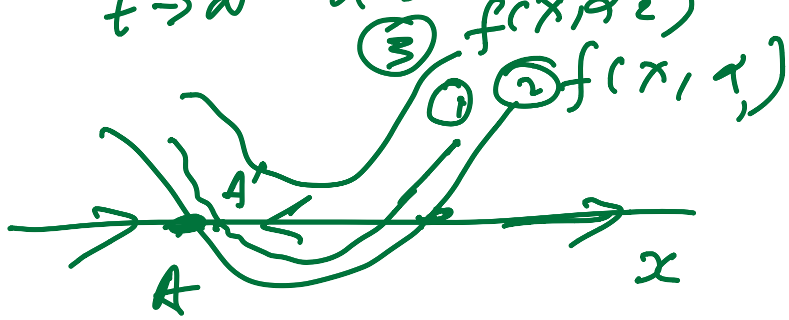
Παρασκευή 2 Απριλίου

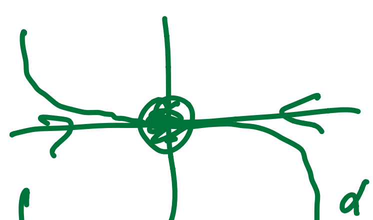
→ $\dot{x} = f(x,\alpha)$ $\qquad$ $f(x,\alpha)$ C^1

$x_e(\alpha)$

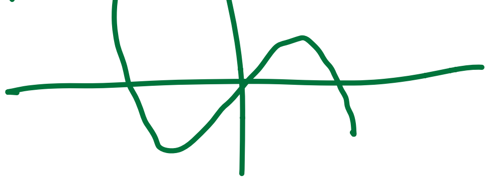
→ $x(t,\alpha)$ συνεχής t, α

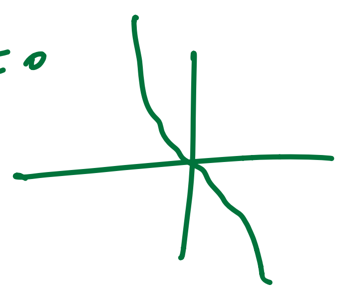
$$\lim_{t\to\infty}\lim_{\alpha\to\alpha_c} x(t,\alpha) \neq \lim_{\alpha\to\alpha_c}\lim_{t\to\infty} x(t,\alpha)$$



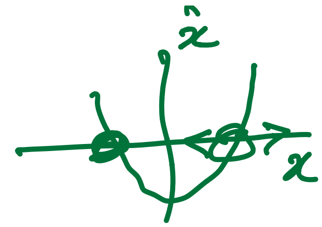


$\alpha > 0$ $\qquad$ $\alpha = 0$



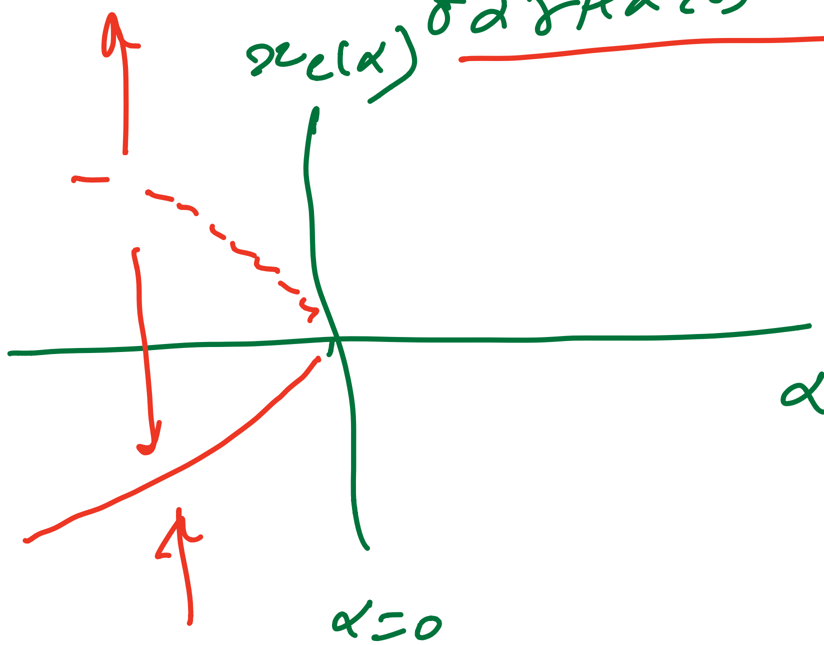


Δομική ευστάθεια



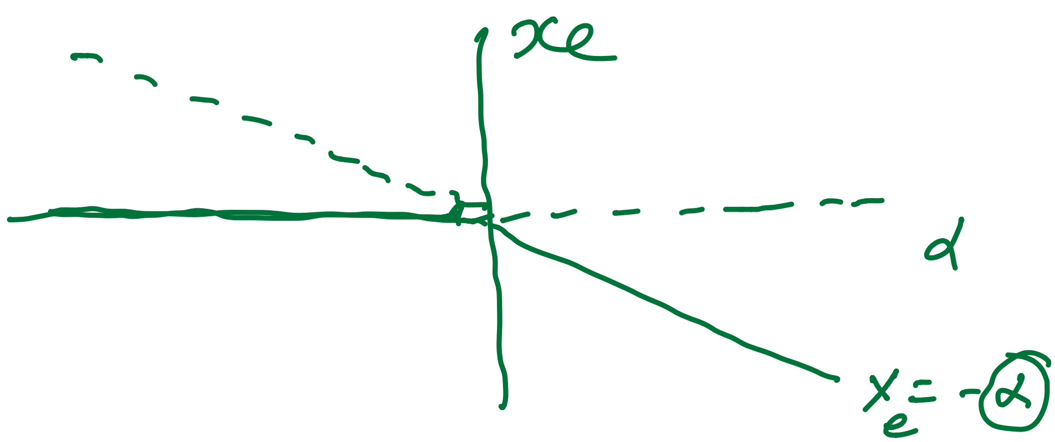
$$\dot{x} = \alpha + x^2$$

σάγματος - κόμβου



κατοπτρισμός

$$\dot{x} = \alpha x + x^2$$

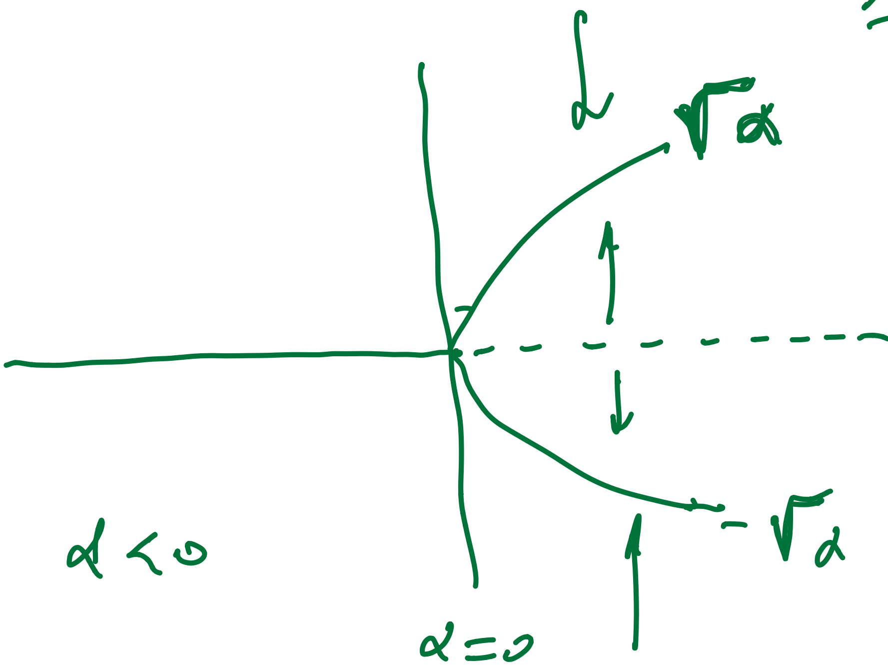


pitchfork δικρανική

$$\longrightarrow \dot{x} = x(\alpha - x^2) = -\frac{d}{dx}V$$

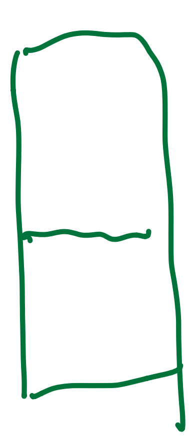
υπερκρίσιμη δικρανική

supercritical



$$\dot{x} = \varepsilon + x(\alpha - x^2)$$

critical opalescence
κρίσιμη διαφάνεια

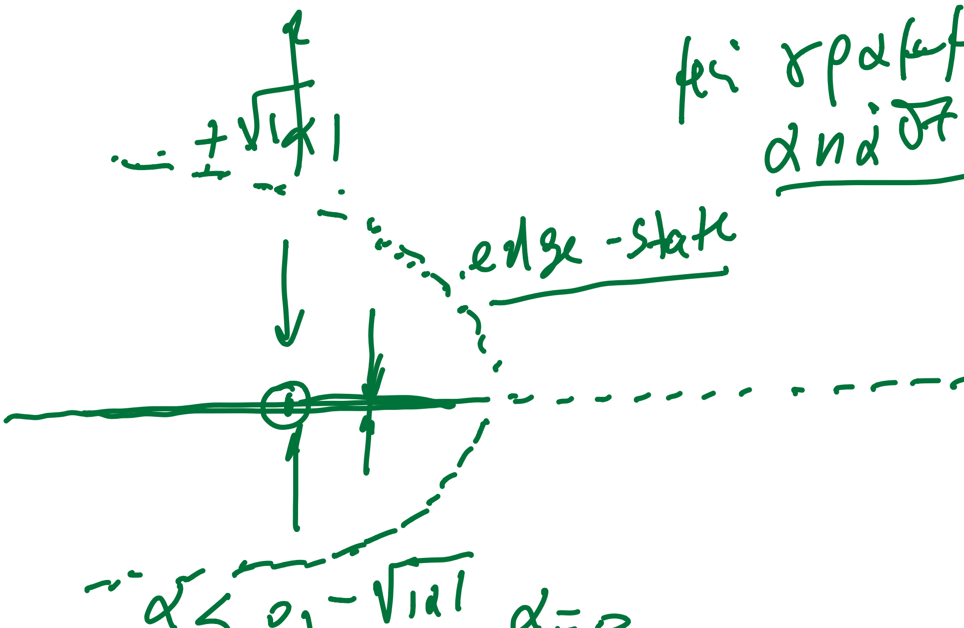


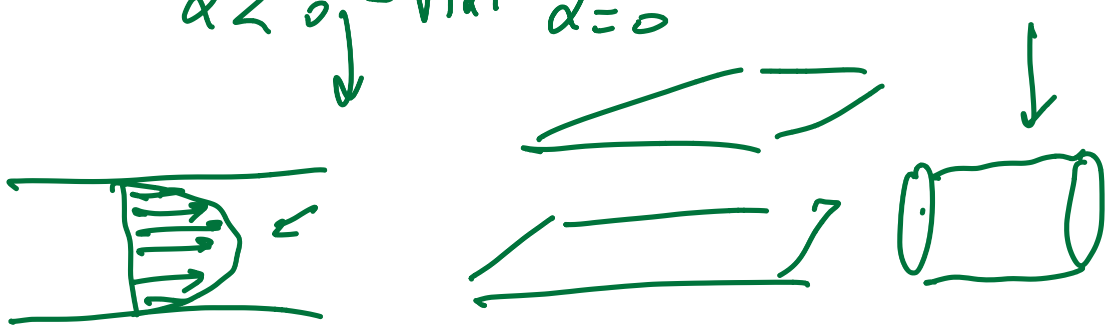
θαμηνική

υποκρίσιμη διαφανικής

$$\dot{x} = x(\alpha + x^2)$$

μη γραμμικής ανάστιξ





$$\dot{x} = f(x) \quad (1)$$

$x_e \qquad f(x_e) = 0$

$x(t) = x_e \quad \forall\, t$ είναι λύση

$f \in C^1 \qquad x(t) = x_e + \eta(t)$

$$(1) \to \dot{\eta} = f(x_e + \eta)$$

$$= \underbrace{f(x_e)}_{=0} + f'(x_e)\eta + O(\eta^2)$$

Εάν $f'(x_e) \neq 0$ αρκούντως μικ.

$\eta \ll 1$

$$\dot{\eta} = f'(x_e)\,\eta$$

ελέγχει τη γραμμική ευστάθεια

$$\eta(t) = e^{f'(x_e)t}\,\eta(0)$$

είναι ευσταθής εάν

$$f'(x_e) < 0$$

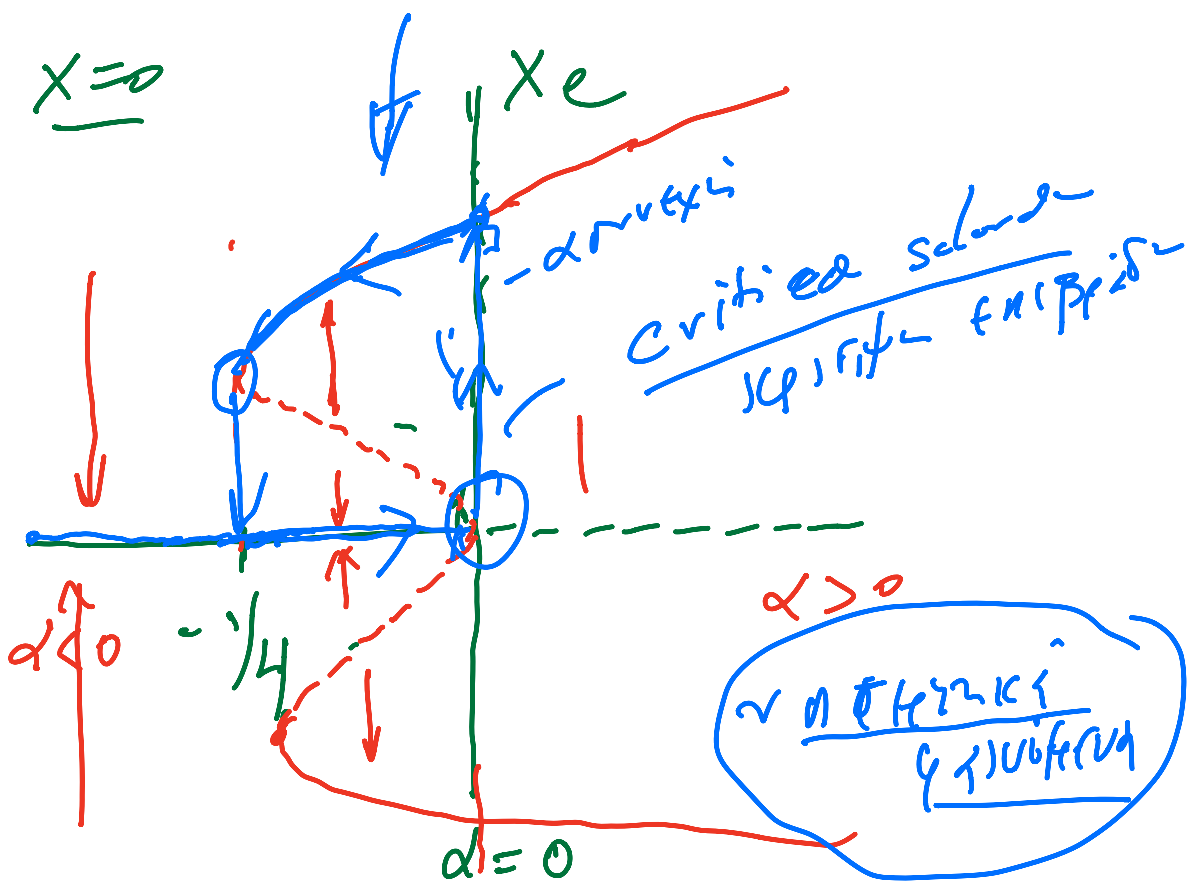
είναι ασταθής εάν $f'(x_e) > 0$

εάν η $f'(x_e) \neq 0$

τότε η ανάλυση και
γραμμική έχει και
γραμμική και ισχύει στη
μια περιοχή γύρω από
σημείο ισορροπίας

$$\dot{x} = x(\alpha + x^2) - x^5$$

$$= x(\alpha + x^2 - x^4)$$



$$x^4 - x^2 - \alpha = 0$$

$$x^2 = \frac{1}{2} \pm \sqrt{\frac{1}{4} + \alpha}$$

(i) $\alpha < -1/4$

(ii) $\underline{\alpha > 0}$

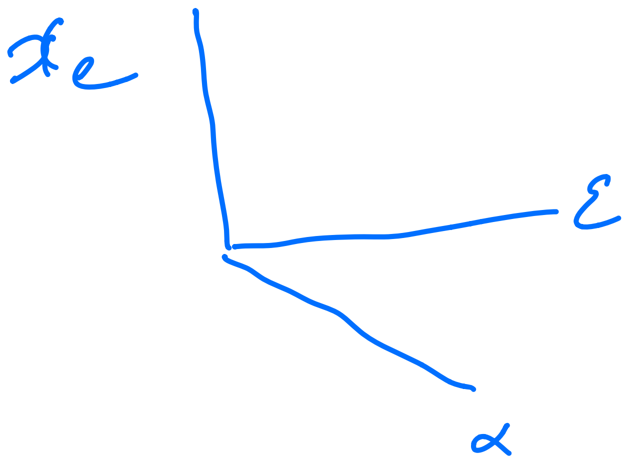
$$x = \pm\sqrt{1/2 + \sqrt{1/4 + \alpha}}$$

[illegible]

(iii) $-\frac{1}{4} < \alpha < 0$ $\quad x = \pm\sqrt{\frac{1}{2} \pm \sqrt{\frac{1}{4}+\alpha}}$

$$\dot{x} = \varepsilon + x(\alpha - x^2), \quad \varepsilon > 0$$

codimension 2

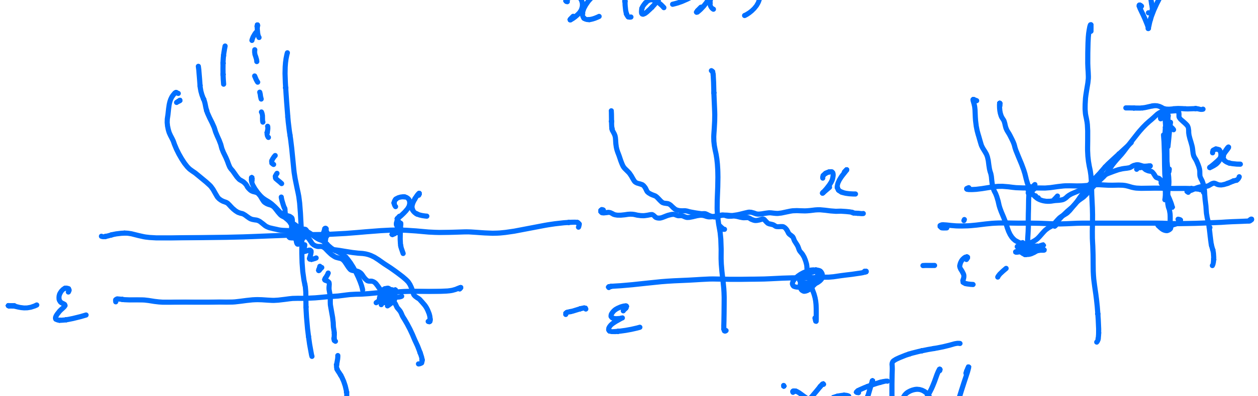


$\varepsilon > 0$ σταθερό

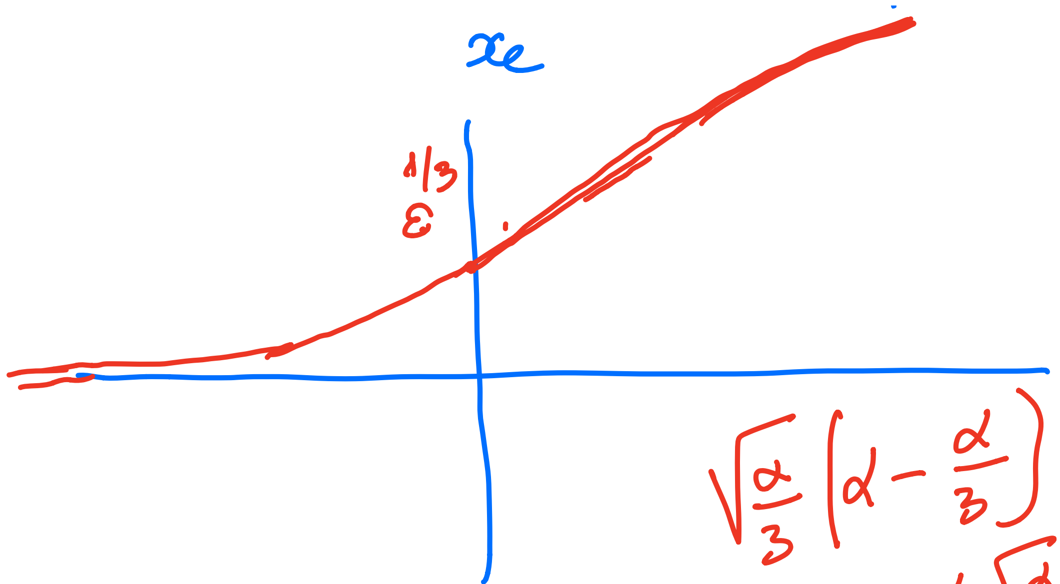
α

$\alpha < 0$ $\qquad \alpha = 0$ $\qquad \alpha > 0$

$x(\alpha - x^2)$

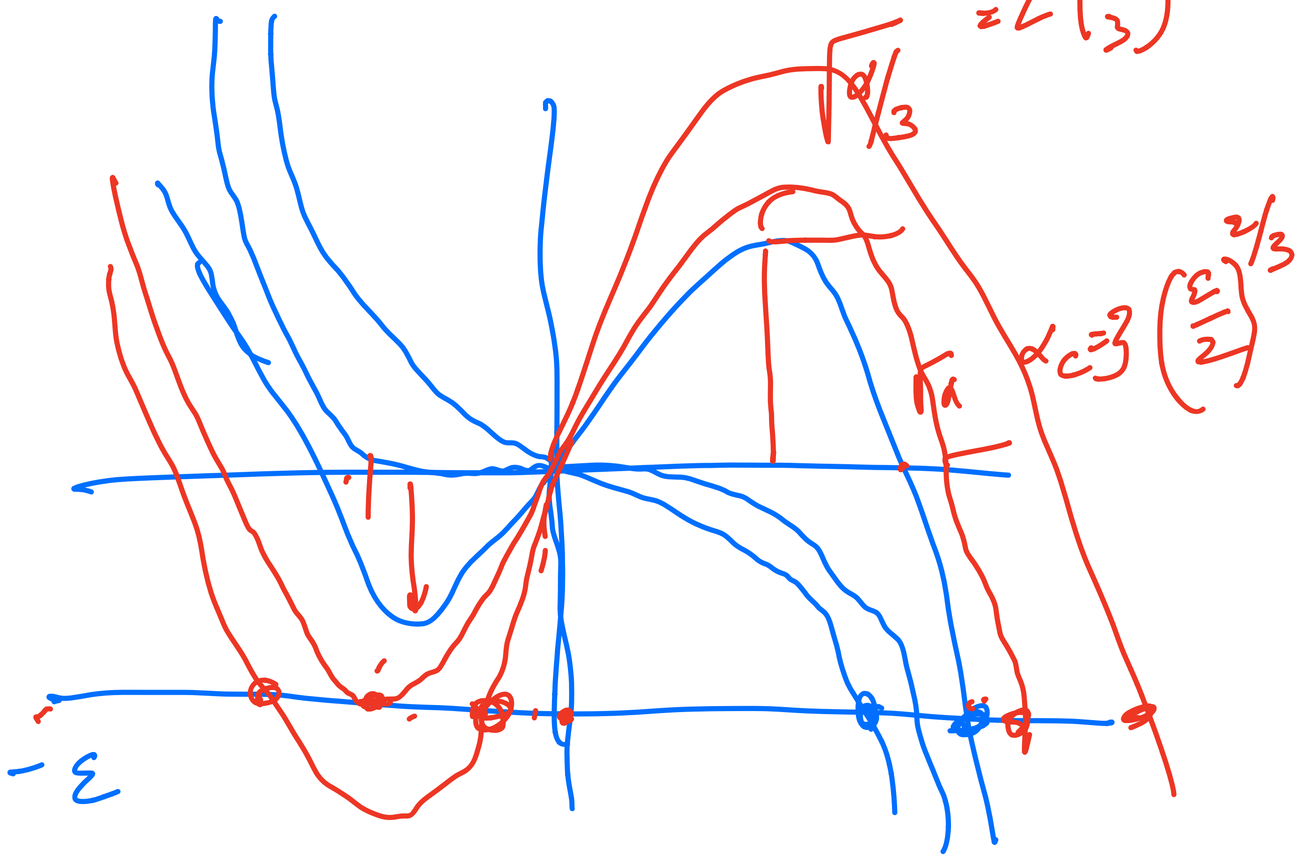


$x = \pm\sqrt{\alpha/3}$



$\alpha = 0$

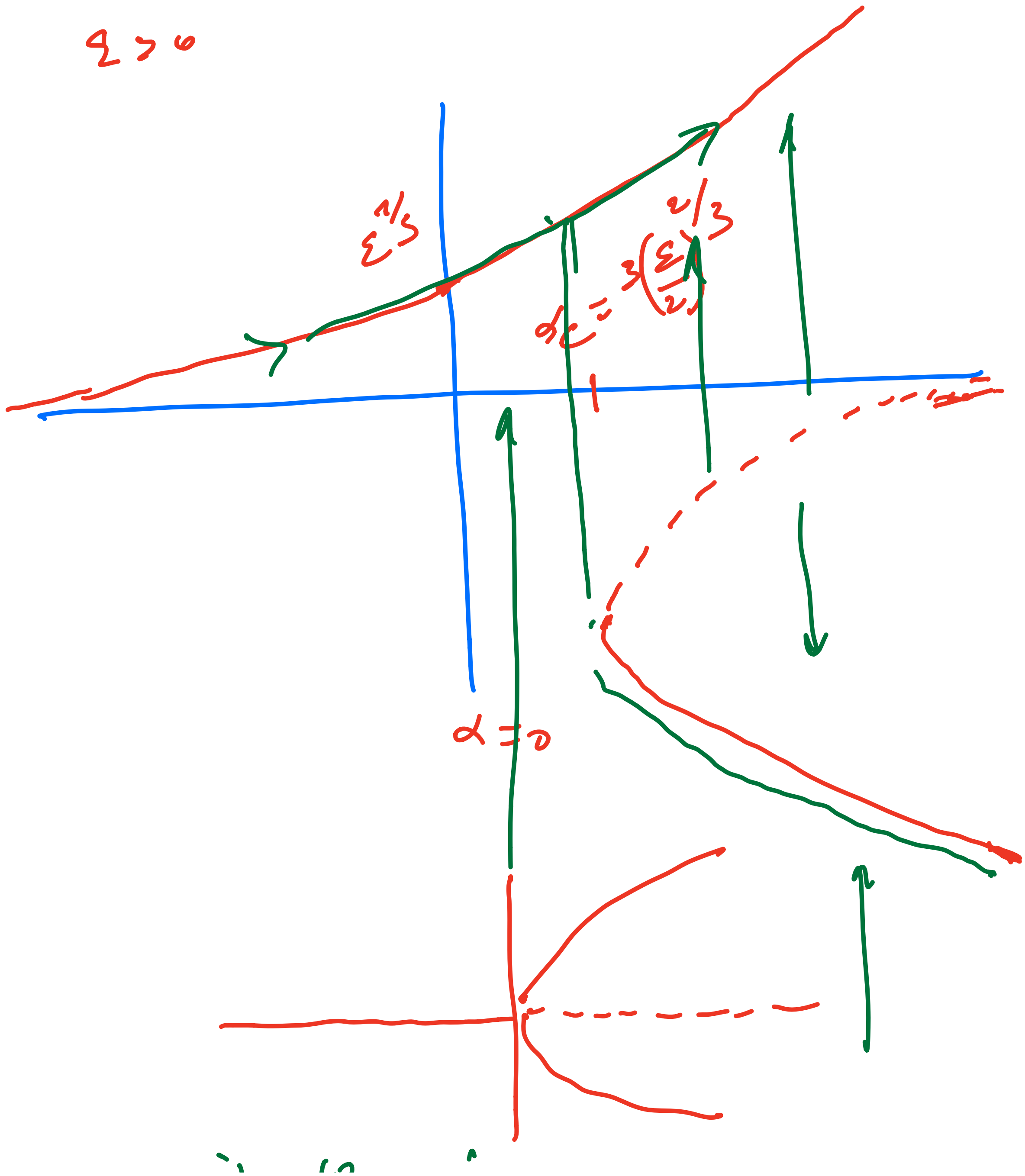
$$\sqrt{\frac{\alpha}{3}}\left(\alpha - \frac{\alpha}{3}\right) = \frac{2\alpha}{3}\sqrt{\frac{\alpha}{3}} = 2\left(\frac{\alpha}{3}\right)^{3/2}$$



$\alpha_c \qquad 2\left(\frac{\alpha_c}{3}\right)^{3/2} > \varepsilon$

T:pf fxl TPA pifn

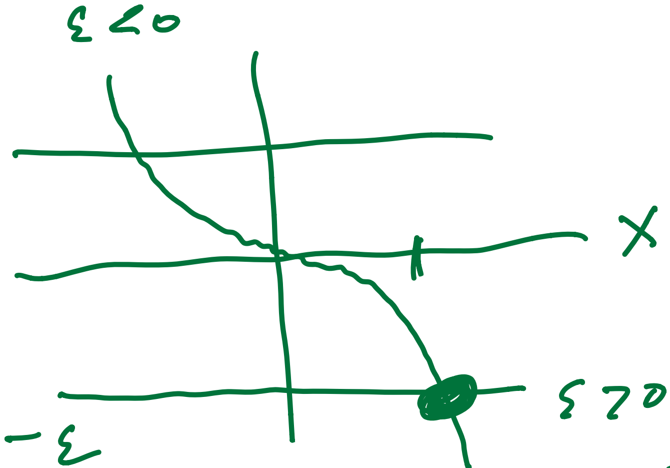
$\varepsilon > 0$



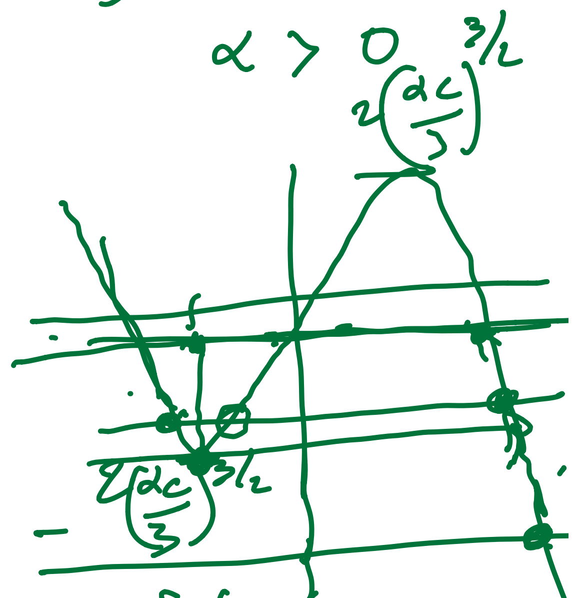
[illegible] 2

ε $\qquad$ $\dot{x} = x(\alpha - x^2)$

$\alpha < 0$



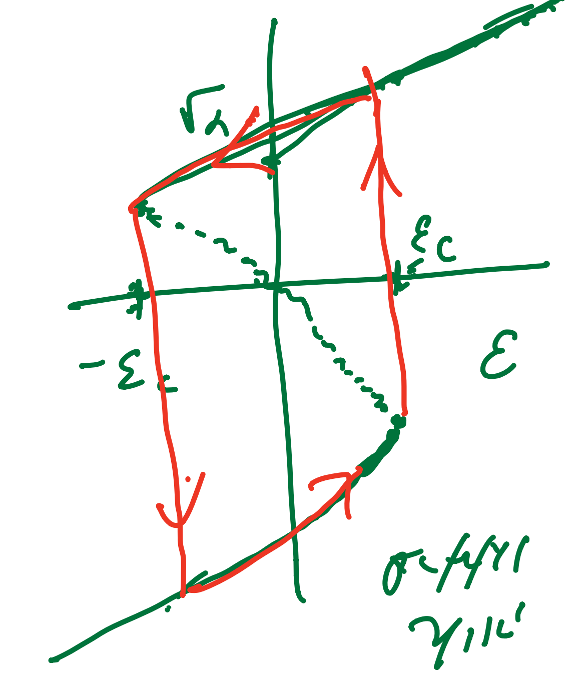
$\alpha > 0$



$x_e(\alpha,\varepsilon)$

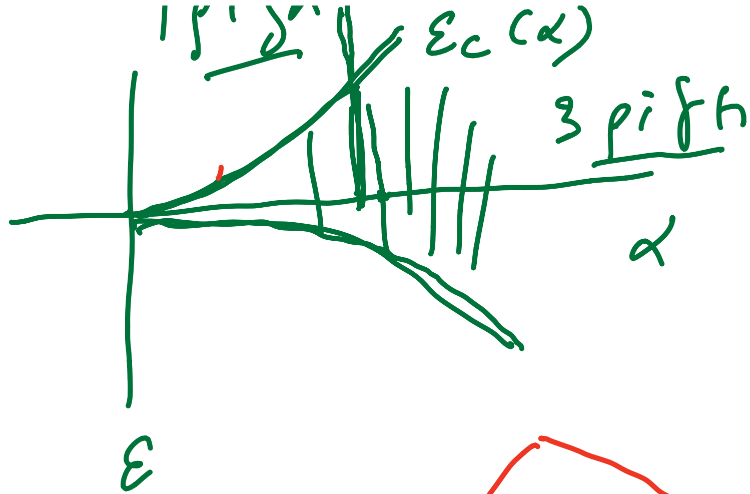
$\varepsilon = 0$

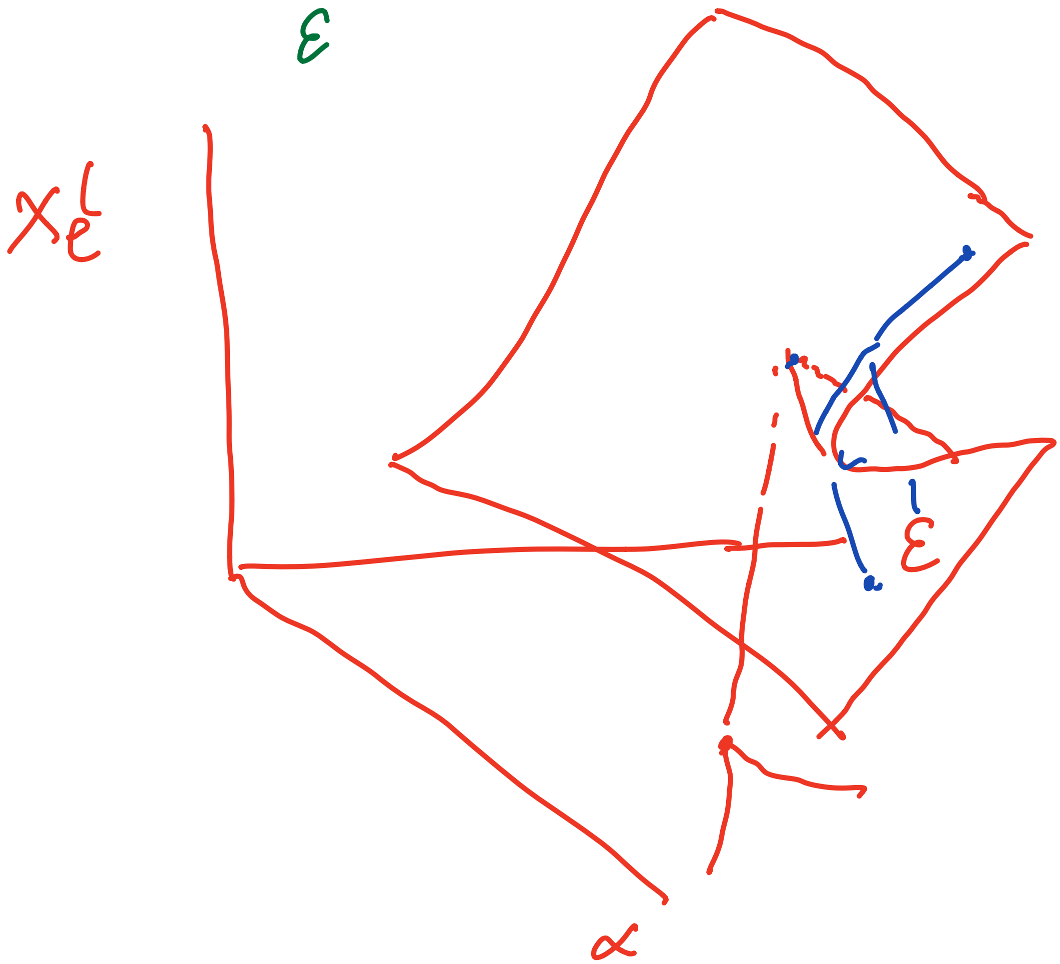
[illegible]



[illegible]

$$\varepsilon_c = 2\left(\frac{\alpha_c}{3}\right)^{3/2}$$


$\varepsilon_c(\alpha)$
α
ε


α
ε