec{f}(t)\cdot\vec{n}(t)\,dt = 0 \qquad \forall\, \vec{n}(t) : \vec{n}(t_1) = \vec{n}(t_2) = \vec{0}$$

Δσό : $\vec{f}(t) = \vec{0}$ ϗ μόνο αυτή απόδειξη:

Έστω $\vec{f}(t) \neq \vec{0} \Rightarrow \exists\, [\tau_1, \tau_2] \in [t_1, t_2]$:

$$\vec{f}(t) \neq \vec{0} \quad \forall\, t \in [\tau_1, \tau_2]$$

$$f_i(t) > 0 \quad \forall\, t \in \ ,,$$

i : κάποια συνιστώσα της $\vec{f}$

Επιλέγω $n_\alpha(t) = \begin{cases} 0 & \alpha \neq i \quad \forall\, t \\ 0 & \alpha = i \quad t \notin (\tau_1, \tau_2) \\ > 0 & \alpha = i \quad t \in (\tau_1, \tau_2) \end{cases}$

$$\vec{f} \cdot \vec{n} = f_i n_i = \begin{cases} 0 & t \notin [\tau_1, \tau_2] \\ > 0 & t \in [\tau_1, \tau_2] \end{cases}$$

$$\int_{t_1}^{t_2} \vec{f} \cdot \vec{n}\, dt > 0 \quad \text{άτοπο} \Rightarrow \vec{f}(t) = \vec{0}$$

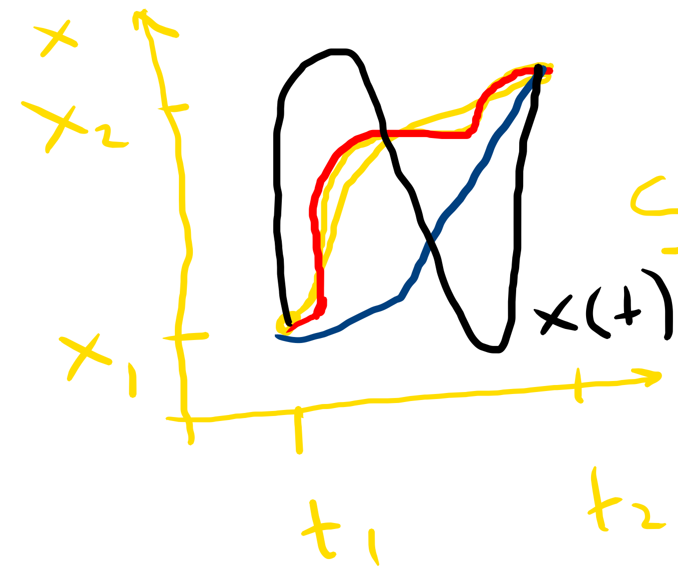
$$\int_{t_1}^{t_2} f(t)\,\dot{n}(t)\,dt = 0 \qquad \forall n(t) \Rightarrow ?$$

$$n(t_1) = n(t_2) = 0$$

$$\int_{t_1}^{t_2} f(t)\,n(t)\,dt = \tilde{f} \cdot \tilde{n} \qquad \tilde{f} = \begin{bmatrix} f(t_1) \\ f(t_1+\delta) \\ f(t_1+2\delta) \\ \vdots \\ f(t_2) \end{bmatrix}$$

$$\downarrow \; \uparrow \; \delta \to 0$$

$$\delta\left[f(t_1)n(t_1) + f(t_1+\delta)\,n(t_1+\delta) + \dots f(t_2)n(t_2) \right] = f(t_1)\,n(t_1)\,\delta + f(t_1+\delta)\,n(t_1+\delta)\,\delta + \dots + f(t_1+(n-1)\delta)\cdot n(t_1+(n-1)\delta)\,\delta$$



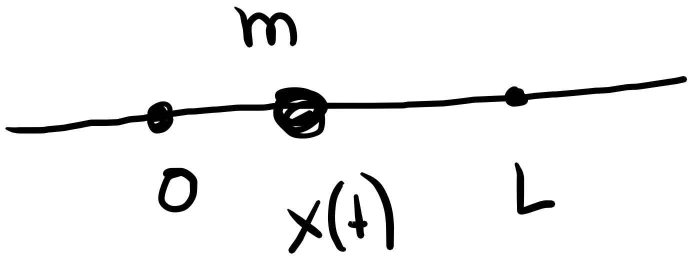
$$S - S \simeq \Theta(\varepsilon^2)$$

$$\int_{t_1}^{t_2} L(t)\,dt = S[x(t)]$$

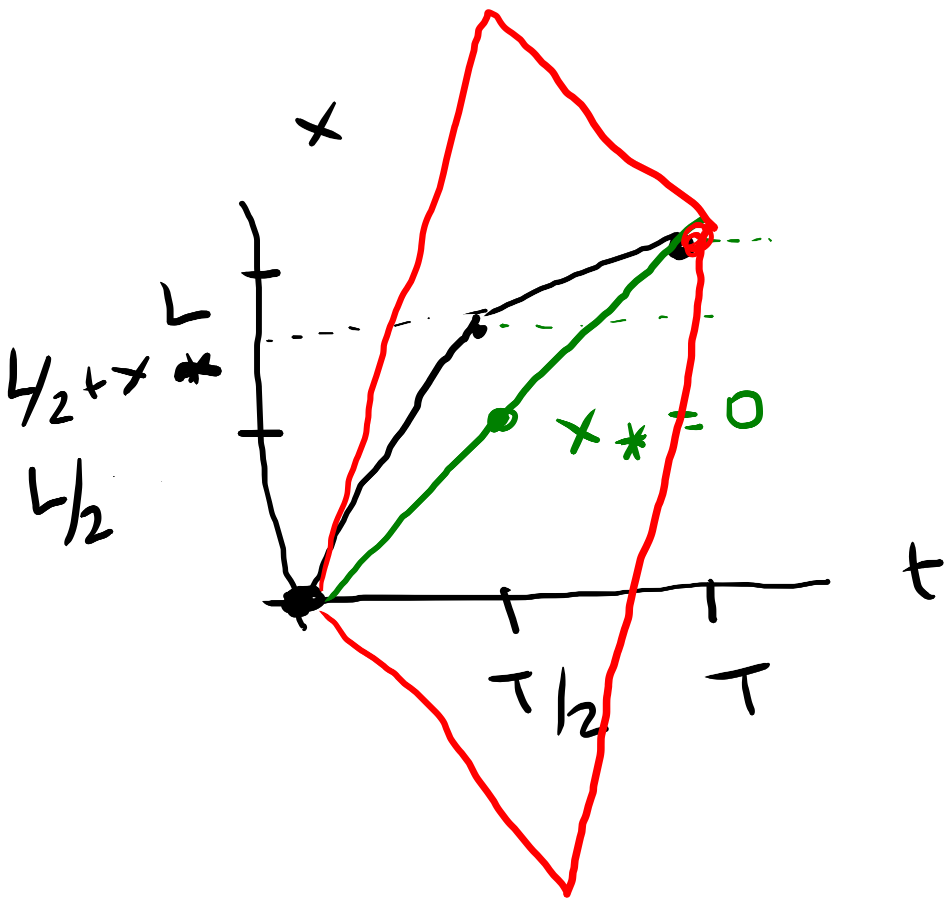
$$\Psi[x(t)] = e^{\frac{i S[x(t)]}{\hbar}}$$

$$\Psi_{\text{πραγμ}}\left(\text{μετάβαση από } x_1, t_1, x_2, t_2\right)$$

$$= \sum_{x(t)} \Psi[x(t)]$$



$$x(0) = 0 \qquad x(T) = L$$



$$S[x(t)] = \frac{1}{2} m \int_0^T \dot{x}^2 \, dt$$

$$= \frac{1}{2} m \left[\left(\frac{x_* + L/2}{T/2} \right)^2 T/2 + \left(\frac{L/2 - x_*}{T/2} \right)^2 \frac{T}{2} \right]$$

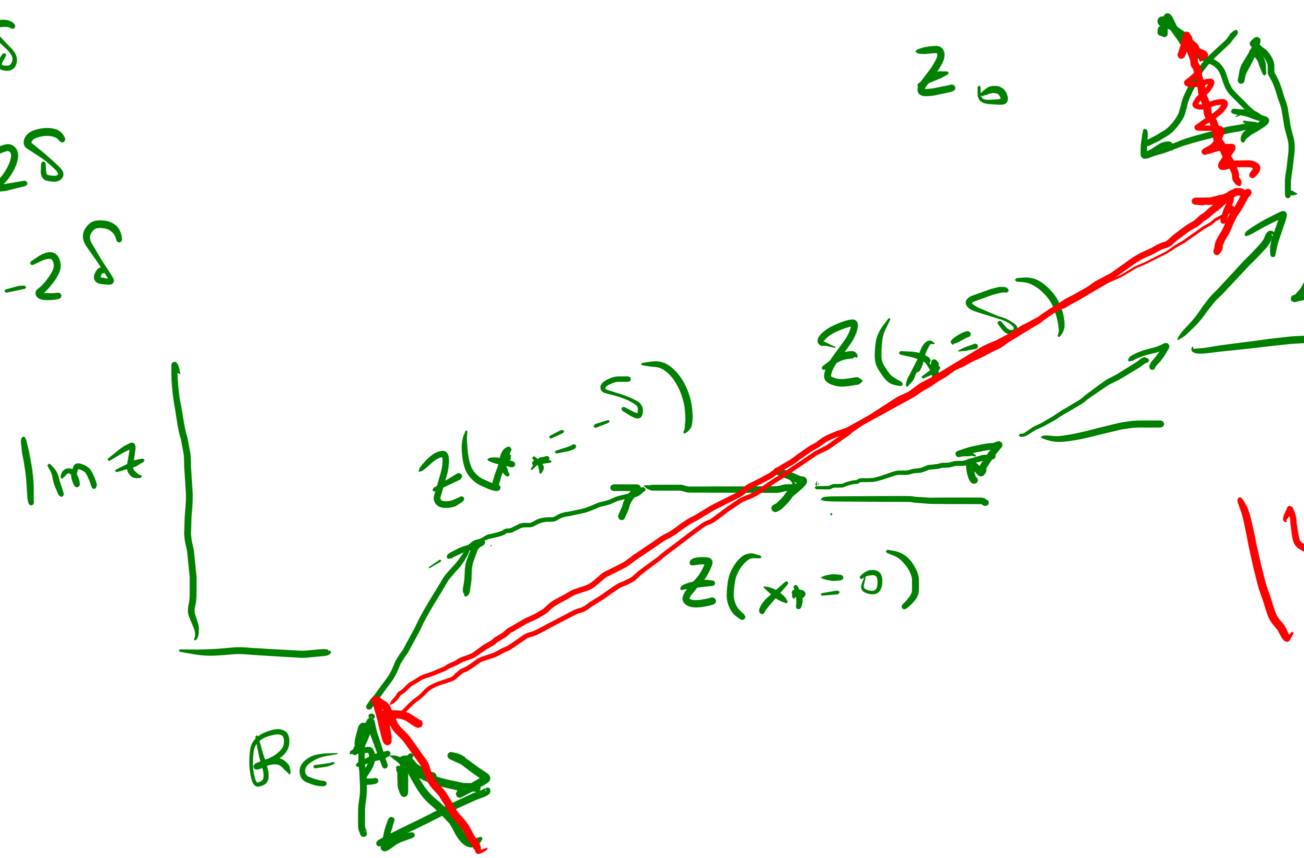
$$= \frac{1}{2} m \left(\frac{x_*^2}{T^2/4} T/2 + \frac{x_*^2}{T^2/4} T/2 + \left(\frac{L}{T} \right)^2 \left(\frac{T}{2} + \frac{T}{2} \right) \right)$$

$$S = \frac{mL^2}{2T} + \frac{2m x_*^2}{T} = S_{cl} + \frac{2m x_*^2}{T}$$

$$\Psi[x(t);x_*] = \underbrace{e^{i\frac{S_{kl}}{\hbar}}}_{z_0}\;\underbrace{e^{i\frac{2m x_+^2}{T\hbar}}}_{z(x_*)}$$

$$e^{i(\alpha+\beta)} = e^{i\alpha}e^{i\beta}$$

$$\sum_{\substack{x_+=0\\ x_+=\delta\\ x_+=-\delta\\ x_+=2\delta\\ x_+=-2\delta}} \Psi[x(t),x_+] = \underbrace{e^{i\frac{S_{kl}}{\hbar}}}_{z_0}\Big(z(x_+=0)+z(x_+=\delta) + z(x_+=-\delta)+\dots\Big)$$



$$\frac{2m x_+^2}{T\hbar}$$

$$|\Psi|^2 \gg |\Phi|^2$$

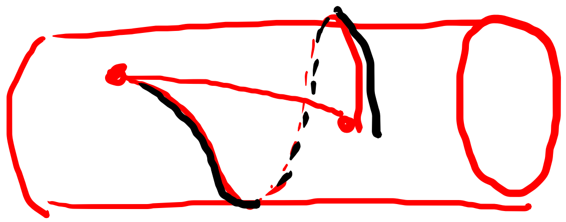
$$\frac{2m x_*^2}{T\hbar} \simeq 1 \Rightarrow x_* \cong \sqrt{\frac{T\hbar}{2m}}$$

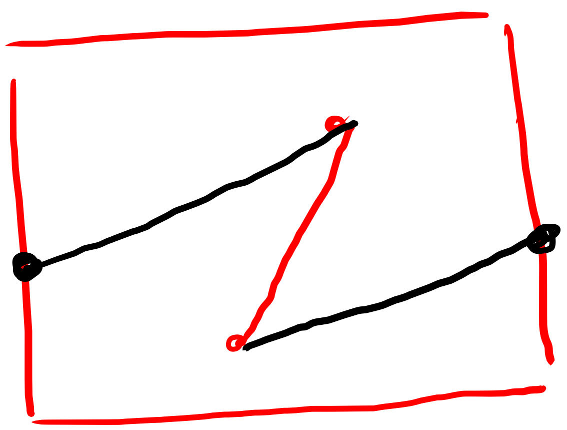
Παίρνουμε με $T = 1\,s$, $L = 1\,m$, $m_e = 10^{-30}\,kg$

$\hbar \simeq 10^{-34}\,kg\,m^2/s$

$$x_* = \sqrt{\frac{1 \cdot 10^{-34}}{2 \cdot 10^{-30}}} \cong 10^{-2}\,m$$

Κλασσικά $m = 1\,kg$ $\quad x_* \simeq \sqrt{\hbar} = 10^{-17}\,m$





$L(x, \dot{x}, t)$ $\qquad$ $x(t)$ $\qquad$ $\dot{x} = \dfrac{dx}{dt}$

διαφορ.

$$S[x(t)] = \int_{t_1}^{t_2} L\, dt$$

Ποια $x(t)$: οδηγεί την S σε "ακρότατο"

σταθερότητα

Ποια $x(t)$

$$S[x(t) + \varepsilon \eta(t)] - S[x(t)] = O(\varepsilon^2) \quad \forall\, \eta(t)$$

$\downarrow$

$O(\varepsilon) = 0$ $\qquad$ $\eta(t_1) = \eta(t_2) = 0$

ταυτοτικά

$$L(x,\dot{x},t) \quad qv \quad x = x + \varepsilon\eta$$

$$f(x+\delta) \cong f(x) + \delta \frac{df}{dx}$$

$$L(x+\varepsilon\eta, \dot{x}+\varepsilon\dot{\eta}, t) = L(x,\dot{x},t) + \varepsilon\left[\frac{\partial L}{\partial x}\eta + \frac{\partial L}{\partial \dot{x}}\dot{\eta}\right] + \varepsilon^2 \cdots\cdots$$

$$S[x+\varepsilon\eta] - S[x] = \underset{\int L(x,\dot{x},t)\,dt - \int L(x,\dot{x},t)\,dt}{\overset{}{0}} + \varepsilon\int_{t_1}^{t_2}\left(\frac{\partial L}{\partial x}\eta + \frac{\partial L}{\partial \dot{x}}\dot{\eta}\right)dt + \varepsilon^2 \cdots$$

שלבה

$$\int_{t_1}^{t_2}\Big(\frac{\partial L}{\partial x}\eta+\underbrace{\frac{\partial L}{\partial \dot x}\dot\eta}_{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot x}\eta\right)-\eta\frac{d}{dt}\left(\frac{\partial L}{\partial \dot x}\right)}\Big)dt=0$$

$$0=\int_{t_1}^{t_2}\left[\frac{\partial L}{\partial x}\eta+\frac{d}{dt}\left(\frac{\partial L}{\partial \dot x}\eta\right)-\eta\frac{d}{dt}\left(\frac{\partial L}{\partial \dot x}\right)\right]dt$$

$$=\underbrace{\cancel{\int_{t_1}^{t_2}d\left(\frac{\partial L}{\partial \dot x}\eta\right)}}_{=0}+\int_{t_1}^{t_2}\eta\left(\frac{\partial L}{\partial x}-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot x}\right)\right)dt$$

$\eta(t_1)=\eta(t_2)$

$\forall\eta(t)$