

$$A \begin{cases} \dot{C}_1(t) = \frac{i\mathcal{E}_0}{2\hbar} e^{i\Delta t} C_2(t) \\ \dot{C}_2(t) = \frac{i\mathcal{E}_0}{2\hbar} e^{-i\Delta t} C_1(t) \end{cases}$$

$\Delta\Sigma$  μετά την RWA

$$\Omega_R = \frac{\mathcal{E}_0}{\hbar}$$

$$\Delta = \omega - \Omega$$

Συναλλακτική λύση των διαφορικών εξισ. με Χ.Ε.

$$A' \begin{cases} \dot{C}_1(t) = \frac{i\Omega_R}{2} e^{i\Delta t} C_2(t) \\ \dot{C}_2(t) = \frac{i\Omega_R}{2} e^{-i\Delta t} C_1(t) \end{cases}$$

$$\begin{aligned} \ddot{C}_1(t) &= \frac{i\Omega_R}{2} i\Delta e^{i\Delta t} C_2(t) + \frac{i\Omega_R}{2} e^{i\Delta t} \dot{C}_2(t) \\ &= \frac{i\Omega_R}{2} i\Delta e^{i\Delta t} \frac{i\Omega_R}{2} e^{i\Delta t} C_1(t) + \frac{i\Omega_R}{2} e^{i\Delta t} \frac{i\Omega_R}{2} e^{-i\Delta t} C_1(t) \Rightarrow \end{aligned}$$

$$\ddot{C}_1(t) = i\Delta \dot{C}_1(t) - \frac{\Omega_R^2}{4} C_1(t) \Rightarrow \boxed{\ddot{C}_1(t) - i\Delta \dot{C}_1(t) + \frac{\Omega_R^2}{4} C_1(t) = 0} \quad (1)$$

$$\begin{aligned} \ddot{C}_2(t) &= \frac{i\Omega_R}{2} (-i\Delta) e^{-i\Delta t} C_1(t) + \frac{i\Omega_R}{2} e^{-i\Delta t} \dot{C}_1(t) \\ &= \frac{i\Omega_R}{2} (-i\Delta) e^{-i\Delta t} \frac{i\Omega_R}{2} e^{-i\Delta t} C_2(t) + \frac{i\Omega_R}{2} e^{-i\Delta t} \frac{i\Omega_R}{2} e^{i\Delta t} C_2(t) \Rightarrow \end{aligned}$$

$$\ddot{C}_2(t) = (-i\Delta) \dot{C}_2(t) - \frac{\Omega_R^2}{4} C_2(t) \Rightarrow \boxed{\ddot{C}_2(t) + i\Delta \dot{C}_2(t) + \frac{\Omega_R^2}{4} C_2(t) = 0} \quad (2)$$

$$\Delta\lambda M \quad C_k(t) = u_k e^{i\mu_k t}$$

$$\dot{C}_k(t) = u_k i\mu_k e^{i\mu_k t}$$

$$\ddot{C}_k(t) = u_k (i\mu_k)^2 e^{i\mu_k t}$$

$\Delta\lambda M =$  Δοκιμάζω λύσεις της μορφής

$$(1) \Rightarrow u_1 (i\mu_1)^2 e^{i\mu_1 t} - i\Delta u_1 i\mu_1 e^{i\mu_1 t} + \frac{\Omega_R^2}{4} u_1 e^{i\mu_1 t} = 0 \Rightarrow$$

$$\boxed{-\mu_1^2 + \Delta\mu_1 + \frac{\Omega_R^2}{4} = 0} \quad (XE1)$$

≠ συνθήκη για το  $u_1$

$$(2) \Rightarrow u_2 (i\mu_2)^2 e^{i\mu_2 t} + i\Delta u_2 i\mu_2 e^{i\mu_2 t} + \frac{\Omega_R^2}{4} u_2 e^{i\mu_2 t} = 0 \Rightarrow$$

$$\boxed{-\mu_2^2 - \Delta\mu_2 + \frac{\Omega_R^2}{4} = 0} \quad (XE2)$$

≠ συνθήκη για το  $u_2$

$$\Delta_1 = \Delta^2 - 4(-1) \frac{\Omega_R^2}{4} = \Delta^2 + \Omega_R^2$$

$$\lambda_{2,1} = \pm \frac{\sqrt{\Delta^2 + \Omega_R^2}}{2} = \pm \lambda \quad (2)$$

$$\mu_1 = \frac{-\Delta \pm \sqrt{\Delta^2 + \Omega_R^2}}{-2} \Rightarrow \mu_1 = \frac{\Delta}{2} \pm \frac{\sqrt{\Delta^2 + \Omega_R^2}}{2} \Rightarrow \mu_1 = \frac{\Delta}{2} \pm \lambda$$

$$\Delta_2 = (-\Delta)^2 - 4(-1) \frac{\Omega_R^2}{4} = \Delta^2 + \Omega_R^2$$

$$\mu_2 = \frac{+\Delta \pm \sqrt{\Delta^2 + \Omega_R^2}}{-2} \Rightarrow \mu_2 = -\frac{\Delta}{2} \pm \frac{\sqrt{\Delta^2 + \Omega_R^2}}{2} \Rightarrow \mu_2 = -\frac{\Delta}{2} \pm \lambda$$

"Αρα, οι λύσεις θα είναι

$$\begin{cases} C_1(t) = a e^{\frac{i\Delta}{2}t} e^{i\lambda t} + \beta e^{\frac{i\Delta}{2}t} e^{-i\lambda t} \\ C_2(t) = \gamma e^{-\frac{i\Delta}{2}t} e^{i\lambda t} + \delta e^{-\frac{i\Delta}{2}t} e^{-i\lambda t} \end{cases}$$

$$\begin{cases} C_1(t) = e^{\frac{i\Delta}{2}t} [a e^{i\lambda t} + \beta e^{-i\lambda t}] \\ C_2(t) = e^{-\frac{i\Delta}{2}t} [\gamma e^{i\lambda t} + \delta e^{-i\lambda t}] \end{cases} \xrightarrow{\Delta=0} \begin{cases} C_1(t) = a e^{i\lambda t} + \beta e^{-i\lambda t} \\ C_2(t) = \gamma e^{i\lambda t} + \delta e^{-i\lambda t} \end{cases}$$

• Τις δοκιμάζουμε τώρα στις (Α') για  $\Delta=0$ , οπότε  $\lambda = \frac{\Omega_R}{2}$

$$a i \lambda e^{i\lambda t} + \beta (-i\lambda) e^{-i\lambda t} = \frac{i\Omega_R}{2} (\gamma e^{i\lambda t} + \delta e^{-i\lambda t}) \Rightarrow \gamma = a \quad \beta = -\delta$$

$$\gamma i \lambda e^{i\lambda t} + \delta (-i\lambda) e^{-i\lambda t} = \frac{i\Omega_R}{2} (a e^{i\lambda t} + \beta e^{-i\lambda t}) \Rightarrow \gamma = a \quad \beta = -\delta$$

$$\begin{cases} C_1(t) = a e^{i\frac{\Omega_R}{2}t} + \beta e^{-i\frac{\Omega_R}{2}t} \\ C_2(t) = a e^{i\frac{\Omega_R}{2}t} - \beta e^{-i\frac{\Omega_R}{2}t} \end{cases}$$

"Έστωσαν οι αρχικές συνθήκες (Α.Σ.)  $C_1(0) = 1, C_2(0) = 0$

$$\begin{cases} 1 = a + \beta \\ 0 = a - \beta \Rightarrow \beta = a \end{cases} \quad 2a = 1 \Rightarrow a = \frac{1}{2}$$

$$C_1(t) = \frac{1}{2} e^{i\frac{\Omega_R}{2}t} + \frac{1}{2} e^{-i\frac{\Omega_R}{2}t} \Rightarrow C_1(t) = \cos\left(\frac{\Omega_R}{2}t\right)$$

$$C_2(t) = \frac{1}{2} e^{i\frac{\Omega_R}{2}t} - \frac{1}{2} e^{-i\frac{\Omega_R}{2}t} \Rightarrow C_2(t) = i \sin\left(\frac{\Omega_R}{2}t\right)$$



• Τις δοκιμάζουμε τώρα στις (A') για  $\Delta \neq 0$   $\lambda = \frac{\sqrt{\Delta^2 + \Omega_R^2}}{2} \Rightarrow 4\lambda^2 = \Delta^2 + \Omega_R^2$  (3)

$$i \frac{\Delta}{2} e^{i \frac{\Delta}{2} t} [a e^{i \lambda t} + \beta e^{-i \lambda t}] + e^{i \frac{\Delta}{2} t} [a i \lambda e^{i \lambda t} + \beta (-i \lambda) e^{-i \lambda t}] =$$

$$\frac{\Omega_R}{2} e^{i \lambda t} e^{-i \frac{\Delta}{2} t} [\gamma e^{i \lambda t} + \delta e^{-i \lambda t}] \Rightarrow$$

$$\frac{\Delta}{2} a + a \lambda = \frac{\Omega_R}{2} \gamma \quad \text{και} \quad \frac{\Delta}{2} \beta - \beta \lambda = \frac{\Omega_R}{2} \delta$$

$$\Rightarrow \frac{\Delta a + 2a \lambda}{2} = \frac{\Omega_R}{2} \gamma \quad \text{και} \quad \frac{\Delta \beta - 2\beta \lambda}{2} = \frac{\Omega_R}{2} \delta$$

$$\boxed{\gamma = \frac{\Delta + 2\lambda}{\Omega_R} a}$$

$$\text{και} \quad \boxed{\delta = \frac{\Delta - 2\lambda}{\Omega_R} \beta}$$

$$-i \frac{\Delta}{2} e^{-i \frac{\Delta}{2} t} [\gamma e^{i \lambda t} + \delta e^{-i \lambda t}] + e^{-i \frac{\Delta}{2} t} [\gamma i \lambda e^{i \lambda t} + \delta (-i \lambda) e^{-i \lambda t}] =$$

$$\frac{\Omega_R}{2} e^{-i \lambda t} e^{i \frac{\Delta}{2} t} [a e^{i \lambda t} + \beta e^{-i \lambda t}] \Rightarrow$$

$$-\frac{\Delta}{2} \gamma + \gamma \lambda = \frac{\Omega_R}{2} a \quad \text{και} \quad -\frac{\Delta}{2} \delta - \delta \lambda = \frac{\Omega_R}{2} \beta$$

$$\frac{-\Delta \gamma + 2\gamma \lambda}{2} = \frac{\Omega_R}{2} a \quad \text{και} \quad \frac{-\Delta \delta - 2\delta \lambda}{2} = \frac{\Omega_R}{2} \beta$$

$$\boxed{a = \frac{-\Delta + 2\lambda}{\Omega_R} \gamma}$$

$$\text{και} \quad \boxed{\beta = \frac{-\Delta - 2\lambda}{\Omega_R} \delta}$$

$$\gamma = \frac{(2\lambda + \Delta)}{\Omega_R} \cdot \frac{(2\lambda - \Delta)}{\Omega_R} \gamma \Rightarrow \frac{4\lambda^2 - \Delta^2}{\Omega_R^2} = 1 \quad \text{το οποίο ισχύει}$$

$$\delta = \frac{(\Delta - 2\lambda)}{\Omega_R} \cdot \frac{-(\Delta + 2\lambda)}{\Omega_R} \delta \Rightarrow -\frac{\Delta^2 - 4\lambda^2}{\Omega_R^2} = 1 \quad \text{το οποίο ισχύει}$$

$$\begin{cases} C_1(t) = e^{i \frac{\Delta}{2} t} [a e^{i \lambda t} + \beta e^{-i \lambda t}] \\ C_2(t) = e^{-i \frac{\Delta}{2} t} \left[ a \frac{\Delta + 2\lambda}{\Omega_R} e^{i \lambda t} + \frac{\Delta - 2\lambda}{\Omega_R} \beta e^{-i \lambda t} \right] \end{cases}$$

"Εξίσωση of άρχικες συνθήκες (Α.Σ)  $C_1(0)=1$ ,  $C_2(0)=0$

(4)

$$1 = \alpha + \beta$$

$$0 = \alpha \frac{\Delta + 2\lambda}{\Omega_R} + \beta \frac{\Delta - 2\lambda}{\Omega_R} \Rightarrow \alpha(\Delta + 2\lambda) = \beta(2\lambda - \Delta) \Rightarrow \left. \begin{array}{l} 1 = \alpha + \beta \\ 0 = \alpha \frac{\Delta + 2\lambda}{\Omega_R} + \beta \frac{\Delta - 2\lambda}{\Omega_R} \end{array} \right\} \Rightarrow$$

$$\beta = \frac{2\lambda + \Delta}{2\lambda - \Delta} \alpha$$

$$1 = \alpha + \frac{2\lambda + \Delta}{2\lambda - \Delta} \alpha = \alpha \frac{2\lambda - \Delta + 2\lambda + \Delta}{2\lambda - \Delta} \Rightarrow (2\lambda - \Delta) = \alpha 4\lambda$$

$$\Rightarrow \boxed{\alpha = \frac{2\lambda - \Delta}{4\lambda}} \quad \beta = \frac{2\lambda + \Delta}{2\lambda - \Delta} \frac{2\lambda - \Delta}{4\lambda} \Rightarrow \boxed{\beta = \frac{2\lambda + \Delta}{4\lambda}}$$

$$C_1(t) = e^{i\frac{\Delta}{2}t} \left[ \frac{2\lambda - \Delta}{4\lambda} e^{i\lambda t} + \frac{2\lambda + \Delta}{4\lambda} e^{-i\lambda t} \right]$$

$$C_2(t) = e^{-i\frac{\Delta}{2}t} \left[ \frac{\Delta + 2\lambda}{\Omega_R} \frac{2\lambda - \Delta}{4\lambda} e^{i\lambda t} + \frac{\Delta - 2\lambda}{\Omega_R} \frac{2\lambda + \Delta}{4\lambda} e^{-i\lambda t} \right]$$

$$\frac{4\lambda^2 - \Delta^2}{4\Omega_R \lambda} = \frac{\Delta^2 + \Omega_R^2 - \Delta^2}{4\Omega_R \lambda} = \frac{\Omega_R}{4\lambda}$$

$$\frac{\Delta^2 - 4\lambda^2}{4\Omega_R \lambda} = \frac{\Delta^2 - \Delta^2 - \Omega_R^2}{4\Omega_R \lambda} = \frac{-\Omega_R}{4\lambda}$$

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$$C_2(t) = e^{-i\frac{\Delta}{2}t} \left[ \frac{\Omega_R}{4\lambda} e^{i\lambda t} - \frac{\Omega_R}{4\lambda} e^{-i\lambda t} \right] \Rightarrow C_2(t) = e^{-i\frac{\Delta}{2}t} \frac{\Omega_R}{4\lambda} 2i \sin(\lambda t)$$

'Αρα  $P_2(t) = |C_2(t)|^2 = \frac{\Omega_R^2}{\Delta^2 + \Omega_R^2} \sin^2(\lambda t)$

$$P_1(t) = |C_1(t)|^2 = 1 - \frac{\Omega_R^2}{\Omega_R^2 + \Delta^2} \sin^2(\lambda t)$$