(62.)

a) $w = z + (1-2i)$

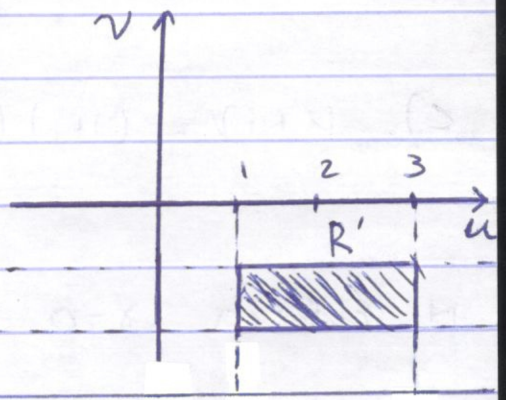
b) $w = \sqrt{2}\, e^{i\frac{\pi}{4}} z$

c) $w = \sqrt{2}\, e^{i\frac{\pi}{4}} z + (1-2i)$

Λύση: a) $w = z + (1-2i) \Rightarrow u + iv = (x+1) + i(y-2) \Rightarrow$

$$\begin{cases} u = x+1 \\ v = y-2 \end{cases}$$

Η ευθεία $x=0 \to u = 0+1 = 1$
– II – $x = 2 \to u = 2+1 = 3$
– II – $y = 0 \to v = -2$
– II – $y = 1 \to v = -1$



b) $w = \sqrt{2}\, e^{i\frac{\pi}{4}} z = \cancel{\sqrt{2}}\left(\frac{1}{\cancel{\sqrt{2}}} + \frac{i}{\cancel{\sqrt{2}}}\right)\cdot(x+iy) = (1+i)\cdot(x+iy) \Rightarrow$

$u + iv = x - y + i(x+y) \Rightarrow \begin{cases} u = x-y \\ v = x+y \end{cases}$

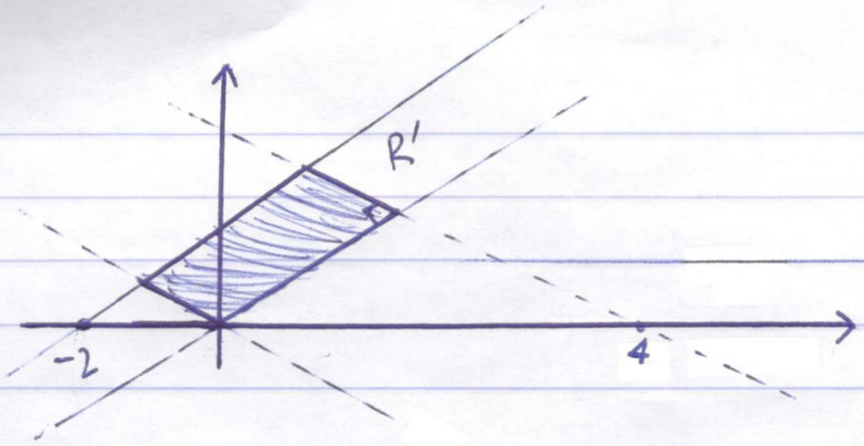
Η ευθεία $x = 0 \to \begin{cases} u = -y \\ v = +y \end{cases} \Rightarrow v = -u$

– II – $x = 2 \to \begin{cases} u = 2-y \\ v = 2+y \end{cases} \Rightarrow v = -u + 4$

– II – $y = 0 \to \begin{cases} u = x \\ v = x \end{cases} \Rightarrow v = u$

– II – $y = 1 \to \begin{cases} u = x-1 \\ v = x+1 \end{cases} \Rightarrow v = u + 2$

63.



$W = \sqrt{2} e^{i\frac{\pi}{4}} z = f(z)$

$f'(z) = \sqrt{2} e^{i\frac{\pi}{4}}$, $|f'(z)|^2 = 2$

$$\left.\begin{matrix} u_x = 1, & u_y = -1 \\ v_x = 1, & v_y = 1 \end{matrix}\right\} \Rightarrow J = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix} = 2$$

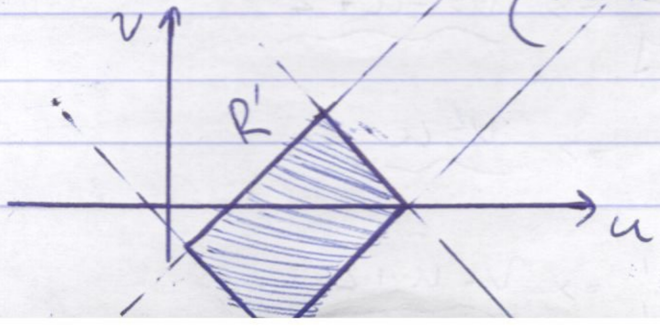
c) $u + iv = (1+i)\cdot(x+iy) + (1-2i) \Rightarrow \begin{cases} u = x - y + 1 \\ v = x + y - 2 \end{cases}$

Η ευθεία $x = 0 \to \begin{cases} u = -y + 1 \\ v = y - 2 \end{cases} \Rightarrow v = -u - 1$

— ιι — $x = 2 \to \begin{cases} u = 3 - y \\ v = y \end{cases} \Rightarrow v = -u + 3$

— ιι — $y = 0 \to \begin{cases} u = x + 1 \\ v = x - 2 \end{cases} \Rightarrow v = u - 3$

— ιι — $y = 1 \to \begin{cases} u = x \\ v = x - 1 \end{cases} \Rightarrow v = u - 1$



3. Αντιστροφή : $\boxed{w = \frac{1}{z}}$

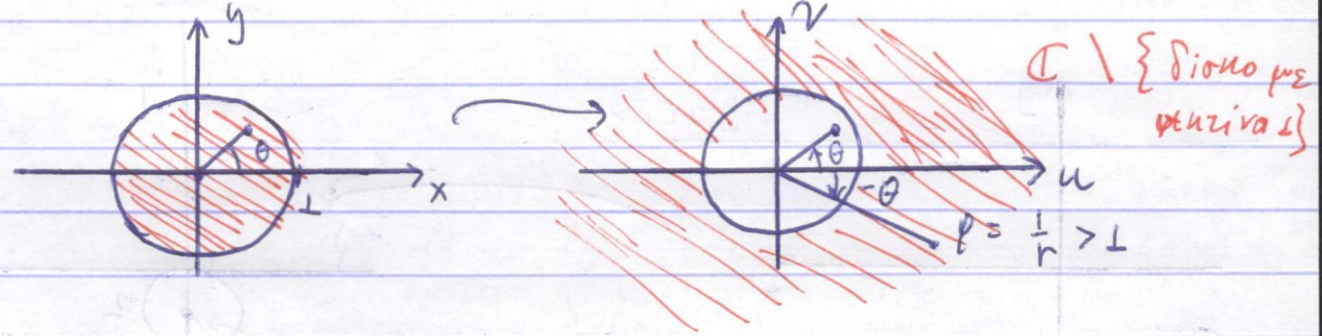
Αν $w = \rho e^{i\varphi}$ και $z = re^{i\theta}$, τότε.

$$\rho e^{i\varphi} = \frac{1}{r} e^{-i\theta} \Rightarrow \begin{cases} \rho = \frac{1}{r} \\ \varphi = -\theta \end{cases}$$

$\rho = \frac{1}{r}$ → αντιστροφή του μέτρου

$\varphi = -\theta$ → "ανάκλαση" του ορίσματος ως προς τον άξονα x.

π.χ.



Πως απεικονίζονται καμπύλες :

$$w = f(z) = u + iv = \frac{1}{x+iy} = \frac{x-iy}{(x+iy)\cdot(x-iy)} \Rightarrow \begin{array}{l} u = \frac{x}{x^2+y^2} \\ v = \frac{-y}{x^2+y^2} \end{array}$$

και $x = \frac{u}{u^2+v^2}$

$y = \frac{-v}{u^2+v^2}$

Κύκλος στο επίπεδο z :

$$C(0,r): \; x^2+y^2 = r \Rightarrow \frac{u^2}{(u^2+v^2)^2} + \frac{v^2}{(u^2+v^2)^2} = r^2 \Rightarrow$$

$$\Rightarrow u^2+v^2 = \frac{1}{r^2} = \rho^2$$

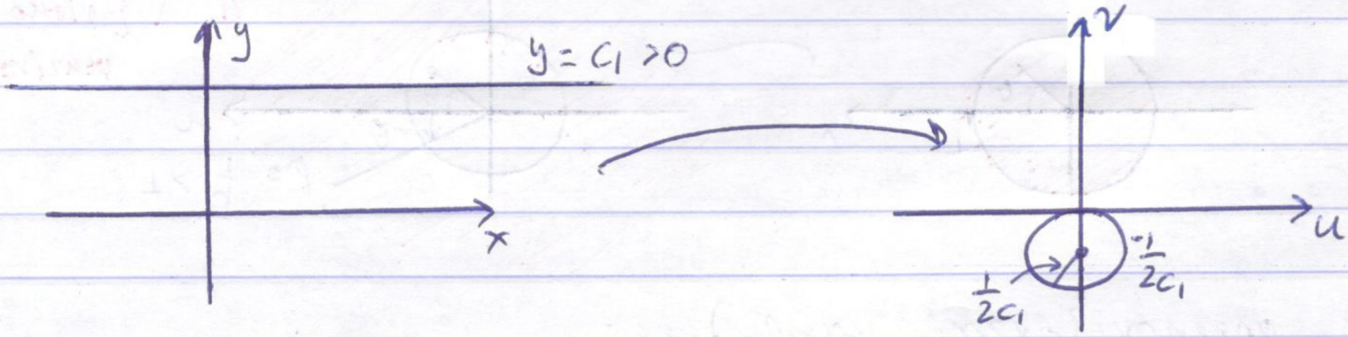
! Κύκλος στο επίπεδο z → κύκλος στο επ. w !

Ευθεία στο επ. z:

$$y=c_1 \Rightarrow -\frac{v}{u^2+v^2}=c_1 \Rightarrow u^2+v^2+\frac{v}{c_1}=0 \Rightarrow$$

$$u^2+v^2+2\frac{v}{2c_1}+\left(\frac{1}{2c_1}\right)^2=\left(\frac{1}{2c_1}\right)^2 \Rightarrow u^2+\left(v+\frac{1}{2c_1}\right)^2=\left(\frac{1}{2c_1}\right)^2$$

Εξίσωση κύκλου στο επ. w με κέντρο $\left(0,-\frac{1}{2c_1}\right)$ και ακτίνα $\frac{1}{2c_1}$



Ευθεία στο επ. z → Κύκλο στο επ. w

2.) Ο μετασχηματισμός $w=z^2$

Αν $z=re^{i\theta}$ και $w=\rho e^{i\varphi}$, τότε

$$\rho e^{i\varphi}=r^2e^{i2\theta} \Rightarrow \begin{cases}\rho=r^2\\ \varphi=2\theta\end{cases}$$

π.χ.