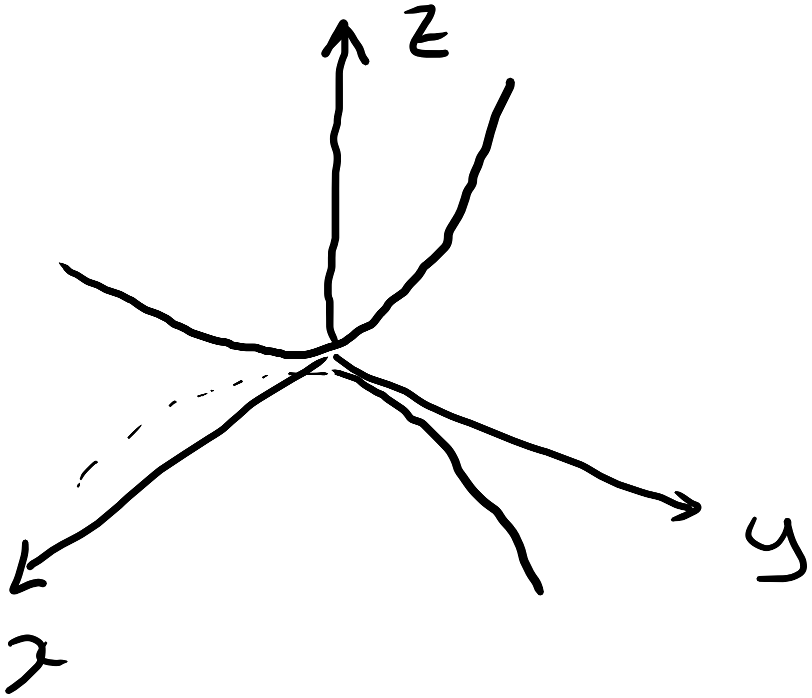
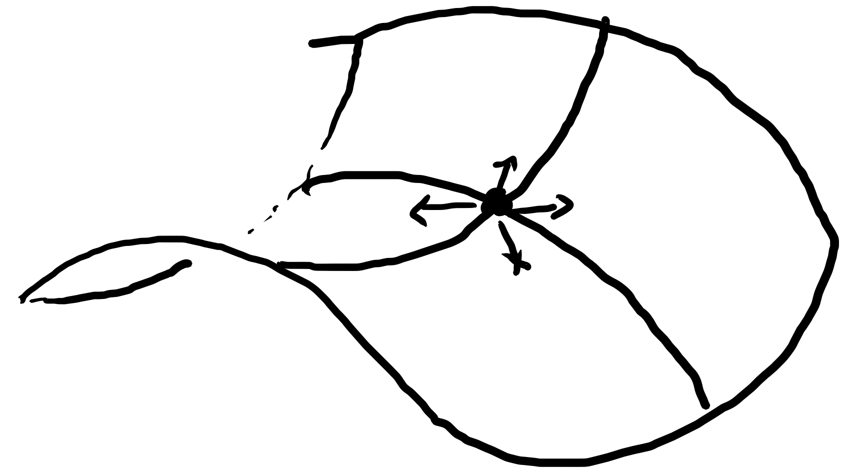
$$z = \frac{x^2}{2} - \frac{y^2}{2}$$

$$\left.\frac{\partial z}{\partial x}\right|_{x_0 y_0} = \left.\frac{\partial z}{\partial y}\right|_{x_0 y_0} = 0 \qquad x_0 = -y_0 = 0$$





$$z_{x_0, y_0} = \text{στάσιμο}$$

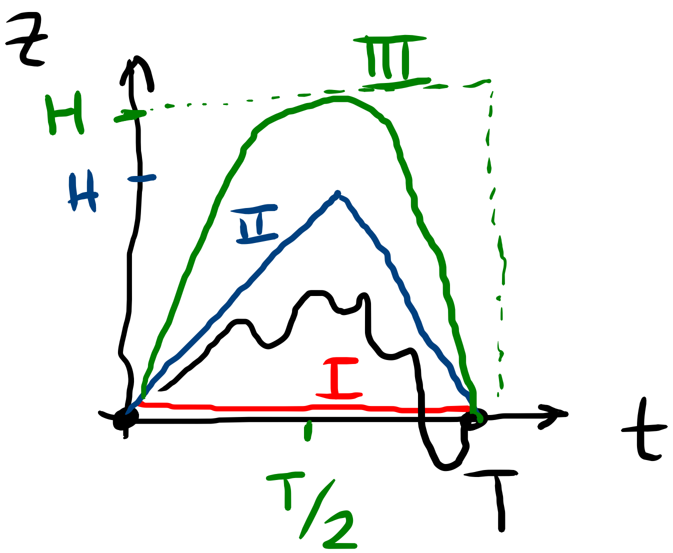
$$z(x \simeq x_0, y \simeq y_0) = \frac{x^2}{2} - \frac{y^2}{2} = \frac{(x_0 + \varepsilon \delta_x)^2}{2} - \frac{(y_0 + \varepsilon \delta_y)^2}{2}$$

$$= \frac{x_0^2}{2} - \frac{y_0^2}{2} + \varepsilon \underbrace{(x_0 \delta x - y_0 \delta y)}_{=0} + O(\varepsilon^2)$$

$$x_0 = y_0 = 0$$

σώματος που κινείται κατακόρ. σε g

$$L = \frac{1}{2} m v^2 - mgz \qquad v = \dot{z}$$



$z = 0 \quad v = 0 \quad L = 0$

$$S_I = \int_0^T 0 \, dt = 0$$

$$z = \begin{cases} \dfrac{Ht}{T/2} & t \leq T/2 \\ H - H\dfrac{(t - T/2)}{T/2} & t > T/2 \end{cases}$$

$$v = \pm \frac{2H}{T}$$

$$L = \frac{1}{2} m \left(\frac{2H}{T}\right)^2 - m g z(t)$$

$$S_{II} = \frac{1}{2} m \frac{4H^2}{T^2} \int_0^T dt - m g \frac{HT}{2}$$

$$= \frac{2mH^2}{T} - m g \frac{HT}{2}$$

$$\frac{dS_{II}}{dH} = 0 = \frac{4mH}{T} - m g \frac{T}{2}$$

$$H = \frac{gT^2}{8}$$

$$= \frac{2m}{T}\left(\frac{gT^2}{8}\right)^2 - m g \frac{T}{2} g \frac{T^2}{8}$$

$$= - \frac{m g^2 T^3}{32}$$

$$z(t) = \frac{t(T-t)}{T^2/4} H \qquad v = \dot{z} = \frac{T-2t}{T^2/4} H$$

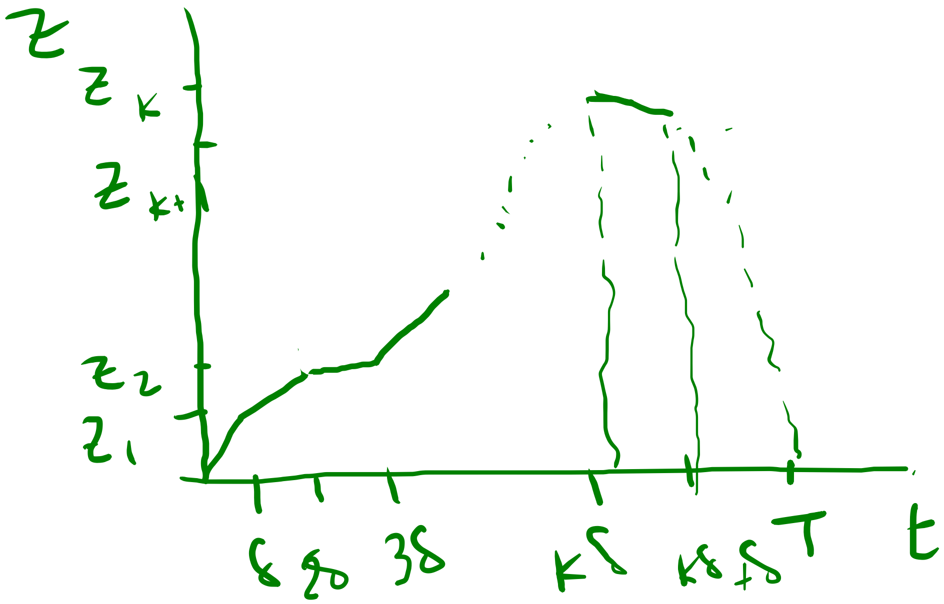
$$S_{III} = \frac{1}{2} m \int_0^T \left(\frac{T-2t}{T^2/4} H\right)^2 dt - mg \underbrace{\int_0^T z(t)\,dt}_{\frac{2}{3}HT}$$

$$= \frac{8m}{T^4} H^2 \int_0^T \underbrace{\left(T^2 - 4tT + 4t^2\right)}_{T^3 - 2T^3 + 4\frac{T^3}{3}} dt - \frac{2}{3} mgHT$$

$$= \frac{8}{3} \frac{mH^2}{T} - \frac{2}{3} mgHT \rightarrow \frac{dS_{III}}{dH} = 0 = \frac{16mH}{3T} - \frac{2}{3} mgT$$

$$\Rightarrow H = g\,T^2/8 \,!$$

$$S_{III_{min}} = -\frac{1}{24} m g^2 \tau^3$$



$$S = \int_0^T dt\, L = \sum_{k=0}^{N-1} S_k$$

$$S_k = \int_{k\delta}^{(k+1)\delta} dt \left\{ \frac{1}{2} m \left(\frac{z_{k+1} - z_k}{\delta} \right)^2 - mg \left[z_k + \frac{z_{k+1} - z_k}{\delta} (t - k\delta) \right] \right\}$$

$$= \frac{1}{2} m \left(\frac{z_{k+1} - z_k}{\delta} \right)^2 - mg\,\delta\, \frac{z_k + z_{k+1}}{2}$$

$$z_N = 0 = z_0$$

$$N\delta = T$$

$$S = S(z_1, z_2, z_3, \ldots, z_{N-1})$$

$S \to$ στάσιμη

$$\frac{\partial S}{\partial z_1} = \frac{\partial S}{\partial z_i} = \cdots = \frac{\partial S}{\partial z_{N-1}} = 0$$

$$\frac{\partial S}{\partial z_\lambda} = \frac{\partial S_n}{\partial z_\lambda} + \frac{\partial S_{\lambda-1}}{\partial z_\lambda} = 0$$

$$\frac{1}{2} m \frac{1}{\delta}\left[-2(z_{\lambda+1} - z_\lambda)\right] + \frac{1}{2} m \frac{1}{\delta}\left[2(z_\lambda - z_{\lambda-1})\right]$$

$$-mg\frac{\delta}{2}(1+1) = 0 = \frac{m}{\delta}(z_\lambda - z_{\lambda+1} + z_\lambda - z_{\lambda-1}) - mg\delta$$

$$\frac{(z_{\lambda+1} - z_\lambda) - (z_\lambda - z_{\lambda-1})}{\delta^2} = -g = \frac{\frac{z_{\lambda+1} - z_\lambda}{\delta} - \frac{z_\lambda - z_{\lambda-1}}{\delta}}{\delta}$$

$\ddot{z}$ $\delta \to 0$

σωματίδιο σε 3-D σε δυναμ. ενέργεια $V(\vec{x})$

$$L = \frac{1}{2} m (\vec{u})^2 - V(\vec{x})$$

η L είναι βαθμωτό μέγεθος

και η S βαθμωτό

$\vec{x}_0(t)$: την φυσική διαδρομή

τη φυσική κίνηση του συστήματος

$$S[\vec{x}_0(t) + \varepsilon \vec{n}(t)] = S[\vec{x}_0(t)] + O(\varepsilon^2)$$

$O(\varepsilon) = 0$

$$S[\vec{x}_0 + \varepsilon\vec{\eta}] = \int_{t_1}^{t_2} \left[\frac{1}{2} m \left(\dot{\vec{x}}_0 + \varepsilon\dot{\vec{\eta}}\right)^2 - V(\vec{x}_0 + \varepsilon\vec{\eta})\right] dt$$

$\vec{x}_0(t_1) = \vec{x}_1$ ✓

$\vec{x}_0(t_2) = \vec{x}_2$ ✓

t_1, t_2 ✓

$\vec{\eta}(t_1) = \vec{\eta}(t_2) = \vec{0}$

$$\underbrace{\int_{t_1}^{t_2} dt \left[\frac{1}{2} m \left(\dot{\vec{x}}_0\right)^2 - V(\vec{x}_0)\right]}_{S[\vec{x}_0]} +$$

$$\varepsilon \int_{t_1}^{t_2} \left(\frac{1}{2} m\, 2\dot{\vec{x}}_0 \cdot \dot{\vec{\eta}} - \vec{\nabla} V\Big|_{\vec{x}_0} \cdot \vec{\eta}\right) dt + \varepsilon^2 \cdots$$

$$V(\vec{x} + \varepsilon\vec{\eta}) \underset{\varepsilon \simeq 0}{\simeq} V(\vec{x}) + \varepsilon\, \vec{\eta} \cdot \vec{\nabla} V\Big|_{\vec{x}} + \varepsilon^2 \cdots$$

$$\int_{t_1}^{t_2} dt \left(m \underbrace{\dot{\vec{x}}_0 \cdot \dot{\vec{n}}} - \vec{\nabla} V\big|_{\vec{x}_0} \cdot \vec{n} \right) = 0 \quad \forall \vec{n}$$

$$\vec{n}(t_1) = \vec{n}(t_2) = \vec{0}$$

$$\vec{n}(t_1) = \vec{n}(t_2) = \vec{0}$$

$$\frac{d}{dt}\left(\dot{\vec{x}}_0 \cdot \vec{n}\right) - \vec{n} \cdot \frac{d^2\vec{x}_0}{dt^2}$$

$$V(\vec{x}_0) = (\vec{x}_0)^2$$

$$\vec{\nabla} V\big|_{\vec{x}_0} = 2\vec{x}_0$$

$$\Rightarrow \int_{t_1}^{t_2} -\vec{n} \cdot \underbrace{\left(m \frac{d^2\vec{x}_0}{dt^2} + \vec{\nabla} V\big|_{\vec{x}_0} \right)}_{= \vec{0}} dt = 0$$

$$\rightarrow m \ddot{\vec{x}}_0 = -\vec{\nabla} V\big|_{\vec{x}_0}$$