

ΣΥΝΟΨΗ ΧΑΜΙΛΤΟΝΙΑΣ

ηροθ

$$\hat{H}_{HM,m} = \hbar\omega_m (\hat{a}_m^\dagger \hat{a}_m + \frac{1}{2}) = \hbar\omega (\hat{N}_m + \frac{1}{2})$$

3 τρόποι HM πεδίου στην κοιλότητα

κι άγνωστες των όρων $\frac{\hbar\omega_m}{2}$

$$\hat{H}_{HM,m} = \hbar\omega_m \hat{a}_m^\dagger \hat{a}_m = \hbar\omega_m \hat{N}_m$$

$$\hat{H}_{\Delta\Sigma} = E_2 \hat{S}_+ \hat{S}_- + E_1 \hat{S}_- \hat{S}_+ \quad E_2 - E_1 = \hbar\Omega$$

δισταδυικά συστήματα

και θέτουμε $E_1 \equiv 0$

$$\hat{H}_{\Delta\Sigma} = \hbar\Omega \hat{S}_+ \hat{S}_-$$

$$\hat{U}_{Em} = \hbar g_m (\hat{S}_+ + \hat{S}_-) (\hat{a}_m^\dagger + \hat{a}_m) = \hat{H}_{\Delta\Sigma - HMm}$$

άλλη ενέργεια $\Delta\Sigma$ - 3 τρόποι HM πεδίου στην κοιλότητα

(\hat{H}_{AF} ερωτική φυσική)
 ↓
 atom field

$$\hbar g_m := e x_{12} \left(\frac{\hbar\omega_m}{\epsilon_0 V} \right)^{1/2} \sin\left(\frac{m\pi z}{L}\right)$$

$$\hbar |g_m| = |e x_{12}| \left(\frac{4\hbar\omega_m n_m}{\epsilon_0 V} \right)^{1/2} \left| \sin\left(\frac{m\pi z}{L}\right) \right| \frac{1}{2\sqrt{n_m}}$$

$|g|$ $E_{0m}(z)$

$$\Omega_{Rm} = \frac{|g| E_{0m}}{\hbar}$$

συντόμια Rabi τού 3 τρόπου

$$\Omega_{Em} := 2\sqrt{n_m} g_m$$

$|\uparrow, n_m\rangle, |\downarrow, n_m\rangle$ οι $\sqrt{\text{καταστάσεις τής}}$ $\hat{H}_{HM,m} + \hat{H}_{\Delta\Sigma}$

Ενώ, η συνολική Χαμιλτονιανή γράφεται

$$\hat{H}_{Rm} = \hbar\omega_m \hat{a}_m^\dagger \hat{a}_m + \hbar\Omega \hat{S}_+ \hat{S}_- + \hbar g_m (\hat{S}_+ + \hat{S}_-) (\hat{a}_m^\dagger + \hat{a}_m)$$

δημογραφία συντόμια Χαμιλτονιανή Rabi

Άγνωστες τους όρους $\hat{S}_+ \hat{a}_m^\dagger$ & $\hat{S}_- \hat{a}_m$, οι οποίοι γίνονται "παράλογοι" όταν υπάρχει ένας μόνο τρόπος!

$$\hat{H}_{JCM} = \hbar\omega_m \hat{a}_m^\dagger \hat{a}_m + \hbar\Omega \hat{S}_+ \hat{S}_- + \hbar g_m (\hat{S}_+ \hat{a}_m + \hat{S}_- \hat{a}_m^\dagger)$$

δημογραφία συντόμια Χαμιλτονιανή Jaynes-Cummings

ΑΣΚΗΣΗ (Παραλείψατε για διφύση των δεικτών των HM τριών m)

⊙

Ⓐ Βρείτε τι κάνουν οι όροι

$\hat{a}^{\dagger}\hat{a}$, $\hat{a}\hat{a}^{\dagger}$, $\hat{S}_+\hat{S}_-$, $\hat{S}_-\hat{S}_+$, $\hat{S}_+\hat{a}^{\dagger}$, $\hat{S}_+\hat{a}$, $\hat{S}_-\hat{a}^{\dagger}$, $\hat{S}_-\hat{a}$ επί καταστάσεις $|\uparrow, n_m\rangle$ ή $|\downarrow, n_m\rangle$

Ⓑ Υπολογίστε τα

$\langle \hat{a}^{\dagger}\hat{a} \rangle$, $\langle \hat{a}\hat{a}^{\dagger} \rangle$, $\langle \hat{S}_+\hat{S}_- \rangle$, $\langle \hat{S}_-\hat{S}_+ \rangle$, $\langle \hat{S}_+\hat{a}^{\dagger} \rangle$, $\langle \hat{S}_+\hat{a} \rangle$, $\langle \hat{S}_-\hat{a}^{\dagger} \rangle$, $\langle \hat{S}_-\hat{a} \rangle$
επί $|\uparrow, n_m\rangle$, $|\downarrow, n_m\rangle$

Ⓐ $\hat{a}^{\dagger}\hat{a}|\uparrow, n\rangle = n|\uparrow, n\rangle$ $\hat{a}^{\dagger}\hat{a}|\downarrow, n\rangle = n|\downarrow, n\rangle$

$[\hat{a}, \hat{a}^{\dagger}] = 1 \Leftrightarrow \hat{a}\hat{a}^{\dagger} = 1 + \hat{a}^{\dagger}\hat{a}$

$\hat{a}\hat{a}^{\dagger}|\uparrow, n\rangle = (1 + \hat{a}^{\dagger}\hat{a})|\uparrow, n\rangle = (n+1)|\uparrow, n\rangle$

$\hat{a}\hat{a}^{\dagger}|\downarrow, n\rangle = (1 + \hat{a}^{\dagger}\hat{a})|\downarrow, n\rangle = (n+1)|\downarrow, n\rangle$

$\hat{S}_+\hat{S}_-|\uparrow, n\rangle = |\uparrow, n\rangle$ $\hat{S}_+\hat{S}_-|\downarrow, n\rangle = |\emptyset, n\rangle$

$\hat{S}_-\hat{S}_+|\uparrow, n\rangle = |\emptyset, n\rangle$ $\hat{S}_-\hat{S}_+|\downarrow, n\rangle = |\downarrow, n\rangle$

$\hat{S}_+\hat{a}^{\dagger}|\uparrow, n\rangle = \sqrt{n+1}|\emptyset, n+1\rangle$ $\hat{S}_+\hat{a}^{\dagger}|\downarrow, n\rangle = \sqrt{n+1}|\uparrow, n+1\rangle$

$\hat{S}_+\hat{a}|\uparrow, n\rangle = \sqrt{n}|\emptyset, n-1\rangle$ $\hat{S}_+\hat{a}|\downarrow, n\rangle = \sqrt{n}|\uparrow, n-1\rangle$

$\hat{S}_-\hat{a}^{\dagger}|\uparrow, n\rangle = \sqrt{n+1}|\downarrow, n+1\rangle$ $\hat{S}_-\hat{a}^{\dagger}|\downarrow, n\rangle = \sqrt{n+1}|\emptyset, n+1\rangle$

$\hat{S}_-\hat{a}|\uparrow, n\rangle = \sqrt{n}|\downarrow, n-1\rangle$ $\hat{S}_-\hat{a}|\downarrow, n\rangle = \sqrt{n}|\emptyset, n-1\rangle$

Ⓑ $\langle \uparrow, n | \hat{a}^{\dagger}\hat{a} | \uparrow, n \rangle = n$

$\langle \downarrow, n | \hat{a}^{\dagger}\hat{a} | \downarrow, n \rangle = n$

$\langle \uparrow, n | \hat{a}\hat{a}^{\dagger} | \uparrow, n \rangle = n+1$

$\langle \downarrow, n | \hat{a}\hat{a}^{\dagger} | \downarrow, n \rangle = n+1$

$\langle \uparrow, n | \hat{S}_+\hat{S}_- | \uparrow, n \rangle = 1$

$\langle \downarrow, n | \hat{S}_+\hat{S}_- | \downarrow, n \rangle = \langle \downarrow, n | \emptyset, n \rangle = 0$

$\langle \uparrow, n | \hat{S}_-\hat{S}_+ | \uparrow, n \rangle = \langle \uparrow, n | \emptyset, n \rangle = 0$

$\langle \downarrow, n | \hat{S}_-\hat{S}_+ | \downarrow, n \rangle = \langle \downarrow, n | \downarrow, n \rangle = 1$

$\langle \hat{a}^\dagger \hat{a} \rangle_{ \uparrow n\rangle} = n$	$\langle \hat{a}^\dagger \hat{a} \rangle_{ \downarrow n\rangle} = n$
$\langle \hat{a} \hat{a}^\dagger \rangle_{ \uparrow n\rangle} = n+1$	$\langle \hat{a} \hat{a}^\dagger \rangle_{ \downarrow n\rangle} = n+1$
$\langle \hat{S}_+ \hat{S}_- \rangle_{ \uparrow n\rangle} = 1$	$\langle \hat{S}_+ \hat{S}_- \rangle_{ \downarrow n\rangle} = 0$
$\langle \hat{S}_- \hat{S}_+ \rangle_{ \uparrow n\rangle} = 0$	$\langle \hat{S}_- \hat{S}_+ \rangle_{ \downarrow n\rangle} = 1$

$\hat{a}^\dagger |\uparrow n\rangle = \sqrt{n+1} |\uparrow n+1\rangle$
 $\hat{a} |\downarrow n\rangle = \sqrt{n} |\downarrow n-1\rangle$
 $\hat{a} |\uparrow n\rangle = \sqrt{n} |\downarrow n-1\rangle$
 $\hat{a}^\dagger |\downarrow n\rangle = \sqrt{n+1} |\uparrow n+1\rangle$

$$\langle \hat{S}_+ \hat{a}^\dagger \rangle_{|\uparrow n\rangle} = 0 \quad \langle \hat{S}_+ \hat{a}^\dagger \rangle_{|\downarrow n\rangle} = 0$$

$$\langle \hat{S}_+ \hat{a} \rangle_{|\uparrow n\rangle} = 0 \quad \langle \hat{S}_+ \hat{a} \rangle_{|\downarrow n\rangle} = 0$$

$$\langle \hat{S}_- \hat{a}_m^\dagger \rangle_{|\uparrow n\rangle} = 0 \quad \langle \hat{S}_- \hat{a}_m^\dagger \rangle_{|\downarrow n\rangle} = 0$$

$$\langle \hat{S}_- \hat{a} \rangle_{|\uparrow n\rangle} = 0 \quad \langle \hat{S}_- \hat{a} \rangle_{|\downarrow n\rangle} = 0$$

... Survival

$$\langle \uparrow n | \hat{S}_+ \hat{a}^\dagger | \uparrow n \rangle = \sqrt{n+1} \langle \uparrow n | 0, n+1 \rangle = 0$$

$$\langle \downarrow n | \hat{S}_+ \hat{a}^\dagger | \downarrow n \rangle = \sqrt{n+1} \langle \downarrow n | \uparrow n+1 \rangle = 0$$

$$\langle \uparrow n | \hat{S}_+ \hat{a} | \uparrow n \rangle = \sqrt{n} \langle \uparrow n | 0, n-1 \rangle = 0$$

$$\langle \downarrow n | \hat{S}_+ \hat{a} | \downarrow n \rangle = \sqrt{n} \langle \downarrow n | \uparrow n-1 \rangle = 0$$

$$\langle \uparrow n | \hat{S}_- \hat{a}^\dagger | \uparrow n \rangle = \sqrt{n+1} \langle \uparrow n | \downarrow n+1 \rangle = 0$$

$$\langle \downarrow n | \hat{S}_- \hat{a}^\dagger | \downarrow n \rangle = \sqrt{n+1} \langle \downarrow n | 0, n+1 \rangle = 0$$

$$\langle \uparrow n | \hat{S}_- \hat{a} | \uparrow n \rangle = \sqrt{n} \langle \uparrow n | \downarrow n-1 \rangle = 0$$

$$\langle \downarrow n | \hat{S}_- \hat{a} | \downarrow n \rangle = \sqrt{n} \langle \downarrow n | 0, n-1 \rangle = 0$$

ΜΕΣΕΣ (ΑΝΑΜΕΝΟΜΕΝΕΣ) ΤΙΜΕΣ ΜΕΓΕΘΩΝ
για τη Χαμιλτονιανή Jaynes - Cummings

Για την κατάσταση: (A) $|\psi_A(t)\rangle = c_1(t)|\downarrow, n\rangle + c_2(t)|\uparrow, n-1\rangle = |A\rangle$

(B) $|\psi_B(t)\rangle = c_1(t)|\uparrow, n\rangle + c_2(t)|\downarrow, n\rangle = |B\rangle$

$$\begin{aligned} \langle \hat{a}^\dagger \hat{a} \rangle_{|A\rangle} &= \left(c_1(t)^* \langle \downarrow, n| + c_2(t)^* \langle \uparrow, n-1| \right) \hat{a}^\dagger \hat{a} \left(c_1(t) |\downarrow, n\rangle + c_2(t) |\uparrow, n-1\rangle \right) \\ &= |c_1(t)|^2 \langle \downarrow, n| \hat{a}^\dagger \hat{a} |\downarrow, n\rangle + c_1^*(t) c_2(t) \langle \downarrow, n| \hat{a}^\dagger \hat{a} |\uparrow, n-1\rangle \\ &\quad + c_2^*(t) c_1(t) \langle \uparrow, n-1| \hat{a}^\dagger \hat{a} |\downarrow, n\rangle + |c_2(t)|^2 \langle \uparrow, n-1| \hat{a}^\dagger \hat{a} |\uparrow, n-1\rangle \\ &= |c_1(t)|^2 n \langle \downarrow, n | \downarrow, n \rangle + c_1^*(t) c_2(t) (n-1) \langle \downarrow, n | \uparrow, n-1 \rangle \\ &\quad + c_2^*(t) c_1(t) n \langle \uparrow, n-1 | \downarrow, n \rangle + |c_2(t)|^2 (n-1) \langle \uparrow, n-1 | \uparrow, n-1 \rangle \\ &= |c_1(t)|^2 n + |c_2(t)|^2 n - |c_2(t)|^2 \Rightarrow \langle \hat{a}^\dagger \hat{a} \rangle_{|A\rangle} = n - |c_2(t)|^2 \end{aligned}$$

$$\begin{aligned} \langle \hat{a} \hat{a}^\dagger \rangle_{|A\rangle} &= \left(c_1(t)^* \langle \downarrow, n| + c_2(t)^* \langle \uparrow, n-1| \right) \hat{a} \hat{a}^\dagger \left(c_1(t) |\downarrow, n\rangle + c_2(t) |\uparrow, n-1\rangle \right) \\ &= |c_1(t)|^2 \langle \downarrow, n| \hat{a} \hat{a}^\dagger |\downarrow, n\rangle + c_1^*(t) c_2(t) \langle \downarrow, n| \hat{a} \hat{a}^\dagger |\uparrow, n-1\rangle \\ &\quad + c_2^*(t) c_1(t) \langle \uparrow, n-1| \hat{a} \hat{a}^\dagger |\downarrow, n\rangle + |c_2(t)|^2 \langle \uparrow, n-1| \hat{a} \hat{a}^\dagger |\uparrow, n-1\rangle = \\ &= |c_1(t)|^2 (n+1) + c_1^*(t) c_2(t) n \langle \downarrow, n | \uparrow, n-1 \rangle \\ &\quad + c_2^*(t) c_1(t) (n+1) \langle \uparrow, n-1 | \downarrow, n \rangle + |c_2(t)|^2 n = \\ &= |c_1(t)|^2 n + |c_1(t)|^2 + |c_2(t)|^2 n \Rightarrow \langle \hat{a} \hat{a}^\dagger \rangle_{|A\rangle} = n + |c_1(t)|^2 \end{aligned}$$

$$\langle \hat{a} \hat{a}^\dagger \rangle_{|A\rangle} - \langle \hat{a}^\dagger \hat{a} \rangle_{|A\rangle} = 1$$

Συμμετρικά άνοδευμένα...

$$\langle \hat{S}_+ \hat{S}_- \rangle_{|A\rangle} = |c_2(t)|^2$$

$$\langle \hat{S}_- \hat{S}_+ \rangle_{|A\rangle} = |c_1(t)|^2$$

$$\langle \hat{S}_+ \hat{S}_- \rangle_{|A\rangle} + \langle \hat{S}_- \hat{S}_+ \rangle_{|A\rangle} = 1$$

$$\langle \hat{a}^\dagger \hat{a} \rangle_{|A\rangle} + \langle \hat{S}_+ \hat{S}_- \rangle_{|A\rangle} = n$$

$$\langle \hat{S}_+ \hat{a} \rangle_{|A\rangle} = c_2^*(t) c_1(t) \sqrt{n}$$

$$\langle \hat{S}_- \hat{a}^\dagger \rangle_{|A\rangle} = c_1^*(t) c_2(t) \sqrt{n}$$

$$\langle \hat{S}_+ \hat{a}^\dagger \rangle_{|A\rangle} = 0$$
$$\langle \hat{S}_- \hat{a} \rangle_{|A\rangle} = 0$$

Για την κατάσταση $|E\rangle = c_1(t) |\uparrow n\rangle + c_2(t) |\downarrow n+1\rangle = |E\rangle$ Συμμετρικά άνοδευμένα

$$\langle \hat{a}^\dagger \hat{a} \rangle_{|E\rangle} =$$

$$\langle \hat{a} \hat{a}^\dagger \rangle_{|E\rangle} =$$

$$\langle \hat{a} \hat{a}^\dagger \rangle_{|E\rangle} - \langle \hat{a}^\dagger \hat{a} \rangle_{|E\rangle} =$$

$$\langle \hat{S}_+ \hat{S}_- \rangle_{|E\rangle} =$$

$$\langle \hat{S}_- \hat{S}_+ \rangle_{|E\rangle} =$$

$$\langle \hat{S}_+ \hat{S}_- \rangle_{|E\rangle} + \langle \hat{S}_- \hat{S}_+ \rangle_{|E\rangle} =$$

$$\langle \hat{a}^\dagger \hat{a} \rangle_{|E\rangle} + \langle \hat{S}_+ \hat{S}_- \rangle_{|E\rangle} =$$

$$\langle \hat{S}_+ \hat{a} \rangle_{|E\rangle} =$$

$$\langle \hat{S}_+ \hat{a}^\dagger \rangle_{|E\rangle} =$$

$$\langle \hat{S}_- \hat{a}^\dagger \rangle_{|E\rangle} =$$

$$\langle \hat{S}_- \hat{a} \rangle_{|E\rangle} =$$

$$\begin{aligned}
\langle \hat{S}_+ \hat{S}_- \rangle_{|A\rangle} &= (c_1(t)^* \langle \downarrow n | + c_2(t)^* \langle \uparrow n-1 |) \hat{S}_+ \hat{S}_- (c_1(t) |\downarrow n\rangle + c_2(t) |\uparrow n-1\rangle) \\
&= |c_1(t)|^2 \langle \downarrow n | \hat{S}_+ \hat{S}_- | \downarrow n \rangle + \\
&\quad c_1(t)^* c_2(t) \langle \downarrow n | \hat{S}_+ \hat{S}_- | \uparrow n-1 \rangle + \\
&\quad c_2(t)^* c_1(t) \langle \uparrow n-1 | \hat{S}_+ \hat{S}_- | \downarrow n \rangle + \\
&\quad |c_2(t)|^2 \langle \uparrow n-1 | \hat{S}_+ \hat{S}_- | \uparrow n-1 \rangle = \\
&= |c_1(t)|^2 \langle \downarrow n | \cancel{\uparrow n} \rangle + \\
&\quad c_1(t)^* c_2(t) \langle \downarrow n | \cancel{\uparrow n-1} \rangle + \\
&\quad c_2(t)^* c_1(t) \langle \uparrow n-1 | \cancel{\downarrow n} \rangle + \\
&\quad |c_2(t)|^2 \langle \uparrow n-1 | \uparrow n-1 \rangle = |c_2(t)|^2
\end{aligned}$$

$$\begin{aligned}
\langle \hat{S}_- \hat{S}_+ \rangle_{|A\rangle} &= (c_1(t)^* \langle \downarrow n | + c_2(t)^* \langle \uparrow n-1 |) \hat{S}_- \hat{S}_+ (c_1(t) |\downarrow n\rangle + c_2(t) |\uparrow n-1\rangle) \\
&= |c_1(t)|^2 \langle \downarrow n | \hat{S}_- \hat{S}_+ | \downarrow n \rangle + \\
&\quad c_1(t)^* c_2(t) \langle \downarrow n | \hat{S}_- \hat{S}_+ | \uparrow n-1 \rangle + \\
&\quad c_2(t)^* c_1(t) \langle \uparrow n-1 | \hat{S}_- \hat{S}_+ | \downarrow n \rangle + \\
&\quad |c_2(t)|^2 \langle \uparrow n-1 | \hat{S}_- \hat{S}_+ | \uparrow n-1 \rangle = \\
&= |c_1(t)|^2 \langle \downarrow n | \downarrow n \rangle + \\
&\quad c_1(t)^* c_2(t) \langle \downarrow n | \cancel{\uparrow n-1} \rangle + \\
&\quad c_2(t)^* c_1(t) \langle \uparrow n-1 | \cancel{\downarrow n} \rangle + \\
&\quad |c_2(t)|^2 \langle \uparrow n-1 | \cancel{\downarrow n-1} \rangle = |c_1(t)|^2
\end{aligned}$$

$$\begin{aligned}
\langle \hat{S}_+ \hat{a} \rangle_{|A\rangle} &= (c_1^*(t) \langle \downarrow n | + c_2^*(t) \langle \uparrow n-1 |) \hat{S}_+ \hat{a} (c_1(t) |\downarrow n\rangle + c_2(t) |\uparrow n-1\rangle) \textcircled{\text{IB}} \\
&= |c_1(t)|^2 \langle \downarrow n | \hat{S}_+ \hat{a} | \downarrow n \rangle + \\
&\quad c_1^*(t) c_2(t) \langle \downarrow n | \hat{S}_+ \hat{a} | \uparrow n-1 \rangle + \\
&\quad c_2^*(t) c_1(t) \langle \uparrow n-1 | \hat{S}_+ \hat{a} | \downarrow n \rangle + \\
&\quad |c_2(t)|^2 \langle \uparrow n-1 | \hat{S}_+ \hat{a} | \uparrow n-1 \rangle = \\
&= |c_1(t)|^2 \sqrt{n} \langle \downarrow n | \uparrow n-1 \rangle + \\
&\quad c_1^*(t) c_2(t) \sqrt{n-1} \langle \downarrow n | \uparrow n-2 \rangle + \\
&\quad c_2^*(t) c_1(t) \sqrt{n} \langle \uparrow n-1 | \uparrow n-1 \rangle + \\
&\quad |c_2(t)|^2 \sqrt{n-1} \langle \uparrow n-1 | \uparrow n-2 \rangle = c_2^*(t) c_1(t) \sqrt{n}
\end{aligned}$$

$$\begin{aligned}
\langle \hat{S}_- \hat{a}^\dagger \rangle_{|A\rangle} &= (c_1(t) \langle \downarrow n | + c_2(t) \langle \uparrow n-1 |) \hat{S}_- \hat{a}^\dagger (c_1(t) |\downarrow n\rangle + c_2(t) |\uparrow n-1\rangle) \\
&= |c_1(t)|^2 \langle \downarrow n | \hat{S}_- \hat{a}^\dagger | \downarrow n \rangle + \\
&\quad c_1^*(t) c_2(t) \langle \downarrow n | \hat{S}_- \hat{a}^\dagger | \uparrow n-1 \rangle + \\
&\quad c_2^*(t) c_1(t) \langle \uparrow n-1 | \hat{S}_- \hat{a}^\dagger | \downarrow n \rangle + \\
&\quad |c_2(t)|^2 \langle \uparrow n-1 | \hat{S}_- \hat{a}^\dagger | \uparrow n-1 \rangle = \\
&= |c_1(t)|^2 \sqrt{n+1} \langle \downarrow n | \uparrow n+1 \rangle + \\
&\quad c_1^*(t) c_2(t) \sqrt{n} \langle \downarrow n | \downarrow n \rangle + \\
&\quad c_2^*(t) c_1(t) \sqrt{n+1} \langle \uparrow n-1 | \uparrow n+1 \rangle + \\
&\quad |c_2(t)|^2 \sqrt{n} \langle \uparrow n-1 | \downarrow n \rangle = c_1^*(t) c_2(t) \sqrt{n}
\end{aligned}$$

$\hat{H}_R = \hbar \omega_m \hat{a}_m^\dagger \hat{a}_m + \hbar \Omega \hat{S}_+ \hat{S}_- + \hbar g^m (\hat{S}_+ + \hat{S}_-) (\hat{a}_m^\dagger + \hat{a}_m)$ Rabi Hamiltonian describing the interaction of a two-level atom with a single-mode harmonic field

$\hbar g^m (\hat{S}_+ \hat{a}_m^\dagger + \hat{S}_- \hat{a}_m)$ the so-called counter-rotating terms (ἀγροσύνη)

$\hat{H}_{JC} = \hbar \omega_m \hat{a}_m^\dagger \hat{a}_m + \hbar \Omega \hat{S}_+ \hat{S}_- + \hbar g^m (\hat{S}_+ \hat{a}_m + \hat{S}_- \hat{a}_m^\dagger)$ Jaynes-Cummings Hamiltonian

Να υπολογίσουμε τα $\langle \hat{a}_m^\dagger \hat{a}_m \rangle$, $\langle \hat{S}_+ \hat{S}_- \rangle$, $\langle \hat{S}_+ \hat{a}_m \rangle$, $\langle \hat{S}_- \hat{a}_m^\dagger \rangle$ για τις καταστάσεις

Ⓐ $|\Psi_A(t)\rangle = c_1(t) |\downarrow, n\rangle + c_2(t) |\uparrow, n-1\rangle$ θα χρησιμοποιήσουμε $c_1(0) = 1, c_2(0) = 0$
 με άρχικες συνθήκες $c_1(0) = 1, c_2(0) = 0$

Ⓑ $|\Psi_B(t)\rangle = c_1(t) |\downarrow, n+1\rangle + c_2(t) |\uparrow, n\rangle$ $c_1(0) = 0, c_2(0) = 1$

Ⓐ

$$\begin{aligned} \langle \hat{a}_m^\dagger \hat{a}_m \rangle_{\text{Ⓐ}} &= \langle \Psi_A(t) | \hat{a}_m^\dagger \hat{a}_m | \Psi_A(t) \rangle = \{ c_1^* \langle \downarrow, n | + c_2^* \langle \uparrow, n-1 | \} \hat{a}_m^\dagger \hat{a}_m \{ c_1 |\downarrow, n\rangle + c_2 |\uparrow, n-1\rangle \} \\ &= |c_1|^2 \langle \downarrow, n | \hat{a}_m^\dagger \hat{a}_m | \downarrow, n \rangle + c_1^* c_2 \langle \downarrow, n | \hat{a}_m^\dagger \hat{a}_m | \uparrow, n-1 \rangle \\ &\quad + c_2^* c_1 \langle \uparrow, n-1 | \hat{a}_m^\dagger \hat{a}_m | \downarrow, n \rangle + |c_2|^2 \langle \uparrow, n-1 | \hat{a}_m^\dagger \hat{a}_m | \uparrow, n-1 \rangle \\ &= |c_1|^2 \sqrt{n} \sqrt{n} \langle \downarrow, n | \downarrow, n \rangle + c_1^* c_2 \sqrt{n-1} \sqrt{n-1} \langle \downarrow, n | \uparrow, n-1 \rangle \\ &\quad + c_2^* c_1 \sqrt{n} \sqrt{n} \langle \uparrow, n-1 | \downarrow, n \rangle + |c_2|^2 \sqrt{n-1} \sqrt{n-1} \langle \uparrow, n-1 | \uparrow, n-1 \rangle = \\ &= n |c_1|^2 + c_1^* c_2 (n-1) \cdot 0 + c_2^* c_1 n \cdot 0 + (n-1) |c_2|^2 = \\ &= n |c_1|^2 + n |c_2|^2 - |c_2|^2 = n (|c_1|^2 + |c_2|^2) - |c_2|^2 = n - |c_2|^2 \Rightarrow \end{aligned}$$

$$\langle \hat{a}_m^\dagger \hat{a}_m \rangle_{\text{Ⓐ}} = n - |c_2(t)|^2 \checkmark$$

$$\begin{aligned} \langle \hat{S}_+ \hat{S}_- \rangle_{\text{Ⓐ}} &= \langle \Psi_A(t) | \hat{S}_+ \hat{S}_- | \Psi_A(t) \rangle = \{ c_1^* \langle \downarrow, n | + c_2^* \langle \uparrow, n-1 | \} \hat{S}_+ \hat{S}_- \{ c_1 |\downarrow, n\rangle + c_2 |\uparrow, n-1\rangle \} \\ &= |c_1|^2 \langle \downarrow, n | \hat{S}_+ \hat{S}_- | \downarrow, n \rangle + c_1^* c_2 \langle \downarrow, n | \hat{S}_+ \hat{S}_- | \uparrow, n-1 \rangle \\ &\quad + c_2^* c_1 \langle \uparrow, n-1 | \hat{S}_+ \hat{S}_- | \downarrow, n \rangle + |c_2|^2 \langle \uparrow, n-1 | \hat{S}_+ \hat{S}_- | \uparrow, n-1 \rangle = \\ &= |c_1|^2 \cdot 0 + c_1^* c_2 \langle \downarrow, n | \uparrow, n-1 \rangle + c_2^* c_1 \cdot 0 + |c_2|^2 \langle \uparrow, n-1 | \uparrow, n-1 \rangle \Rightarrow \end{aligned}$$

$$\langle \hat{S}_+ \hat{S}_- \rangle_{\text{Ⓐ}} = |c_2(t)|^2 \checkmark$$

ΑΡΑ $\langle \hat{a}_m^\dagger \hat{a}_m \rangle_{\text{Ⓐ}} + \langle \hat{S}_+ \hat{S}_- \rangle_{\text{Ⓐ}} = n \checkmark$

$$\begin{aligned}
 \langle \hat{S}_+ \hat{a}_m \rangle_A &= \langle \psi_A(t) | \hat{S}_+ \hat{a}_m | \psi_A(t) \rangle = \{ c_1^* \langle \downarrow, n | + c_2^* \langle \uparrow, n-1 | \} \hat{S}_+ \hat{a}_m \{ c_1 | \downarrow, n \rangle + c_2 | \uparrow, n-1 \rangle \} \\
 &= |c_1|^2 \langle \downarrow, n | \hat{S}_+ \hat{a}_m | \downarrow, n \rangle + c_1^* c_2 \langle \downarrow, n | \hat{S}_+ \hat{a}_m | \uparrow, n-1 \rangle + \\
 &+ c_2^* c_1 \langle \uparrow, n-1 | \hat{S}_+ \hat{a}_m | \downarrow, n \rangle + |c_2|^2 \langle \uparrow, n-1 | \hat{S}_+ \hat{a}_m | \uparrow, n-1 \rangle = \\
 &= |c_1|^2 \sqrt{n} \langle \downarrow, n | \uparrow, n-1 \rangle + c_1^* c_2 \cdot \sqrt{n-1} \cdot \langle \downarrow, n | \hat{S}_+ | \uparrow, n-2 \rangle \\
 &+ c_2^* c_1 \sqrt{n} \langle \uparrow, n-1 | \uparrow, n-1 \rangle + |c_2|^2 \sqrt{n-1} \langle \uparrow, n-1 | \hat{S}_+ | \uparrow, n-2 \rangle \Rightarrow \\
 \langle \hat{S}_+ \hat{a}_m \rangle_A &= c_2^*(t) c_1(t) \cdot \sqrt{n}
 \end{aligned}$$

$$\begin{aligned}
 \langle \hat{S}_- \hat{a}_m^\dagger \rangle_A &= \langle \psi_A(t) | \hat{S}_- \hat{a}_m^\dagger | \psi_A(t) \rangle = \{ c_1^* \langle \downarrow, n | + c_2^* \langle \uparrow, n-1 | \} \hat{S}_- \hat{a}_m^\dagger \{ c_1 | \downarrow, n \rangle + c_2 | \uparrow, n-1 \rangle \} \\
 &= |c_1|^2 \langle \downarrow, n | \hat{S}_- \hat{a}_m^\dagger | \downarrow, n \rangle + c_1^* c_2 \langle \downarrow, n | \hat{S}_- \hat{a}_m^\dagger | \uparrow, n-1 \rangle + \\
 &+ c_2^* c_1 \langle \uparrow, n-1 | \hat{S}_- \hat{a}_m^\dagger | \downarrow, n \rangle + |c_2|^2 \langle \uparrow, n-1 | \hat{S}_- \hat{a}_m^\dagger | \uparrow, n-1 \rangle = \\
 &= |c_1|^2 \sqrt{n+1} \langle \downarrow, n | \hat{S}_- | \downarrow, n+1 \rangle + c_1^* c_2 \sqrt{n} \langle \downarrow, n | \downarrow, n \rangle \\
 &+ c_2^* c_1 \langle \uparrow, n-1 | \hat{S}_- | \uparrow, n+1 \rangle \sqrt{n+1} + |c_2|^2 \langle \uparrow, n-1 | \downarrow, n \rangle \sqrt{n} \Rightarrow \\
 \langle \hat{S}_- \hat{a}_m^\dagger \rangle_A &= c_1^*(t) c_2(t) \sqrt{n}
 \end{aligned}$$

(E)

$$\begin{aligned}
 \langle \hat{a}_m^\dagger \hat{a}_m \rangle_{\text{E}} &= \langle \psi_{\text{E}}(t) | \hat{a}_m^\dagger \hat{a}_m | \psi_{\text{E}}(t) \rangle = \{ c_1^* \langle \downarrow, n+1 | + c_2^* \langle \uparrow, n | \} \{ \hat{a}_m^\dagger \hat{a}_m \} \{ c_1 | \downarrow, n+1 \rangle + c_2 | \uparrow, n \rangle \} \\
 &= |c_1|^2 \langle \downarrow, n+1 | \hat{a}_m^\dagger \hat{a}_m | \downarrow, n+1 \rangle + c_1^* c_2 \langle \downarrow, n+1 | \hat{a}_m^\dagger \hat{a}_m | \uparrow, n \rangle \\
 &\quad + c_2^* c_1 \langle \uparrow, n | \hat{a}_m^\dagger \hat{a}_m | \downarrow, n+1 \rangle + |c_2|^2 \langle \uparrow, n | \hat{a}_m^\dagger \hat{a}_m | \uparrow, n \rangle \\
 &= |c_1|^2 \sqrt{n+1} \sqrt{n+1} \langle \downarrow, n+1 | \downarrow, n+1 \rangle + c_1^* c_2 n \langle \downarrow, n+1 | \uparrow, n \rangle \\
 &\quad + c_2^* c_1 (n+1) \langle \uparrow, n | \downarrow, n+1 \rangle + |c_2|^2 n \langle \uparrow, n | \uparrow, n \rangle = \\
 &= |c_1|^2 (n+1) + n |c_2|^2 = n (|c_1|^2 + |c_2|^2) + |c_1|^2 \Rightarrow
 \end{aligned}$$

$$\langle \hat{a}_m^\dagger \hat{a}_m \rangle_{\text{E}} = n + |c_1(t)|^2 \quad (4.60)$$

$$\begin{aligned}
 \langle \hat{S}_+ \hat{S}_- \rangle_{\text{E}} &= \langle \psi_{\text{E}}(t) | \hat{S}_+ \hat{S}_- | \psi_{\text{E}}(t) \rangle = \{ c_1^* \langle \downarrow, n+1 | + c_2^* \langle \uparrow, n | \} \{ \hat{S}_+ \hat{S}_- \} \{ c_1 | \downarrow, n+1 \rangle + c_2 | \uparrow, n \rangle \} \\
 &= |c_1|^2 \cdot 0 + c_1^* c_2 \langle \downarrow, n+1 | \uparrow, n \rangle + c_2^* c_1 \cdot 0 + |c_2|^2 \langle \uparrow, n | \uparrow, n \rangle \Rightarrow
 \end{aligned}$$

$$\langle \hat{S}_+ \hat{S}_- \rangle_{\text{E}} = |c_2(t)|^2 \quad (4.60)$$

$$\begin{aligned}
 \langle \hat{S}_+ \hat{a}_m \rangle_{\text{E}} &= \langle \psi_{\text{E}}(t) | \hat{S}_+ \hat{a}_m | \psi_{\text{E}}(t) \rangle = \{ c_1^* \langle \downarrow, n+1 | + c_2^* \langle \uparrow, n | \} \{ \hat{S}_+ \hat{a}_m \} \{ c_1 | \downarrow, n+1 \rangle + c_2 | \uparrow, n \rangle \} \\
 &= |c_1|^2 \langle \downarrow, n+1 | \hat{S}_+ \hat{a}_m | \downarrow, n+1 \rangle + c_1^* c_2 \langle \downarrow, n+1 | \hat{S}_+ \hat{a}_m | \uparrow, n \rangle + \\
 &\quad - c_2^* c_1 \langle \uparrow, n | \hat{S}_+ \hat{a}_m | \downarrow, n+1 \rangle + |c_2|^2 \langle \uparrow, n | \hat{S}_+ \hat{a}_m | \uparrow, n \rangle \\
 &= |c_1|^2 \langle \downarrow, n+1 | \uparrow, n \rangle \sqrt{n+1} + c_1^* c_2 \cdot 0 + c_2^* c_1 \langle \uparrow, n | \uparrow, n \rangle \sqrt{n+1} + |c_2|^2 \cdot 0
 \end{aligned}$$

$$\langle \hat{S}_+ \hat{a}_m \rangle_{\text{E}} = c_2^*(t) c_1(t) \cdot \sqrt{n+1}$$

$$\begin{aligned}
 \langle \hat{S}_- \hat{a}_m^\dagger \rangle_{\text{E}} &= \langle \psi_{\text{E}}(t) | \hat{S}_- \hat{a}_m^\dagger | \psi_{\text{E}}(t) \rangle = \{ c_1^* \langle \downarrow, n+1 | + c_2^* \langle \uparrow, n | \} \{ \hat{S}_- \hat{a}_m^\dagger \} \{ c_1 | \downarrow, n+1 \rangle + c_2 | \uparrow, n \rangle \} \\
 &= |c_1|^2 \langle \downarrow, n+1 | \hat{S}_- \hat{a}_m^\dagger | \downarrow, n+1 \rangle + c_1^* c_2 \langle \downarrow, n+1 | \hat{S}_- \hat{a}_m^\dagger | \uparrow, n \rangle \\
 &\quad + c_2^* c_1 \langle \uparrow, n | \hat{S}_- \hat{a}_m^\dagger | \downarrow, n+1 \rangle + |c_2|^2 \langle \uparrow, n | \hat{S}_- \hat{a}_m^\dagger | \uparrow, n \rangle = \\
 &= |c_1|^2 \langle \downarrow, n+1 | \uparrow, n+2 \rangle \sqrt{n+2} + c_1^* c_2 \langle \downarrow, n+1 | \downarrow, n+1 \rangle \sqrt{n+1} \\
 &\quad + c_2^* c_1 \cdot 0 + |c_2|^2 \langle \uparrow, n | \downarrow, n+1 \rangle \sqrt{n+1} \Rightarrow
 \end{aligned}$$

$$\langle \hat{S}_- \hat{a}_m^\dagger \rangle_{\text{E}} = c_1^*(t) c_2(t) \cdot \sqrt{n+1}$$

$$\text{APA} \quad \langle \hat{a}_m^\dagger \hat{a}_m \rangle_{\text{E}} + \langle \hat{S}_+ \hat{S}_- \rangle_{\text{E}} = n+1 \quad (4.61)$$