Μαρτίου 11 2024

$$\dot{x} = -x \log x \qquad \ln$$

$\dot{x} = 0 \qquad x = 0$

$$\rightarrow \quad X = x \ln\left(\frac{k}{X}\right)$$

$$\dot{x} = x(1-x)$$

Άλγεβρα

$2 \times 2 \qquad \alpha \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \beta \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \gamma \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + \delta \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} =$

$A^0 \quad A \quad A^2 \quad A^3 \quad \ldots\ldots$

I

$$\det \begin{vmatrix} \alpha-\lambda & \beta \\ \gamma & \delta-\lambda \end{vmatrix} = 0 \qquad \lambda^2 - \underbrace{(\alpha+\delta)}_{\text{Trace}(A)}\lambda + \underbrace{(\alpha\delta-\beta\gamma)}_{\det A} = 0$$

$$\boxed{A^2 - (\alpha+\delta) A + (\alpha\delta - \beta\gamma) I = 0}$$

Cayley-Hamilton

I, A

A^2 δεν είναι γραμμικώς A

$$A^3 - \text{Trace}(A)\, A^2 + \det(A)\, A = 0$$

$$\alpha A + \beta I$$

$$e^{At} = I + At + \frac{A^2 t^2}{2!}$$

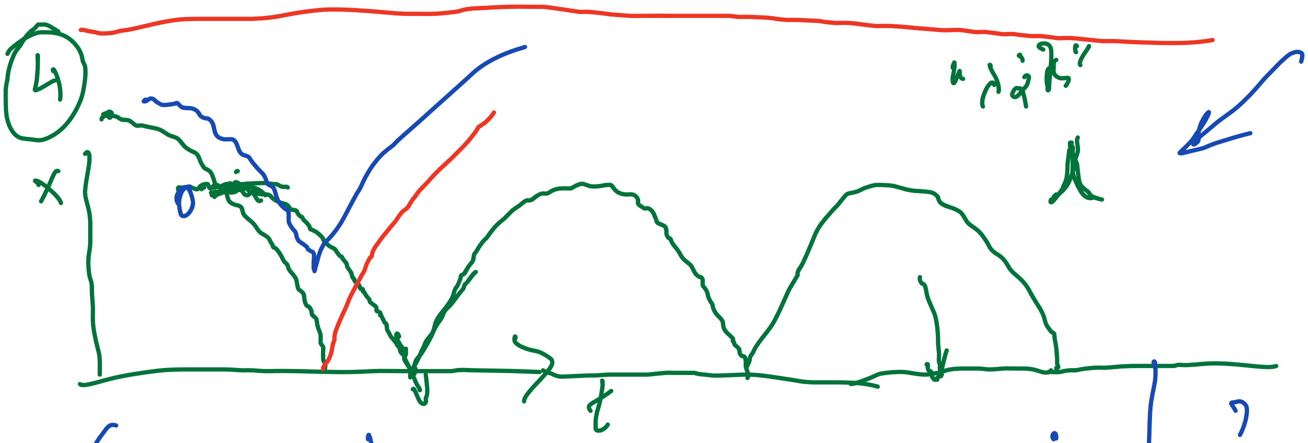
$$= \alpha A + \beta I$$

$$A = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \lambda_1^2 & 0 \\ 0 & \lambda_2^2 \end{bmatrix}$$

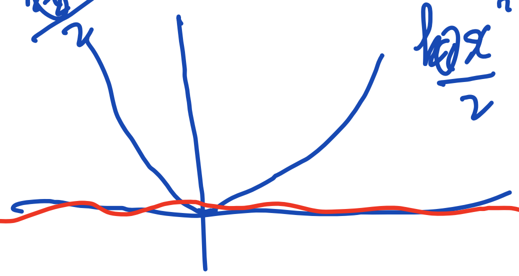
$$e^{\varepsilon \frac{d}{dt}} f(t) = f(t+\varepsilon)$$

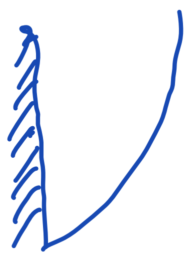
4



$T(E) \longrightarrow$ μη γραμμική !?

λάθη





μη γραμμική

$U(x) = V(-x)$

3 $\quad 0 \cdot \ddot{x} = -1 \cdot 0 \quad x(0)=0 \quad \dot{x}(0)=0$

$\dot{x} = c\sqrt{x} \quad x(0)=0$

$\dot{x} = x(1-x)$ $\quad x \geq 0$ σύστημα δυναμικό [illegible]

$$\boxed{x_{n+1} = x_n + \delta x_n (1-x_n)}$$ Euler

διατηρεί τα αποτ[illegible]

Ελκυστής (ν-σύνολο) $t \to \infty$ 1

$\delta < 2$

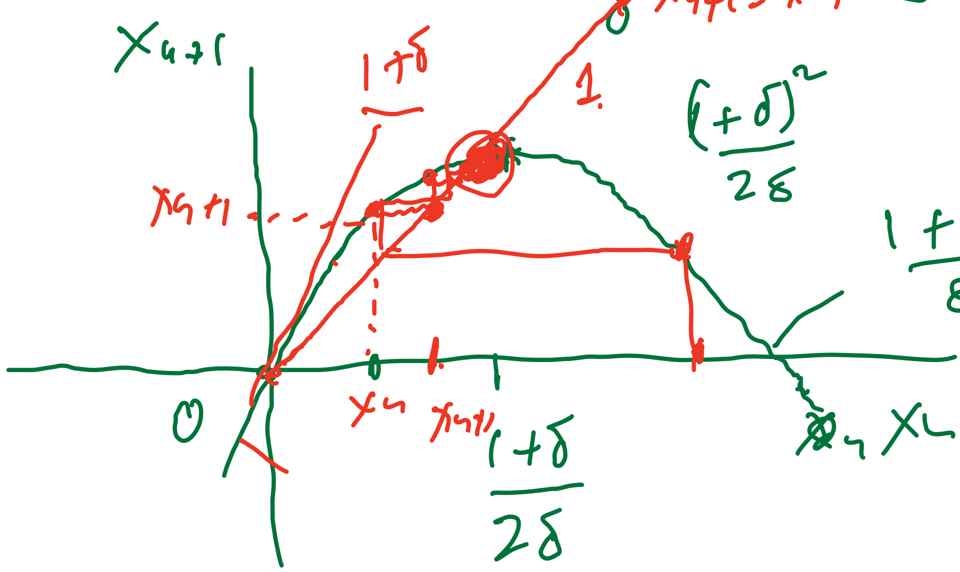
$\wedge$

$\delta > 2$

1

περιοδική λύση

$$x_{n+1} = \delta x_n \left[\frac{1+\delta}{\delta} - x_n \right] = f(x_n)$$



$\frac{1+\delta}{2\delta}(1+\delta)$

$\frac{1+\delta}{\delta} = 1 + \frac{1}{\delta}$

$\boxed{f(x_n) = x_n}$

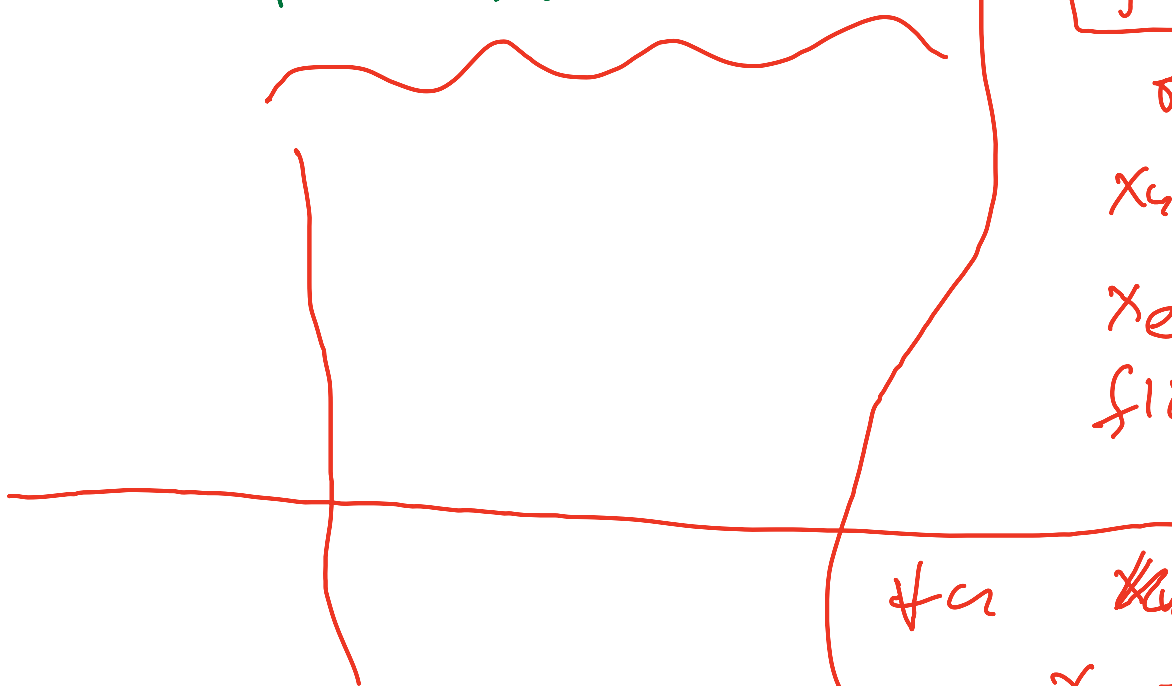
σταθερό

x_{n+1}

x_e $\quad x_1 = x_e$

$f(x_e) = x_e$

$\forall n$ $\quad x_n = x_e$



σταθερό σημείο

$\delta \quad x_e = 0 \qquad f(0) = 0$

$$\delta x_e \left(\frac{1+\delta}{\delta} - x_e \right) = x_e$$

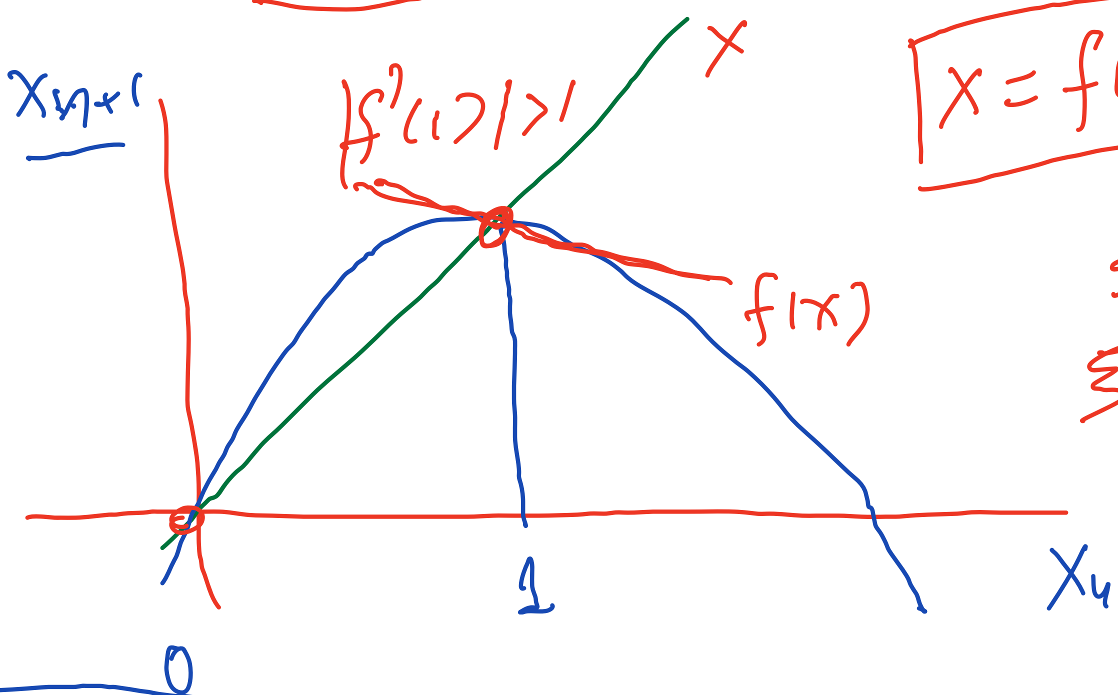
$$(1+\delta)\, x_e - \delta x_e^2 = x_e$$

$$\delta x_e = \delta x_e^2$$

$$\delta \; x_e (1 - x_e) = 0$$

σταθερά σημεία της διακριτά σύστ.

$$x_e = 0, \qquad x_e = 1$$



$$\boxed{x = f(x)}$$

x

$\xi = x - x_e$

$$x_{n+1} = f(x_n)$$

x_e $\quad \forall n$

$$\boxed{\xi_n = x_n - x_e}$$

$$\boxed{x_e = f(x_e)}$$

f:

$$x_n = x_e + \xi_n \qquad x_{n+1} = x_e + \xi_{n+1}$$

$$\underbrace{x_e + \xi_{n+1}}_{x_{n+1}} = f(x_e + \xi_n)$$

$$= \underset{x_e}{f(x_e)} + f'(x_e)\xi_n + O(\xi_n^2)$$

$$\boxed{\xi_{n+1} = f'(x_e)\,\xi_n}$$

[illegible] $|f'(x_e)| > 1$

$$\dot{\xi} = f'(x_e)\,\xi$$

[illegible] > 0

$$\xi(t) = e^{t f'(x_e)}\,\xi(0)$$

$f'(x_e) > 0$

$f'(x_e) < 0$

$$f(x) = (1+\delta)x - \delta x^2 \qquad \delta > 0$$

$$f'(x) = (1+\delta) - 2\delta x$$

$x=0$: $f'(0) = 1+\delta > 1$

[illegible] $x=0$ [illegible]

$x=1$: $f'(1) = 1+\delta - 2\delta = 1-\delta$

$|1-\delta| > 1 \qquad \delta > 2$

$$\xi_{n+1} = \xi_n + \varepsilon f'(x_e)\xi_n$$

$$= (1 + \varepsilon f_e')\,\xi_n$$

$f_e' > 0$

$\delta > 0$

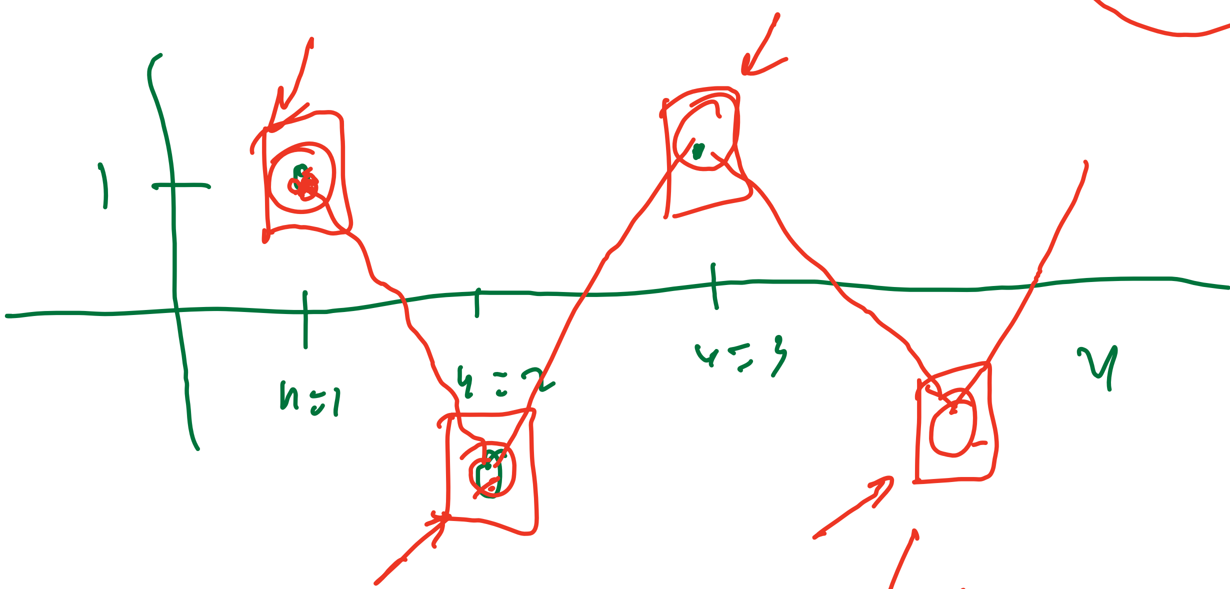
$$\boxed{\delta = 2 \quad \text{τι γίνεται}}$$

$$x_{n+2} = f(f(x_n))$$

$$\boxed{f(x_e) = x_e}$$

$$\boxed{x_e = f(f(x_e))}$$

Περιοδική για 2



$$\left|\big(f(f(x_e))\big)'\right| = \left|f'(f(x_e))\, f'(x_e)\right| > 1$$

$$x_e = f(f \cdots (f(x_e))))))$$

$$\boxed{e^{At}}$$

$$f(x_e, y_e) = g(x_e, y_e) = 0$$

$$\dot{x} = f(x,y)$$

$$\dot{y} = g(x,y)$$

$$\dot{\xi} = A\,\xi$$

$$A = \begin{pmatrix} \partial f/\partial x & \partial f/\partial y \\ \partial g/\partial x & \partial g/\partial y \end{pmatrix}_{x_e, y_e}$$

A αν $\exists_\lambda$, $\mathrm{Re}(\lambda) > 0$ ασταθές

$\forall \lambda$ $\mathrm{Re}(\lambda) < 0$ ευσταθές

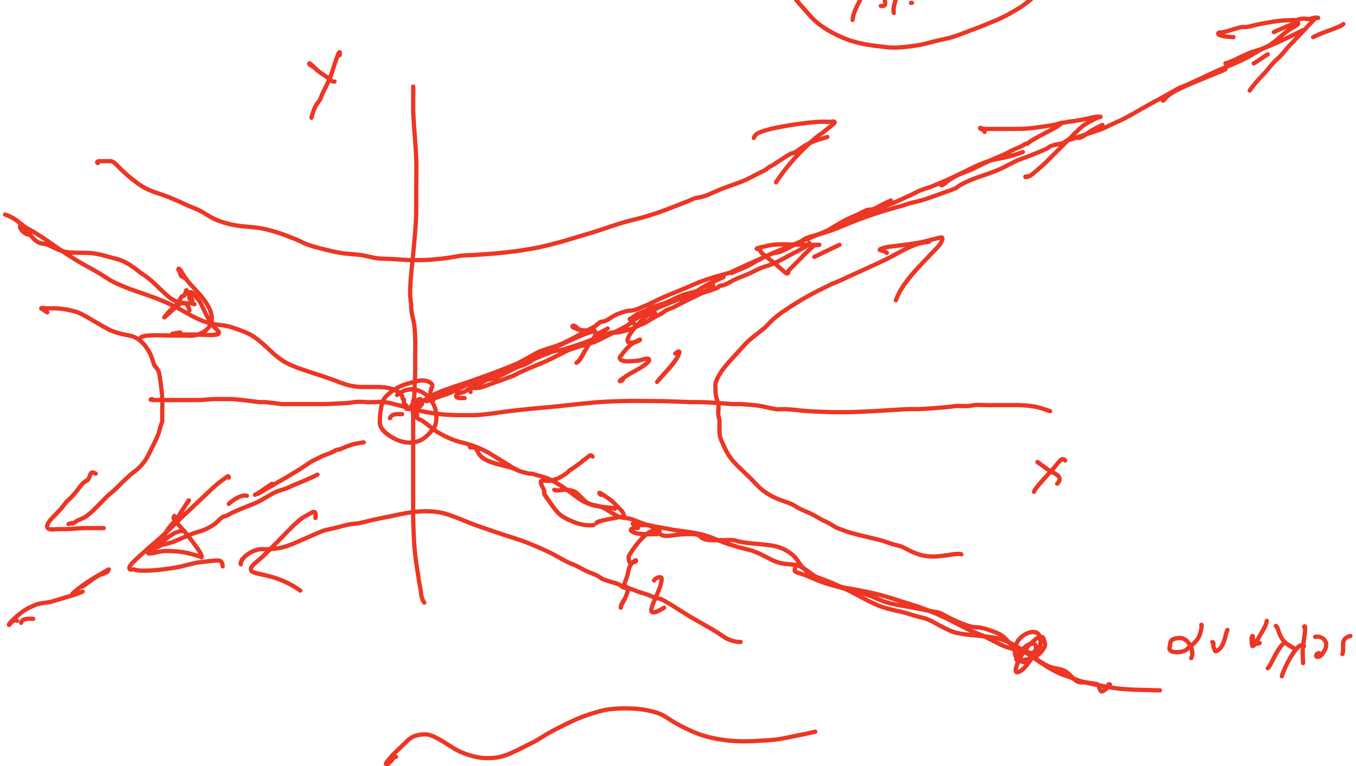
ξ_1 είναι ιδιοκατα ξ_2

$A\xi_1 = \lambda_1 \xi_1$ | $A\xi_2 = \lambda_2 \xi_2$

$e^{At}\xi_1 = e^{\lambda_1 t}\xi_1$ | $e^{At}\xi_2 = e^{\lambda_2 t}\xi_2$

$\lambda_1 > 0$

$\lambda_2 <$



αν ακολουθει