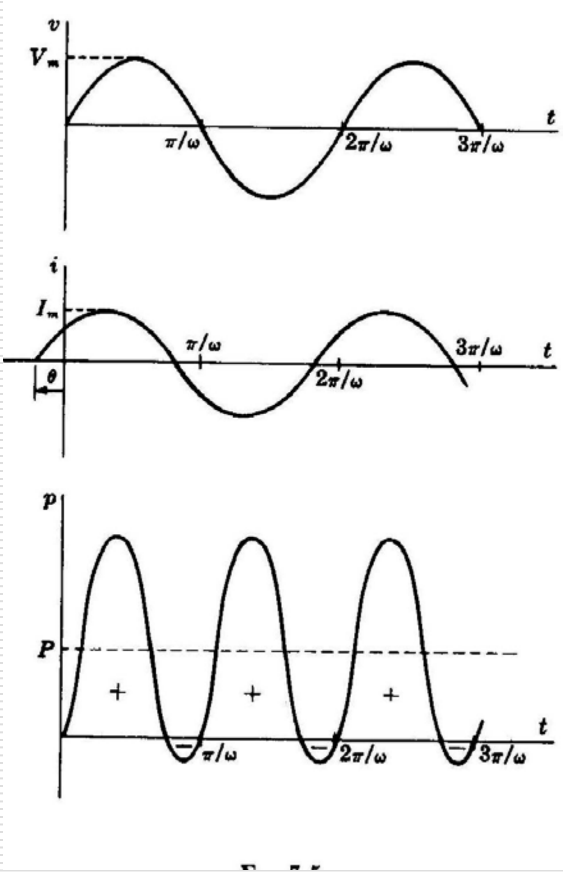
Ισχύς στη γενική περίπτωση

$$v = V_0 \sin \omega t \qquad i = I_0 \sin(\omega t - \theta)$$

$$p = vi = I_0 V_0 \sin \omega t \sin(\omega t - \theta)$$

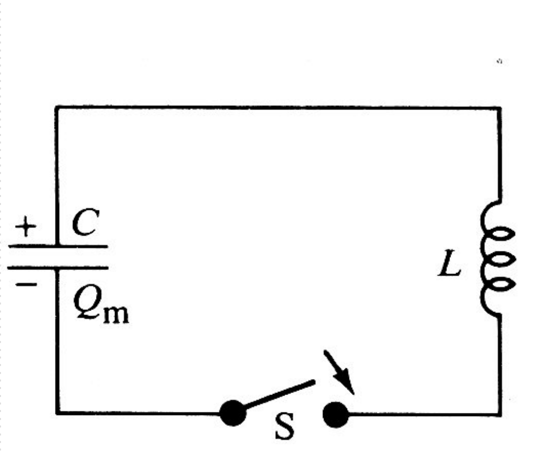
$$p = \tfrac{1}{2} I_0 V_0 (\cos\theta - \cos(2\omega t - \theta))$$

$$P = -\tfrac{1}{2} I_0 V_0 \cos\theta$$



$$P = I_{\varepsilon\nu} V_{\varepsilon\nu} \cos\theta$$

Ταλάντωση L, C



$$v_C + v_L = 0$$

$$v_C = \frac{q}{C}$$

$$v_L = L\frac{di}{dt} = L\frac{d^2 q}{dt^2}$$

$$L\frac{d^2 q}{dt^2} + \frac{q}{C} = 0 \rightarrow$$

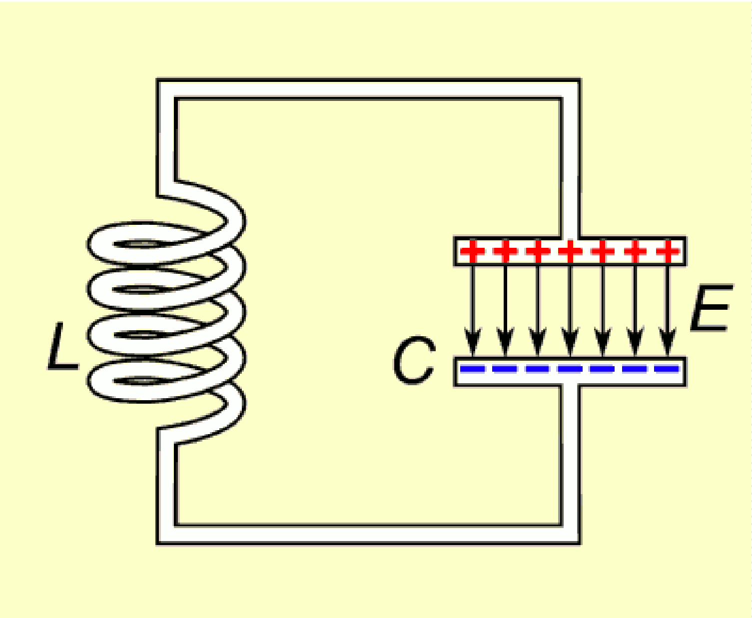
$$\frac{d^2 q}{dt^2} + \frac{q}{LC} = 0 \rightarrow \omega_0 = \sqrt{LC}$$

Η εξίσωση είναι εξίσωση ταλάντωσης με συχνότητα ω_0. $q = q_0 \cos(\omega_0 t + \varphi)$

Το ω_0 ονομάζεται ιδιοσυχνότητα του κυκλώματος

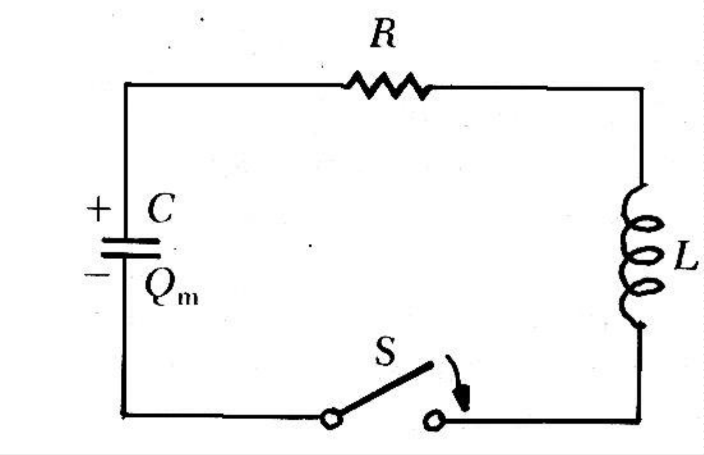
Ταλάντωση L, C

Θεωρούμε ότι το πηνίο έχει μηδενική και ο πυκνωτής άπειρη αντίσταση.



Η ενέργεια του ηλεκτρικού πεδίου μετατρέπεται σε ενέργεια μαγνητικού πεδίου και αντίστροφα.

Ταλάντωση R,L,C σε σειρά.



$$v_R + v_L + v_C = 0$$

$$v_R = Ri = R\frac{dq}{dt}$$

$$v_C = \frac{q}{C}$$

$$v_L = L\frac{di}{dt} = L\frac{d^2q}{dt^2}$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0 \rightarrow$$

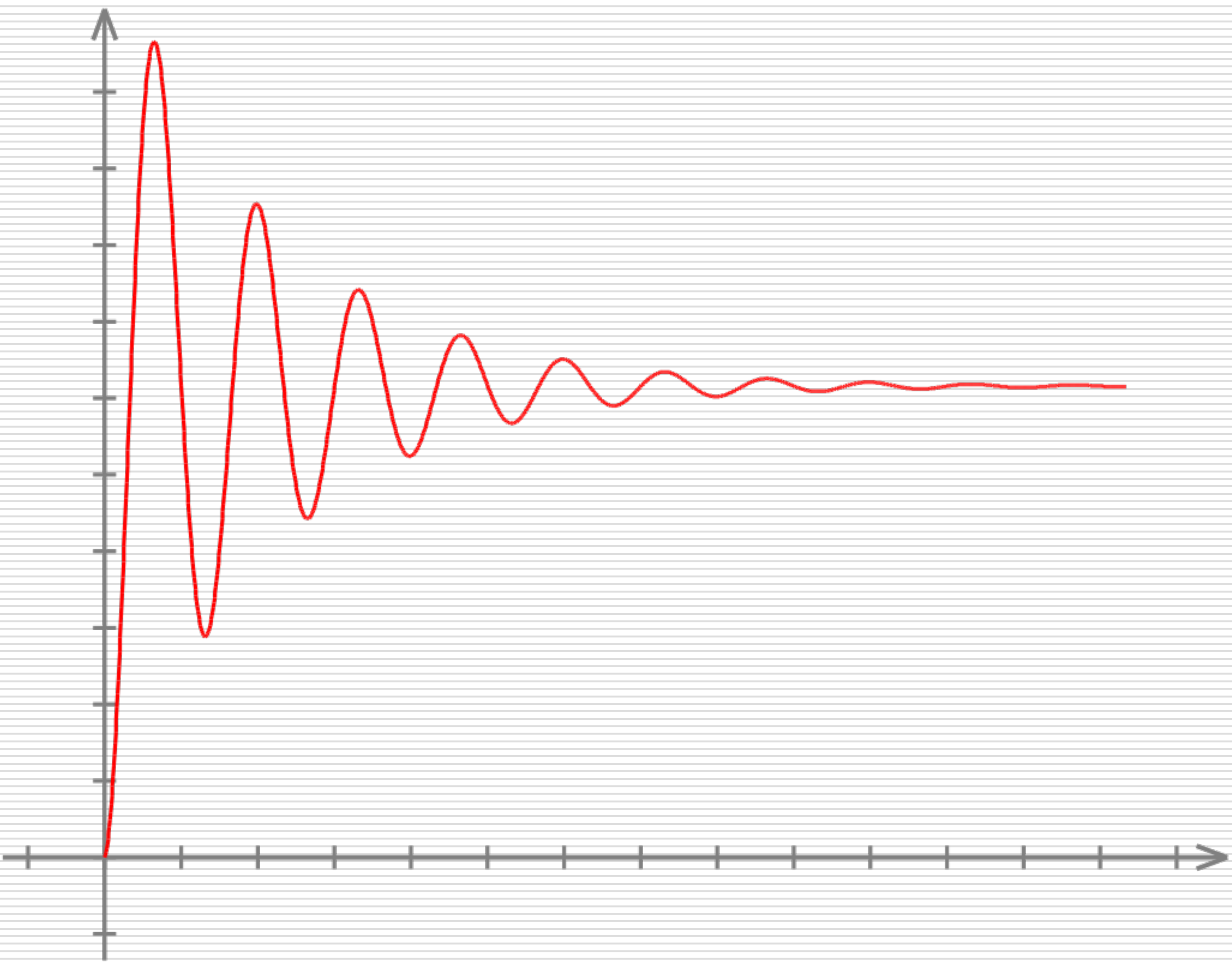
$$\frac{d^2q}{dt^2} + \frac{R}{LC}\frac{dq}{dt} + \frac{q}{LC} = 0$$

$$\omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

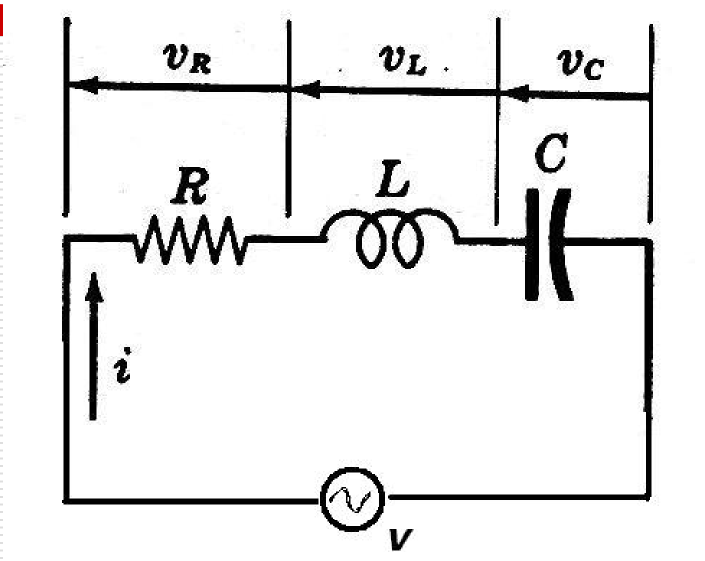
$$\omega = \sqrt{\omega_0 - \left(\frac{R}{2L}\right)^2}$$

$$q = q_0 e^{-\frac{R}{2L}t} \cos(\omega t + \varphi)$$

Μεταβατική κατάσταση



Συντονισμός για κύκλωμα R-L-C σε σειρά.



$$v = v_R + v_L + v_C \qquad v = V_0 \sin \omega t$$

$$I_{\varepsilon\nu} = V_{\varepsilon\nu} / Z$$

$$I_{\varepsilon\nu} = \frac{V_{\varepsilon\nu}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$X_L = X_C \Rightarrow \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

$$P = I_{\varepsilon\nu}^2 R = \frac{V_{\varepsilon\nu}^2 R}{Z^2} = \frac{V_{\varepsilon\nu}^2 R}{R^2 + (X_L - X_C)^2}$$

Συντονισμός για κύκλωμα R-L-C σε σειρά

Η ισχύς που παράγεται (ενεργός ισχύς) σαν συνάρτηση της συχνότητας.

$$(X_L - X_C)^2 = (\omega L - \frac{1}{\omega C})^2 = \frac{L^2}{\omega^2}(\omega^2 - \omega_0^2)^2$$

$$P = \frac{V_{\varepsilon\nu}^2 R \omega^2}{R^2 \omega^2 + L^2(\omega^2 - \omega_0^2)^2}$$

$$\omega = \omega_0 \Rightarrow P = \frac{V_{\varepsilon\nu}^2}{R}$$

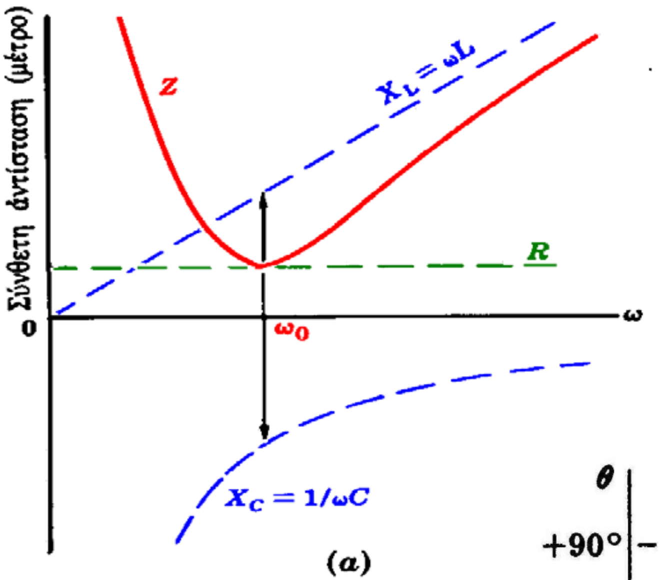
$$\tan\varphi = \frac{X_L - X_C}{R} = \frac{\omega L - 1/\omega C}{R}$$

$$\omega = \omega_0 \rightarrow \varphi = 0$$

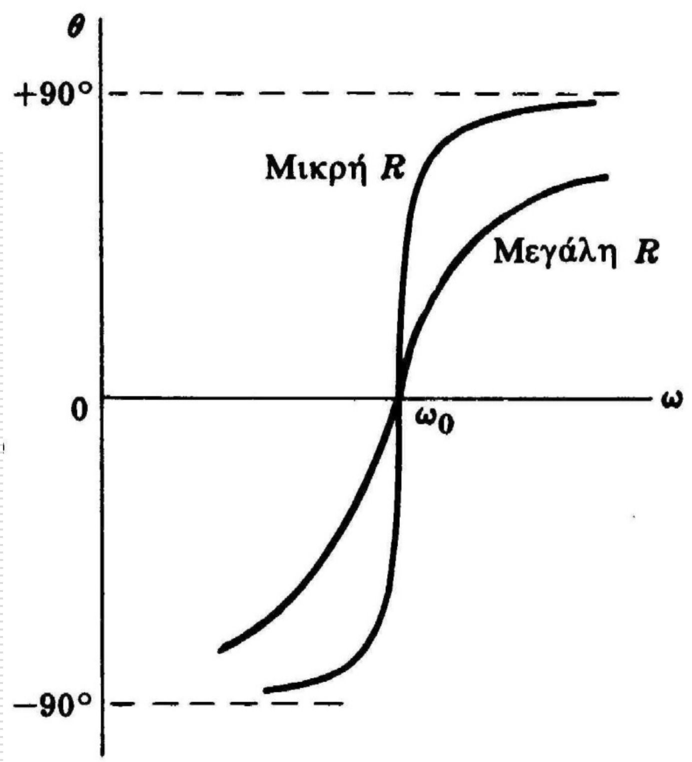
Η διαφορά φάσης μεταξύ τάσης και ρεύματος γίνεται μηδέν.

Η παραγόμενη ισχύς γίνεται μέγιστη.

Συντονισμός σε Κύκλωμα RLC σε σειρά.

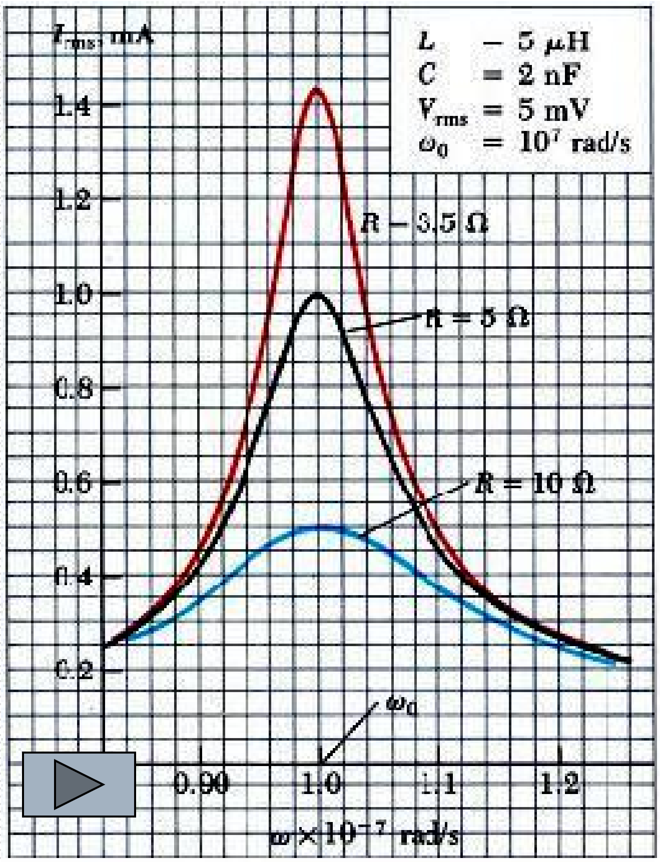


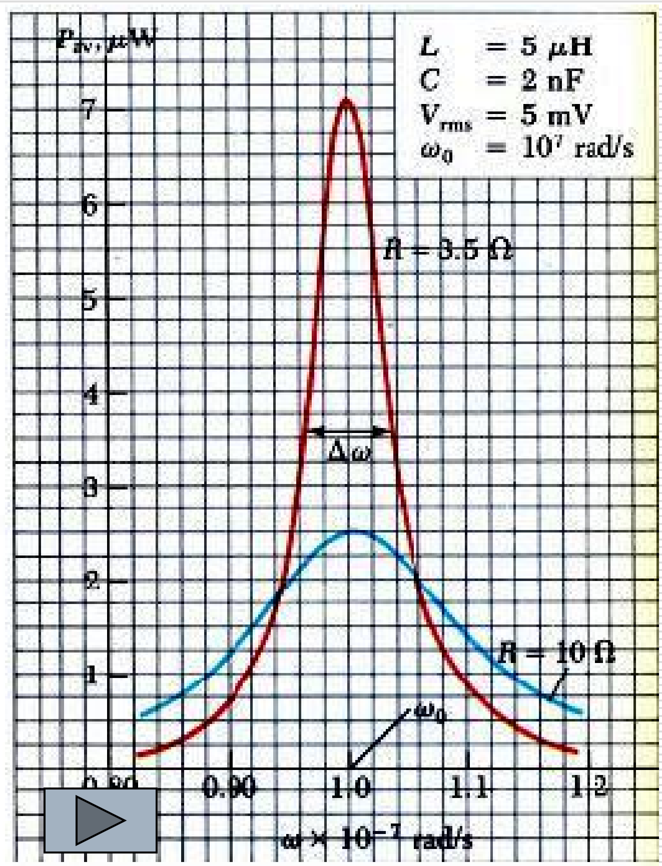
Σύνθετη αντίσταση



Διαφορά φάσης

Καμπύλη συντονισμού, ρεύμα

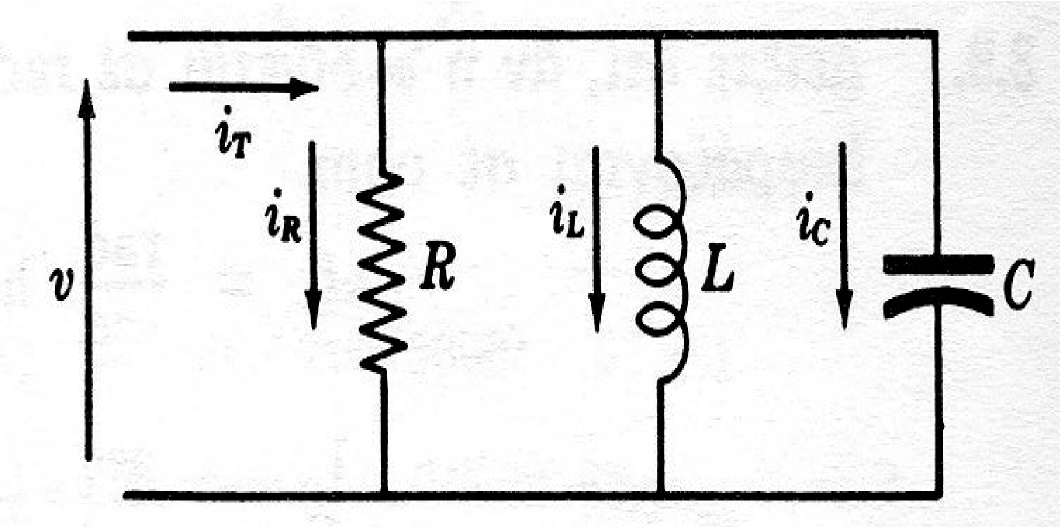




Παράγοντας ποιότητας Q

$$Q = 2\pi \frac{\text{Μέγιστη Αποθηκευμένη Ενέργεια}}{\text{Απώλεια} / T}$$

Συντονισμός σε κύκλωμα RLC Παράλληλα

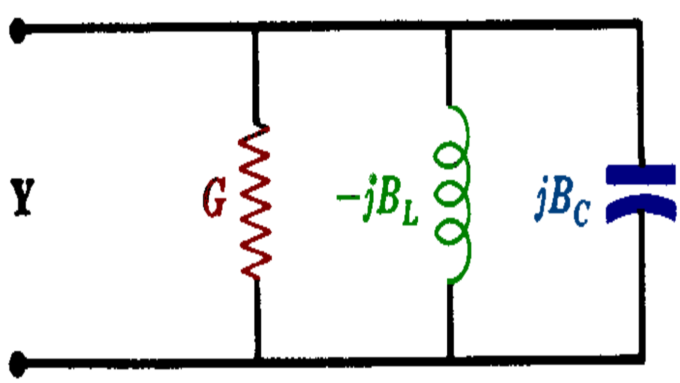


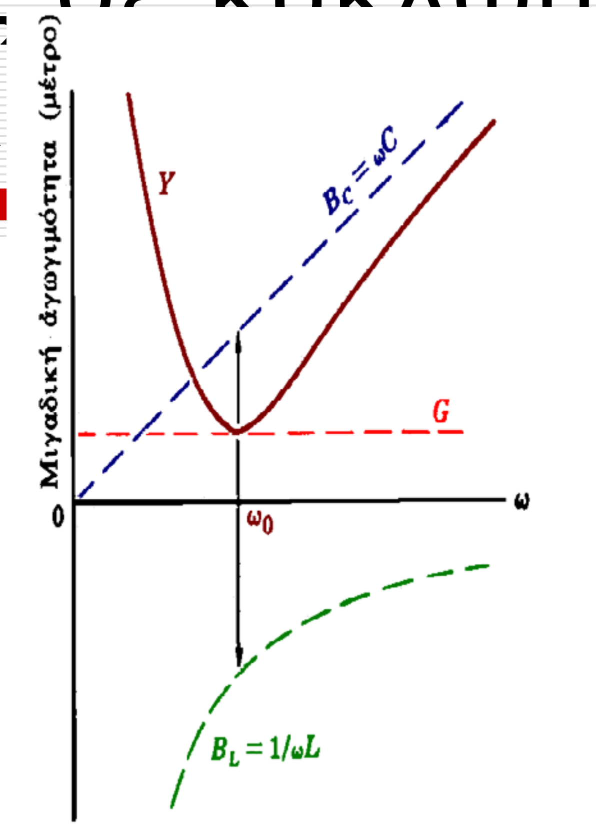
$$i = i_R + i_L + i_C$$

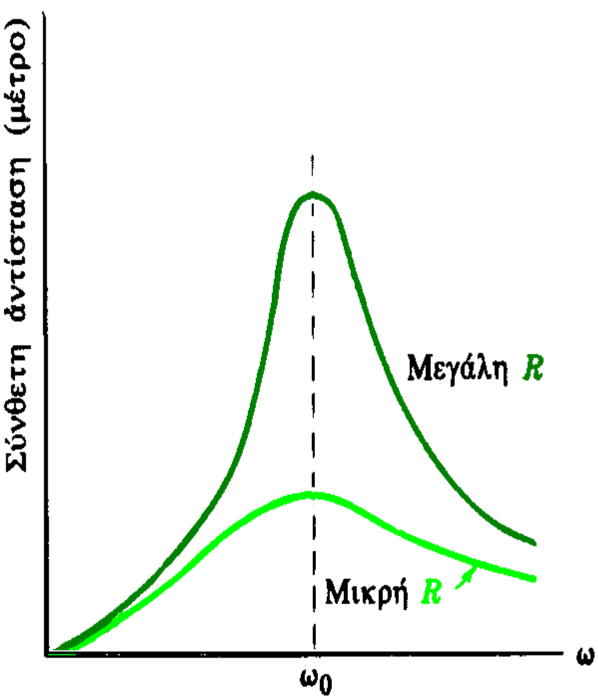
$$\frac{1}{Z} = \sqrt{\frac{1}{R} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)} = \sqrt{\frac{1}{R} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

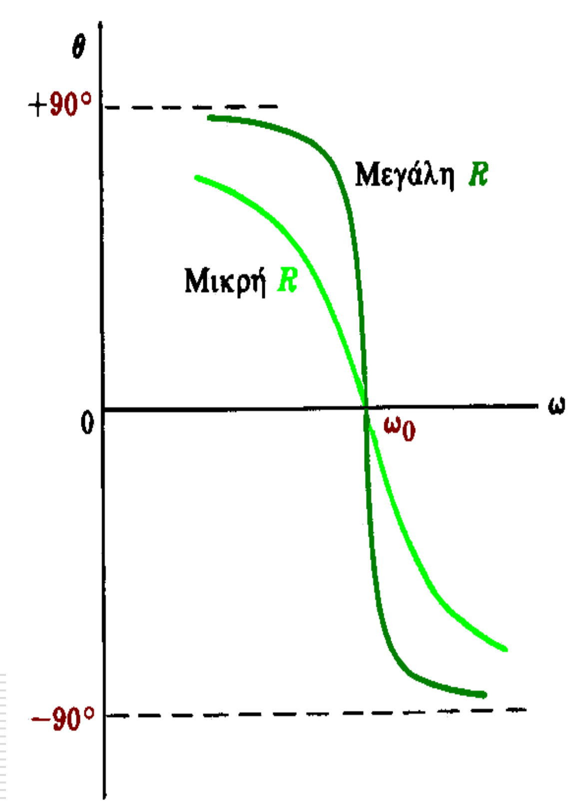
$$\omega_0 = \frac{1}{\sqrt{LC}} \rightarrow Z = R$$

Συντονισμός σε κύκλωμα RLC Παράλλ



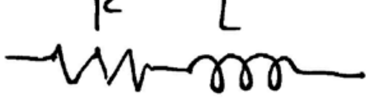






Συντελεστής Ποιότητας

$$Q = 2\pi \frac{\text{Μεγ. Αποθ. Ενέργεια}}{\text{Καταναλ. Ενέργεια/περίοδο}}$$

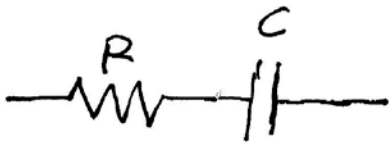


L: $U_{max} = \frac{1}{2} L I_0^2$

R: $W = \frac{1}{2} R I_0^2 \cdot T$

$$Q = 2\pi \frac{\frac{1}{2} L I_0^2}{\frac{1}{2} R I_0^2 T} =$$

$$Q = \frac{\omega L}{R}$$



$$U_C = \frac{1}{2} C V_0^2 = \frac{1}{2} \frac{I_0^2}{\omega^2 C}$$

$$V_0 = I_0 \cdot X_C = \frac{I_0}{\omega C}$$

$$Q = \frac{2\pi \frac{1}{2} I_0^2 / \omega^2 C}{\frac{1}{2} R I_0^2 T} = \frac{1}{\omega C R}$$

Εύρος Ζώνης (BW)

$f_1, f_2 \rightarrow \frac{1}{2} P_{max}$

$f_2 - f_1 = BW \qquad Q = \frac{f_0}{f_2 - f_1} = \frac{f_0}{BW}$

Απόδειξη :

γιατί;

$$P_{max} = \frac{1}{2} I_0^2 R$$
$$P_1 = \frac{1}{2}\left(\frac{1}{2} I_0^2 R\right)$$
$$P_1 = \frac{1}{2} P_{max}$$

$$I_1 = \frac{1}{\sqrt{2}} I_0$$

$$\Rightarrow \frac{1}{\sqrt{2}} \frac{V_0}{R} = \frac{V_0}{Z}$$

$$\Rightarrow \sqrt{2}\, R = \sqrt{R^2 + (X_L - X_C)^2}$$

$$R = X_L - X_C$$

① Χωρητική : $\frac{1}{\omega_1 C} - \omega_1 L = R$

② Επαγωγική $\omega_2 L - \frac{1}{\omega_2 C} = R$

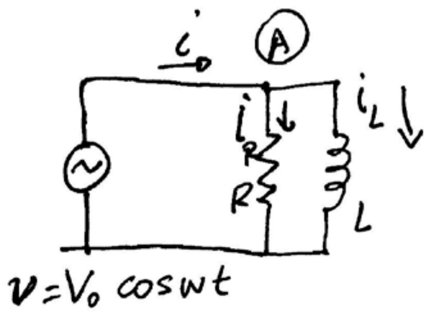
$$\omega_1 = \frac{-R + \sqrt{R^2 + 4L/C}}{2L}$$
$$\omega_2 = \frac{R + \sqrt{R^2 + 4L/C}}{2L}$$

$$2\pi \cdot BW = \omega_2 - \omega_1 = \frac{R}{L}$$

$$BW = \frac{1}{2\pi} \frac{R}{L}$$

$$Q = \frac{f_0}{f_2 - f_1} = \omega_0 \frac{L}{R}$$

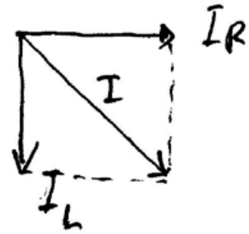
Παράλληλα Στοιχεία



Ⓐ $i = i_R + i_L$

$$i_R = \frac{V_0}{R}\cos\omega t$$

$$i_L = \frac{V_0}{\omega L}\cos\left(\omega t - \frac{\pi}{2}\right)$$

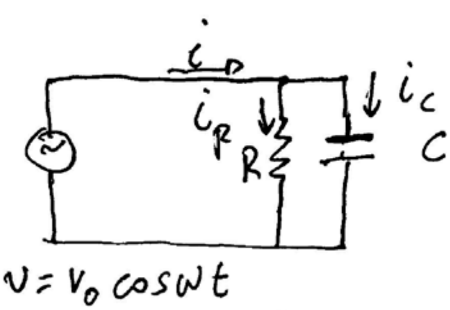


$$I_{0R} = \frac{V_0}{R} \qquad I_{0L} = \frac{V_0}{\omega L}$$

$$I_0 = \sqrt{I_R^2 + I_L^2}$$

$$I_0 = V_0\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L}\right)^2}$$

$$\rightarrow \quad \frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L}\right)^2} \qquad \varphi = \tan^{-1}\frac{R}{\omega L}$$



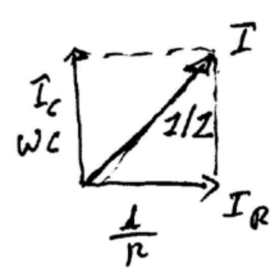
$$i = i_R + i_C$$

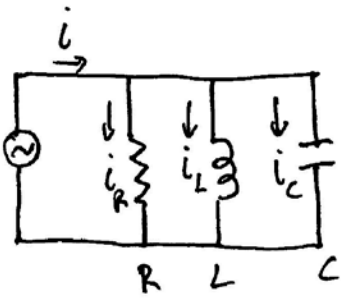
$$i_R = \frac{V_0}{R}\cos\omega t$$

$$i_C = \omega C V_0 \cos\left(\omega t + \frac{\pi}{2}\right)$$

$$\frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + (\omega C)^2}$$

$$\varphi = \tan^{-1}\frac{\omega C}{R}$$





$$i = i_R + i_L + i_C$$

Πλάτη:

$$I_R = \frac{V_0}{R}$$

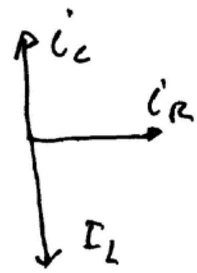
$$I_L = \frac{V_0}{\omega L} \qquad I_C = \omega C V_0$$

$$v = V_0 \cos\omega t$$

$$i_R = \frac{V_0}{R}\cos\omega t$$

$$i_L = \frac{V_0}{\omega L}\cos\left(\omega t - \frac{\pi}{2}\right)$$

$$i_C = \omega C V_0 \cos\left(\omega t + \frac{\pi}{2}\right)$$



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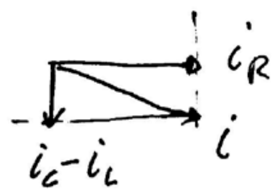

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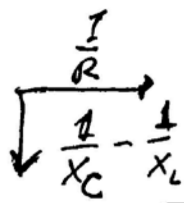
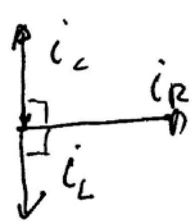


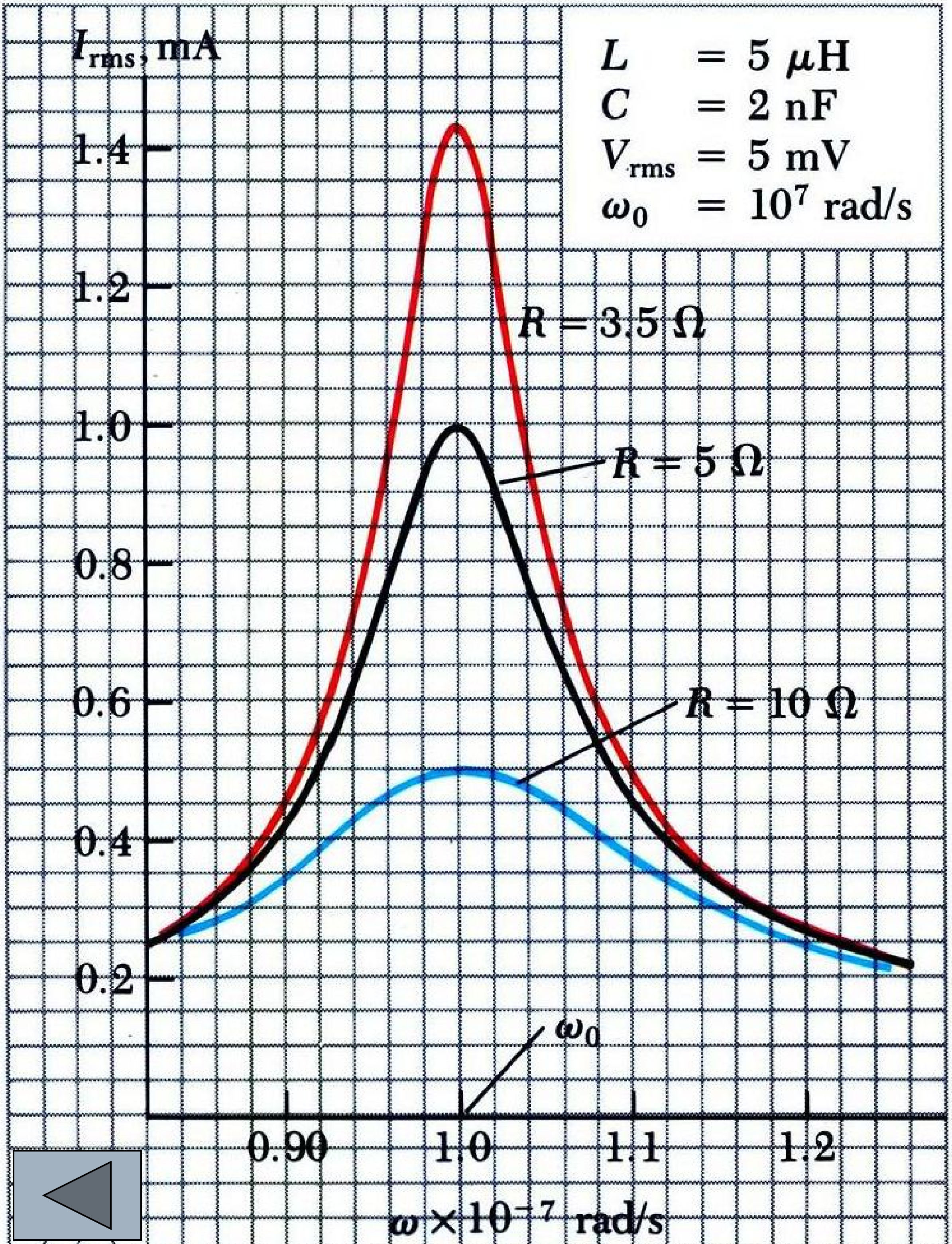
$$\frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}$$

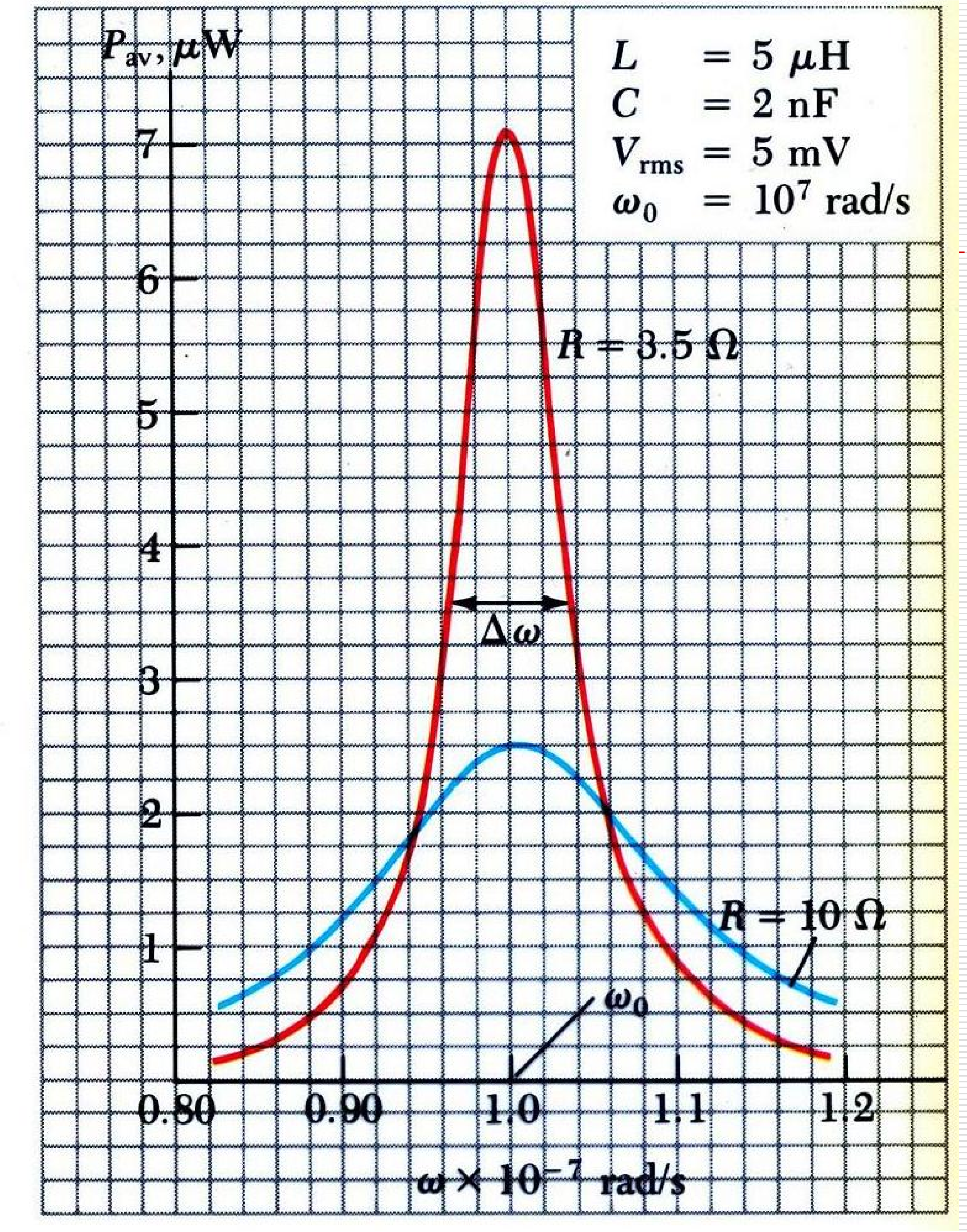
$$\rightarrow \frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L} - \omega C\right)^2}$$

$$\tan\varphi = \frac{\omega C - \frac{1}{\omega L}}{1/R}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \rightarrow Z = R$$




I_{rms}, mA
$L = 5\ \mu H$
$C = 2$ nF
$V_{rms} = 5$ mV
$\omega_0 = 10^7$ rad/s
1.4
1.2
1.0
0.8
0.6
0.4
0.2
$R = 3.5\ \Omega$
$R = 5\ \Omega$
$R = 10\ \Omega$
ω_0
0.90
1.0
1.1
1.2
$\omega \times 10^{-7}$ rad/s



P_{av}, μW
L = 5 μH
C = 2 nF
V_{rms} = 5 mV
ω_0 = 10^7 rad/s
7
6
5
4
3
2
1
R = 3.5 Ω
Δω
R = 10 Ω
ω_0
0.80
0.90
1.0
1.1
1.2
$\omega \times 10^{-7}$ rad/s