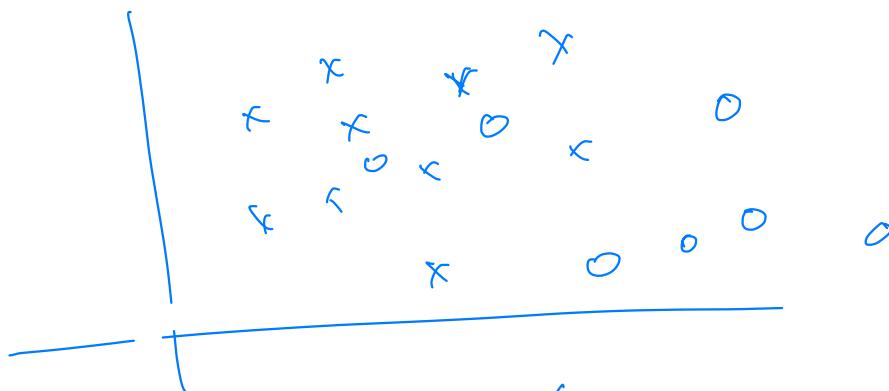


2021-12-08

Movzeno Support Vector Classifier για με διακυρώσεις ερωτήσεις



$$\max_{\|b\|=1} M$$

$$y_i^T (x_i^T b + b_0) \geq M \quad \forall i$$

F εγίρει από την

$$F = \emptyset$$

ανέγκειον

Xεράψων

$$\begin{aligned} & \max_{b, b_0, \xi_1, \xi_2, \dots, \xi_N} M \\ & \|b\|=1 \\ & y_i (x_i^T b + b_0) \geq M - \xi_i \quad i=1, \dots, N \\ & \xi_i \geq 0 \end{aligned}$$

a)

$$\begin{aligned} & \min \sum \xi_i \\ & \|b\|=1 \\ & y_i (x_i^T b + b_0) \geq M \\ & \xi_i \geq 0 \end{aligned}$$

(b)

$$\begin{aligned} & \max M \\ & \|b\|=1 \\ & y_i (x_i^T b + b_0) \geq M - \xi_i, i=1, \dots, N \\ & \xi_i \geq 0 \\ & \sum_{i=1}^n \xi_i \leq K \end{aligned}$$

μη κερδισμένης
μορφής
 $\|b\|=1$

$$\max M$$

$$y_i (x_i^T b + b_0) \geq M \|b\| - \underline{\xi_i \|b\|}$$

Evaffacerie porcje

$$\max M$$

$$y_i (x_i^T b + b_0) \stackrel{\|b\|=1}{\geq} M - M \xi_i = M(1-\xi_i)$$

$$\xi_i \geq 0$$

$$\sum_{i=1}^N \xi_i \leq K$$



$$\max M$$

$$\text{Dzwie } \|b\| = \frac{1}{M}$$

$$y_i (x_i^T b + b_0) \geq M \|b\| (1-\xi_i)$$

$$\xi_i \geq 0$$

$$\sum \xi_i \leq K$$



$$\max \frac{1}{\|b\|}$$

$$y_i (x_i^T b + b_0) \geq 1 - \xi_i$$

$$\xi_i \geq 0$$

$$\sum \xi_i \leq K$$

SVC

Підпорядковані
загальним
проприєтетам

$$\begin{aligned} & \min \frac{1}{2} \|b\|^2 \\ & y_i (x_i^T b + b_0) \geq 1 - \xi_i \\ & \xi_i \geq 0 \\ & \sum_{i=1}^N \xi_i \leq K \end{aligned}$$

Lagrangian primal-dual formulation

Εφευρετής πρωτότυπης ηλεγκτικής προσαρμογής : $Z_P = \min f(x)$
 $g_i(x) \geq 0, i=1, \dots, M$

Primal Lagrangian: $F_p = \{x : g_i(x) \geq 0, i=1, \dots, M\}$ επικεκυτής πρώτης προσαρμογής

$$L_p(x, \mu) = f(x) - \sum_{i=1}^M \mu_i g_i(x) \quad (\mu \geq 0)$$

Εφευρετής $L_D(\mu)$ $\boxed{\min_x L_p(x, \mu)}$

$$\min_x L_p(x, \mu) \leq \min_{x \in F_p} L_p(x, \mu)$$

$$\forall x \in F_p \Rightarrow g_i(x) \geq 0 \quad \text{Ati} \quad \left. \begin{array}{l} \mu_i \geq 0 \\ \text{Ati} \end{array} \right\} \mu_i g_i(x) \geq 0 \quad \forall i$$

$$\Rightarrow L_p(x, \mu) = f(x) - \underbrace{\sum \mu_i g_i(x)}_{\geq 0} \leq f(x) \quad \forall x \in F_p$$

$$\Rightarrow \min_{x \in F_p} L_p(x, \mu) \leq \min_{x \in F_p} f(x) = Z_P$$

$$\Rightarrow L_D(\mu) \leq Z_P \quad \forall \mu \geq 0$$

$$\Rightarrow \boxed{\max_{\mu \geq 0} L_D(\mu) \leq z_p} \Rightarrow \boxed{z_D \leq z_p}$$

δυϊκό^ό
προβλήμα.

↑
ασθένης
δυϊκότητα.

ιοχυρη δυϊκότητα

$$z_D = z_p$$

Συμπληρωματικότητα

Σε δεύτερη αύγον, x^*, μ^* .

$$\mu_i^* g_i(x^*) = 0 \quad \forall i = 1, \dots, M.$$

(αν $g_i(x^*) > 0 \Rightarrow \mu_i^* = 0$,
 $\mu_1^* > 0 \Rightarrow g_1(x^*) = 0$)

$$\min \frac{1}{2} \|b\|^2$$

$$y_i(x_i^T b + b_0) \geq 1 - \xi_i$$

$$\xi_i \geq 0$$

$$\sum \xi_i \leq K.$$

$$\min_{b, b_0, \xi_i} \frac{1}{2} \|b\|^2 + C \cdot \sum_{i=1}^N \xi_i$$

$$y_i(x_i^T b + b_0) \geq 1 - \xi_i$$

$$\xi_i \geq 0 \quad i=1, \dots, N$$

not for Lagrange

$$\left\{ \begin{array}{l} \alpha_i \quad y_i(x_i^T b + b_0) - (1 - \xi_i) \geq 0 \quad i=1, \dots, N \\ \mu_i \quad \xi_i \geq 0, \quad i=1, \dots, N \end{array} \right.$$

$$L_p(b, b_0, \xi_i, \alpha, \mu) =$$

$$\nabla_b x_i^T b = x_i$$

$$\frac{1}{2} b^T b + C \sum_{i=1}^N \xi_i - \sum_{i=1}^N \alpha_i [y_i(x_i^T b + b_0) - (1 - \xi_i)] - \sum_{i=1}^N \mu_i \xi_i$$

$$L_D(\alpha, \mu) = \min_{b, b_0, \xi_i} L_p(b, b_0, \xi_i, \alpha, \mu)$$

$$\nabla_b L_p = b - \sum_{i=1}^N \alpha_i y_i x_i = 0 \Rightarrow b = \sum_{i=1}^N \alpha_i y_i x_i$$

$$\frac{\partial L_p}{\partial b_0} = \sum_{i=1}^N \alpha_i y_i = 0$$

$$\frac{\partial L}{\partial \xi_i} = C - \alpha_i - \mu_i = 0 \Rightarrow \alpha_i = C - \mu_i \quad \forall i=1, \dots, N$$

$$\max_{\alpha, \mu \geq 0} L_D(\alpha, \mu)$$

Auf der rechten

$$b = \sum_i \alpha_i y_i x_i$$

$$\mu_i = C - \alpha_i$$

$$b^T b = \left(\sum_i \alpha_i y_i x_i \right)^T \cdot \left(\sum_{i'} \alpha_{i'} y_{i'} x_{i'}^T \right)$$

$$= \sum_{i, i'=1}^N \alpha_i \alpha_{i'} y_i y_{i'}^T \underbrace{x_i^T x_{i'}}_{} \quad \text{?}$$

$$\text{Einsetzen} \quad \left[\sum_{i=1}^N \alpha_i y_i x_i^T \right] b = b^T b$$

$$\left(\sum_{i=1}^N \alpha_i \right) \left(\sum_{j=1}^P b_j \right) \neq \sum_{i=1}^N \alpha_i b_i$$

=
The diagram illustrates the distributive property of multiplication over summation. It shows two separate sums: $\sum_{i=1}^N \alpha_i$ and $\sum_{j=1}^P b_j$. A curved arrow from the first sum points to a large oval enclosing the second sum. Inside the oval, every term b_j is multiplied by every term α_i , resulting in a double sum where each term is $\alpha_i b_j$.

$$L_D = -\frac{1}{2} \sum_{i,i'} \alpha_i \alpha_{i'}^T y_i y_{i'} x_i^T x_{i'}^T$$

$$+ C \sum_{i=1}^N \xi_i + \sum_i \alpha_i (1 - \xi_i) - \sum \mu_i \xi_i$$

$$= -\frac{1}{2} \sum_{i,i'} \dots -$$

$$+ \sum_{i=1}^N \alpha_i + \sum_{i=1}^N (C - \alpha_i - \mu_i) \xi_i \xrightarrow{= 0}$$

$$L_D(\alpha) = \sum_{i=1}^N \alpha_i - \frac{1}{2} \sum_{i,i'} \alpha_i \alpha_{i'}^T y_i y_{i'}^T x_i^T x_{i'}^T$$

complementarity

$$\dots \Rightarrow \delta^+$$

