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CONSTRUCTION OF MATHEMATICAL KNOWLEDGE THROUGH WHOLE-CLASS DISCUSSION

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Abstract

A word problem that requires addition of fractions with different denominators was presented to 11 classes of 4th- or 5th-graders, 10 years of age. It had three answer alternatives: adding the denominators and numerators separately, transforming fractions into decimals before adding them, and the standard, most appropriate solution. Students in each class were required to choose an alternative themselves, state their reason for the choice, and discuss which alternative would be right. After whole-class discussion, they were asked to choose an alternative again and nominate the student who had offered the most plausible idea. Then students in six classes were informed which alternative was the most appropriate, whereas those in five classes were not. Finally, the students, irrespective of the presence/absence of feedback, were required to solve the initial problem without the alternatives as the post-test, and two new problems as the transfer test.

The results showed that (a) the students could offer more or less plausible arguments for or against each alternative, which in most classes included the correct explanation; (b) both vocal and silent students could write a mathematical expression for the post-test problem and manipulated it correctly, incorporating other students' ideas presented during the discussion, with or without the teacher's feedback, though their generalization was rather limited; and (c) the students could recognize and memorize reasonable explanations offered by other students in the discussion.

In recent years it has been agreed that mathematical knowledge is acquired by construction, but its acquisition is constrained by sociocultural contexts (Hatano, 1996). Thus, a growing number of researchers have been interested in studying the learning of mathematics as a collective enterprise in sociocultural contexts, rather than as a process occurring only within an individual mind (e.g. Cobb & Bauersfeld, 1995). To put it differently, these

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researchers assume that students' mathematical ideas develop through their communicative practices (Roschelle, 1996) or dialogical interactions (Wertsch, 1985) with other members of the classroom.

Many of these researchers of the sociogenetic orientation have taken it for granted that much mathematics learning occurs through the discourse between a teacher and students. Although the conventional classroom discourse often takes the form of the teacher initiation–student response–teacher evaluation (IRE) sequence, reflecting a transmissionist view, there have been constructivistic attempts to go beyond it. For example, the teacher can clarify what a student says by questioning; she can articulate a student's idea by paraphrasing or asking him to rephrase it; she can even develop it by requesting an explanation or posing an apparently disconfirming example (Lampart, 1990; Wells, 1993). These attempts at teacher "revoicing" (O'Connor & Michaels, 1993), unlike the IRE sequence, leave a student some room for negotiation. A number of discourse-analytic studies have shown that the target student elaborates his idea in such a dialogue, in other words, through the negotiation and joint expansion of meanings (see O'Connor, 1994 for review).

Another mode of the teacher's follow-up of a student's response is, instead of giving feedback herself, to invite another student or all other students to examine and evaluate the response (O'Connor & Michaels, 1996). The teacher may ask other students to elaborate on the original response or to criticize it. Alternatively, she can just encourage students to argue among themselves by treating students' utterances as "thinking devices or objects to which one can respond" (Wertsch & Bivens, 1992). She may thus be able to organize whole-class discussion, through which students enhance their mathematical understanding by assimilating similar ideas into their own and/or contrasting their ideas with opposing ones (Orsolini & Pontecorvo, 1992).

Facilitating student–student interactions in the classroom is, as recommended by the National Council of Teachers of Mathematics (1989) in the USA, considered to be a highly desirable way for students to construct mathematical knowledge. The teacher still plays a very important role in whole-class discussion, but she no longer treats all students individually. In other words, a mathematics class becomes a community for joint learning activity that is organized by the teacher.

Student–student interactions taking place in the whole class would probably work better than those occurring in small groups, the effectiveness of which depends heavily on the constituent members' ability (Webb, 1991). Whereas in whole-class discussion, the teacher can indirectly intervene when needed, for example, by connecting students' arguments and by providing them with an evaluative criterion, activities in a number of small groups may go beyond the teacher's control and result in unproductive outcomes. As many investigators who adopt the sociogenetic approach assume, the teacher's intervention is essential for students' learning, and it is fully compatible with the emphasis on students' construction of mathematical knowledge.

However, there have been far fewer studies on student–student discursive interactions in the classroom than teacher–student ones. In this sense, many mathematics educators and researchers seem to assume that students cannot readily learn from other students' proposed ideas, which, unlike the teacher's, may be erroneous or uninformative. They seem to have three related concerns about the effectiveness of other students' utterances for mathematics learning. The first is that none of the students may offer the correct solution or interpretation, or its critical components. In other words, students, especially those who are academically unsophisticated, may not be able to explore the hypothesis space extensively enough to offer reasonable alternatives. Second, because students' utterances may include incorrect but apparently plausible answers, they may mislead other students, who may not be able to extract promising ideas from those offered in order to reach the correct answer. Third, because in a large group many students have to remain silent, the silent students may lose interest in the discourse and thus fail to learn.

In contrast, Hatano & Inagaki (1991) argue for student–student discursive interaction in the classroom, and suggest answers to the above concerns. They claim that discussion among students provides good opportunities for them to actively construct target knowledge, because they are supported by social-motivational factors in the classroom.

First, although individual students may have difficulty exploring the hypothesis space thoroughly by themselves, the division of labor, which occurs quite naturally in a large group, distributes the task work among the members, making it easier to perform. In a sense, the more varied the group membership, the more likely it is that the correct idea will be presented by one of the members due to their different interests and knowledge bases. In other words, because a class consisting of tens of students has a much richer database than does any one of its individual members, we can expect the correct answer to be offered during a discussion or a collective attempt to solve a problem. Especially when the whole group is divided into a small number of 'parties,' students are supposed to seek good arguments for their own party and arguments against others.

Second, Hatano & Inagaki (1991) argue, students can pick out plausible or promising ideas and avoid being misled by incorrect ideas offered by other students by relying on social as well as cognitive cues. It is true that many of the ideas initially proposed by students who have not yet learned the subject of discussion, may be inaccurate. Some of them may be completely wrong. However, these weak ideas seldom impress the whole class. Even academically less sophisticated students of elementary school age can individually evaluate the plausibility of arguments and selectively pay attention to convincing ones to some extent. They can be helped socially, if they carefully observe whether each argument encourages its proponents, silences its opponents, or attracts many 'third-party' students. In such cases, students can learn a great deal through student–student interaction by accurately evaluating ideas offered in whole-class discussion and incorporating them into their own.

Third, although many people are concerned that whole-class discussion is likely to produce many silent participants, who may not learn anything, Hatano & Inagaki (1991) assert that even silent students are often involved actively in whole-class discussion, by identifying a student who acts as an 'agent' or 'spokesperson' for them. Many silent participants can nominate peers whose ideas they agree with and can learn through these peers' utterances.

Although Hatano & Inagaki (1991) provided some preliminary data revealing that both vocal and silent participants could learn from whole-class discussion about a topic in biology, many more studies are needed before we can conclude that whole-class discussion is effective for the construction of mathematical knowledge. First, we have to examine whether Hatano and Inagaki's findings can be generalized from science to mathematics lessons, because the nature of persuasive arguments is supposedly different between these two disciplines. In this connection, it is worth examining how much students can learn in mathematics through student–student interaction without being given the correct answer by the teacher, because mathematics learning involves logical necessity and consistency with already learned procedures and principles, rather than on the empirical confirmation of facts, which is vital in science.

Second, we need to correlate features of the discussion with learning outcomes by observing whole-class discussion in a fairly large number of classes. Hatano & Inagaki (1991) could not provide this sort of data, because of a limited number of classes they dealt with.

Third, we need fine-grained analyses of whole-class discussion in mathematics, for example, what arguments students offer, how a student elaborates his or her idea through attempts to incorporate other students' compatible ideas or by refuting opposing ideas, and what types of arguments induce conversions in antagonists. We also need to examine how well students distinguish strong arguments from weaker ones and remember the major points of strong arguments that may be used in future problem solving. It may be too hard for students to memorize all different arguments, but they may store plausible ones that might be used later, assuming that they pay attention only to important statements (Stigler & Fernandez, 1995).

Thus in the present study, we investigated processes and products of whole-class discussion in mathematics in 11 elementary school classes. Students had to solve a new arithmetic word problem by expanding their prior knowledge through whole-class discussion. We used the format of whole-class discussion that was similar to the one in the Hypothesis–Experiment–Instruction (HEI) method proposed by Itakura (1967) who had originally proposed this method for science education. In this method students are presented with a problem with three or four answer alternatives (which leads them later to recognize the distribution of choices), and then encouraged to discuss a variety of ideas for and against these alternatives. We expected that this procedure, unlike that of more open-ended group discussion, could structure the discussion (Moschkovich, 1996) and simplify our analysis of it.

Specifically, we primarily examined the following questions:

1. Can students offer plausible arguments, which include correct (or partially correct) and persuasive ones? Can they elaborate their arguments in whole-class discussion?
2. When an appropriate solution procedure or interpretation is offered in the discussion, can students, both vocal and silent, incorporate it, change their original ideas, and apply the elaborated idea obtained through the discussion to other problems, with or without the teacher's feedback? To put it differently, can students' learning outcomes be explained in terms of features of the discussion?
3. Can students, not only vocal participants but also silent ones, evaluate the reasonableness of explanations offered by other students in the discussion and recall the content of the most plausible ones?

Method

A word problem that requires addition of fractions with different denominators was presented, with three answer alternatives, to 11 classes of 4th- or 5th-graders. Students in each class were required to choose an alternative themselves, state their reason for the choice, and discuss which alternative would be right. After whole-class discussion, they were asked to choose an alternative again and nominate the student who had offered the most plausible ideas. Then, students in about half the classes were informed which alternative was correct, and those in the other half were not. Finally, the students, irrespective of the presence/absence of feedback, were required to solve the initial problem without the alternatives as the post-test, and two new problems as the transfer test.

Participants

Six classes of 4th-graders ($N = 155$) in the last quarter of the school year and five classes of 5th-graders ($N = 143$) in the first quarter, from different public elementary schools in Osaka, participated in the study. (See Table 2 for the number of participants per class.) The former students were 10 years 4 months old on average and the latter, 10 years 9 months old, and both of them were mostly from lower-middle-class families. The numbers of boys and girls in each class were almost the same.

The students had been taught how to add fractions with a common denominator and also how to add decimals, but not yet how to add fractions with different denominators. They had not been taught either how to reduce fractions, except for the simple cases of those reducible to 1/2 and 1/3, nor how to change fractions into decimals, except for fractions having denominators of 10 or 100.

Target Problem

A word problem concerning the addition of fractions with different denominators was used in the discussion as the target: Taro drinks 1/2 liter of milk at breakfast and 1/5 liter at supper. How many liters of milk does he drink a day? X. $1/2 + 1/5 = 2/7$, Answer is 2/7 liter; Y. $0.5 + 0.2 = 0.7$, Answer is 0.7 liter; Z. $1/2 + 1/5 = 7/10$, Answer is 7/10 liter.

We constructed the three alternatives based on the solutions observed in Takahashi's classroom (Takahashi, 1992). We assumed that almost all students would write a mathematical expression involving addition for this word problem, but would have difficulty finding the correct answer. We also made additional assumptions: The alternative X is a typical error that many students would make. The alternative Y is the response that would seldom occur spontaneously because it deals with decimals, instead of fractions. We included it among the three options, because decimals were more familiar than fractions for children and thus this alternative might be appealing when it was given. However, we did not treat it as the most appropriate solution, because, although it works for this problem, it is only an approximation for many fractions, such as 1/3. Alternative Z is universally correct, and in this sense it is the most appropriate answer. We thus recommended alternative Z in the feedback provided at the end of the discussion in 6 of the 11 classes.

Procedure

A male experimenter (the third author) who had a diploma for elementary school teaching gave one lesson in each school during one of the regular lessons, using the following procedures: (1) students were given the target problem individually in the form of a paper-and-pencil test, and asked to choose one of the three answer alternatives and to write the reasons for their choices; (2) the students' choices, counted by a show of hands, were tabulated on the blackboard; (3) after one or two students who supported each alternative were invited to state reasons for their choices, all of the students were encouraged to discuss them; (4) to examine the effect of the discussion, students were individually given the initial problem with the three answer alternatives in the paper-and-pencil format and asked to choose one once again. They were allowed to change their choices. They were also asked to give the name of the student who had stated the most plausible ideas in the discussion, and recall these ideas as much as possible; (5) the students were informed by the experimenter that alternative Z was the most appropriate answer without further explanation; (6) immediately after the step (5), as the post-test, they were asked to solve the initial word problem in the open-ended form, in other words, they were required to write a mathematical expression and find the answer for it; (7) as the transfer test, they were asked to solve two computation problems that also asked for the addition of fractions with different denominators, i.e. $1/2 + 1/3 =$ and $1/4 + 2/5 =$. The computation problems were used as the transfer test, instead of word problems, to save time; posing computation problems would suffice, because our subjects' difficulty was not in writing a mathematical expression for the word problem but in finding out how to run the addition of fractions with different denominators. Thus, these transfer problems would serve to clarify what knowledge the students had acquired, because both problems involved fractions that cannot be transformed into those having 10 as the denominator, and the numerator of the second problem's sum cannot be obtained by adding the two denominators, as in the original problem.

In the discussion (i.e. step 3), the experimenter took the role of chairperson and repeated or clarified students' ideas, as Japanese teachers often do in lessons. However, he avoided giving any evaluation of the ideas or any suggestion of the correct answer. He also tried not to elaborate or advance a student's utterance. When no additional opinions seemed to be forthcoming from the class, the experimenter ended the discussion. When the discussion, excluding the initial statements of the reasons for choices, continued for more than 15 min, the experimenter ended it at the earliest break in the conversation.

In five classes (three 4th-graders and two 5th-graders) step 5 was omitted. That is, students were not given any information about the correct answer by the experimenter. These no-feedback classes were G–K, while feedback classes were A–F. All of the steps were videotaped, and step 3, the discussion period, was transcribed.

Coding of Students' Utterances

Those students' utterances that were publicly made in the statement of reasons or the discussion were divided into arguments. Here 'utterance' is defined as the total speech given by a single student until the experimenter–teacher or another student took the floor, and an 'argument' is defined as a distinct idea for or against one of the alternatives. Thus, for instance, if a student explained why alternative Z was correct and alternative X must be incorrect, his or her utterance was treated as involving two arguments. If he or she repeated the same argument, it was counted as one.

The arguments were classified into either for or against each alternative (X, Y, or Z), and then classified into subcategories in terms of the major reasoning. See Table 1 for details. Two categories for X ($1/2 + 1/5 = 2/7$): X-simple addition, and X-others. Three

Table 1
Classification Categories of Students' Arguments

For X (1/2 + 1/5 = 2/7)
X-simple addition—numbers are simply to be added after being grouped into denominators and numerators.
X-others—other arguments for X.
Against X
NX-semantic—the answer of 2/7 is smaller than 1/2 (one half) despite the fact that another number (1/5) is added to 1/2.
NX-same denominator—in the previous lesson on the addition of fractions with the same denominators, the denominators are not added.
NX-others—other arguments against X.
For Y (0.5 + 0.2 = 0.7)
Y-how—explaining how 1/2 can be changed to 0.5 and 1/5 to 0.2.
Y-correspond—1/2 is 0.5 and 1/5 is 0.2 (without any explanation).
Y-others—other arguments for Y.
Against Y
NY-irrelevant—because the original problem is about fractions, Y is irrelevant.
NY-others—other arguments against Y.
For Z (1/2 + 1/5 = 7/10)
Z-why—explaining why making denominators common is needed before adding.
Z-how—explaining how to make denominators common (by referring to the least common multiple or by multiplying both the denominator and the numerator by the same number).
Z-how-1/10—explaining how to make denominators 10 (by using 1/10 as a unit).
Z-correspond—1/2 corresponds to 5/10 and 1/5 to 2/10 (without further explanation).
Z-others—other arguments for Z.
Against Z
NZ-wrong answer—the numerator of 7 cannot be obtained, though the denominator of 10 can be.
NZ-others—other arguments against Z.
For both Y and Z
YZ-right—both Y and Z are correct, because 0.7 equals to 7/10.

categories against X: NX-semantic, NX-same denominator, and NX-others. Three for Y (0.5 + 0.2 = 0.7): Y-how, Y-correspond, and Y-others. Two against Y: NY-irrelevant, and NY-others. Five for Z (1/2 + 1/5 = 7/10): Z-why, Z-how, Z-how-1/10, Z-correspond, and Z-others. Two against Z: NZ-wrong answer, and NZ-others. One for both Y and Z: YZ-right.

Two raters independently coded all utterances first into arguments and then into the above categories, based on a written manual including definitions and an example. The percentage of agreements for the classification into the subcategories was 92.7. Cases of initial disagreement were negotiated.

Results

This section is divided into three parts. First, we describe how whole-class discussion proceeded in each class, in other words, what arguments were offered and how they were elaborated. The description is mostly qualitative in nature. Second, we analyze learning outcomes and their relationships with features of the whole-class discussion, both class by class and comparing vocal and silent participants. This part represents more or less conventional analyses of process–product correlations. Finally, we examine, among the vocal students, who was nominated as presenting the most plausible ideas, and whether both vocal and silent students could recall what the ideas had been.

How Whole-Class Discussion Proceeded

On average, 19 min (range, 15–25 min) were spent for the discussion including the statement of reasons for choices; the actual time spent varied from class to class depending on the course of the discussion. In most classes, 6–9 students stated their explanations or arguments once or twice during this period; these speakers (referred to as 'vocal' students hereinafter) represented one-third to one-fifth of the entire students in the class.

Table 2 shows frequencies of the major categories of arguments for and against alternatives X, Y, and Z. It summarizes the discussion in each class, providing with the bases for interpreting students' learning outcomes. In six classes (D, E, F, I, J, and K), one or more correct, and widely applicable, explanation(s) for the valid procedure (that is, Z-why or Z-how) was offered. In three more classes (A, B and H), explanation(s), which was correct for the target problem though not generally applicable (Z-how-1/10), was

Table 2
Frequencies of Coded Arguments For and Against Each Alternative

Classes	A	B	C	D	E	F	G	H	I	J	K
No. of participants	33	18	14	20	33	28	25	33	32	32	30
No. of vocal students	6	7	7	6	9	6	9	11	7	8	7
Total number of utterances	8	11	8	9	13	6	10	18	10	15	8
For X (1/2 + 1/5 = 2/7)											
X-simple addition	2	1	2	1	1	1	4	1	1	2	2
For Y (0.5 + 0.2 = 0.7)											
Y-how	1	2	0	0	0	0	0	1	1	2	1
Y-correspond	0	1	0	1	1	1	1	1	2	0	1
For Z (1/2 + 1/5 = 7/10)											
Z-why	0	0	0	0	1	1	0	0	1	1	1
Z-how	0	0	0	1	2	1	0	0	0	3	2
Z-how-1/10	1	1	0	0	0	1	0	1	6	2	0
Z-correspond	0	3	0	5	0	0	0	1	0	0	0
For Y and Z											
YZ-right	0	0	0	0	3	0	0	1	1	5	2
Against X											
NX-semantic	3	0	0	1	2	1	1	0	0	1	2
NX-same denominator	0	0	0	0	1	1	0	5	0	1	0
Against Y											
NY-irrelevant	0	0	0	1	2	0	3	0	2	1	2
Against Z											
NZ-wrong answer	0	0	1	0	0	0	3	0	0	0	2

Categories of 'others' were excluded from this table.
See Table 1 for what we mean by X-simple addition, Y-how, Y-correspond, Z-why, Z-how, Z-how-1/10, Z-correspond, NX-semantic, NX-same denominator, NY-irrelevant, NZ-wrong answer, and YZ-right.
Classes A, B, C, G, H, and I belonged to the 4th-grade; others, the 5th-grade.

presented. Only in two classes (C and G) out of the 11, students failed to offer effective explanations about how to get the answer 7/10. Although the development of discussion varied, depending upon the distribution of students' prior responses, the number of expressive students in a class, and so on, we can conclude that a majority of the classes could offer correct arguments at least a few times during whole class discussion.

Arguments For and Against Each Alternative

How persuasive were students' arguments? Arguments for and against each alternative, if given at all, were highly similar in content across the classes. Proponents of alternative X ($1/2 + 1/5 = 2/7$) in all 11 classes asserted that because this problem involved addition, the numbers had to be added, after they had been grouped into denominators and numerators [coded as X-simple addition]. An auxiliary, less frequent argument was that when two fractions had the same denominator, that denominator would remain the same after the fractions were added, but when denominators were different, there would be no other way than adding them. In contrast, opponents of this alternative gave counter arguments, relying on either 'semantics' (i.e. whether the solution makes sense), or consistency (i.e. whether the solution is consistent with the procedure that they have learned), or both. That is, X could not be correct because its answer 2/7 was smaller than 1/2, though another number (1/5) was added to it [NX-semantic]; when they had learned the addition of fractions with the same denominators, denominators were not added, and if these were added, the answer would not make sense, as shown, say, in the case of $1/2 + 1/2$ [NX-same denominator].

Supporters of Y ($0.5 + 0.2 = 0.7$) in all classes except class C stated that 1/2 is 0.5 and 1/5 is 0.2, and that 0.5 and 0.2 combined equals to 0.7, either explaining the fraction–decimal correspondence by drawing a figure or referring to division [Y-how], or just asserting the correspondence 1/2 to 0.5 and 1/5 to 0.2 without any explanation [Y-correspond]. This argument was rejected by opponents of Y, because the original problem was about fractions, not about decimals [NY-irrelevant]. This counter argument against Y was not refuted by the supporter of Y except in classes E and J where a substantial number of students asserted that both alternatives Y and Z were correct. These refuters claimed that the wording of 'how many liters of milk' should permit both the answer in decimals as well as the one in fractions. Although 12 students in five classes concluded that both alternatives Y and Z were correct (because $0.7 = 7/10$) [YZ-right], nobody pointed out that the procedure relying on the transformation into decimals works only as an approximation for many fractions. This is probably because the discussion time was short and because the procedure using decimals worked for the present problem.

Supporters of alternative Z ($1/2 + 1/5 = 7/10$), which was the most appropriate solution, explicitly or implicitly referred to the necessity of transforming fractions with different denominators into those with the common denominator, and offered explanations for obtaining the common denominator, 10 in this case. The Z-why argument, which was observed in five classes, indicated why making the denominators common is needed before adding. The Z-how argument, which was observed also in five classes, explained how to make denominators common by using the least common multiple, or by multiplying both the denominator and the numerator by the same number. These two types of arguments are considered to be more advanced than others for Z, because they can be applied to any addition of fractions with different denominators. Z-how-1/10, seen in six classes, explained how to make the common denominator of 10, by using 1/10 as a unit, that is, 1/2 can be expressed as 5/10 (1/10 × 5) and 1/5, 2/10 (1/10 × 2). This explanation was correct in the target problem, but was not always applicable to other fractions. Nine students in three classes gave Z-correspond, indicating that 1/2 corresponds to 5/10 and 1/5 to 2/10 without further explanation. Arguments against alternative Z, which were observed in three classes, were based on some misunderstanding of Z; a few students complained that they could not get the numerator of 7 in the sum, though they could get the denominator of 10.

Arguments Inducing Public Conversions

How persuasive these arguments were for other students can be examined by analyzing cases of public conversion. During the discussion, 22 students in 10 classes indicated that they had changed their mind. Out of these 22 public conversions, 10 in 9 classes were from X (1/2 + 1/5 = 2/7) to Z (1/2 + 1/5 = 7/10), 5 in 4 classes were from Y(0.5 + 0.2 = 0.7) to Z, and 3 in 2 classes were from X, Y, or Z to the claim that both Y and Z are right. Five of these 10 conversions from X to Z occurred just after one of the supporters of Z had pointed out that X could not be correct because its answer, 2/7, was smaller than 1/2, though another number (1/5) was added to 1/2 [NX-semantic], whereas the other four occurred without a speaker who expressed his or her opinion against X just before the conversion. The latter four students referred to a supporter of Z who explained how to obtain the common denominator. In class G, the argument of NX-semantic was offered toward the end of the discussion, but no public conversion occurred, except that one of the students cried, “That’s a sudden complete reversal!” This is probably because there was no effective explanation about how to obtain the common denominator in this class, due to the lack of time. This suggests that the argument of NX-semantic may not be sufficient to induce public conversion, although it seems to be a persuasive argument.

Four of the five conversions from Y (solution by decimals) to Z (solution by fraction) occurred through reference to the Z supporter’s explanations about how to get the common denominator. The other occurred through reference to the argument that the original problem was about fractions, not about decimals.

Conversion to the conclusion that both Y and Z were correct occurred just after the argument of YZ-right was offered, and these converts pointed out that 0.7 is equal to 7/10; for example, “Since we learned before that 1/10 is equal to 0.1, I think 7/10 will be equal to 0.7, and as *Tanaka* (another student’s name) said, we can also change 0.7 into 7/10, so I think both Y and Z are right.”

Two conversions from Z to Y occurred in class J, where the supporters of Y explained how to change the fractions into decimals and the supporters of Z explained how to obtain common denominators by relying on the least common multiple. These converts stated that the explanations using decimals were more understandable to them than those using the least common multiple. One of them stated that although both Y and Z were correct, the procedure of Y was better because it did not require something like the least common multiple, which was novel to them.

Two conversions from Y (0.5 + 0.2 = 0.7) to X (1/5 + 1/2 = 2/7) occurred only in class G, where there was no supporter of Z at the first choice, and thus no effective explanation about why and how to make the common denominator in the discussion. These converts seemed to have been influenced by the X supporter's assertion that when two fractions had the same denominator, the denominator would be the same in the sum of the fractions, but when the denominators were different, they must be added; one of the converts stated, "At first I thought that Y was right, but in the addition of these new fractions that we will learn in the 5th-grade, we may add the denominators."

Elaborations of Arguments During Discussion: An Example

Although our experimenter–teacher almost never revoiced a student's utterance, students sometimes not only changed their choices as mentioned above but also elaborated their arguments of why X is wrong and why both Y and Z are right, by incorporating other students' ideas. This process could be seen most clearly when they were involved in extended discussion. They often referred to other students' utterances, either for or against them, saying, for instance, "I am arguing against *Sato* (another student name) that..." or "I do not think *Sato* is right, because..." in the case of denial and "As *Tanaka* said, I think..." or "Let me add to what *Suzuki* said that..." in the case of support.

We present, as an illustrative example, the sequence of students' utterances in class E below. We select it for three reasons: (a) the three alternatives were chosen at similar rates before discussion; (b) the time spent and the number of vocal students were about the average; but (c) it revealed respectable learning outcomes. In actuality, the teacher's utterances, mostly nominating the next student and repeating or clarifying the preceding student's utterance, were inserted, but we have removed them in the protocol so that the connections among the students' utterances can be seen more evidently.

Class E

1. S1, a supporter of X: I got the answer by adding the upper ones (numerators) and lower ones (denominators) separately.
2. S2, a supporter of Y: One half is 0.5 in decimals, and one-fifth is 0.2. So I got 0.7 by changing them into decimals.
3. S3: I think, S2's solution is OK, though different (from mine), but X is wrong.
4. S4, a supporter of Z: A half is one of the one divided into two, and one-fifth is one of the whole divided into five. Because we cannot handle numerators unless denominators are the same, we make them the same, 10, and for this, we multiply two by five, so this (1/2) becomes 5/10, and five is multiplied by two, so this (1/5) becomes 2/10. 5/10 + 2/10, that is, there are seven units of the one divided into 10. We have 7/10.
5. S5: I change from Y to Z. Removing the decimal point, this one, Y, is almost the same as X.
6. S1: We are studying fractions today, so decimals are irrelevant.
7. S6: Calculation is correct in Y, but the problem is given in fractions, so we have to give the answer in fractions.
8. S4: S6 said that we have to change the answer (obtained in Y) into fractions, but I think both Y and Z are right.
9. S7: I choose Z, but adding to S4, I think both are OK because 7/10 is equal to 0.7.
10. S8: In X, 2/7 is smaller than a half of one, that is 1/2.

11. S9: If so, a number may become bigger by a subtraction.
12. S1: I have changed from X to Z. I thought lower ones (denominators) could be added, but if we do so, the answer would not make sense. In a previous lesson, for example, 1/6 added to 1/6 equals to 2/6, so we did not add lower ones.
13. S10: I choose Z, but argue against X. A half is 0.5 in decimals, and one-fifth is 0.2, 0.2 and 0.5 combined is equal to 0.7, and 0.7 liter is the same as 7/10 liter. Only the answer in X is different and thus wrong.
14. S11: S4's solution seems OK, but I propose my own. I get 10 by multiplying 2 by 5. (Numerators are) 5 by 1×5 and 2 by 1×2. As we did for 1/6 added to 1/6, we add just the upper numbers, not the lower ones, 10, so we get 7/10.
15. S5: I believe Z is correct, and do not want to shift to X. 2/2 is one liter, and 5/5 is one liter. 2/7 is much smaller than 1/2 and 1/5 combined.

As can be seen, a group of vocal students in the above example has developed an understanding of why X is incorrect. S1 (utterance no. 12) referred to an earlier lesson in which denominators were not added when he converted from position X to Z, probably influenced by the argument of S8 (utterance no. 10) and other students that the answer, 2/7, was too small. Thus, at the end, all of the students seemed convinced that X could not be right because of its absurd answer and lack of consistency with previous lessons. Moreover, at least some of the students correctly recognized through their exchange of ideas that both Y and Z are correct, although they are apparently different solutions. Although they did not elaborate on how to obtain the common denominator, they seemed to grasp that it was necessary to do so before adding fractions with different denominators.

Learning Outcomes and Their Relationships with Features of Discussion

How often could the students, both vocal and silent, acquire through whole class discussion the correct procedure for adding fractions with different denominators? Could their learning performances be explained in terms of the arguments offered by other students during the discussion? In this part we will examine these questions.

Change of Choice Responses After the Discussion

Table 3 shows response frequencies before and after the discussion but before feedback was given by the experimenter (i.e. the first and second choices) in each class. In six classes (A, B, D, E, I, and K) the supporters of Z ($1/5 + 1/2 = 7/10$) persuaded almost all the others: 90% or more were supporters of Z after the discussion. However, in the remaining classes, its supporters could not clearly win. In three classes (F, H and J), the supporters of alternative Y ($0.5 + 0.2 = 0.7$), which is correct and appropriate for the original problem but cannot readily be used for addition of fractions in general, maintained or even increased their number. This is probably because the argument that the solution relying on the decimal was irrelevant was not offered (F and H) or because it was refuted by one of the supporters of Y, who were a majority (J). In two other classes (C and G), the supporters of X ($1/2 + 1/5 = 2/7$) constituted the largest group before discussion. Their number decreased after the discussion in both classes, but 20% (3/14) and 40% (10/25) of the students, respectively, still supported alternative X as the appropriate

Table 3
Choice Response Distributions Before and After Discussion

Classes	A	B	C	D	E	F	G	H	I	J	K
	Bef.→ Aft.	Bef.→ Aft.	Bef.→ Aft.	Bef.→ Aft.	Bef.→ Aft.	Bef.→ Aft.	Bef.→ Aft.	Bef.→ Aft.	Bef.→ Aft.	Bef.→ Aft.	Bef.→ Aft.
X. 1/2 + 1/5 = 2/7	12→0	3→0	9→3	7→2	11→0	17→1	15→10	13→0	5→0	13→1	8→0
Y. 0.5 + 0.2 = 0.7	6→1	7→0	3→3	8→0	9→3	7→4	10→0	5→6	12→1	14→19	10→1
Z. 1/2 + 1/5 = 7/10	15→32	8→18	2→8	5→18	13→30	4→23	0→15	15→27	15→31	5→12	12→29

solution after the discussion. This is probably because, as described above, no plausible explanations for alternative Z were given in these classes.

Post-Test Performances

Table 4 shows students' performances on the original problem at the post-test, which was given without answer alternatives. The first row for the feedback group or no-feedback group shows the number of correct solutions for it, and the second, the number of students who could write a mathematical expression and manipulated it correctly, in other words, spontaneously wrote the intermediate step of the solution, such as, $1/2 + 1/5 = 5/10 + 2/10 = 7/10$, or $1/2 + 1/5 = 0.5 + 0.2 = 0.7 = 7/10$. We did not include into this category such a response as $1/2 + 1/5 = 2/10 + 5/10 = 7/10$, because students may have taken 1/2 for 2/10; and did not include either responses without an intermediate step of manipulation, such as $1/2 + 1/5 = 7/10$.

Almost all students, except for those in class G, solved the target problem correctly, irrespective of the presence or absence of feedback. It should be reminded that students in classes G, H, I, J, and K did not get feedback that Z is the most appropriate solution. In class G, as described above, no effective solutions (Z-why, Z-how, or Z-how-1/10) were offered during the discussion, and thus a substantial number of students continued to choose X ($1/2 + 1/5 = 2/7$) at the second choice after the discussion. The percentage of the correct solution on the post-test was 100% in class C (which had three supporters of X at the second choice) where feedback was given, but 48% in class G (which had 10 supporters of X) where no feedback was given after the discussion. This suggests that feedback is effective when good ideas are not offered in discussion, though its effect seems limited, since only three of the 14 students (21%) of class C wrote a mathematical expression and manipulated it correctly. When plausible ideas are proposed in discussion, teacher's feedback may not be necessary, because the students in classes H, I, J, and K displayed good post-test performances in terms of percentages of the correct solution and writing and manipulating a mathematical expression correctly. There were no statistically significant differences between feedback and no-feedback groups as a whole by the Mann–Whitney U-test in either measure ($P > 0.10$). Considering that, as feedback, the experimenter–teacher told the students without further explanation only that alternative Z is most appropriate, and that many students spontaneously wrote a mathematical expression involving an intermediate step for the target problem at the post-test, it is strongly suggested that the students, irrespective of the presence or absence of feedback, learned how to solve the target problem from the discussion, not just memorized the correct answer presented as one of the answer alternatives.

Let us compare the result of the post-test taken by vocal versus silent participants. Although those who failed to solve the target problem at the post-test were primarily silent participants (i.e. 19/23), it should be noted that almost all the students except for those in class G, correctly solved the problem at the post-test. Six of the 12 students in class G who correctly solved the target problem were also silent participants. Mean percentages of correctly writing and manipulating a mathematical expression were 69.9 for the vocal participants versus 55.8 for the silent, and there was no statistically significant difference between these two groups by the Mann–Whitney U-test ($P > 0.10$). These findings indicate

Table 4
Performances on the Post-Test and on the Transfer Test

	Feedback groups						Non-feedback groups				
	A	B	C	D	E	F	G	H	I	J	K
No. of students	[33]	[18]	[14]	[20]	[33]	[28]	[25]	[33]	[32]	[32]	[30]
Post-test problem											
% Correct answer	91	100	100	85	100	96	48	94	100	97	100
% Correct expression/manipulation	82	100	21	65	76	68	0	64	66	66	40
Transfer problems											
Correct solutions	3	17	29	25	42	39	0	33	31	34	60
Correct denominators only	6	17	36	25	15	11	0	9	6	6	13
Transformation-into-10	52	11	7	10	15	11	8	15	9	13	0
Simple addition	6	11	7	10	0	7	44	9	0	13	0
No answer	24	44	21	30	21	11	24	21	47	28	27
Others	9	0	0	0	6	21	24	12	6	6	0

'Correct expression/manipulation' means that students wrote a mathematical expression and manipulated it correctly, e.g. writing an intermediate step, such as, $1/2 + 1/5 = 5/10 + 2/10 = 7/10$, or $1/2 + 1/5 = 0.5 + 0.2 = 0.7 = 7/10$.
Figures show percentages.

that not only vocal but also silent participants learned how to solve the target problem through the whole-class discussion.

Performances on Transfer Test Items

Students' solutions of the transfer problems (1/2 + 1/3 = and 1/4 + 2/5 =) were classified into one of seven categories: *correct solution* when both problems were solved correctly; *correct-denominators-only* when the student solved both problems through obtaining a common denominator by multiplying both denominators but the sum of numerators by adding denominators (e.g. 1/4 + 2/5 = (5 + 4)/(4 × 5)). This was an incorrect procedure that had happened to work for the original problem; *transformation-into-10* when the student approximated fractions to be added always by fractions with 10 as the denominator (e.g. 1/4 + 2/5 = 2/10 + 4/10), which was a procedure that also had happened to work for the original problem (a procedure that relied on decimals for both problems was included in this category); *simple-addition* when the student added denominators as well as numerators as in alternative X for the target problem [e.g. 1/4 + 2/5 = (1 + 2)/(4 + 5)]; *no answer* when the student gave no answer for either or both of the two problems; *others* when solutions that did not fit with any of the above categories or when inconsistent solutions between the two problems were observed.

Table 4 also shows the results from the transfer test. Few students in each class adopted the simple-addition strategy (i.e. the same solution as alternative X for the target problem) for the two transfer problems, except for class G. This indicates that the students learned that they should not add numbers after grouping them into denominators and numerators.

The mean occurrence of correct solutions was 44% in four classes (E, F, J, and K) where effective explanations, i.e., both Z-why and Z-how, were offered in the discussion, whereas it was 16% in five other classes (A, B, C, G, and H) where students proposed Z-how-1/10, which was not always applicable to all fractions, and/or Z-correspond, which did not explicitly explain how to make denominators common, or neither of these explanations. The other two classes (D and I) were intermediate in that Z-how and Z-correspond, or Z-why and Z-how-1/10 were proposed; the mean percentage of correct solutions was 28%. We ran the Kruskal–Wallis one-way analysis of variance by ranks for these three types of classes and found that these were significantly different ($H = 7.25$, $P < 0.01$). This indicates that students who successfully solved the two transfer items were more likely to come from classes in which effective solutions were offered than from classes in which no such solutions were proposed.

Whereas in most classes Z-how-1/10 arguments were observed with such arguments as Z-why, Z-how, or Z-correspond, in class A the Z-how-1/10 argument was offered only once without these supporting arguments (see Table 2). It should be noted that 52% (17/33) of the students in class A showed the solution strategy of transformation-into-10 at the transfer test, while the students in the other classes showed this solution strategy at about 10% on the average (range, 0–16%). These results suggest that the influence of a given argument on students' learning may be mediated by other arguments for the same alternative offered in discussion.

Table 5 shows the percentages of correct solutions of vocal and silent participants in each class at the transfer test. Although the mean percentage was apparently higher among the vocal participants than the silent ones (41.1% versus 23.7%), there was no statistically

Table 5
Proportions of Correct Solutions of Vocal and Silent Participants in Each Class on Transfer Test

Classes	A	B	C	D	E	F	G	H	I	J	K
Vocal students	0	42.9	28.9	33.3	66.7	50.0	0	27.3	42.9	75.0	85.7
Silent students	3.7	0	28.6	21.4	33.3	36.4	0	36.4	28.0	20.8	52.2

significant difference between these two groups by the Mann–Witney U-test ($P > 0.10$). This finding suggests that silent participants, as well as vocal ones, can learn, through whole-class discussion, a solution strategy that can be applied to new problems.

Evaluation and Recall of the Remarks Offered During the Discussion

Could the students evaluate a variety of arguments or explanations offered during the discussion in terms of their reasonableness? Who would they nominate as the student who had given the most plausible idea in the discussion? Could they recall more or less accurately what the plausible idea was? We will examine these questions below.

Who the Students Perceived Had Proposed the Most Plausible Idea

In the discussion, a total of 83 students publicly spoke once or twice (very few of them spoke three times). The vocal students who had been supporters of alternatives X ($1/2 + 1/5 = 2/7$) or Y ($0.5 + 0.2 = 0.7$) at the beginning often converted publicly to Z ($1/2 + 1/5 = 7/10$), as described above. At the final tally thus 14 were vocal supporters of X, 16 of Y, and 53 supported Z (including eight students who supported both Y and Z). Out of these 83 speakers, only 15 (18%) nominated themselves. Table 6 shows the patterns of nominations of speakers as proponents of the most plausible idea. As clearly shown, there were very few who did not nominate any speaker (percentages of nominations ranged from 82 to 100%).

The nomination concentrated on only a few speakers in each class. That is, in seven classes, about 80% or more of the nominations named the same one or two speakers, and in the remaining four (G, H, I, J) about 50% or more did so. When we take the third-most-popular students into account, about 70% of the nominations were covered in these four classes.

The most ‘popular’ nominees (i.e. those yielding the largest number of nominations) in these 11 classes were, with one exception, vocal supporters of Z (the most appropriate solution) or students who declared that they had become supporters of Z or both Y and Z during the discussion. Moreover, supporters of Z were on the average nominated more often than the supporters of X or Y. We computed for each vocal student the proportion of nominations (the number of yielded nominations divided by the number of peers in the class), and calculated mean proportions across classes. The mean proportion for the Z supporters was 16.2% (SD, 22.7), that for the Y supporters was 6.3% (SD, 8.8), and that for the X supporters was 2.9% (SD, 5.6). The differences in the mean proportions were significant, $F(2, 80) = 3.61$, $P < 0.05$, assuming the independence of each proportion

Table 6
Patterns of Nominations of Speakers as Proponents of the Most Plausible Idea

Classes	A	B	C	D	E	F	G	H	I	J	K
No. of participants	33	18	14	20	33	28	25	33	32	32	30
No. of students who nominated a speaker	33	18	14	20	33	26	22	27	28	29	31
	[100]	[100]	[100]	[100]	[100]	[93]	[88]	[82]	[88]	[91]	[97]
No. of vocal students	6	7	7	6	9	6	9	11	7	8	7
Frequencies of nominations for the most popular nominee	15	13	8	18	25	13	9	7	10	8	24
	[45]	[72]	[57]	[90]	[76]	[46]	[36]	[21]	[31]	[25]	[80]
Frequencies of nominations for the 2nd most popular Nominee	14	2	3	1	3	10	5	6	7	7	2
	[42]	[11]	[21]	[5]	[9]	[36]	[20]	[18]	[22]	[22]	[7]

Figures in square brackets show percentages.

nominated. This suggests that the students evaluated the arguments made by the supporters of Z as more plausible than those made by the supporters of X or Y.

Next we examined what types of arguments made by the supporters of Z were evaluated by the participants as most plausible. We compared occurrence rates of Z-why and Z-how between the more popular nominees who obtained nominations from more than 20% of their peers and the less popular nominees who were nominated by less than 10% of their peers. Out of 13 more-popular nominees, nine (69.2%) gave Z-why and/or Z-how arguments, while 4 of 31 less-popular nominees (12.9%) made such arguments; the difference was highly significant, χ^2 (1, $N = 44$) $= 13.96$, $P < 0.001$. This indicates that the participants accurately evaluated arguments in the discussion in terms of their plausibility, even among vocal supporters of Z.

Recall of the Plausible Ideas

How accurately did the students recall the ideas of the vocal participants that they nominated as the proponents of reasonable ideas? To examine this question, we coded students' recalled contents, relying on the coding scheme that was applied to speakers' utterances in the discussion (see Table 1) with the following three modifications: (a) Y-how and Y-correspond were combined into one category, and Z-how and Z-how-1/10, into another, because it was hard to distinguish these ideas in the recall data; (b) A comment or recall of impression alone, such as "*Sato*'s explanation was easy to understand," was coded as 'no recall'; (c) When a student's description was incomplete, it was noted and marked minus(−), such as Z-how(−). Two raters coded all the students' recalled contents independently. The percentage of agreement was 93.3. Cases of initial disagreement were negotiated.

We divided all the students' recalled contents thus coded into the following four categories, by comparing them with the coded categories of the corresponding nominees: 'accurate recall'—the coded categories were the same (when a nominee offered multiple arguments, we judged it as accurate recall if the nominator recalled the gist of at least one of them); 'simplified/incomplete recall'—the nominator's description was a simpler or incomplete version of the argument of their nominees', for example, while the nominee's explanation was Z-how, the nominator's recalled content was Z-correspond or Z-how(−); 'no recall/erroneous recall'—students could not recall their nominee's utterance, or recalled the argument of a different vocal student from the student that they nominated; 'others'—cases that did not apply to the above categories, including the case in which nobody was nominated.

Results were as follows. First, there were very few students (12.1%) who nominated someone but could not recall at all or recalled erroneous contents (the latter case was observed in only three students in all). About 60% of the students recalled their nominees' idea more or less accurately, and about 20% recalled it in a simplified or incomplete form. Second, a great majority of the students recalled only one argument of their nominees' utterance containing two or more arguments. That is, 19 vocal students in nine classes made utterances consisting of two or more arguments, and they were nominated by 160 students. Ten of these 160 students recalled two or more arguments of their nominee's utterance, but the others recalled only one argument. A majority of the students who recalled one argument from two or more, such as [Z-why and Z-how], or [Z-how, NX-same denominator, and NY-irrelevant], recalled Z-how much more often than Z-why, and Z-how rather than NX-same denominator or NY-irrelevant. This suggests that the students selected and incorporated more important aspects of speakers' utterances in terms of solving the target problem.

We also compared recalled contents of vocal participants with those of silent participants. Table 7 shows accuracies of recall by vocal and silent participants, when 15 vocal students who nominated themselves were excluded. The vocal participants tended to recall more accurately than the silent ones. Chi-square analysis for 'accurate recall,' 'simplified/incomplete recall,' and 'no/erroneous recall' by the two types of participants, excluding the column of 'others,' indicated a significant difference in the distributions, χ^2 $(2, N = 259) = 7.92$, $P < 0.05$. However, we would like to emphasize that more than 3/4 of the silent participants recalled vocal participants' ideas, though some of their recall contents tended to be simplified. This suggests that silent participants also attended to vocal participants' utterances and tried to incorporate their arguments.

Discussion

The present study showed that (a) upper elementary school children could offer in whole-class discussion plausible arguments for or against each of the alternative solutions of the target problem, which in most classes included the correct explanation; (b) both vocal and silent students tended to shift through the discussion toward the most appropriate solution of the target problem with or without the teacher's feedback, incorporating ideas from other students, though their generalization was rather limited; and (c) the students could recognize and memorize reasonable explanations offered by other students in the discussion. In this section, we will present our preliminary model of students' mathematical knowledge construction through whole-class discussion, interpret the above results in terms of the model, and discuss cross-cultural and methodological issues.

The Construction of Knowledge Through Discussion

We assume that individual students, through participating in whole-class discussion, take as well as give a set of interesting and promising ideas that serve as the material for constructing new knowledge (Resnick, 1987). Unlike ideas presented by the teacher or a textbook, students' ideas may be weak, inaccurate, or even false. Nevertheless, social and

Table 7
Accuracies of Recall by Vocal and Silent Participants

	Vocal students (68)	Silent students (215)
Accurate recall	72.1 (49)	54.0 (116)
Simplified/incomplete recall	10.3 (7)	24.2 (52)
No recall/erroneous recall	10.3 (7)	13.0 (28)
Others	7.4 (5)	8.8 (19)

Figures show occurrence rates (%) and figures in parentheses indicate the number of students.
Fifteen vocal students who nominated themselves were excluded from the table.

cognitive constraints enable students to focus on and incorporate plausible ideas only. Students are helped a great deal by social cues, for example, whether ideas are offered by students who are known to be high achievers in mathematics, whether their peers agree with the ideas, etc. They also tend to acquire pieces of knowledge that are consistent with their prior understanding and useful for solving the target problem. Because whole-class discussion is a collaborative attempt to solve the target problem and to understand why the solution is valid, participating in the discussion enables students to readily recognize the pragmatic relevance of the presented ideas.

In the discussion in a large group, many participants have to remain silent. We assume, however, that many silent participants actively try to find ‘agent(s)’ who ‘speak for’ them in the discussion, and if they can, they participate in the discussion vicariously and enhance their understanding as a result. Even when silent participants cannot find such an agent, some of them may respond to the proponents’ and opponents’ arguments in their mind. In other words, active participation is the prerequisite for the construction of knowledge, but it may take forms other than speaking out.

The results of the present experiment, summarized above, can be interpreted using this model. A majority of the students in this study seemed to learn that, when fractions are to be added, they should first make the denominators common, and not add denominators and numerators separately. Some of them even learned how to make denominators of fractions common through participating in the discussion (See Table 4). We must admit, however, that not many arguments offered by the students were informative for learning how to make denominators common or why it is a legitimate procedure. In other words, the whole-class discussion we organized elicited a small number of plausible ideas embedded in many ideas that were less plausible, sometimes even incomprehensible, misleading, or erroneous (see Table 2).

The secret for the success of whole-class discussion can be found in students’ active and socially sensitive minds. Students somehow picked out the most plausible ideas from those offered either by their proponents or opponents. They somehow recognized them as plausible, and memorized the part that was most useful for solving the target problem (see the results regarding the most popular nominees and what the students recalled). In a sense, students’ active and socially sensitive minds allowed the infrequent reasonable ideas to survive and influence themselves. In addition, the students interpreted the successful solution of the target problem as the one they could generally apply to other problems. Although their generalization was rather limited in the present study using the single target problem, we can expect that students will make better use of the solution of prior problems as the number of problems they have solved increases.

Japanese Sociocultural Contexts

The results reported above may have been due to the sociocultural contexts of the Japanese classroom and/or to the students’ Japanese mentalities. It has been pointed out that the practice of teaching/learning mathematics differs between the United States and Asian countries including Japan (e.g. Stevenson & Stigler, 1992).

Hatano & Inagaki (1998) have proposed that two tendencies of Japanese students are particularly important in creating distinctly Japanese educational practice in mathematics that heavily depends on whole class discussion. The first characteristic is the students’ involvement in class. The Japanese classroom often constitutes a community of learners (Brown, 1994) or, more precisely, a caring community, where “all children, regardless of their academic achievements, are genuinely valued members” (Lewis, 1995, p. 177). In this classroom every child can and should expect moral support for her serious attempt to learn, because enhancing individual students’ learning is a supreme goal of lessons in that community. It also implies that a student has to offer her ideas in detail, even when the ideas may be erroneous, because such an attempt at sharing contributes to the group’s successful comprehension.

The second tendency might be called socialization for listenership. Japanese children are good listeners, trained to listen to significant others eagerly and carefully, whereas they are not good at expressing their ideas clearly and persuasively. They are not frustrated nor do they become inattentive even when they have little opportunity for verbally expressing their own ideas, as long as they have some sense of participation. This may be an important prerequisite for an extended whole-class discussion in Japan.

Similarly, as noted by O’Connor & Michaels (1996), Japanese students may tend to refer to other students often. However, as far as the present experiment is concerned, students’ frequent reference to other students was probably caused by the form of the target problem—that it was presented with three answer alternatives. This form, even without the experimenter–teacher’s explicit attempt to connect students’ utterances, structured the discussion so that every argument was either for or against each alternative. We predict that similar patterns of utterances will be observed in other countries, as long as the lesson is organized as in Hypothesis–Experiment–Instruction, including the form of presentation of the target problem. Teachers may collect different solutions from their students, instead of using prearranged set of solutions; this strategy works equally well in terms of students learning (Morita & Inagaki, 1997), though it may complicate analyses from the researchers’ point of view.

Methodological Issues: A Two-Level Analysis

The present experiment was based on what might be called the sociocultural-constraints approach. Although we have stressed the importance of the social—participation in discussion, peers’ ideas, social cues, etc.—in the construction of knowledge, the analysis based on this approach is, and must be, individualistic, because individuals are supposed to remain individuals. In other words, although participants influence and are influenced by others, they are not supposed to construct a collective understanding as a product of a series of negotiations.

A better strategy for analyzing the relationship between collective problem solving and comprehension activity on the one hand and individual understanding on the other would be a two-level analysis of activity (Hatano & Inagaki, 1994). This analysis assumes that through interactions, some discussed and negotiated meanings or understandings are first constructed collectively, and then participants incorporate this ‘information’ individually for generating, elaborating, and revising their comprehension.

We propose, in a Vygotskian fashion, to investigate the target phenomenon of collective comprehension activity with individual outcomes as a collective or intermental process, as well as an individual or intramental process (as reflecting the intermental process). On the one hand, we should observe what occurs in a group as a whole, i.e. how members collaborate for joint understanding. At the same time, however, we should examine cognitive changes in the constituent members, because what has been achieved collectively may not be shared individually. More concretely, we might investigate in future studies (a) how collective problem solving and comprehension activity successfully takes place among those participants who are academically unsophisticated, and (b) how each individual member deepens her comprehension by assimilating information offered in the collective activity. We can examine the former by asking the participants to make a joint summary that all of them think reflects the preceding collective activity as well as by analyzing a transcription of the collective activity. We can investigate the latter by tracing how individual members actively participate in the collective activity and incorporate ideas offered during the activity, through their individual review of the discussion as well as their performance on test items. We believe such innovative methodologies will enable us to understand more deeply how students construct mathematical knowledge through whole-class discussion.

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References

Brown, A. L. (1994). The advancement of learning. *Educational Researcher*, *23*(8), 4–12.

Cobb, P., & Bauersfeld, H. (Eds.) (1995). *The emergence of mathematical meaning: interaction in classroom cultures*. Hillsdale, NJ: Erlbaum.

Hatano, G. (1996). A conception of knowledge acquisition and its implications for mathematics education. In L. Steffe, P. Nesher, P. Cobb, G.A. Goldin, & B. Greer (Eds.), *Theories of mathematical learning* (pp. 197–217). Hillsdale, NJ: Erlbaum.

Hatano, G., & Inagaki, K. (1991). Sharing cognition through collective comprehension activity. In L.B. Resnick, J. M. Levine, & S. D. Teasley (Eds.), *Perspectives on socially shared cognition* (pp. 331–348). Washington, DC: American Psychological Association.

Hatano, G., & Inagaki, K. (1994). *A two-level analysis of collective comprehension activity*. Paper presented at the Symposium, Integrating the Cognitive and Social in the Construction of Mathematical and Scientific Knowledge, American Educational Research Association, New Orleans.

Hatano, G., & Inagaki, K. (1998). Cultural contexts of schooling revisited: A review of the learning gap from a cultural psychology perspective. In S. G. Paris, & H. M. Wellman (Eds.), *Global prospects for education: Development, culture and schooling* (pp. 79–104). Washington, DC: American Psychological Association.

Itakura, K. (1967). Instruction and learning of concept 'force' in static based on Kasetsu–Jikken–Jigyo(Hypothesis–Experiment–Instruction): A new method of science teaching (in Japanese). *Bulletin of National Institute for Educational Research*, *52*, 1–121.

Lampart, M. (1990). When the problem is not the question and the solution is not the answer: mathematical knowing and teaching. *American Educational Research Journal*, *27*, 29–63.

Lewis, C. C. (1995). *Educating hearts and minds: Reflections on Japanese preschool and elementary education*. Cambridge: Cambridge University Press.

Morita, E., & Inagaki, K. (1997). Organizing whole-class discussion in mathematics lessons: Effects of presenting a problem with answer alternatives (in Japanese with an English summary). *Japanese Journal of Educational Psychology*, *45*, 129–139.

Moschkovich, J. N. (1996). Moving up and getting steeper: Negotiating shared descriptions of linear graphs. *Journal of Learning Sciences*, *5*, 239–277.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

O'Connor, M. C. (1994). *The role of language and discourse in the emergence of mathematical understanding*. Paper presented at the Conference, Communication for Understanding in Mathematics Classrooms, University of Wisconsin, Madison.

O'Connor, M. C., & Michaels, S. (1993). Aligning academic task and participation status through revoicing: Analysis of a classroom discourse strategy. *Anthropology and Education Quarterly*, *24*, 318–335.

O'Connor, M. C., & Michaels, S. (1996). Shifting participant frameworks: Orchestrating thinking practices in group discussion. In D. Hicks (Ed.), *Discourse, learning, and schooling* (pp. 63–103). Cambridge: Cambridge University Press.

Orsolini, M., & Pontecorvo, C. (1992). Children's talk in classroom discussions. *Cognition and Instruction*, *9*, 113–136.

Resnick, L. B. (1987). Constructing knowledge in school. In L. S. Liben (Ed.), *Development and learning: conflict or congruence?* (pp. 19–50). Hillsdale, NJ: Erlbaum.

Roschelle, J. (1996). Guest editor's introduction. *Journal of Learning Sciences*, *5*, 169–171.

Stevenson, H. W., & Stigler, J. W. (1992). *The learning gap*. New York: Summit Books.

Stigler, J. W., & Fernandez, C. (1995). Learning mathematics from classroom instruction: Cross-cultural and experimental perspectives. In C. A. Nelson (Ed.), *Basic and applied perspectives on learning, cognition, and development*. *The Minnesota symposia on child psychology* (Vol. 28, pp. 103–130).

Takahashi, M. (1992). Addition and subtraction of fractions (in Japanese). *Studies in Learning How to Learn*, January, 22–24.

Webb, N. M. (1991). Task-related verbal interaction and mathematical learning in small groups. *Journal for Research in Mathematics Education*, *22*, 366–389.

Wells, G. (1993). Reevaluating the IRF sequence: A proposal for the articulation of theories of activity and discourse for the analysis of teaching and learning in the classroom. *Linguistics and Education*, *5*, 1–37.

Wertsch, J. (1985). *Vygotsky and the social formation of mind*. Cambridge, MA: Harvard University Press.

Wertsch, J., & Bivens, J. A. (1992). The social origins of individual mental functioning: alternatives and perspectives. *The Quarterly Newsletter of the Laboratory of Comparative Human Cognition*, *14*, 35–44.