ΕΛΛΗΝΙΚΗ ΔΗΜΟΚΡΑΤΙΑ
Εθνικόν και Καποδιστριακόν Πανεπιστήμιον Αθηνών
ΙΔΡΥΘΕΝ ΤΟ 1837

Σχολή Θετικών Επιστημών
Τμήμα Ιστορίας και Φιλοσοφίας της Επιστήμης

Βασίλης Σιώχος
Μαθηματικός
ΜΔΕ Επιστήμη της Πληροφορίας
Υπ. Διδάκτωρ ΙΦΕ

Mathematical Markup Language (MathML)

Version 3.0 2nd Edition

Δεκέμβριος 2020

Σύνδεση με TEI

- Mathematical and chemical formula pose problems similar to those posed by tables in that rendition may be of great significance and hard to disentangle from content.
- Functionality is provided by:
  - OpenMath (https://www.openmath.org/standard/om20-2004-06-30/) and
  - MathML (https://www.w3.org/TR/MathML3/)
    - OMDoc (Open Mathematical Documents) is a format for encoding STEM documents and knowledge (http://omdoc.org/)

OMDoc (http://omdoc.org/)

- An extension of the OpenMath standard that supplies markup for structures such as axioms, theorems, proofs, definitions, texts (mixing formal content with mathematical text).

TEI Element <formula>

- Contains a mathematical or other formula.
- Makes no attempt to represent the internal structure of formula.
- By default, a <formula> is assumed to contain character data which is not validated in any way. The notation used may however be named, using the notation attribute provided by the att.notated class.
  - <formula notation="TeX">$e=mc^2$</formula>

OpenMath vs MathML

- OpenMath and MathML have certain common aspects.
- They both use prefix operators, both are XML-based and they both construct their objects by applying certain rules recursively.
- Such similarities facilitate mapping between the two standards.
- There are also some key differences between MathML and OpenMath.

OpenMath vs MathML

- OpenMath does not provide support for presentation of mathematical objects and its scope of semantically-oriented elements is much broader that of MathML, with the expressive power to cover virtually all areas of computational mathematics.
- In fact, a particular set of Content Dictionaries, the ‘MathML CD Group’, covers the same areas of mathematics as the Content Markup elements of MathML 2.0.

OpenMath vs MathML

OpenMath	MathML
<OMA>	<interval>, <set>, <list>, <matrix>,
	<vector>, <apply>, <lambda>, <reln>.
<OMATTR>	*attributes associated to a tag*
<OMI>, <OMF>	<cn>
<OMV>	<ci>
<OMSTR>	*not supported*
<OMBIND>	*not supported*
not supported	<declare>

Εκδόσεις της MathML

- MathML Version 3.0 2nd Edition is an ISO/IEC International Standard (ISO/IEC 40314:2015)
- 2010
  - MathML 3.0
- 2003
  - MathML 2.0 (2nd ed.)
- 1999
  - MathML 1.01
- 1998
  - MathML 1.0

Στόχοι σχεδιασμού της MathML (1/2)

- Encode mathematical material suitable for all educational and scientific communication.
- Encode both mathematical notation and mathematical meaning.
- Facilitate conversion to and from other mathematical formats, both presentational and semantic. Output formats should include:
  - graphical displays
  - speech synthesizers
  - input for computer algebra systems
  - other mathematics typesetting languages, such as TEX
  - plain text displays, e.g. VT100 emulators
  - international print media, including braille

Στόχοι σχεδιασμού της MathML (1/2)

- Support efficient browsing of lengthy expressions.
- Provide for extensibility.
- Be well suited to templates and other common techniques for editing formulas.
- Be legible to humans, and simple for software to generate and process.

MathML

- MathML is a markup language for describing mathematics.
- It is usually expressed in XML syntax, although HTML and other syntaxes are possible.
- Two main strains of markup:
  - Presentation markup
  - Content markup

MathML Presentation Markup - Attribute Values

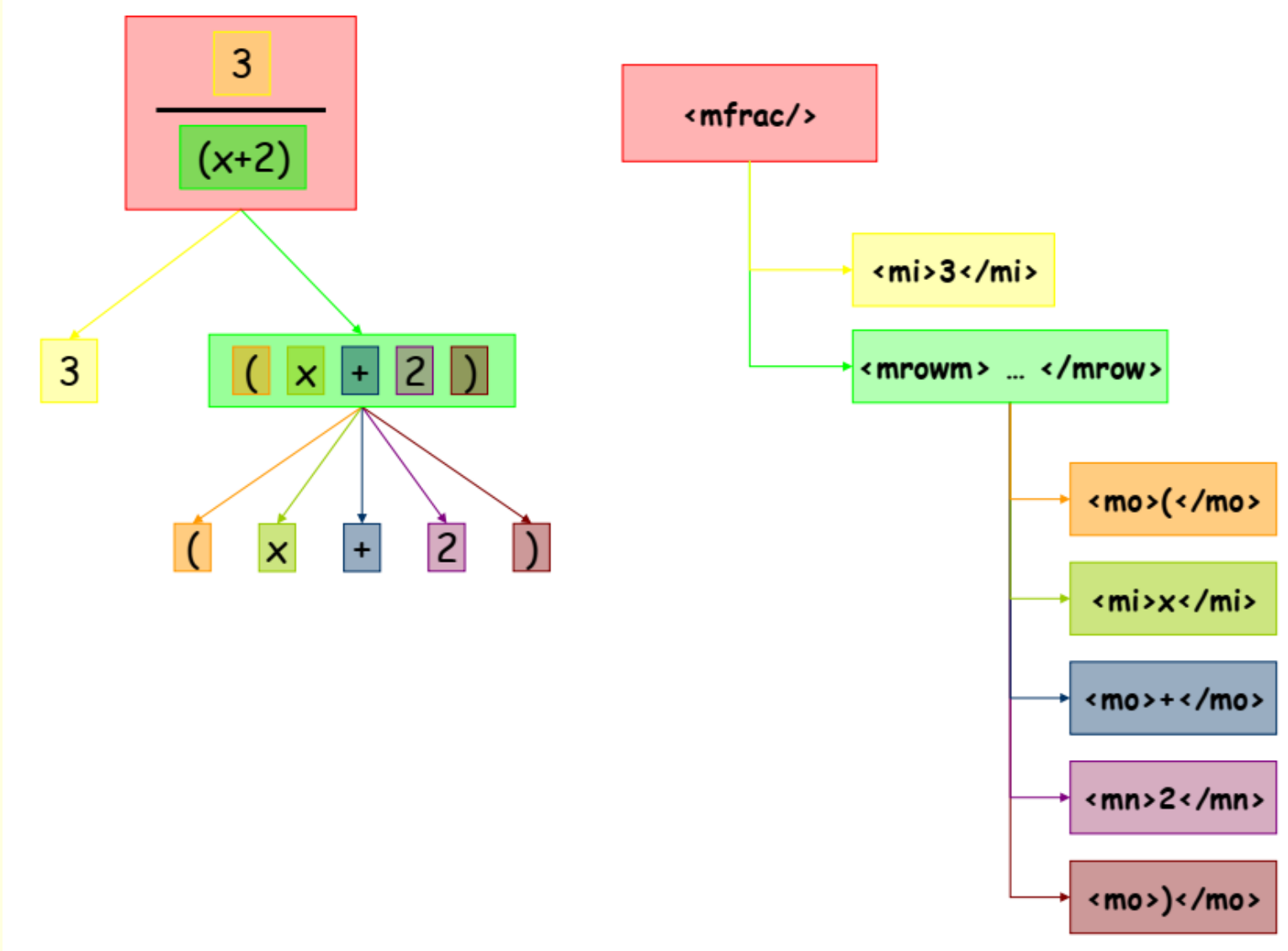
- Syntax notation
  - decimal-digit, hexadecimal-digit, unsigned-integer, positive-integer, integer, unsigned-number, number, character, string, length, unit, namedlength, color, id, idref, URI, italicized word, "literal“
- Length Valued Attributes
  - Number, number unit, namedspace
- Color Valued Attributes
  - #RGB, #RRGGBB, html-color-name
- Default values of attributes
- Attributes Shared by all MathML Elements
  - Id, xref, class, style, href

MathML – Top level Element

- Top-Level <math> Element

```
<math>
    <mrow>
        <mo> ( </mo>
            <mrow>
                <mi> a </mi>
                <mo> + </mo>
                <mi> b </mi>
            </mrow>
        <mo> ) </mo>
    </mrow>
</math>
```

Expression Tree



Token Elements

mi	identifier
mn	number
mo	operator, fence, or separator
mtext	text
mspace	space
ms	string literal

```
1  <mrow>
2    <mo> ( </mo>
3    <mrow>
4      <mi> a </mi>
5      <mo> + </mo>
6      <mi> b </mi>
7    </mrow>
8    <mo> ) </mo>
9  </mrow>
10
```

$$(a+b)$$

General Layout Schemata

mrow	group any number of sub-expressions horizontally
mfrac	form a fraction from two sub-expressions
msqrt	form a square root (radical without an index)
mroot	form a radical with specified index
mstyle	style change
merror	enclose a syntax error message from a preprocessor
mpadded	adjust space around content
mphantom	make content invisible but preserve its size
mfenced	surround content with a pair of fences
menclose	enclose content with a stretching symbol such as a long division sign.

```
<mrow>
  <msup>
    <mfenced open = '(' close = ')'>
      <mrow>
        <mtext>α</mtext>
        <mo>+</mo>
        <mtext>β</mtext>
      </mrow>
    </mfenced>
    <mn>2</mn>
  </msup>
  <mtext> </mtext>
  <mo>=</mo>
  <mtext> </mtext>
  <msup>
    <mtext>α</mtext>
    <mn>2</mn>
  </msup>
  <mtext> </mtext>
  <mo>+</mo>
  <mtext> </mtext>
  <mn>2</mn>
  <mo lspace='thinmathspace' rspace='thinmathspace'>&sdot;</mo>
  <mtext>α</mtext>
  <mo lspace='thinmathspace' rspace='thinmathspace'>&sdot;</mo>
  <mtext>β </mtext>
  <mo>+</mo>
  <msup>
    <mtext>β</mtext>
    <mn>2</mn>
  </msup>
  <mtext> </mtext>
</mrow>
```

$$(\alpha+\beta)^2 = \alpha^2 + 2\cdot\alpha\cdot\beta + \beta^2$$

$$f(x) = (x+1)^4$$
$$= x^4 + 4x^3 + 6x^2$$
$$+ 4x + 1$$

```
<mrow>
 <mrow>  <mi>f</mi> <mo>&#x2061;<!--FUNCTION APPLICATION--></mo> <mo>(</mo> <mi>x</mi> <mo>)</mo>  </mrow>

 <mo id='eq1-equals'>=</mo>
 <mrow>
  <msup>
   <mrow> <mo>(</mo> <mrow> <mi>x</mi> <mo>+</mo> <mn>1</mn> </mrow> <mo>)</mo> </mrow>
   <mn>4</mn>
  </msup>
  <mo linebreak='newline' linebreakstyle='before'
                  indentalign='id' indenttarget='eq1-equals'>=</mo>
  <mrow>
   <msup> <mi>x</mi> <mn>4</mn> </msup>
   <mo id='eq1-plus'>+</mo>
   <mrow>  <mn>4</mn> <mo>&#x2062;<!--INVISIBLE TIMES--></mo> <msup> <mi>x</mi> <mn>3</mn> </msup>  </mrow>
   <mo>+</mo>
   <mrow>  <mn>6</mn> <mo>&#x2062;<!--INVISIBLE TIMES--></mo> <msup> <mi>x</mi> <mn>2</mn> </msup>  </mrow>

   <mo linebreak='newline' linebreakstyle='before'
                   indentalignlast='id' indenttarget='eq1-plus'>+</mo>
   <mrow>  <mn>4</mn> <mo>&#x2062;<!--INVISIBLE TIMES--></mo> <mi>x</mi>  </mrow>
   <mo>+</mo>
   <mn>1</mn>
  </mrow>
 </mrow>
</mrow>
```

Tables and Matrices

mtable	table or matrix
mlabeledtr	row in a table or matrix with a label or equation number
mtr	row in a table or matrix
mtd	one entry in a table or matrix
maligngroup and malignmark	alignment markers

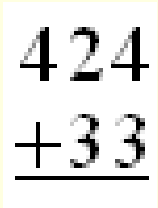
```
<mrow>
  <mo> ( </mo>
  <mtable>
    <mtr>
      <mtd> <mn>1</mn> </mtd>
      <mtd> <mn>0</mn> </mtd>
      <mtd> <mn>0</mn> </mtd>
    </mtr>
    <mtr>
      <mtd> <mn>0</mn> </mtd>
      <mtd> <mn>1</mn> </mtd>
      <mtd> <mn>0</mn> </mtd>
    </mtr>
    <mtr>
      <mtd> <mn>0</mn> </mtd>
      <mtd> <mn>0</mn> </mtd>
      <mtd> <mn>1</mn> </mtd>
    </mtr>
  </mtable>
  <mo> ) </mo>
</mrow>
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

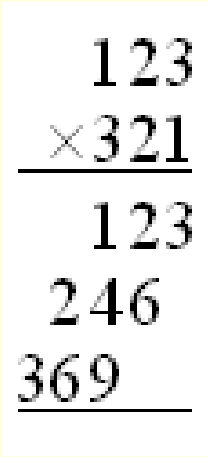
Elementary Math Layout

mstack	columns of aligned characters
mlongdiv	similar to msgroup, with the addition of a divisor and result
msgroup	a group of rows in an mstack that are shifted by similar amounts
msrow	a row in an mstack
mscarries	row in an mstack that whose contents represent carries or borrows
mscarry	one entry in an mscarries
msline	horizontal line inside of `mstack`

```
<mstack>
  <mn>424</mn>
  <msrow> <mo>+</mo> <mn>33</mn> </msrow>
  <msline/>
</mstack>
```

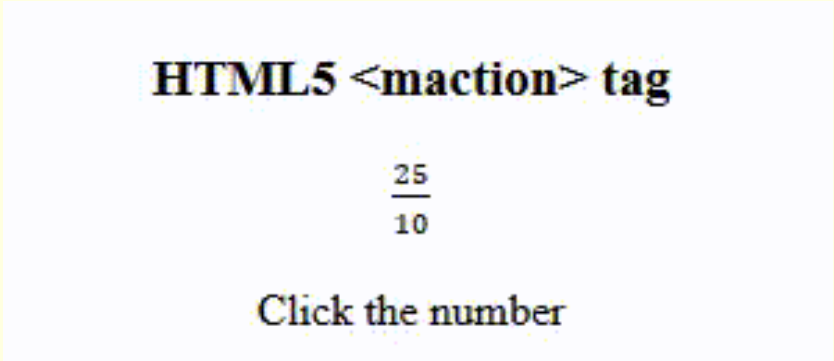



```
<mstack>
  <msgroup>
    <mn>123</mn>
    <msrow><mo>&#xD7;<!--MULTIPLICATION SIGN--></mo><mn>321</mn></msrow>
  </msgroup>
  <msline/>
  <msgroup shift="1">
    <mn>123</mn>
    <mn>246</mn>
    <mn>369</mn>
  </msgroup>
  <msline/>
</mstack>
```



Enlivening Expressions

maction	bind actions to a sub-expression



https://www.geeksforgeeks.org/html5-mathml-maction-tag/

```
<mrow>
  <mi>x</mi>
  <mo>=</mo>
  <mfrac>
    <mrow>
      <mrow>
        <mo>-</mo>
        <mi>b</mi>
      </mrow>
      <mo>&#xB1;<!--PLUS-MINUS SIGN--></mo>
      <msqrt>
        <mrow>
          <msup>
            <mi>b</mi>
            <mn>2</mn>
          </msup>
          <mo>-</mo>
          <mrow>
            <mn>4</mn>
            <mo>&#x2062;<!--INVISIBLE TIMES--></mo>
            <mi>a</mi>
            <mo>&#x2062;<!--INVISIBLE TIMES--></mo>
            <mi>c</mi>
          </mrow>
        </mrow>
      </msqrt>
    </mrow>
    <mrow>
      <mn>2</mn>
      <mo>&#x2062;<!--INVISIBLE TIMES--></mo>
      <mi>a</mi>
    </mrow>
  </mfrac>
</mrow>
```

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Κλάσμα

Τετραγωνική Ρίζα

MathML Content Markup

- The intent of Content Markup is to provide an explicit encoding of the underlying mathematical meaning of an expression.
- Content MathML represents mathematical objects as *expression trees*.
  - *Strict Content MathML is a subset of MAthML, which* uses a minimal, but sufficient, set of elements to represent the meaning of a mathematical expression in a uniform structure,
  - Content MathML grammar is backward compatible with MathML 2.0, and generally tries to strike a more pragmatic balance between verbosity and formality.

Strict Content MathML

- Content MathML provides a large number of predefined functions encoded as empty elements (e.g. sin, log, etc.) and a variety of constructs for forming compound objects (e.g. set, interval, etc.).
- Strict Content MathML uses a single element (csymbol) with an attribute pointing to an external definition in extensible content dictionaries to represent all functions

Strict Content MathML encoding elements

- basic expressions, i.e. Numbers, string literals, encoded bytes, Symbols, and Identifiers.
- derived expressions, i.e. function applications and binding expressions, and
- semantic annotations
- error markup

Numbers <cn>

	Schema Fragment (Strict)		Schema Fragment (Full)	
Class	Cn		Cn	
Attributes	CommonAtt, type		CommonAtt, DefEncAtt, type?, base?	
`type` **Attribute Values**	"integer" | "real" | "double" | "hexdouble"		"integer" | "real" | "double" | "hexdouble" | "e-notation" | "rational" | "complex-cartesian" | "complex-polar" | "constant" | text	default is real
`base` **Attribute Values**			integer	default is 10
Content	text		(text | mglyph | sep | PresentationExpression)*	

- <cn type="hexdouble">7F800000</cn>

Content Identifiers <ci>

- Content identifiers represent "mathematical variables" which have properties, but no fixed value.

	Schema Fragment (Strict)	Schema Fragment (Full)
Class	Ci	Ci
Attributes	CommonAtt, type?	CommonAtt, DefEncAtt, type?
type **Attribute Values**	"integer"| "rational"| "real"| "complex"| "complex-polar"| "complex-cartesian"| "constant"| "function"| "vector"| "list"| "set"| "matrix"	string
Qualifiers		BvarQ, DomainQ, degree, momentabout, logbase
Content	text	text | mglyph | PresentationExpression

Content Identifiers <ci>

```
<ci type="integer">n</ci>
```

```
<semantics>
    <ci>n</ci>
    <annotation-xml cd="mathmltypes" name="type"
    encoding="MathML-Content">
        <csymbol cd="mathmltypes">integer_type</csymbol>
     </annotation-xml>
</semantics>
```

Content Symbols <csymbol>

- A csymbol is used to refer to a specific, mathematically-defined concept with an external definition (OpenMath Society repository of Content Dictionaries (CDs)).

	Schema Fragment (Strict)	Schema Fragment (Full)
Class	Csymbol	Csymbol
Attributes	CommonAtt, cd	CommonAtt, DefEncAtt, type?, cd?
Content	SymbolName	text | mglyph | PresentationExpression
Qualifiers		BvarQ, DomainQ, degree, momentabout, logbase

```
<csymbol type="T">symbolname</csymbol>
```

```
<semantics>
  <csymbol>symbolname</csymbol>
  <annotation-xml cd="mathmltypes" name="type" encoding="MathML-Content">
    <ci>T</ci>
  </annotation-xml>
</semantics>
```

String Literals <cs>

- The cs element encodes "string literals" which may be used in Content MathML expressions.

	Schema Fragment (Strict)	Schema Fragment (Full)
Class	Cs	Cs
Attributes	CommonAtt	CommonAtt, DefEncAtt
Content	text	text


```
<set>
        <cs>A</cs> <cs>B</cs> <cs>  </cs>
</set>
```

Function Application <apply>

- the apply element is used to build an expression tree that represents the application a function or operator to its arguments, specifying exactly the scope of any operator or function.

	Schema Fragment (Strict)	Schema Fragment (Full)
Class	Apply	Apply
Attributes	CommonAtt	CommonAtt, DefEncAtt
Content	ContExp+	ContExp+ | (ContExp, BvarQ, Qualifier?, ContExp*)

(x + y) might be encoded as:

```
<apply><csymbol cd="arith1">plus</csymbol><ci>x</ci><ci>y</ci></apply>
```

Other elements

- Bindings and Bound Variables <bind> and <bvar>
  - The bvar element is used to denote the bound variable of a binding expression, e.g. in sums, products, and quantifiers or user defined functions.
  - Binding expressions are represented as MathML expression trees using the bind element.
- Structure Sharing <share>
  - The share element has an href attribute used to to reference a MathML expression tree.

Other elements

- Attribution via semantics
  - MathML uses the semantics element to wrap the annotated element and the annotation-xml and annotation elements used for representing the annotations themselves.
- Error Markup <cerror>
  - A content error expression is made up of a csymbol followed by a sequence of zero or more MathML expressions.
- Encoded Bytes <cbytes>
  - The content of cbytes represents a stream of bytes as a sequence of characters in Base64 encoding

Content MathML for Specific Operators and Constants

Functions and Inverses
- Interval <interval>
- Inverse <inverse>
- Lambda <lambda>
- Function composition <compose/>
- Identity function <ident/>
- Domain <domain/>
- codomain <codomain/>
- Image <image/>
- Piecewise declaration <piecewise>, <piece>, <otherwise>

Arithmetic, Algebra and Logic
- Quotient <quotient/>
- Factorial <factorial/>
- Division <divide/>
- Maximum <max/>
- Minimum <min/>
- Subtraction <minus/>
- Addition <plus/>
- Exponentiation <power/>
- Remainder <rem/>
- Multiplication <times/>
- Root <root/>
- Greatest common divisor <gcd/>
- And <and/>
- Or <or/>
- Exclusive Or <xor/>
- Not <not/>
- Implies <implies/>
- Universal quantifier <forall/>
- Existential quantifier <exists/>
- Absolute Value <abs/>
- Complex conjugate <conjugate/>
- Argument <arg/>
- Real part <real/>
- Imaginary part <imaginary/>
- Lowest common multiple <lcm/>
- Floor <floor/>
- Ceiling <ceiling/>

Relations
- Equals <eq/>
- Not Equals <neq/>
- Greater than <gt/>
- Less Than <lt/>
- Greater Than or Equal <geq/>
- Less Than or Equal <leq/>
- Equivalent <equivalent/>
- Approximately <approx/>
- Factor Of <factorof/>

Calculus and Vector Calculus
- Integral <int/>
- Differentiation <diff/>
- Partial Differentiation <partialdiff/>
- Divergence <divergence/>
- Gradient <grad/>
- Curl <curl/>
- Laplacian <laplacian/>

Theory of Sets
- Set <set>
- List <list>
- Union <union/>
- Intersect <intersect/>
- Set inclusion <in/>
- Set exclusion <notin/>
- Subset <subset/>
- Proper Subset <prsubset/>
- Not Subset <notsubset/>
- Not Proper Subset <notprsubset/>
- Set Difference <setdiff/>
- Cardinality <card/>
- Cartesian product <cartesianproduct/>

Sequences and Series
- Sum <sum/>
- Product <product/>
- Limits <limit/>
- Tends To <tendsto/>

Content MathML for Specific Operators and Constants

Elementary classical functions

- Common trigonometric functions <sin/>, <cos/>, <tan/>, <sec/>, <csc/>, <cot/>
- Common inverses of trigonometric functions <arcsin/>, <arccos/>, <arctan/>, <arcsec/>, <arccsc/>, <arccot/>
- Common hyperbolic functions <sinh/>, <cosh/>, <tanh/>, <sech/>, <csch/>, <coth/>
- Common inverses of hyperbolic functions <arcsinh/>, <arccosh/>, <arctanh/>, <arcsech/>, <arccsch/>, <arccoth/>
- Exponential <exp/>
- Natural Logarithm <ln/>
- Logarithm <log/> , <logbase>

Statistics

- Mean <mean/>
- Standard Deviation <sdev/>
- Variance <variance/>
- Median <median/>
- Mode <mode/>
- Moment <moment/>, <momentabout>

Linear Algebra

- Vector <vector>
- Matrix <matrix>
- Matrix row <matrixrow>
- Determinant <determinant/>
- Transpose <transpose/>
- Selector <selector/>
- Vector product <vectorproduct/>
- Scalar product <scalarproduct/>
- Outer product <outerproduct/>

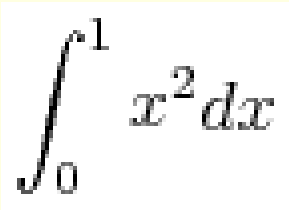
Constant and Symbol Elements

- integers <integers/>
- reals <reals/>
- Rational Numbers <rationals/>
- Natural Numbers <naturalnumbers/>
- complexes <complexes/>
- primes <primes/>
- Exponential e <exponentiale/>
- Imaginary i <imaginaryi/>
- Not A Number <notanumber/>
- True <true/>
- False <false/>
- Empty Set <emptyset/>
- pi <pi/>
- Euler gamma <eulergamma/>
- infinity <infinity/>

Ολοκλήρωμα

- Content MathML

```
<apply>
    <int/> <bvar><ci>x</ci></bvar>
    <lowlimit><cn>0</cn></lowlimit>
    <uplimit><cn>1</cn></uplimit>
    <apply>
        <power/><ci>x</ci><cn>2</cn>
    </apply>
</apply>
```



- Sample Presentation

```
<mrow>
    <msubsup>
        <mi>&#x222b;<!--INTEGRAL--></mi><mn>0</mn><mn>1</mn>
    </msubsup>
    <msup><mi>x</mi><mn>2</mn></msup>
    <mi>d</mi> <mi>x</mi>
</mrow>
```

Annotation Framework

- MathML provides a general framework for annotation in order to represent associations between :
  - presentation and content markup forms for an expression
  - MathML expressions and data of other kinds

- A MathML expression may be decorated with a sequence of pairs made up of a symbol that indicates:
  - the kind of annotation, known as: the *annotation key*, and
  - associated data, known as the: *annotation value*.

Annotation elements

- The semantics, annotation, and annotation-xml elements are used together to represent annotations in MathML.
  - <semantics> element provides the container for a expression and its annotations
  - <annotation> element is the container for text annotations
  - <annotation-xml> element is used for structured annotations

```
<semantics>
        ……(math expression)…
        <annotation encoding="">
                …
        </annotation>
        <annotation-xml encoding="…">
                …
        </annotation-xml>
</semantics>
```

<semantics> elements

- The semantics element is the container element that associates annotations with a MathML expression.

Name	values	default
definitionURL	*URI*	*none*
	The location of an external source for semantic information	
encoding	*string*	*none*
	The encoding of the external semantic information	

```
<semantics>
  <mrow>
    <mrow>
      <mi>sin</mi>
      <mo>&#x2061;<!--FUNCTION APPLICATION--></mo>
      <mfenced><mi>x</mi></mfenced>
    </mrow>
    <mo>+</mo>
    <mn>5</mn>
  </mrow>
  <annotation-xml cd="mathmlkeys" name="contentequiv" encoding="MathML-Content">
    <apply>
      <plus/>
      <apply><sin/><ci>x</ci></apply>
      <cn>5</cn>
    </apply>
  </annotation-xml>
  <annotation encoding="application/x-tex">
    \sin x + 5
  </annotation>
</semantics>
```

<annotation> element

The annotation element is the container element for a semantic annotation whose representation is parsed character data in a non-XML format.

Name	values	default
definitionURL	*URI*	*none*
	The location of the annotation key symbol	
encoding	*string*	*none*
	The encoding of the semantic information in the annotation	
cd	*string*	mathmlkeys
	The content dictionary that contains the annotation key symbol	
name	*string*	alternate-representation
	The name of the annotation key symbol	
src	*URI*	*none*
	The location of an external source for semantic information	

```
<annotation encoding="image/png"
src="333/formula56.png"/>
```

<annotation-xml> element

- Is the container element for a semantic annotation whose representation is structured markup.
- Should contain the markup elements, attributes, and character data for the annotation.

Name	values	default
definitionURL	*URI*	*none*
	The location of the annotation key symbol	
encoding	*string*	*none*
	The encoding of the semantic information in the annotation	
cd	*string*	mathmlkeys
	The content dictionary that contains the annotation key symbol	
name	*string*	alternate-representation
	The name of the annotation key symbol	
src	*URI*	*none*
	The location of an external source for semantic information	

```
<annotation-xml encoding="application/openmath+xml">
 <OMA xmlns="http://www.openmath.org/OpenMath">
  <OMS cd="arith1" name="plus"/>
  <OMA><OMS cd="transc1" name="sin"/><OMV name="x"/></OMA>
  <OMI>5</OMI>
 </OMA>
</annotation-xml>
```

Annotation elements

```
<semantics>
 <mrow>
  <mrow>
   <mi>sin</mi>
   <mo>&#x2061;<!--FUNCTION APPLICATION--></mo>
   <mfenced><mi>x</mi></mfenced>
  </mrow>
  <mo>+</mo>
  <mn>5</mn>
 </mrow>
 <annotation encoding="application/x-tex">
  \sin x + 5
 </annotation>
 <annotation-xml encoding="application/openmath+xml">
  <OMA xmlns="http://www.openmath.org/OpenMath">
   <OMS cd="arith1" name="plus"/>
   <OMA><OMS cd="transc1" name="sin"/><OMV name="x"/></OMA>
   <OMI>5</OMI>
  </OMA>
 </annotation-xml>
</semantics>
```

Annotation keys

- Specify only the logical nature of the relationship between an expression and an annotation
- Are defined as symbols in Content Dictionaries, and are specified using of the cd and name attributes on the *annotation* and *annotation-xml* elements

Alternate representations

Alternate representations

```
<semantics>
  <apply>
    <plus/>
    <apply><sin/><ci>x</ci></apply>
    <cn>5</cn>
  </apply>
  <annotation-xml encoding="MathML-Presentation">
    <mrow>
      <mrow>
        <mi>sin</mi>
        <mo>&#x2061;<!--FUNCTION APPLICATION--></mo>
        <mfenced open="(" close=")"><mi>x</mi></mfenced>
      </mrow>
      <mo>+</mo>
      <mn>5</mn>
    </mrow>
  </annotation-xml>
  <annotation encoding="application/x-maple">
    sin(x) + 5
  </annotation>
  <annotation encoding="application/vnd.wolfram.mathematica">
    Sin[x] + 5
  </annotation>
  <annotation encoding="application/x-tex">
    \sin x + 5
  </annotation>
  <annotation-xml encoding="application/openmath+xml">
    <OMA xmlns="http://www.openmath.org/OpenMath">
      <OMA>
        <OMS cd="arith1" name="plus"/>
        <OMA><OMS cd="transc1" name="sin"/><OMV name="x"/></OMA>
        <OMI>5</OMI>
      </OMA>
    </OMA>
  </annotation-xml>
</semantics>
```

Content equivalents

- the contentequiv annotation key should be used to make an explicit assertion that the annotation provides a definitive content markup equivalent for an expression.

```
<semantics>
  <mrow>
    <mrow>
      <mi>a</mi>
      <mfenced open="(" close=")">
        <mrow><mi>x</mi><mo>+</mo><mn>5</mn></mrow>
      </mfenced>
    </mrow>
  </mrow>
  <annotation-xml cd="mathmlkeys" name="contentequiv"
                  encoding="MathML-Content">
    <apply>
      <ci>a</ci>
      <apply><plus/><ci>x</ci><cn>5</cn></apply>
    </apply>
  </annotation-xml>
</semantics>
```

Annotation references

- A semantics element may contain empty annotation and annotation-xml elements that provide encoding and src attributes to specify an external location for the annotation value associated with the annotation.

```
<semantics>
  <mfrac><mi>a</mi><mrow><mi>a</mi><mo>+</mo><mi>b</mi></mrow></mfrac>
  <annotation encoding="image/png" src="333/formula56.png"/>
  <annotation encoding="application/x-maple" src="333/formula56.ms"/>
</semantics>
```

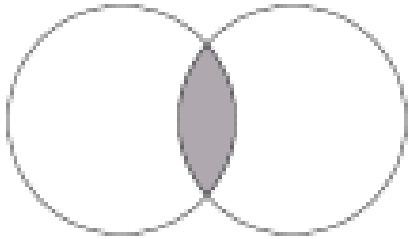
Επικοινωνία με άλλα περιβάλλοντα - html

```
<!DOCTYPE html>
<html lang="en">
 <head>
  <title>The quadratic formula</title>
 </head>
 <body>
  <h1>The quadratic formula</h1>
  <p>
          <math>
           <mi>x</mi>
           <mo>=</mo>
           <mfrac>
            <mrow>
                 <mo form="prefix">−</mo> <mi>b</mi>
                 <mo>±</mo>
                 <msqrt> <msup> <mi>b</mi> <mn>2</mn> </msup>
                 <mo>−</mo> <mn>4</mn> <mo>⁢</mo> <mi>a</mi> <mo>⁢</mo> <mi>c</mi> </msqrt>
            </mrow>
            <mrow> <mn>2</mn> <mo>⁢</mo> <mi>a</mi>  </mrow>
           </mfrac>
          </math>
  </p>
 </body>
</html>
```

MathML and Graphical Markup

- The annotation-xml elements are used to indicate alternative representations of the MathML-Content depiction of the intersection of two sets:
  - SVG format (Scalable Vector Graphics)
  - XHTML img element embedded as an XHTML fragment

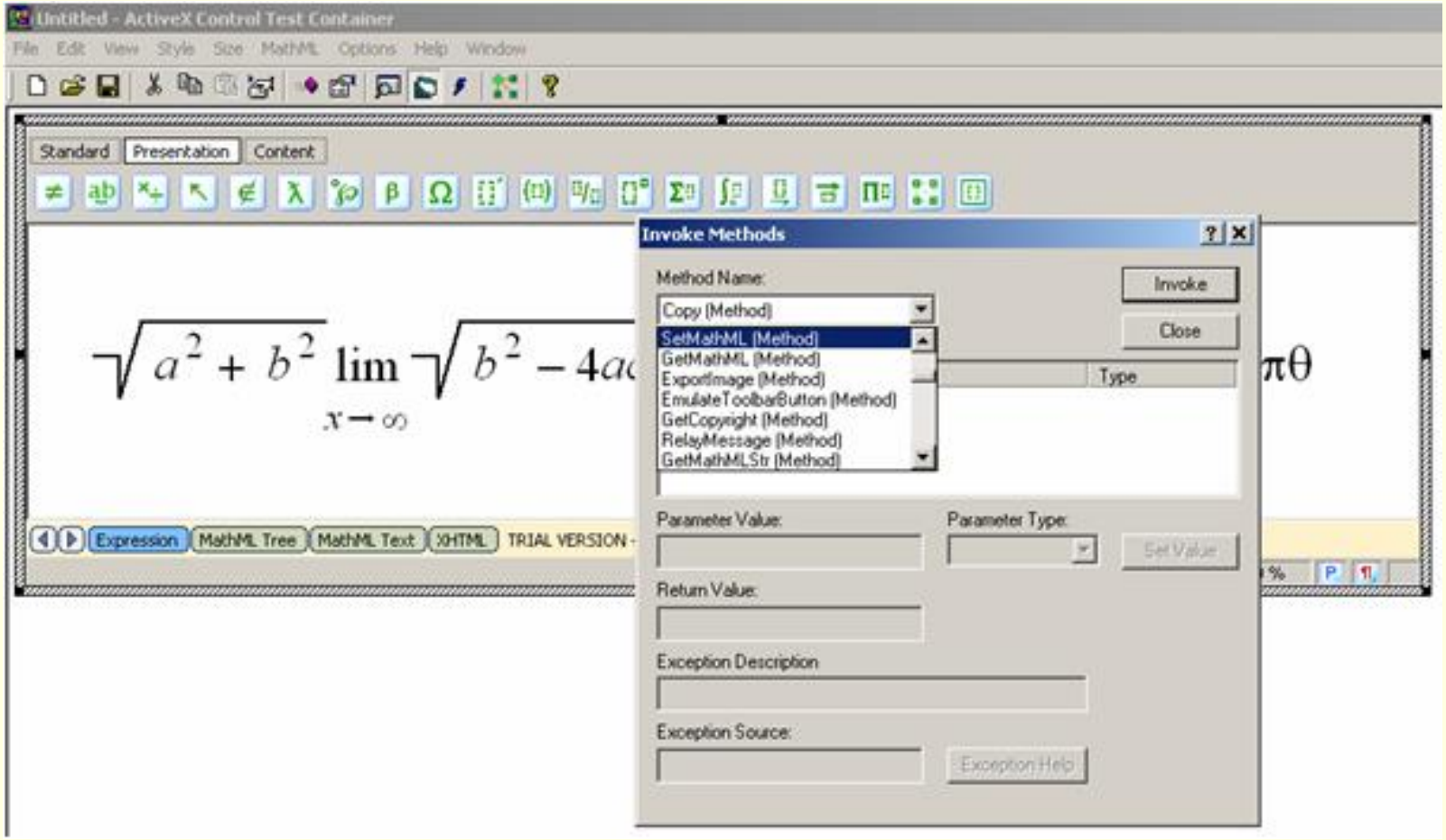
MathML and Graphical Markup



```
<semantics>
  <apply>
    <intersect/>
    <ci>A</ci>
    <ci>B</ci>
  </apply>
  <annotation-xml encoding="image/svg+xml">
    <svg xmlns="http://www.w3.org/2000/svg"  viewBox="0 0 290 180">
      <clipPath id="a">
      <circle cy="90" cx="100" r="60"/>
      </clipPath>
      <circle fill="#AAAAAA" cy="90" cx="190"
              r="60" style="clip-path:url(#a)"/>
      <circle stroke="black" fill="none" cy="90" cx="100" r="60"/>
      <circle stroke="black" fill="none" cy="90" cx="190" r="60"/>
    </svg>
  </annotation-xml>
  <annotation-xml encoding="application/xhtml+xml">
    <img xmlns="http://www.w3.org/1999/xhtml"
         src="intersect.gif" alt="A intersect B"/>
  </annotation-xml>
</semantics>
```

Formulator

- http://www.mmlsoft.com/index.php/products/activex-control



MathML και… Γεωμετρία??

- MathML
  - Λογικοί τελεστές (and, or, xor, not, implies)
  - Ποσοτικοδείκτες (forall, exists)
- Naciri, H., & Rideau, L. (2001). “The Marriage of MathML and Theorem Proving”.

Σύμβολα Γεωμετρίας στην MathML…

MathML Symbol	HTML Entity	Hex Code	Description
°	°	°	To specify degrees
∠	∠	∠	To specify angle
∡	∡	∡	To specify measured angle
∟	∟	∟	To specify right angle
⦜	⦜	⦜	To specify right angle with square
◿	⊿	⊿	To specify right triangle
○	○	○	To specify circle
△	△	△	To specify triangle
□	□	□	To specify square
▱	▱	▱	To specify parallelogram
∥	∥	∥	To specify parallel
∦	∦	∦	To specify not parallel
⊥	⊥	⊥	To specify perpendicular
≅	≅	≅	To specify congruent
→	→	→	To specify ray (used with <mover>)
↔	↔	↔	To specify line (used with <mover>)
-	(n/a)	-	To specify line segment (used with <mover>)

Βιβλιογραφία

- Ausbrooks, R., Buswell, S., Carlisle, D., & Chavchanidze, G. (2014). Mathematical Markup Language (MathML) Version 3.0 2nd Edition. W3C Recommendation 10 April 2014. World Wide Web Consortium (W3C).
- Mathematical Markup Language (MathML) Version 3.0 2nd Edition, (2014), https://www.w3.org/TR/MathML3/ .
- The OpenMath Standard, (2000), http://www.openmath.org.
- Kohlhase, M., & Rabe, F. (2012). Semantics of OpenMath and MathML 3. Mathematics in Computer Science, 6(3), 235-260.
- Fateman, R. (2001). A Critique of OpenMath and Thoughts on Encoding Mathematics, January, 2001. University of California, Berkeley.
- Kohlhase, M. (2009). An open markup format for mathematical documents. *International University of Bremen*.