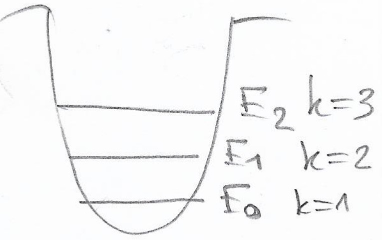
ΤΡΙΣΤΑΘΜΙΚΟ ΜΕ ΑΑΤ

$$\dot{C}_{k'} = -\frac{i}{\hbar}\sum_k C_k(t)\, e^{i(\Omega_{k'}-\Omega_k)t}\, U_{\varepsilon k'k}(t)$$

1

ΑΑΤ $\boxed{E_n = \hbar\Omega\left(n+\frac{1}{2}\right)}$ $\quad E_{n+1} - E_n = \hbar\Omega$



$$E_0 = \frac{\hbar\Omega}{2}, \quad E_1 = \frac{3\hbar\Omega}{2} \quad E_2 = \frac{5\hbar\Omega}{2}$$

$\boxed{k = n+1}$

$$\boxed{\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\Omega^2}{2}\hat{x}^2}$$

✱ ΔΕΙΤΕ 1'

Υποτίθεται πως

$$\Phi_n(\vec{r}) = X_1(x)\, Y_1(y)\, Z_n(z)$$

Ας θεωρήσουμε ότι μπορούμε να περιορίσουμε στις 3 κατώτερες στάθμες του ΑΑΤ

$$U_{\varepsilon k'k}(t) = e\mathcal{E}_0 \cos\omega t\, z_{k'k}$$

$$a = \left(\frac{\hbar}{m\Omega}\right)^{1/2}$$

$\overrightarrow{34}$

$$z_{k'k} = \int d^3r\, \Phi^*_{k'}(\vec{r})\, z\, \Phi_k(\vec{r})$$

$$Z_n(z) = u_n(z)\exp\left(-\frac{m\Omega z^2}{2\hbar}\right)$$

$$z_{kk} = \int d^3r\, \underbrace{|\Phi_k(\vec{r})|^2}_{A}\, \underbrace{z}_{\Pi} = 0$$

k		n	$u_n(z)$
1	A	0	$(1/a\sqrt{\pi})^{1/2}$
2	Π	1	$(1/a\sqrt{\pi})^{1/2}\, 2\frac{z}{a}$
3	A	2	$(1/8a\sqrt{\pi})^{1/2}\left[2-4\left(\frac{z}{a}\right)^2\right]$
4	Π	3	$(1/48a\sqrt{\pi})^{1/2}\left[12\left(\frac{z}{a}\right)-8\left(\frac{z}{a}\right)^3\right]$
		n	$(1/n!\,2^n a\sqrt{\pi})^{1/2} H_n\left(\frac{z}{a}\right)$

πολ. Hermite

$$z_{12} = \int d^3r\, \underset{A}{\Phi^*_1(\vec{r})}\, \underset{\Pi}{z}\, \underset{\Pi}{\Phi_2(\vec{r})} \neq 0$$

$\|$

$$z_{21} = \int d^3r\, \Phi^*_2(\vec{r})\, z\, \Phi_1(\vec{r}) \neq 0$$

$$z_{13} = \int d^3r\, \underbrace{\underset{A}{\Phi^*_1(\vec{r})}\, \underset{\Pi}{z}\, \underset{A}{\Phi_3(\vec{r})}}_{\Pi} = 0$$

$$z_{31} = \dots = 0$$

γιατί;

$$z_{23} = \int d^3r\, \underbrace{\underset{\Pi}{\Phi^*_2(\vec{r})}\, \underset{\Pi}{z}\, \underset{A}{\Phi_3(\vec{r})}}_{A} \neq 0$$

$$z_{12} = z_{21} \neq z_{23} = z_{32}$$

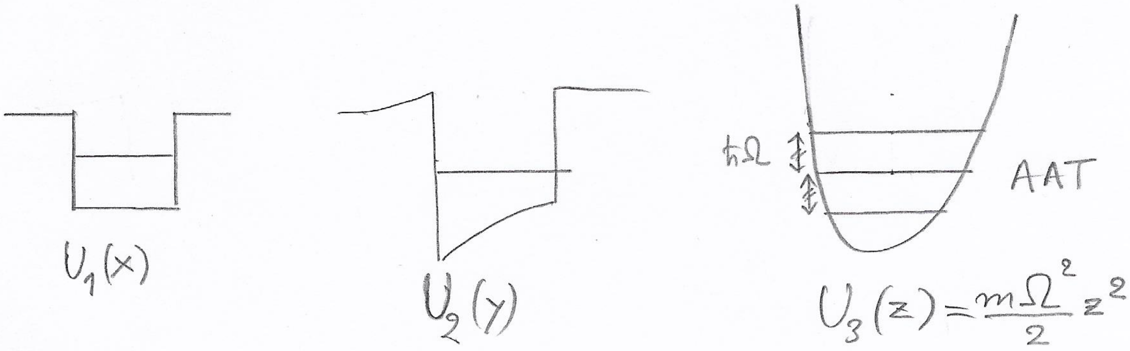
$$z_{32} = z_{23}$$

$$U_{\varepsilon 12}(t) = e\mathcal{E}_0\cos\omega t\, z_{12} = -p_{z12}\cdot\mathcal{E}_0\cos\omega t$$

$$\boxed{\begin{gathered} -e z_{12} = -e z_{21} \\ p_{z12} = p_{z21} := p \end{gathered}}$$

$$U_{\varepsilon 21}(t) = e\mathcal{E}_0\cos\omega t\, z_{21} = -p_{z21}\mathcal{E}_0\cos\omega t$$

π.χ



$$U(\vec{r}) = U_1(x) + U_2(y) + U_3(z)$$

χωρέ 1 μόνο στάθμη   χωρέ 1 μόνο στάθμη   ΑΑΤ ...

$$\Phi_n(\vec{r}) = X_1(x)\,Y_1(y)\,Z_n(z)$$

ΑΣΚΗΣΗ Να υπολογιστεί ο λόγος $\frac{P}{P'} = \frac{\Omega_R}{\Omega_k}$ σε αυτό το σύστημα.

$$\boxed{\begin{aligned}\wp_{z23} &= \wp_{z32} := \wp' \\ &= -e z_{23} = -e z_{32}\end{aligned}} \longrightarrow \text{όμοιων}$$

②

$$\dot{c}_1(t) = -\frac{i}{\hbar} c_1(t) e^{i(\Omega_1-\Omega_1)t} \cancel{U_{\Sigma 11}(t)}^{0} - \frac{i}{\hbar} c_2(t) e^{i(\Omega_1-\Omega_2)t} U_{\Sigma 12}(t)$$

$$-\frac{i}{\hbar} c_3(t) e^{i(\Omega_1-\Omega_3)t} \cancel{U_{\Sigma 13}(t)}^{0}$$

$$\boxed{\dot{c}_1(t) = +\frac{i}{\hbar} c_2(t) e^{-i\Omega t} \wp \mathcal{E}_0 \cos\omega t} \quad ①$$

$$\dot{c}_2(t) = -\frac{i}{\hbar} c_1(t) e^{i(\Omega_2-\Omega_1)t} U_{\Sigma 21}(t) - \frac{i}{\hbar} c_2(t) e^{i(\Omega_2-\Omega_2)t} \cancel{U_{\Sigma 22}(t)}^{0}$$

$$-\frac{i}{\hbar} c_3(t) e^{i(\Omega_2-\Omega_3)t} U_{\Sigma 23}(t)$$

$$\boxed{\dot{c}_2(t) = +\frac{i}{\hbar} c_1(t) e^{i\Omega t} \wp \mathcal{E}_0 \cos\omega t + \frac{i}{\hbar} c_3(t) e^{-i\Omega t} \wp' \mathcal{E}_0 \cos\omega t} \quad ②$$

$$\dot{c}_3(t) = -\frac{i}{\hbar} c_1(t) e^{i(\Omega_3-\Omega_1)t} \cancel{U_{\Sigma 31}(t)}^{0} - \frac{i}{\hbar} c_2(t) e^{i(\Omega_3-\Omega_2)t} U_{\Sigma 32}(t)$$

$$-\frac{i}{\hbar} c_3(t) e^{i(\Omega_3-\Omega_3)t} \cancel{U_{\Sigma 33}(t)}^{0}$$

$$\boxed{\dot{c}_3(t) = +\frac{i}{\hbar} c_2(t) e^{i\Omega t} \wp' \mathcal{E}_0 \cos\omega t} \quad ③$$

μετασχηματισμός (M)
$$\begin{cases} c_1(t) = \mathbb{C}_1(t)\, e^{\frac{i\Delta t}{2}} \\ c_2(t) = \mathbb{C}_2(t)\, e^{-\frac{i\Delta t}{2}} \\ c_3(t) = \mathbb{C}_3(t)\, e^{\frac{3i\Delta t}{2}} \end{cases}$$

$$\cos\omega t = \frac{e^{i\omega t} + e^{-i\omega t}}{2}$$

ώστε να κάνουμε RWA

① ⇒ $\dot{C}_1(t) = \frac{i}{\hbar}\wp\mathcal{E}_0\, C_2(t)\, e^{-i\Omega t}\,\frac{e^{i\omega t}+e^{-i\omega t}}{2}$ ③

$\Delta := \omega - \Omega$

$\Omega_R = \frac{\wp\mathcal{E}_0}{\hbar} > 0$ για $\wp > 0$ ...

RWA

$$\dot{C}_1(t) = \frac{i}{2\hbar}\wp\mathcal{E}_0\, C_2(t)\, e^{i(\omega-\Omega)t} \Rightarrow \boxed{\dot{C}_1(t) = \frac{i}{2}\Omega_R\, C_2(t)\, e^{i\Delta t}} \quad (1')$$

② ⇒ $\dot{C}_2(t) = \frac{i}{\hbar}\wp\mathcal{E}_0\, C_1(t)\, e^{i\Omega t}\,\frac{e^{i\omega t}+e^{-i\omega t}}{2}$

$+\frac{i}{\hbar}\wp'\mathcal{E}_0\, C_3(t)\, e^{-i\Omega t}\,\frac{e^{i\omega t}+e^{-i\omega t}}{2}$

$\Omega_R = \frac{-\wp\mathcal{E}_0}{\hbar}$ μπλε για $\wp < 0$

$\Omega_R' = \frac{-\wp'\mathcal{E}_0}{\hbar}$

κλπ...

RWA

$$\dot{C}_2(t) = \frac{i\wp\mathcal{E}_0}{2\hbar}\, C_1(t)\, e^{-i(\omega-\Omega)t} + \frac{i}{2\hbar}\wp'\mathcal{E}_0\, C_3(t)\, e^{i(\omega-\Omega)t}$$

$\Omega_R' = \frac{\wp'\mathcal{E}_0}{\hbar} > 0$ για $\wp' > 0$ ...

$$\boxed{\dot{C}_2(t) = \frac{i}{2}\Omega_R\, C_1(t)\, e^{-i\Delta t} + \frac{i}{2}\Omega_R'\, C_3(t)\, e^{i\Delta t}} \quad (2')$$

Η 2 είναι δ διαμεσολαβητής.

άν ορίζουμε τις Ω_R, Ω_R' θετικές

③ ⇒ $\dot{C}_3(t) = \frac{i\wp'\mathcal{E}_0}{\hbar}\, C_2(t)\, e^{i\Omega t}\,\frac{e^{i\omega t}+e^{-i\omega t}}{2}$

RWA

$$\boxed{\dot{C}_3(t) = \frac{i\wp'\mathcal{E}_0}{2\hbar}\, C_2(t)\, e^{-i\Delta t}} \quad (3')$$

(1') (2') (3')

οι παράγωγοι των συναρτήσεων σχετίζονται με τις συναρτήσεις με χρονικώς εξαρτώμενους συντελεστές

$$
\left.\begin{aligned}
\dot{c}_1(t) &= \dot{\mathbb{C}}_1(t)\, e^{\frac{i\Delta t}{2}} + \mathbb{C}_1(t)\,\frac{i\Delta}{2}\, e^{\frac{i\Delta t}{2}} \\
\dot{c}_2(t) &= \dot{\mathbb{C}}_2(t)\, e^{-\frac{i\Delta t}{2}} + \mathbb{C}_2(t)\left(-\frac{i\Delta}{2}\right) e^{-\frac{i\Delta t}{2}} \\
\dot{c}_3(t) &= \dot{\mathbb{C}}_3(t)\, e^{-3\frac{i\Delta t}{2}} + \mathbb{C}_3(t)\left(-3\frac{i\Delta}{2}\right) e^{-3\frac{i\Delta t}{2}}
\end{aligned}\right\} \Longleftarrow M
$$

1' M

$$
\dot{\mathbb{C}}_1(t)\, \cancel{e^{\frac{i\Delta t}{2}}} + \mathbb{C}_1(t)\,\frac{i\Delta}{2}\, \cancel{e^{\frac{i\Delta t}{2}}} = \frac{i}{2}\Omega_R\, \mathbb{C}_2(t)\, \cancel{e^{-\frac{i\Delta t}{2}}}\, \cancel{e^{i\Delta t}}
$$

$$
\boxed{\dot{\mathbb{C}}_1(t) = -i\frac{\Delta}{2}\,\mathbb{C}_1(t) + i\frac{\bar{\Omega}_R}{2}\,\mathbb{C}_2(t)} \quad 1''
$$

2' M

$$
\dot{\mathbb{C}}_2(t)\, \cancel{e^{-\frac{i\Delta t}{2}}} + \mathbb{C}_2(t)\left(-\frac{i\Delta}{2}\right) \cancel{e^{-\frac{i\Delta t}{2}}} = \frac{i\Omega_R}{2}\,\mathbb{C}_1(t)\, \cancel{e^{\frac{i\Delta t}{2}}}\, \cancel{e^{-i\Delta t}} + \frac{i}{2}\Omega_R'\, \mathbb{C}_3(t)\, \cancel{e^{-3\frac{i\Delta t}{2}}}\, \cancel{e^{i\Delta t}}
$$

$$
\boxed{\dot{\mathbb{C}}_2(t) = i\frac{\bar{\Omega}_R}{2}\,\mathbb{C}_1(t) + i\frac{\Delta}{2}\,\mathbb{C}_2(t) + i\frac{\bar{\Omega}_R'}{2}\,\mathbb{C}_3(t)} \quad 2''
$$

3' M

$$
\dot{\mathbb{C}}_3(t)\, \cancel{e^{-\frac{3i\Delta t}{2}}} + \mathbb{C}_3(t)\left(-\frac{3i\Delta}{2}\right) \cancel{e^{-\frac{3i\Delta t}{2}}} = \frac{i}{2}\Omega_R'\, \mathbb{C}_2(t)\, \cancel{e^{-\frac{i\Delta t}{2}}}\, \cancel{e^{-i\Delta t}}
$$

$$
\boxed{\dot{\mathbb{C}}_3(t) = i\frac{\bar{\Omega}_R'}{2}\,\mathbb{C}_2(t) + i\frac{3\Delta}{2}\,\mathbb{C}_3(t)} \quad 3''
$$

1'' 2'' 3''

οι παράγωγοι των συναρτήσεων
σχετίζονται με τις συναρτήσεις
με χρονικώς ανεξάρτητους συντελεστές

$$
\underbrace{\begin{bmatrix} \dot{c}_1(t) \\ \dot{c}_2(t) \\ \dot{c}_3(t) \end{bmatrix}}_{\dot{\vec{x}}(t)} = \underbrace{\begin{bmatrix} -i\frac{\Delta}{2} & -\frac{i\Omega_R}{2} & 0 \\ -i\frac{\Omega_R}{2} & i\frac{\Delta}{2} & -i\frac{\Omega_R'}{2} \\ 0 & -i\frac{\Omega_R'}{2} & i\frac{3\Delta}{2} \end{bmatrix}}_{\tilde{A}} \underbrace{\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}}_{\vec{x}(t)}
$$

$$\boxed{\tilde{A} = -iA}$$

ΔΛΜ

$$\boxed{\vec{x}(t) = \vec{u}\, e^{\tilde{\lambda} t}}$$

$$\dot{\vec{x}}(t) = \tilde{A}\, \vec{x}(t)$$

$$\vec{u}\, \tilde{\lambda}\, \cancel{e^{\tilde{\lambda} t}} = \tilde{A}\, \vec{u}\, \cancel{e^{\tilde{\lambda} t}}$$

$$\tilde{A}\, \vec{u} = \tilde{\lambda}\, \vec{u} \qquad \boxed{\tilde{\lambda} = -i\lambda}$$

$$\boxed{A\vec{u} = \lambda \vec{u}}$$

$$
A = \begin{bmatrix} \frac{\Delta}{2} & +\frac{\Omega_R}{2} & 0 \\ +\frac{\Omega_R}{2} & -\frac{\Delta}{2} & +\frac{\Omega_R'}{2} \\ 0 & +\frac{\Omega_R'}{2} & -\frac{3\Delta}{2} \end{bmatrix} \qquad (A - \lambda I)\vec{u} = \vec{0}
$$

$$
\det \begin{bmatrix} \frac{\Delta}{2} - \lambda & +\frac{\Omega_R}{2} & 0 \\ +\frac{\Omega_R}{2} & -\frac{\Delta}{2} - \lambda & +\frac{\Omega_R'}{2} \\ 0 & +\frac{\Omega_R'}{2} & -\frac{3\Delta}{2} - \lambda \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
$$

ΛΥΣΗ για $\boxed{\Delta = 0}$

$$\begin{bmatrix} -\lambda & +\frac{\Omega_R}{2} & 0 \\ +\frac{\Omega_R}{2} & -\lambda & +\frac{\Omega_R'}{2} \\ 0 & +\frac{\Omega_R'}{2} & -\lambda \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\det = 0 \Rightarrow -\lambda \begin{vmatrix} -\lambda & +\frac{\Omega_R'}{2} \\ +\frac{\Omega_R'}{2} & -\lambda \end{vmatrix} - \frac{\Omega_R}{2} \begin{vmatrix} +\frac{\Omega_R}{2} & 0 \\ +\frac{\Omega_R'}{2} & -\lambda \end{vmatrix} = 0$$

$$-\lambda\left[\lambda^2 - \frac{\Omega_R'^2}{4}\right] + \frac{\Omega_R}{2}\frac{\Omega_R}{2}\lambda = 0$$ Από εδώ κ' πέρα ίδιο...

$$-\lambda^3 + \lambda\frac{\Omega_R'^2}{4} + \lambda\frac{\Omega_R^2}{4} = 0 \Rightarrow \lambda\left[-\lambda^2 + \frac{\Omega_R'^2}{4} + \frac{\Omega_R^2}{4}\right] = 0$$

$\boxed{\lambda = 0}$ ή $\lambda^2 = \frac{\Omega_R^2 + \Omega_R'^2}{4}$

$$\boxed{\lambda = \pm\frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2}}$$

$$\lambda_1 = -\frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2} \qquad \lambda_2 = 0 \qquad \lambda_3 = \frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2}$$

$$\lambda_1 = -\Lambda < 0 \qquad \lambda_2 = 0 \qquad \lambda_3 = \Lambda > 0$$

$$\bullet\ \lambda_1 = -\frac{\sqrt{\Omega_R^2+\Omega_R'^2}}{2} = -\Lambda < 0$$

$$\begin{bmatrix} \Lambda & -\frac{\Omega_R}{2} & 0 \\ -\frac{\Omega_R}{2} & \Lambda & -\frac{\Omega_R'}{2} \\ 0 & -\frac{\Omega_R'}{2} & \Lambda \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Lambda U_1 - \frac{\Omega_R}{2} U_2 = 0 \Rightarrow \Lambda U_1 = \frac{\Omega_R}{2} U_2 \Rightarrow U_1 = \frac{\Omega_R}{2\Lambda} U_2$$

$$-\frac{\Omega_R}{2} U_1 + \Lambda U_2 - \frac{\Omega_R'}{2} U_3 = 0$$

$$-\frac{\Omega_R'}{2} U_2 + \Lambda U_3 = 0 \Rightarrow \Lambda U_3 = \frac{\Omega_R'}{2} U_2 \Rightarrow U_3 = \frac{\Omega_R'}{2\Lambda} U_2$$

$$-\frac{\Omega_R}{2}\frac{\Omega_R}{2\Lambda} U_2 + \Lambda U_2 - \frac{\Omega_R'}{2}\frac{\Omega_R'}{2\Lambda} U_2 = 0$$

$$\Rightarrow U_2 \left[-\frac{\Omega_R^2}{4\Lambda} + \Lambda - \frac{\Omega_R'^2}{4\Lambda} \right] = 0 \qquad \Lambda = \frac{\sqrt{\Omega_R^2+\Omega_R'^2}}{2}$$

$$\Rightarrow U_2 \underbrace{\left[\frac{-\Omega_R^2 + 4\Lambda^2 - \Omega_R'^2}{4\Lambda} \right]}_{\text{μηδέν}} = 0 \qquad \boxed{\Lambda^2 = \frac{\Omega_R^2+\Omega_R'^2}{4}}$$

(αν $U_2 = 0$ $\Rightarrow U_1 = U_2 = U_3 = 0$) $\Rightarrow U_2$ ό,τι θέλουμε

πχ. $U_2 = 1$

$$\vec{V}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ 1 \\ \frac{\Omega_R'}{2\Lambda} \end{bmatrix}$$

$$\vec{V}_1 = \beta \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ 1 \\ \frac{\Omega_R'}{2\Lambda} \end{bmatrix} \qquad \vec{V}_1 \cdot \vec{V}_1 = 1 \Rightarrow |\beta|^2 \left(\frac{\Omega_R^2}{4\Lambda^2} + 1 + \frac{\Omega_R'^2}{4\Lambda^2} \right) = 1$$

$$|\beta|^2 \frac{\Omega_R^2 + 4\Lambda^2 + \Omega_R'^2}{4\Lambda^2} = 1$$

$$|\beta|^2 \frac{2}{1} = 1 \Rightarrow \text{πχ. } \beta = \frac{1}{\sqrt{2}}$$

• $\lambda_2 = 0$

$$\begin{bmatrix} 0 & -\frac{\Omega_R}{2} & 0 \\ -\frac{\Omega_R}{2} & 0 & -\frac{\Omega_R'}{2} \\ 0 & -\frac{\Omega_R'}{2} & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$$

$$-\frac{\Omega_R}{2} u_2 = 0 \qquad u_2 = 0$$

$$-\frac{\Omega_R}{2} u_1 - \frac{\Omega_R'}{2} u_3 = 0 \quad -\frac{\Omega_R}{\cancel{2}} u_1 = \frac{\Omega_R'}{\cancel{2}} u_3 \Rightarrow \boxed{u_3 = -\frac{\Omega_R}{\Omega_R'} u_1}$$

$$-\frac{\Omega_R'}{2} u_2 = 0 \qquad u_2 = 0$$

$$\vec{V}_2 = \beta \begin{bmatrix} 1 \\ 0 \\ -\frac{\Omega_R}{\Omega_R'} \end{bmatrix} \quad \vec{V}_2 \cdot \vec{V}_2 = 1 \Rightarrow |\beta|^2 \left(1 + \frac{\Omega_R^2}{\Omega_R'^2}\right) = 1 \Rightarrow |\beta|^2 \frac{\Omega_R'^2 + \Omega_R^2}{\Omega_R'^2} = 1$$

$$|\beta|^2 \frac{4\Lambda^2}{\Omega_R'^2} = 1 \Rightarrow \pi\chi.\ \beta = \frac{\Omega_R'\ 2}{2\sqrt{\Omega_R^2 + \Omega_R'^2}} = \frac{\Omega_R'}{\sqrt{\Omega_R^2 + \Omega_R'^2}}$$

$$\vec{V}_2 = \begin{bmatrix} \frac{\Omega_R'}{\sqrt{\Omega_R^2 + \Omega_R'^2}} \\ 0 \\ -\frac{\Omega_R}{\sqrt{\Omega_R^2 + \Omega_R'^2}} \end{bmatrix} = \begin{bmatrix} \frac{\Omega_R'}{2\Lambda} \\ 0 \\ -\frac{\Omega_R}{2\Lambda} \end{bmatrix}$$

• $\lambda_3 = \dfrac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2} = \Lambda > 0$

$$\begin{bmatrix} -\Lambda & -\frac{\Omega_R}{2} & 0 \\ -\frac{\Omega_R}{2} & -\Lambda & -\frac{\Omega_R'}{2} \\ 0 & -\frac{\Omega_R'}{2} & -\Lambda \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$$

$$-\Lambda u_1 - \frac{\Omega_R}{2} u_2 = 0 \quad \Rightarrow -\frac{\Omega_R}{2} u_2 = \Lambda u_1 \Rightarrow u_1 = -\frac{\Omega_R}{2\Lambda} u_2$$

$$-\frac{\Omega_R}{2} u_1 - \Lambda u_2 - \frac{\Omega_R'}{2} u_3 = 0$$

$$-\frac{\Omega_R'}{2} u_2 - \Lambda u_3 = 0 \quad \Rightarrow -\frac{\Omega_R'}{2} u_2 = \Lambda u_3 \Rightarrow u_3 = -\frac{\Omega_R'}{2\Lambda} u_2$$

$$+\frac{\Omega_R}{2}\frac{\Omega_R}{2\Lambda} u_2 - \Lambda u_2 + \frac{\Omega_R'}{2}\frac{\Omega_R'}{2\Lambda} u_2 = 0 \Rightarrow u_2 \left[\frac{\Omega_R^2}{4\Lambda} - \frac{4\Lambda^2}{4\Lambda} + \frac{\Omega_R'^2}{4\Lambda}\right] = 0$$

$$u_2 \Bigg[\underbrace{\frac{\Omega_R^2 + \Omega_R'^2 - 4\Lambda^2}{4\Lambda}}_{\text{μηδέν}}\Bigg] = 0 \quad \Rightarrow \quad u_2 \text{ ξ,η διάλογα}$$

$\pi\chi.\ u_2 = 1$ $\Rightarrow$

$$\vec{V}_3 = \beta \begin{bmatrix} -\frac{\Omega_R}{2\Lambda} \\ 1 \\ -\frac{\Omega_R'}{2\Lambda} \end{bmatrix}$$

$$\vec{V}_3 \cdot \vec{V}_3 = 1 \Rightarrow |\beta|^2 \left(\frac{\Omega_R^2}{4\Lambda^2} + 1 + \frac{\Omega_R'^2}{4\Lambda^2} \right) = 1$$

$$|\beta|^2 \frac{\Omega_R^2 + \Omega_R'^2 + 4\Lambda^2}{4\Lambda^2} = 1 \Rightarrow |\beta|^2 \cdot 2 = 1 \Rightarrow$$

$$\text{αx} \quad \beta = \frac{1}{\sqrt{2}}$$

$$\vec{V}_3 = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ -1 \\ \frac{\Omega_R'}{2\Lambda} \end{bmatrix}$$

γενική λύση

$$\vec{x}(t) = \sum_{k=1}^{3} \sigma_k \vec{v}_k e^{-i\lambda_k t}$$

$$2\Lambda = \sqrt{\Omega_R^2 + \Omega_R'^2}$$

$$4\Lambda^2 = (\Omega_R^2 + \Omega_R'^2)$$

$$16\Lambda^4 = (\Omega_R^2 + \Omega_R'^2)^2$$

$C_1(0)=1 \qquad C_2(0)=0 \qquad C_3(0)=0$ ἔστωσαν οἱ ἀρχ. συνθῆκες (10)

$$\vec{x}(t)=\begin{bmatrix} C_1(t)\, e^{-\frac{i\Delta t}{2}} \\ C_2(t)\, e^{\frac{i\Delta t}{2}} \\ C_3(t)\, e^{\frac{3i\Delta t}{2}} \end{bmatrix} = \frac{\sigma_1}{\sqrt{2}}\begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ 1 \\ \frac{\Omega_R'}{2\Lambda} \end{bmatrix} e^{-i\lambda_1 t} + \sigma_2 \begin{bmatrix} \frac{\Omega_R'}{2\Lambda} \\ 0 \\ -\frac{\Omega_R}{2\Lambda} \end{bmatrix} e^{-i\lambda_2 t} + \frac{\sigma_3}{\sqrt{2}}\begin{bmatrix} \frac{\Omega_R}{2\Lambda} \\ -1 \\ \frac{\Omega_R'}{2\Lambda} \end{bmatrix} e^{-i\lambda_3 t}$$

$$1=\frac{\sigma_1}{\sqrt{2}}\frac{\Omega_R}{2\Lambda}+\sigma_2\frac{\Omega_R'}{2\Lambda}+\frac{\sigma_3}{\sqrt{2}}\frac{\Omega_R}{2\Lambda}$$

$$0=\frac{\sigma_1}{\sqrt{2}}-\frac{\sigma_3}{\sqrt{2}} \Rightarrow \boxed{\sigma_1=\sigma_3=\sigma}$$

$$0=\frac{\sigma_1}{\sqrt{2}}\frac{\Omega_R'}{2\Lambda}-\sigma_2\frac{\Omega_R}{2\Lambda}+\frac{\sigma_3}{\sqrt{2}}\frac{\Omega_R'}{2\Lambda}$$

$$0=\frac{\sigma}{\sqrt{2}}\frac{\Omega_R'}{2\Lambda}-\frac{\sigma_2\Omega_R}{2\Lambda}+\frac{\sigma}{\sqrt{2}}\frac{\Omega_R'}{2\Lambda} \Rightarrow 0=\frac{\sigma\Omega_R'}{\sqrt{2}\Lambda}-\frac{\sigma_2\Omega_R}{2\Lambda} \Rightarrow$$

$$\frac{\sigma_2\Omega_R}{2\Lambda}=\frac{\sigma\Omega_R'}{\sqrt{2}\Lambda} \Rightarrow \boxed{\sigma_2=\sigma\sqrt{2}\frac{\Omega_R'}{\Omega_R}}$$

$$1=\frac{\sigma}{\sqrt{2}}\frac{\Omega_R}{2\Lambda}+\sigma\sqrt{2}\frac{\Omega_R'}{\Omega_R}\frac{\Omega_R'}{2\Lambda}+\frac{\sigma}{\sqrt{2}}\frac{\Omega_R}{2\Lambda}$$

$$2\Lambda=\sigma\left(\frac{\Omega_R}{\sqrt{2}}+\sqrt{2}\frac{\Omega_R'^2}{\Omega_R}+\frac{\Omega_R}{\sqrt{2}}\right)=\sigma\frac{\Omega_R^2+2\Omega_R'^2+\Omega_R^2}{\sqrt{2}\,\Omega_R}=\sigma\frac{2}{\sqrt{2}}\frac{\Omega_R^2+\Omega_R'^2}{\Omega_R}$$

$$\sigma=\frac{\sqrt{\Omega_R^2+\Omega_R'^2}\,\sqrt{2}\,\Omega_R}{2\left(\Omega_R^2+\Omega_R'^2\right)} \Rightarrow \boxed{\sigma=\frac{\Omega_R}{\sqrt{2}\sqrt{\Omega_R^2+\Omega_R'^2}}=\frac{\Omega_R}{\sqrt{2}\,2\Lambda}}$$

$$\sigma=\frac{\Omega_R}{\sqrt{2}\,2\Lambda}$$

$$C_1(t)\, e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R}{2\underbrace{\sqrt{\Omega_R^2+\Omega_R'^2}}_{2\Lambda}} \, \frac{\Omega_R \cancel{2}}{\cancel{2}\underbrace{\sqrt{\Omega_R^2+\Omega_R'^2}}_{2\Lambda}} \, e^{+i\Lambda t}$$

$$+ \frac{\Omega_R}{\cancel{\sqrt{2}}\underbrace{\sqrt{}}_{2\Lambda}} \cdot \frac{\cancel{\sqrt{2}}\, \Omega_R'}{\cancel{\Omega_R}} \, \frac{\Omega_R'}{\underbrace{\sqrt{}}_{2\Lambda}} \, e^{-i0}$$

$$+ \frac{\Omega_R}{\sqrt{2}\underbrace{\sqrt{}}_{2\Lambda}} \cdot \frac{\Omega_R \cancel{2}}{\cancel{2}\underbrace{\sqrt{}}_{2\Lambda}} \, e^{-i\Lambda t}$$

$$C_1(t)\, e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R^2}{2\underbrace{\left(\Omega_R^2+\Omega_R'^2\right)}_{4\Lambda^2}} \cdot e^{i\Lambda t} + \frac{\Omega_R'^2}{\underbrace{\Omega_R^2+\Omega_R'^2}_{4\Lambda^2}} + \frac{\Omega_R^2}{2\underbrace{\left(\Omega_R^2+\Omega_R'^2\right)}_{4\Lambda^2}} \cdot e^{-i\Lambda t} \quad \leftarrow$$

$$C_1(t)\, e^{-\frac{i\Delta t}{2}} = \frac{\cancel{2}\, \Omega_R^2}{\cancel{2}\underbrace{\left(\Omega_R^2+\Omega_R'^2\right)}_{4\Lambda^2}} \cos\Lambda t + \frac{\Omega_R'^2}{\underbrace{\Omega_R^2+\Omega_R'^2}_{4\Lambda^2}} . \quad \leftarrow$$

$$C_2(t)\, e^{\frac{i\Delta t}{2}} = \frac{\Omega_R}{\sqrt{2}\sqrt{2}\sqrt{}}\, 1 \cdot e^{+i\Lambda t} - \frac{\Omega_R}{\sqrt{2}\sqrt{2}\sqrt{}} \cdot e^{-i\Lambda t}$$

$$= \frac{\Omega_R}{2\sqrt{}} \left(e^{+i\Lambda t} - e^{-i\Lambda t} \right)$$

$\cancel{\cos} + i \sin$

$- \cancel{\cos} + i \sin$

$$C_2(t)\, e^{i\frac{\Delta t}{2}} = \frac{\Omega_R}{\cancel{2}\sqrt{}}\, \cancel{2}\, i \sin\Lambda t$$

$$|C_2(t)|^2 = \frac{\Omega_R^2}{\sqrt{\Omega_R^2+\Omega_R'^2}} \, \sin^2(\Lambda t) = \frac{\Omega_R^2}{\Omega_R^2+\Omega_R'^2} \cdot \left(\frac{1}{2} - \frac{\cos(2\Lambda t)}{2} \right)$$

$$\sin^2 x = \frac{1}{2} - \frac{\cos 2x}{2}$$

$$|c_2(t)|^2 = \frac{\Omega_R^2}{\Omega_R^2 + \Omega_R'^2} \cdot \left(\frac{1}{2} - \frac{\cos(2\Lambda t)}{2}\right)$$

$$= \frac{\Omega_R^2}{2\left(\Omega_R^2 + \Omega_R'^2\right)} - \frac{\Omega_R^2}{2\left(\Omega_R^2 + \Omega_R'^2\right)} \cdot \cos(2\Lambda t)$$

$$A_2 = \frac{\Omega_R^2}{\Omega_R^2 + \Omega_R'^2}$$

maximum transfer percentage

μέγιστο ποσοστό μεταφερόμενων

$$T_2 = \frac{2\pi}{2\Lambda} = \frac{2\pi}{\sqrt{\Omega_R^2 + \Omega_R'^2}} = \frac{2\pi}{\sqrt{\Omega_R^2 + \Omega_R'^2}}$$

για $\Omega_R = \Omega_R'$ $\Rightarrow A_2 = \frac{1}{2}$

$$T_2 = \frac{2\pi}{\sqrt{2}\,|\Omega_R|} = \frac{1}{\sqrt{2}} \frac{2\pi}{|\Omega_R|}$$

$\uparrow$

περίοδος αντίστοιχου διστάθμικού

$$2\Lambda = \sqrt{\Omega_R^2 + \Omega_R'^2}$$
$$= \sqrt{2}\,\Omega_R$$

$$|c_2(t)|^2 = \frac{1}{2}\left(\frac{1}{2} - \frac{\cos(\sqrt{2}\,\Omega_R t)}{2}\right)$$

$$|c_2(t)|^2 = \frac{1}{4} - \frac{1}{4} \cdot \cos(\underbrace{\sqrt{2}\,\Omega_R}_{2\omega_1 = \omega_2} t)$$

$$|c_2(t)|^2 = \frac{1}{4} - \frac{1}{4} \cos(\omega_2 t)$$

$$C_1(t)\, e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R^2}{2\cdot 4\Lambda^2} e^{+i\Lambda t} + \frac{\Omega_R'^2}{4\Lambda^2} + \frac{\Omega_R^2}{2\cdot 4\Lambda^2} e^{-i\Lambda t}$$

$$C_1(t)\, e^{-\frac{i\Delta t}{2}} = \frac{\Omega_R^2}{4\Lambda^2}\cos(\Lambda t) + \frac{\Omega_R'^2}{4\Lambda^2}$$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4}\cos^2(\Lambda t) + \frac{\Omega_R'^4}{16\Lambda^4} + 2\,\frac{\Omega_R^2}{4\Lambda^2}\cdot\cos(\Lambda t)\cdot\frac{\Omega_R'^2}{4\Lambda^2}$$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4}\cos^2(\Lambda t) + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2\,\Omega_R^2\Omega_R'^2}{16\Lambda^4}\cdot\cos(\Lambda t)$$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4}\cdot\frac{\cos(2\Lambda t)+1}{2} + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2\,\Omega_R^2\Omega_R'^2}{16\Lambda^4}\cdot\cos(\Lambda t)$$

$$T_A = \frac{2\pi}{2\Lambda} \qquad T_B = \frac{2\pi}{\Lambda} \qquad \frac{T_B}{T_A} = 2 \Rightarrow \text{περιοδική κίνηση}$$

με περίοδο $\boxed{T_1 = \frac{2\pi}{\Lambda} = \frac{2\pi}{\sqrt{\Omega_R^2+\Omega_R'^2}}\cdot 2 = 2T_2}$

$$\left|C_1\left(\frac{2\pi}{\Lambda}\right)\right|^2 = \frac{\Omega_R^4}{16\Lambda^4}\cdot\frac{\cos\left(2\Lambda\frac{2\pi}{\Lambda}\right)+1}{2} + \frac{\Omega_R'^2}{16\Lambda^4} + \frac{2\,\Omega_R^2\Omega_R'^2}{16\Lambda^4}\cdot\cos\left(\Lambda\frac{2\pi}{\Lambda}\right) =$$

$$= \frac{\Omega_R^4}{16\Lambda^4} + \frac{\Omega_R'^2}{16\Lambda^4} + \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4} = \frac{(\Omega_R^2+\Omega_R'^2)^2}{16\Lambda^4} = 1$$

$$\left|C_1\left(\frac{2\pi}{2\Lambda}\right)\right|^2 = \frac{\Omega_R^4}{16\Lambda^4}\cdot\frac{\cos\left(2\Lambda\frac{2\pi}{2\Lambda}\right)+1}{2} + \frac{\Omega_R'^2}{16\Lambda^4} + \frac{2\,\Omega_R^2\Omega_R'^2}{16\Lambda^4}\cdot\cos\left(\Lambda\frac{2\pi}{2\Lambda}\right)$$

$$= \frac{\Omega_R^4}{16\Lambda^4} + \frac{\Omega_R'^2}{16\Lambda^4} - \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4} = \frac{(\Omega_R^2-\Omega_R'^2)^2}{16\Lambda^4} = \frac{(\Omega_R^2-\Omega_R'^2)^2}{(\Omega_R^2+\Omega_R'^2)^2}$$

η τιμή της $|C_1(t)|^2$ στο ήμισυ της περιόδου

$$\frac{d}{dt}|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4}\cdot 2\cos(\Lambda t)\cdot(-\Lambda)\sin(\Lambda t) + \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4}(-\Lambda)\sin(\Lambda t)$$

$$\frac{d}{dt}|C_1(t)|^2 = \frac{(-2\Lambda)\Omega_R^2}{16\Lambda^4}\cdot\sin(\Lambda t)\cdot\left[\Omega_R^2\cdot\cos(\Lambda t) + \Omega_R'^2\right]$$

$$\frac{d^2}{dt^2}|C_1(t)|^2 = \frac{(-2\Lambda)\Omega_R^2}{16\Lambda^4}\cdot\Lambda\cdot\cos(\Lambda t)\cdot\left[\Omega_R^2\cos(\Lambda t) + \Omega_R'^2\right] + \frac{(-2\Lambda)\Omega_R^2}{16\Lambda^4}\sin(\Lambda t)\cdot\Omega_R^2(-\Lambda)\cdot\sin(\Lambda t)$$

$$= \frac{(-2\Lambda)\Omega_R^2}{16\Lambda^4}\cdot\Lambda\cdot\left\{\Omega_R^2\cdot\cos^2(\Lambda t) + \Omega_R'^2\cos(\Lambda t) - \Omega_R^2\sin^2(\Lambda t)\right\}$$

$$\frac{d|c_1(t)|^2}{dt} = \frac{(-2\Lambda)\cdot\Omega_R^2}{16\Lambda^4}\cdot\sin(\Lambda t)\cdot\left[\Omega_R^2\cdot\cos(\Lambda t) + \Omega_R'^2\right]$$

$$\frac{d^2|c_1(t)|^2}{dt^2} = \frac{(-2\Lambda^2\cdot\Omega_R^2)}{16\Lambda^4}\left[\Omega_R^2\sin^2(\Lambda t) - \Omega_R^2\cos^2(\Lambda t) - \Omega_R'^2\cos(\Lambda t)\right]$$

$$\frac{d|c_1(t)|^2}{dt} = 0 \Rightarrow \sin(\Lambda t) = 0 \quad \text{ή} \quad \cos(\Lambda t) = -\frac{\Omega_R'^2}{\Omega_R^2}$$

$\Downarrow$

$\Lambda t = n\pi,\ n\in\mathbb{Z}$ $\boxed{\Pi 1}$

Προσοχή, πρέπει $\Omega_R'^2 \leq \Omega_R^2$ $\boxed{\Pi 2}$

$\boxed{\Pi 2}$

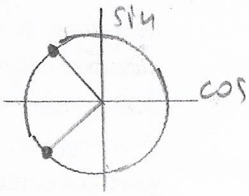
$$\frac{d^2}{dt^2}|c_1(t)|^2 = \frac{2\Lambda^2\Omega_R^2}{16\Lambda^4}\cdot\left[\Omega_R^2\cdot\sin^2(\Lambda t) - \Omega_R^2\cdot\frac{\Omega_R'^4}{\Omega_R^4} + \Omega_R'^2\,\frac{\Omega_R'^2}{\Omega_R^2}\right]$$

$$= \frac{2\Lambda^2}{16\Lambda^4}\left[\Omega_R^4\cdot\sin^2(\Lambda t) - \Omega_R'^4 + \Omega_R'^4\right] = \frac{2\Lambda^2}{16\Lambda^4}\cdot\Omega_R^4\cdot\sin^2(\Lambda t) > 0$$

Δηλ. στην $\boxed{\Pi 2}$ έχουμε ελάχιστα, με τιμή μηδενική διότι τότε:

$$|c_1(t)|^2 = \frac{\cancel{\Omega_R^4}}{16\Lambda^4}\cdot\frac{\Omega_R'^4}{\cancel{\Omega_R^4}} + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2\cancel{\Omega_R^2}\Omega_R'^2}{16\Lambda^4}(-1)\frac{\Omega_R'^2}{\cancel{\Omega_R^2}} = \frac{2\Omega_R'^4}{16\Lambda^4} - \frac{2\Omega_R'^4}{16\Lambda^4} = 0$$

Μάλιστα σε μία περίοδο $T_1 = \frac{2\pi}{\Lambda}$, η οποία είναι και η περίοδος του όρου $\cos(\Lambda t)$, $\exists$ 2 φορές όπου έχουμε $\cos(\Lambda t) = \frac{-\Omega_R'^2}{\Omega_R^2}$ δηλαδή θα έχουμε 2 μηδενισμούς σε μία περίοδο T_1.



Οπότε τότε, $\alpha_1 = 1$ μέγιστο ποσοστό μεταβιβάσεως
maximum transfer percentage

$\boxed{\Pi 1}$

$$\frac{d^2}{dt^2}|c_1(t)|^2 = \frac{2\Lambda^2\Omega_R^2}{16\Lambda^4}\left[-\Omega_R^2\cdot\cos^2(\Lambda t) - \Omega_R'^2\cos(\Lambda t)\right]$$

$\sin(\Lambda t) = 0 \Rightarrow \cos(\Lambda t) = \pm 1$

Π1α $\Lambda t = 0, 2\pi, 4\pi, \ldots$ $\cos(\Lambda t) = 1$ $\rightarrow t = 0, T_1, 2T_1, \ldots$

Π1β $\Lambda t = \pi, 3\pi, 5\pi, \ldots$ $\cos(\Lambda t) = -1$ $\rightarrow t = \frac{T_1}{2}, \frac{3T_1}{2}, \frac{5T_1}{2}, \ldots$

Π1α

$$\frac{d^2|C_1(t)|^2}{dt^2} = \frac{2\Lambda^2\Omega_R^2}{16\Lambda^4}\left(-\Omega_R^2 - \Omega_R'^2\right) < 0 \Rightarrow$$ τοπικό μέγιστο

με τιμή

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4}\cdot 1 + \frac{\Omega_R'^4}{16\Lambda^4} + \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4} = \frac{(\Omega_R^2+\Omega_R'^2)^2}{16\Lambda^4} = 1$$

δηλ. είναι ολικό μέγιστο

Π1β

$$\frac{d^2|C_1(t)|^2}{dt^2} = \frac{2\Lambda^2\Omega_R^2}{16\Lambda^4}\left(-\Omega_R^2 + \Omega_R'^2\right) > 0 \Rightarrow$$ τοπικό ελάχιστο

με τιμή

$$|C_1(t)|^2 = \frac{\Omega_R^4}{16\Lambda^4} + \frac{\Omega_R'^4}{16\Lambda^4} - \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4} = \frac{(\Omega_R^2-\Omega_R'^2)^2}{(\Omega_R^2+\Omega_R'^2)^2}$$

η οποία είναι η τιμή της $|C_1(t)|^2$ στο ήμισυ της περιόδου

$$A_1 = 1 - \frac{(\Omega_R^2-\Omega_R'^2)^2}{(\Omega_R^2+\Omega_R'^2)^2} = \frac{4\cdot\Omega_R^2\cdot\Omega_R'^2}{(\Omega_R^2+\Omega_R'^2)^2}$$

για $\Omega_R = \Omega_R'$

$$T_1 = \frac{2\pi}{\sqrt{2}\,\Omega_R}\cdot 2 = \sqrt{2}\,\frac{2\pi}{\Omega_R}$$

περίοδος αντίστοιχου διστάθμιου

Π2 $\Rightarrow A_1 = 1$ $\quad \cos(\Lambda t) = -1 \Rightarrow \Lambda t = +\pi, 3\pi, \ldots \quad t = \frac{\pi}{\Lambda} = \frac{2\pi}{2\Lambda} =$ Το ήμισυ της περιόδου

Π1 $\Rightarrow A_1 = 1$ στο ήμισυ της περιόδου

1 φορά μηδενισμός...

δηλ. για $\Omega_R = \Omega_R'$ οι Π1, Π2 ταυτίζονται

$$2\Lambda = \sqrt{\Omega_R^2 + \Omega_R'^2} = \sqrt{2}\,\Omega_R$$

$$4\Lambda^2 = 2\Omega_R^2$$

$$16\Lambda^4 = 4\Omega_R^4$$

π.χ. όπως στο GG, GGG

$T_{GG} \cong 20.6783$ fs

$T_{GGG} = 29.2436$ fs $= \sqrt{2}\,T_{GG}$

$T_{31} = 14.6218$ fs $= \frac{T_{GG}}{\sqrt{2}}$

$T_{32} = 29.2436$ fs $= T_{GGG} = \sqrt{2}\,T_{GG}$

$$|C_1(t)|^2 = \frac{\Omega_R^4}{4\Omega_R^4}\cdot\frac{\cos(\sqrt{2}\,\Omega_R t)+1}{2} + \frac{\Omega_R^4}{4\Omega_R^4} + \frac{2\Omega_R^4}{4\Omega_R^4}\cdot\cos\left(\frac{\sqrt{2}}{2}\Omega_R t\right)$$

$$|c_1(t)|^2 = \frac{1}{4}\left(\frac{1}{2}\cdot\cos(\underbrace{\sqrt{2}\Omega_R}_{2\omega_1=\omega_2} t) + \frac{1}{2}\right) + \frac{1}{4} + \frac{1}{2}\cos\left(\underbrace{\frac{\sqrt{2}}{2}\Omega_R}_{\omega_1} t\right)$$

$$\frac{1}{8}\cos(2\omega_1 t) + \frac{1}{8} + \frac{1}{4} + \frac{1}{2}\cos(\omega_1 t) =$$

$$\frac{1}{8}\left(\underbrace{\cos(2\omega_1 t) + 1}_{2\cos^2(\omega_1 t)} + 2 + 4\cos(\omega_1 t)\right) = \frac{1}{8}\left(2\cos^2(\omega_1 t) + 2 + 4\cos(\omega_1 t)\right)$$

$$= \frac{1}{4}\left(\cos^2(\omega_1 t) + 1 + 2\cos(\omega_1 t)\right) = \frac{1}{4}\left(\underbrace{\cos(\omega_1 t) + 1}_{2\cos^2\left(\frac{\omega_1 t}{2}\right)}\right)^2 = \cos^4\left(\frac{\omega_1 t}{2}\right)$$

$$C_3(t)\, e^{3\frac{i\Delta t}{2}} = \frac{\Omega_R}{4\Lambda}\cdot\frac{\Omega_R'}{2\Lambda}\, e^{+i\Lambda t} + \frac{\Omega_R'}{2\Lambda}\,\frac{\Omega_R'}{\cancel{\Omega_R}}(-1)\frac{\Omega_R}{2\Lambda} + \frac{\Omega_R}{4\Lambda}\cdot\frac{\Omega_R'}{2\Lambda}\, e^{-i\Lambda t}$$

$$C_3(t)\, e^{3\frac{i\Delta}{2}t} = \frac{\Omega_R\Omega_R'}{4\Lambda^2}\cos(\Lambda t) - \frac{\Omega_R\Omega_R'}{4\Lambda^2}$$

$$|C_3(t)|^2 = \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\cdot\cos^2(\Lambda t) + \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4} - 2\,\frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\cdot\cos(\Lambda t)$$

$$|C_3(t)|^2 = \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\,\frac{\cos(2\Lambda t)+1}{2} + \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4} - \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4}\cdot\cos(\Lambda t)$$

$T_A = \frac{2\pi}{2\Lambda}$ $\quad T_B = \frac{2\pi}{\lambda}$ $\quad \frac{T_B}{T_A} = 2 \Rightarrow$ η κίνηση είναι περιοδική!!!

με περίοδο

$$\boxed{T_3 = \frac{2\pi}{\Lambda} = \frac{2\pi}{\sqrt{\Omega_R^2+\Omega_R'^2}}\,2 = 2T_2 = T_1}$$

$$\frac{d}{dt}|C_3(t)|^2 = \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\,2\cos(\Lambda t)(-\Lambda)\sin(\Lambda t) - 2\,\frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}(-\Lambda)\sin(\Lambda t)$$

$$\frac{d}{dt}|C_3(t)|^2 = \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\,2(-\Lambda)\left[\cos(\Lambda t)\cdot\sin(\Lambda t) - \sin(\Lambda t)\right]$$

$\frac{d}{dt}|C_3(t)|^2 = 0 \Rightarrow \sin(\Lambda t) = 0$ ή $\cos(\Lambda t) = 1$

$\Rightarrow t = T_3, 2T_3, \dots$

(Π1) $\Lambda t = 0, 2\pi, 4\pi, \dots \Rightarrow \sin(\Lambda t) = 0$ και $\cos(\Lambda t) = 1$

(Π2) $\Lambda t = \pi, 3\pi, 5\pi, \dots \Rightarrow \sin(\Lambda t) = 0$ και $\cos(\Lambda t) = -1$

$\Rightarrow t = \frac{T_3}{2}, \frac{3T_3}{2}, \dots$

(Π1) (Π2) $|C_3(t)|^2 = \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\cdot 1 + \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4} - \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4}(\pm 1) = \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4} \mp \frac{2\Omega_R^2\Omega_R'^2}{16\Lambda^4}$

(Π1) $\Rightarrow |C_3(t)|^2 = 0$ ελάχιστο

(Π2) $\Rightarrow |C_3(t)|^2 = \frac{4\Omega_R^2\Omega_R'^2}{16\Lambda^4}$ μέγιστο διότι...

$$\frac{d^2}{dt^2}|C_3(t)|^2 = \frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}\,2(-\Lambda)\left[\frac{1}{\cancel{2}}\cancel{2}\Lambda\cos(2\Lambda t) - \Lambda\cos(\Lambda t)\right] = \underbrace{\frac{\Omega_R^2\Omega_R'^2}{16\Lambda^4}}_{\vartheta}2\Lambda^2\left(\cos(\Lambda t) - \cos(2\Lambda t)\right)$$

(Π1) $\Rightarrow (\dots) = \cos(2\pi) - \cos(4\pi) = 0$

(Π2) $\Rightarrow (\dots) = \cos(\pi) - \cos(2\pi) = -1-1 = -2 \Rightarrow \frac{d^2}{dt^2}|C_3(t)|^2 < 0$ μέγιστο

Ἀν τα πράγματα εἶναι ἔτσι, τότε (141)

$$\frac{d^3|C_3(t)|^2}{dt^3} = \vartheta\cdot\left(\Lambda(-1)\cdot\sin(\Lambda t) + 2\Lambda\sin(2\Lambda t)\right)$$

$$= \vartheta\Lambda\left(2\sin(2\Lambda t) - \sin(\Lambda t)\right)$$

και για $\Lambda t = 2\pi \overset{\text{ΠΛ}}{\Rightarrow} = \vartheta\Lambda\left(2\sin(4\pi) - \sin(2\pi)\right) = 0$

$$\frac{d^4|C_3(t)|^2}{dt^4} = \vartheta\Lambda\left(2\cdot 2\Lambda\cos(2\Lambda t) - \Lambda\cdot\cos(\Lambda t)\right)$$

$$= \vartheta\Lambda^2\left(4\cdot\cos(2\Lambda t) - \cos(\Lambda t)\right)$$

και για $\Lambda t = 2\pi \overset{\text{ΠΛ}}{\Rightarrow} = \vartheta\Lambda^2\left(4\cdot\underbrace{\cos(4\pi)}_{1} - \underbrace{\cos(2\pi)}_{1}\right) > 0$

ΞΑΝΑ $\frac{d^2}{dt^2}|C_3(t)|^2 = \vartheta\left(\underbrace{\cos(\Lambda t)}_{1} - \underbrace{\cos(2\Lambda t)}_{1}\right) = 0$

για $\Lambda t = 2\pi$ ΠΛ

$$\frac{d^3}{dt^3}|C_3(t)|^2 = \vartheta\left(-\Lambda\underbrace{\sin(\Lambda t)}_{0} + 2\Lambda\underbrace{\sin(2\Lambda t)}_{0}\right) = 0$$

για $\Lambda t = 2\pi$ ΠΛ

$$\frac{d^4}{dt^4}|C_3(t)|^2 = \vartheta\left(-\Lambda^2\cos(\Lambda t) + 2\Lambda\cdot 2\Lambda\cos(2\Lambda t)\right) = \vartheta\Lambda^2\left(4\underbrace{\cos(2\Lambda t)}_{1} - \underbrace{\cos(\Lambda t)}_{1}\right)$$

για $\Lambda t = 2\pi$ ΠΛ $= \vartheta\Lambda^2\cdot 3 > 0$

δηλαδή στα ἐλάχιστα μηδενίζονται ἡ $|C_3(t)|^2$, καθώς και ἡ 1η, 2η και 3η παράγωγός της!

ἐνῶ, ἡ 4η παράγωγός της εἶναι θετική...

μοιάζει κάπως με ἐπιπεδη συνάρτηση (flat function)

[ὅπου μηδενίζονται σε κάποιο σημεῖο ὅλες οἱ παράγωγοι]

Άρα $\boxed{A_3 = \dfrac{4\Omega_R^2\Omega_R'^2}{(\Omega_R^2+\Omega_R'^2)^2}}$ μέγιστο ποσοστό μεταβιβάσεως
maximum transfer percentage

για $\Omega_R' = \Omega_R$ $\quad T_3 = \dfrac{2\pi}{\sqrt{2}\,\Omega_R}\cdot 2 = \sqrt{2}\,\dfrac{2\pi}{\Omega_R}$ $\quad 2\Lambda = \sqrt{2\Omega_R^2} = \sqrt{2}\,\Omega_R$

↓ περίοδος αντιστοίχου δισταθμικού

$$|c_3(t)|^2 = \frac{\Omega_R^4}{4\Omega_R^4}\,\frac{\cos(\sqrt{2}\,\Omega_R t)+1}{2} + \frac{\Omega_R^4}{4\Omega_R^4} - \frac{2\Omega_R^4}{4\Omega_R^4}\cdot\cos\left(\frac{\sqrt{2}\,\Omega_R t}{2}\right)$$

$$|c_3(t)|^2 = \frac{1}{4}\left(\frac{1}{2}\cos(\underbrace{\sqrt{2}\,\Omega_R t}_{2\omega_3 = 2\omega_1 = \omega_2}) + \frac{1}{2} + \frac{1}{4} - \frac{1}{2}\cos\left(\underbrace{\frac{\sqrt{2}\,\Omega_R t}{2}}_{\omega_1 = \omega_3}\right)\right)$$

$$= \frac{1}{8}\cos(2\omega_3 t) + \frac{1}{8} + \frac{1}{4} - \frac{1}{2}\cos(\omega_3 t)$$

$$= \frac{1}{8}\left(\cos(2\omega_3 t) + 1 + 2 - 4\cos(\omega_3 t)\right)$$

$$= \frac{1}{8}\left(2\cos^2(\omega_3 t) + 2 - 4\cos(\omega_3 t)\right)$$

$$= \frac{1}{4}\left(\cos^2(\omega_3 t) + 1 - 2\cos(\omega_3 t)\right)$$

$$= \frac{1}{4}\left(1 - \cos(\omega_3 t)\right)^2$$

$$= \frac{1}{4}\left(2\sin^2\left(\frac{\omega_3 t}{2}\right)\right)^2 = \sin^4\left(\frac{\omega_3 t}{2}\right)$$

Άρα $$\boxed{A_3 = \frac{4\,\Omega_R^2\,\Omega_{R'}^2}{\left(\Omega_R^2+\Omega_{R'}^2\right)^2}}$$

μέγιστος ρυθμός μεταβιβάσεως

maximum transfer rate $1 \to 3$

$$:= \frac{A_3}{T_3/2} = \frac{2A_3}{T_3} = \frac{4\,\Omega_R^2\,\Omega_{R'}^2}{\left(\Omega_R^2+\Omega_{R'}^2\right)^2}\,\frac{\sqrt{\Omega_R^2+\Omega_{R'}^2}}{2\pi\, 2}$$

Μέσες πιθανότητες παρουσίας του ηλεκτρονίου σε κάθε στάθμη

$$\left\langle |C_3(t)|^2 \right\rangle = \frac{\Omega_R^2\,\Omega_{R'}^2}{16\Lambda^4}\,\frac{1}{2} + \frac{\Omega_R^2\,\Omega_{R'}^2}{16\Lambda^4} = \frac{3\,\Omega_R^2\,\Omega_{R'}^2}{2\cdot 16\Lambda^4}$$

$$\left\langle |C_1(t)|^2 \right\rangle = \frac{\Omega_R^4}{16\Lambda^4}\,\frac{1}{2} + \frac{\Omega_{R'}^4}{16\Lambda^4} = \frac{\Omega_R^4 + 2\Omega_{R'}^4}{2\cdot 16\Lambda^4}$$

$$\left\langle |C_2(t)|^2 \right\rangle = \frac{\Omega_R^2}{\Omega_R^2+\Omega_{R'}^2}\,\frac{1}{2} = \frac{\Omega_R^2}{4\Lambda^2}\,\frac{1}{2} = \frac{4\,\Omega_R^2\,\Lambda^2}{2\cdot 16\Lambda^4}.$$

$$\left\langle |C_1(t)|^2 \right\rangle + \left\langle |C_2(t)|^2 \right\rangle + \left\langle |C_3(t)|^2 \right\rangle = \frac{3\Omega_R^2\Omega_{R'}^2 + \Omega_R^4 + 2\Omega_{R'}^4 + 4\Omega_R^2\Lambda^2}{2\cdot 16\Lambda^4}$$

$$= \frac{3\Omega_R^2\Omega_{R'}^2 + \Omega_R^4 + 2\Omega_{R'}^4 + \Omega_R^2\left(\Omega_R^2+\Omega_{R'}^2\right)}{2\cdot 16\Lambda^4} = \frac{4\Omega_R^2\Omega_{R'}^2 + 2\Omega_R^4 + 2\Omega_{R'}^4}{2\cdot 16\Lambda^4} =$$

$$\frac{\Omega_R^4 + 2\Omega_R^2\Omega_{R'}^2 + \Omega_{R'}^4}{\left(\Omega_R^2+\Omega_{R'}^2\right)^2} = 1$$

ορισμός t_{3mean}

$$\frac{3\cancel{\Omega_R^2\Omega_{R'}^2}}{2\cdot\cancel{16\Lambda^4}} = \frac{\cancel{\Omega_R^2\Omega_{R'}^2}}{\cancel{16\Lambda^4}}\cos^2(\Lambda t_{3mean}) + \frac{\cancel{\Omega_R^2\Omega_{R'}^2}}{\cancel{16\Lambda^4}} - \frac{2\cancel{\Omega_R^2\Omega_{R'}^2}}{\cancel{16\Lambda^4}}\cos(\Lambda t_{3mean})$$

ο απαιτούμενος χρόνος για να πιάσει τη μέση τιμή

$$\frac{3}{2} = \cos^2(\Lambda t_{3mean}) + 1 - 2\cos(\Lambda t_{3mean})$$

$$\frac{1}{2} = \cos^2(\Lambda t_{3mean}) - 2\cdot\cos(\Lambda t_{3mean})$$

$$1 = 2\cos^2 x - 4\cos x$$

$2\psi^2 - 4\psi - 1 = 0 \qquad \psi = \cos x$

$\Delta = 16 + 4\cdot 2 = 24$

$$\frac{4 \pm \sqrt{24}}{2\cdot 2} = 1 \pm \frac{\sqrt{24}}{4} = 1 \pm \sqrt{\frac{24}{16}} = 1 \pm \sqrt{\frac{3\cdot \cancel{8}}{2\cdot \cancel{8}}}$$

$$= 1 \pm \frac{\sqrt{3}}{\sqrt{2}} \qquad 1 - \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}} \qquad 1 + \frac{\sqrt{3}}{\sqrt{2}} > 1$$

απορρίπτεται

$$\cos(\Lambda\, t_{3mean}) = 1 - \frac{\sqrt{3}}{\sqrt{2}}$$

$$\cos\left(\frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2}\, t_{3mean}\right) = 1 - \frac{\sqrt{3}}{\sqrt{2}}$$

$$\frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2}\, t_{3mean} = 1.797477\ldots$$

$$t_{3mean} = \frac{2\cdot 1.797477}{\sqrt{\Omega_R^2 + \Omega_R'^2}}$$

mean tranfer rate

<u>μέσος ρυθμός μεταφοράς</u>

$$k := \frac{\langle |c_3(t)|^2 \rangle}{t_{3mean}} = \frac{3\,\Omega_R^2\,\Omega_R'^2}{2\cdot 16\,\Lambda^4}\;\frac{\sqrt{\Omega_R^2 + \Omega_R'^2}}{2\cdot 1.797477}$$

$$\frac{k}{\frac{\Gamma_3}{\Gamma_3}} = \frac{3\,\cancel{\Omega_R^2\,\Omega_R'^2}}{2\,\cancel{(\Omega_R^2+\Omega_R'^2)^2}}\;\frac{\cancel{\sqrt{\Omega_R^2+\Omega_R'^2}}}{2\cdot 1.797477}\cdot\frac{\cancel{(\Omega_R^2+\Omega_R'^2)^2}\; 2\pi\; 2}{4\,\cancel{\Omega_R^2\,\Omega_R'^2}\,\cancel{\sqrt{\Omega_R^2+\Omega_R'^2}}}$$

$$= \frac{3\cdot \pi}{4\cdot 1.797477} \simeq 1.31083\ldots$$

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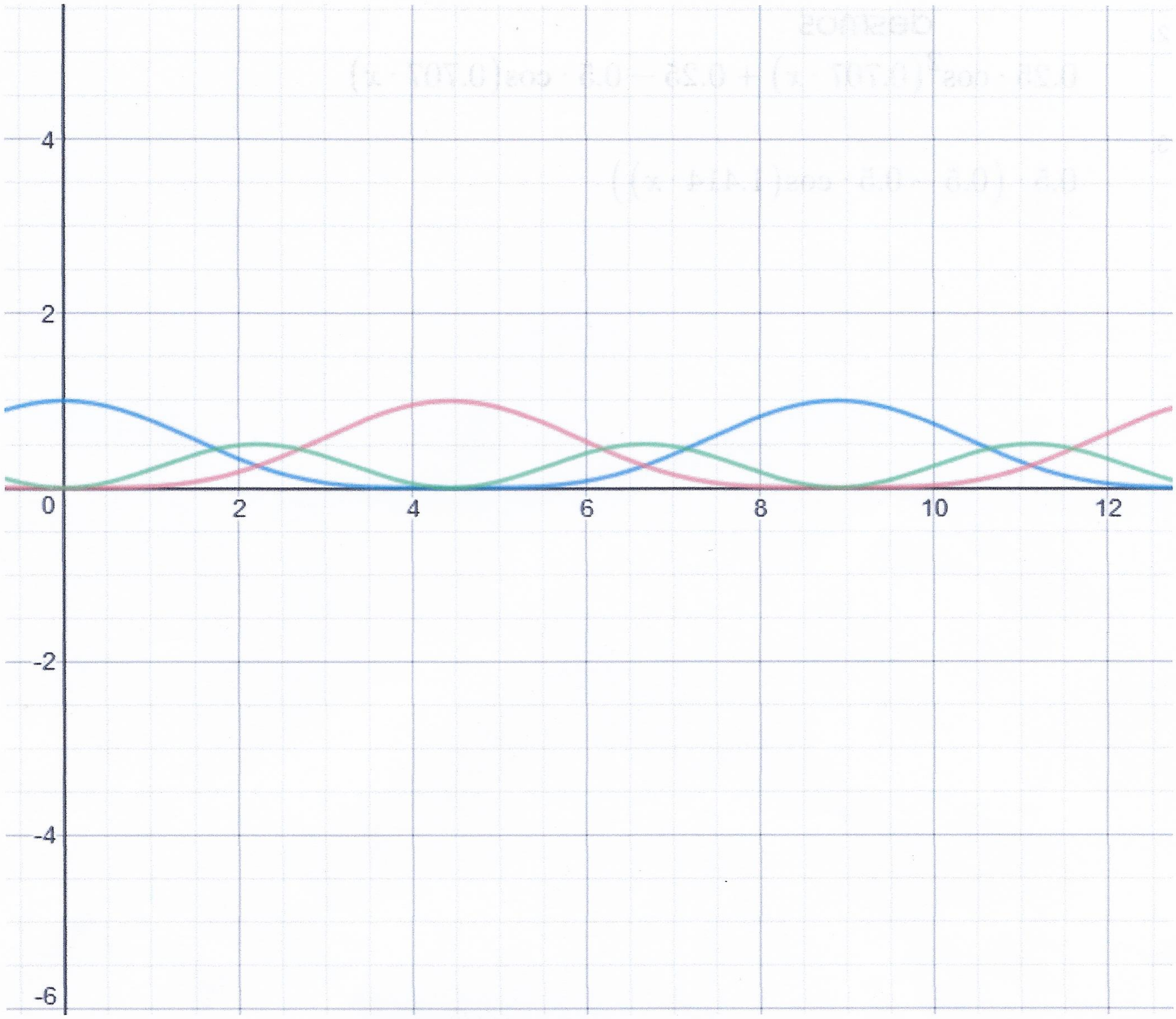

— $0.25 \cdot \cos^2(0.707 \cdot x) + 0.25 + 0.5 \cdot \cos(0.707 \cdot x)$

— $0.25 \cdot \cos^2(0.707 \cdot x) + 0.25 - 0.5 \cdot \cos(0.707 \cdot x)$

— $0.5 \cdot (0.5 - 0.5 \cdot \cos(1.414 \cdot x))$

πράσινο

για

$\Omega_R = \Omega_R' = 1$

$2\Lambda = \sqrt{1^2 + 1^2} = \sqrt{2} \Rightarrow \Lambda = \frac{\sqrt{2}}{2}$

ΑΣΚΗΣΗ (matlab)

Χρησιμοποιώντας το πρόγραμμα Oscillationsim.m να γίνει η γραφική παράσταση των ταλαντώσεων ΔΣ με $|\Delta| = \sqrt{3}\,\Omega_R$ $\left(\Rightarrow \alpha = \frac{1}{4},\ T = \frac{1}{2}\frac{2\pi}{\Omega_R}\right)$

ΑΣΚΗΣΗ (matlab)

Να φτιαχθεί ανάλογο πρόγραμμα για Τριστάθμικό Σύστημα και να γίνει η γραφική παράσταση για $\Omega_R = \Omega_{R'} = 1$ $\quad \Delta = 0$