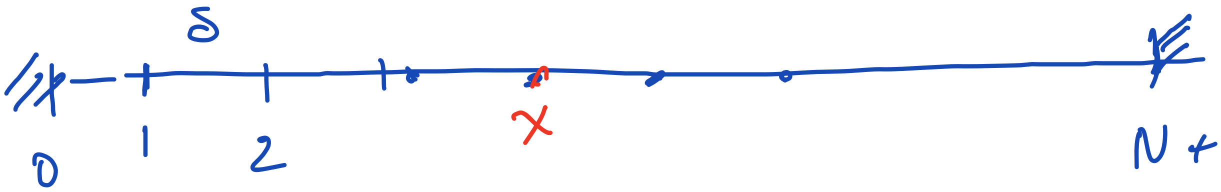
Τετάρτη 3 Απριλίου

$$\ddot{\psi}_n = (\psi_{n+1} + \psi_{n-1} - 2\psi_n) + \alpha\left[(\psi_{n+1}-\psi_n)^2 - (\psi_n - \psi_{n-1})^2\right]$$

$\psi_0 = 0$

$\psi_{N+1} = 0$

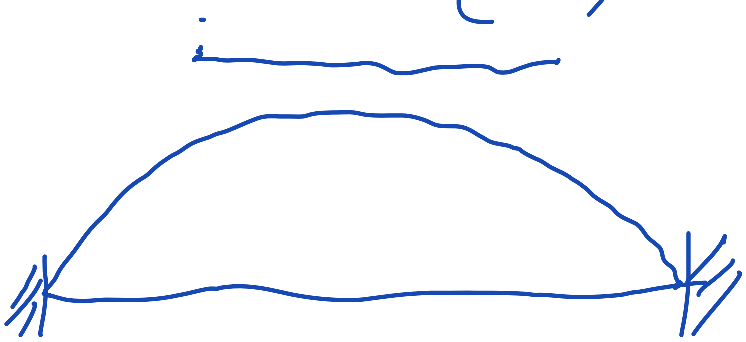


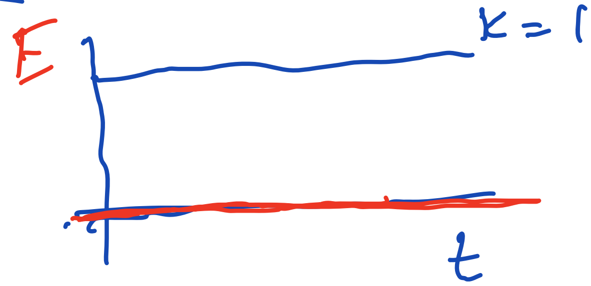
$$\psi_k(n) = \sin\frac{\pi k n}{(N+1)}$$

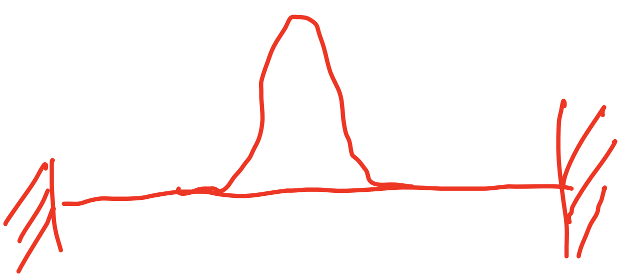
$n = 1 \ \ N$

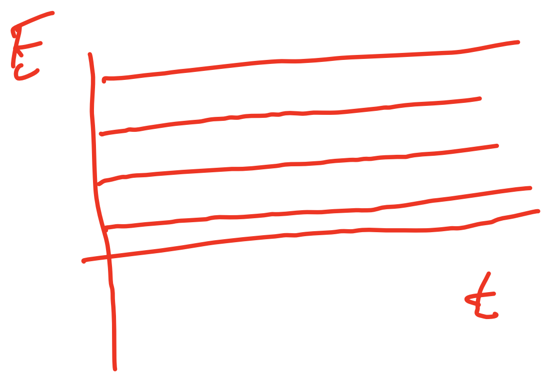
$k = 1, 2, \dots, N$

$\underline{k=1}$ $\psi_1 = \sin\frac{\pi n}{(N+1)}$ ←









$$\psi_k(x) = a_k(t)\sin\frac{\pi k x}{N+1}$$

$x = n$

$n = 1, \dots N$

$$\ddot{\alpha}_k \sin\frac{\pi k n}{N+1} = \alpha_k\left[\sin\frac{\pi k(n+1)}{N+1} + \sin\frac{\pi k(n-1)}{N+1} - 2\sin\frac{\pi k n}{N+1}\right]$$

$$\ddot{\alpha}_k \sin\frac{\pi k n}{N+1} = \alpha_k\left[\sin\frac{\pi k n}{N+1}\cdot\cos\frac{\pi k}{N+1} + \cancel{\cos\frac{\pi k n}{N+1}\sin\frac{\pi k}{N+1}} + \sin\frac{\pi k n}{N+1}\cos\frac{\pi k}{N+1} - \cancel{\cos\frac{\pi k n}{N+1}\sin\frac{\pi k}{N+1}} - 2\sin\frac{\pi k n}{N+1}\right]$$

$$\ddot{\alpha}_K = 2\left(\cos\frac{\pi K}{N+1} - 1\right)\alpha_K$$

$$\ddot{\alpha}_K = -\underbrace{\cancel{4}\sin^2\frac{\pi K}{2(N+1)}}_{\omega_K^2}\alpha_K$$

$$\omega_1^2 = 4\sin^2\frac{\pi}{2(N+1)}, \ \ldots\ldots$$

$$E_K = \frac{1}{2}\left(\dot{\alpha}_K^2 + \omega_K^2\alpha_K^2\right) \leftarrow$$

$$\rightarrow \psi(x) = \sum_{k=1}^{N} \sin\frac{k\pi x}{N+1}\,\alpha_k \qquad x = 1, \ldots, N$$

$$\int_0^{N+1} \sin\frac{k\pi x}{N+1}\,\sin\frac{k'\pi x}{N+1}\,dx = \delta_{kk'}\,\frac{N+1}{2}$$

$\underline{dx = 1}$

$$1 \cdot \sum \sin\frac{k\pi n}{N+1}\,\sin\frac{k'\pi n}{N+1} = \delta_{kk'}\,\frac{N+1}{2}$$

$$\rightarrow \alpha_k = \sum_{n=1}^{N} \frac{2}{N+1}\,\sin\frac{k\pi n}{N+1}\,\psi_n$$

$$S_{nk} = \sin\frac{k\pi n}{N+1}$$

$$\psi_n = S_{nk}\,a_k$$

$$a_k = S^{-1}_{kn}\,\psi_n$$

$$\ddot{\psi}_n = \psi_{n+1} + \psi_{n-1} - 2\psi_n \simeq \frac{\partial^2 \psi}{\partial x^2}$$

$$|\ddot{\psi}\rangle = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_N \end{pmatrix} = \begin{bmatrix} -2 & 1 & 0 & 0 & 0 & \cdots \\ 1 & -2 & 1 & 0 & & \cdots \\ 0 & 1 & - & 1 & & \\ & & & & \ddots & \\ & & & & & -2 \end{bmatrix} |\psi\rangle \qquad \frac{\partial^2 \psi}{\partial t^2} = \frac{\partial^2 \psi}{\partial x^2}$$

$$D^2$$

$$\underline{D^2}$$

$$\ddot{\psi} = D^2 \psi$$

$$\psi = S a$$

$$S \ddot{a} = D^2 S a$$

$$\ddot{a} = \underbrace{(S^{-1} D^2 S)}_{= -\Omega^2} a$$

$$\ddot{\psi} = D^2 \psi$$

$$u = \begin{bmatrix} \psi \\ \varphi \end{bmatrix}$$

$$\dot{\psi} = \varphi$$
$$\dot{\varphi} = D^2 \psi$$

$$\dot{u} = \begin{bmatrix} 0 & 1 \\ D^2 & 0 \end{bmatrix} u$$

$$\begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 0 & I \\ -\Omega^2 & 0 \end{bmatrix}$$