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Book Review

A New History of Greek Mathematics

By Reviel Netz. Cambridge (Cambridge University Press). 2022

A good few months ago, I was tasked with preparing a review for the *British Journal for the History of Science*, focusing on the first volume of *The Cambridge History of Science*. In that review, I penned the following lines: "One of the major challenges that contemporary historians face when called upon to teach an introductory course on Ancient Science is the absence of a collective, updated, and authoritative work of reference…".[1] As one might anticipate, those who find themselves teaching an introductory course on ancient Greek mathematics encounter a problem quite akin to the aforementioned one. This situation arises, in part, due to the outdated nature of the widely popular *A History of Greek Mathematics*, authored by Sir Thomas L. Heath across two volumes in 1921.[2] Despite its decades-long usefulness, Heath's work falls short as a suitable student handbook. The book I am currently reviewing, Reviel Netz's *A New History of Greek Mathematics*, aims to address precisely this issue. Whether this endeavor proves successful remains to be seen.

Let me first provide some insights into the subject of the book, Greek mathematics. Why should non-specialists find interest in such a topic? Well, compared to other ancient mathematical cultures, an intriguing aspect of Greek mathematics is its gradual detachment from the practical requirements of everyday life, customary state administration, and even basic classroom training.[3] By the fifth century BCE, various fields of knowledge, including astronomy, mechanics, and harmonic theory, underwent mathematization; and, as a result of this process, new communities emerged. These groups fostered interaction networks, formulated their own specialized vocabulary, introduced novel theories, and crafted specialized tools. Over a period exceeding twelve centuries, Greek mathematicians generated countless original results, either as individual propositions (e.g., that 'there exist only five regular solids' (Euclid, *Elements* XIII.18) and how 'to square a parabola' (Archimedes, *Quadrature of the Parabola*)) or as entirely fresh theories (e.g., the theory of proportions and irrational magnitudes; the theory of conic sections) and original methods (e.g., the method of analysis and the method of premodern algebra). The revolutionary character of Greek mathematics is most evident on a methodological level, primarily due to the establishment of the axiomatic-and-deductive reasoning; i.e., the concept of formal mathematical proof. This innovation draws a clear demarcation between Greek and non-Greek mathematical traditions, exerting an unparalleled influence on subsequent mathematical development. For these compelling reasons, Greek mathematics presents a captivating subject worthy of exploration not only by specialists, but also by students and the erudite general public.

Let us now turn our attention to the structural features of the book under review. Netz's *A New History of Greek Mathematics* consists of seven substantial chapters (not arranged in strict chronological order), a

[1] Sialaros, Michalis (2021), review of: Jones, Alexander and Taub, Liba (eds.) (2018), *The Cambridge History of Science: Volume 1, Ancient Science* (Cambridge: Cambridge University Press), *British Journal for the History of Science* 54 (1), 124-125.

[2] Heath, Thomas L. (1921), *A History of Greek Mathematics*, 2 vols. (Oxford: Clarendon Press).

[3] Of course, practical mathematics was always in circulation in the Greek world, at least since the 15th century BCE (Linear B).

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ten-page epilogue, and a basic index. While readers should not anticipate an abundance of bibliographical references, each chapter does conclude with an up-to-date list of suggestions for further reading. It is worth noting that the book features eight exquisite images, grouped together between pages 272 and 273; however, incorporating a greater number of visuals and strategically placing them within pertinent sections would undeniably have enhanced the book's visual appeal for students. As a final point, the book's cover features the individual commonly believed to depict Euclid in Raphael's "School of Athens"—a cliché choice for my taste.

The first three chapters delve into the evolution of Greek mathematics, placing particular emphasis on geometry, from its origins to the culmination of the Hellenistic era. Chapter 1, titled: "To the Threshold of Greek Mathematics", provides a contextualized depiction of the genesis of Greek mathematics. Netz asserts that the most important antecedent to the Greeks can be traced back to ancient Mesopotamia, where mathematics developed as an ingenious, yet uncomplicated pursuit intertwined with the training of state bureaucrats. According to Netz's account, Greek mathematics was not the result of gradual evolution but of rapid development which took place around the fifth century BCE. Within this framework, Hippocrates of Chios assumes a pivotal role, while the mathematical accomplishments often ascribed to Thales and Pythagoras are dismissed without much discussion. The subsequent two chapters, titled: "The Generation of Archytas" and "The Generation of Archimedes" respectively, divide the efforts of classical Greek mathematics into two generations: the generation of Archytas, characterized by an ongoing dialogue between philosophy and mathematics; and the generation of Archimedes, during which mathematics evolves into a more autonomous discipline. It is hardly surprising that Archimedes takes center stage in this narrative, not only because he deserves it but also because Netz has extensively published on this subject in the past. The absence of Euclid's name in the chapter titles is a knife in the heart—but this is most probably my own personal bias speaking. Chapter 4, titled "Mathematics in the World", and Chapter 5, titled "Mathematics of the Stars", explore the process of mathematization that occurred within the realms of mechanics and astronomy respectively. In contrast to the preceding chapters, Netz's narrative here departs from a strict chronological order. As a result, these two chapters can be approached as independent introductions to their respective subjects. Notably, Chapter 5 provides a compelling overview of the history of astronomy, spanning from its beginnings to the publication of the most famous astronomical book of all times, Ptolemy's *Almagest*—a complex endeavor that can only be fully appreciated by specialists. Chapter 6, titled "The Canonization of Greek Mathematics", presents Greek mathematical sciences in Late Antiquity, underscoring the renewed interconnectedness of mathematics and Neoplatonic philosophy. Lastly, Chapter 7, titled "Into Modern Science: The Legacy of Greek mathematics", traces the transmission of Greek mathematical knowledge across the Byzantine Empire, the Islamic World, and the Latin West. Within this chapter, Netz presents the idea that the emergence of modern science was not marked by a scientific revolution, but rather a scientific renaissance inspired by ancient Greek mathematics.

This is a fine moment to pose the million-dollar question: "What is 'new' in Netz's *New History of Greek Mathematics*?" In a nutshell, the answer is that this book offers a simple (yet far from simplistic), engaging narrative, exceptionally well-suited for students. Furthermore, the book offers comprehensive historical and historiographical context. Both choices come at a cost. Let us have a more detailed look at these points.

According to Netz, his goal was to produce "a single narrative account, of use for the general interested public, as well as for undergraduate classes and for those graduate students and scholars looking for some entry point into the historical foundations of science" (p.xii). Almost inherently, this decision involves a trade-off: technical analysis. If you seek extensive proofs, intricate reconstructions, or sophisticated chains of deductive reasoning, I would suggest seeking other sources. This does not imply that Netz completely omits detailed examples; however, they are limited in number. Still, it is worth highlighting that when Netz does present such discussions, he does so in a manner that could prove valuable in an educational setting. To be specific, I find his approach to presenting Hippocrates of Chios's method for squaring the lunes (p. 37-47), Archytas's solution to the problem of doubling the cube (p. 65), and the discussion on Apollo-

nius' conics (p. 161-176) to be charmingly paradigmatic. And it is precisely Netz's demonstrated skill in presenting intricate subjects with simplicity that makes the lack of technical discourse more noticeable.

The second concern with the author's decision for "a single narrative account" was not unavoidable; I am referring to a tendency to overlook certain unresolved (and challenging, due to the scarcity of sources) historical questions in favor of simplicity. To provide specific examples, (a) the idea that Thales and Pythagoras engaged in no mathematical pursuits whatsoever, but rather functioned more like a 'Homer' wearing a retrospectively constructed identity (p. 17-32); or (b) the notion that Babylonian mathematics held a more significant role in shaping Greek mathematics than Egyptian mathematics. I am not claiming that Netz is necessarily incorrect in these assertions; but I do believe that these are no easy topics. The complexity of these matters would require a more well-balanced approach, allowing students to recognize that these topics remain open for investigation.

A final concern with the author's narrative approach is that it occasionally involves a level of unnecessary speculation. This range of speculation encompasses ideas that could be classified either as 'grand' (such as that there were likely over a hundred mathematicians from Archytas to Theudius, each possibly averaging no more than two books (p. 58-59)) or as 'minor' (for example, the explanation for the title of Democritus's lost book *On the Contact of Circles and Spheres* (p. 33), the suggestion that Hipparchus probably never read Aristotle's *Physics* (p. 346), or the link between the disappearance of Aristarchus's heliocentric model and Eudoxus's homocentric spheres (p. 312)). These speculations offer little to Netz's greater narrative and, in my opinion, could have been avoided. Be that is it may, tackling the challenge of creating a coherent history of Greek mathematical sciences within just 500 pages is an extremely complex, and thus difficult, task. Netz stands up to the challenge.

Let us now shift our focus to the second innovation in Netz's book, which I have previously described as "historical and historiographical" contextualization. When one reads Heath's work today, it is difficult to avoid the impression that Greek mathematics emerged within a historical, social, and political void, merely constituting a sequence of deductively linked mathematical propositions. To be fair, this is not exclusively Heath's fault. After all, people who wrote histories of mathematics in his time displayed minimal interest in furnishing historical contexts or reflecting on their own methodological approaches and practices. Viewed from this perspective, comparing Netz's *New History of Greek Mathematics* to Heath's *History of Greek Mathematics* seems unfair. Nevertheless, the fact that the two books share almost identical titles invites certain parallels, especially from a methodological perspective.

Firstly, Netz expands the scope of 'mathematics' to align with the Greek understanding of the term 'mathēmata', encompassing what we would today recognize as 'mathematical sciences'. Consequently, he endeavors to bestow special significance to fields such as astronomy and mechanics, extending well beyond pure geometry and arithmetic. In this line of thought, he also incorporates in his study other areas of knowledge, such as the art of warfare, geography, or economics. This choice yields not only a presentation of mathematical concepts but also political and social frameworks for each period of interest (see, for example, the thought-provoking discussion on Athens and Alexandria (p. 123)). Secondly, contextualization is extended to non-Greek epochs as well. In his endeavor to tell a story about Greek mathematics, Netz dedicates a substantial portion of his work to outlining the landscape that preceded and succeeded the Greek era. This discourse is not confined solely to civilizations that directly interacted with the Greeks but offers a rich tapestry of insights from other contexts, such as the realm of mathematics in Latin America or China.

Thirdly, on several occasions, Netz attempts to show the change of historiographical attitudes towards both ancient testimonies, such as Proclus and Aristotle (p. 22), and recent historical accounts, for instance Unguru's critique of geometrical algebra (p. 120). The significant lesson students are expected to glean from this approach is that there have been several shifts in the perspective of mathematical history writing. And this happens because historiography is shaped by the intellectual backgrounds, motivations, and institutional settings of its practitioners. Lastly, Netz provides historical and institutional context, although

not systematically, to some modern historians of Greek mathematics. For example, we gain insight into the quest of Otto Neugebauer—the first, ever, lecturer in a mathematics department to specialize in the history of mathematics—to trace the origins of mathematics, during a time when his colleagues at the University of Göttingen were deeply engrossed in establishing the foundations of their own discipline (p. 20). This contextualization also helps us explain the anachronism present in Neugebauer's work on Babylonian mathematics.

To conclude this brief review, Netz has succeeded in compiling a captivating and comprehensible introduction to Greek mathematics, catering ideally to those without specialized knowledge. This publication does not seek to replace Heath, as the author explicitly clarifies in his preface this was not his intention: "I keep Heath by my side, and I urge you to do so as well" (p. xii). Perhaps, if the book had been given a slightly different title, this question might not have arisen. In any case, Reviel Netz's *A New History of Greek Mathematics* will serve as a greatly anticipated textbook on ancient Greek mathematics, and I warmly welcome its publication.

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