Linear Appoximation / Newton's method

$f(x)$

$x = a$

$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$f(x) \simeq f(a) + (x-a) f'(a)$

$F(x) = 0$

$x - a \simeq -\frac{F(a)}{F'(a)}$

$x = a - \frac{F(a)}{F'(a)}$

Παράδειγμα:

Έστω $\sqrt{9.06} = ?$

$f(x) = \sqrt{x}$ $f'(x) = \frac{1}{2\sqrt{x}}$

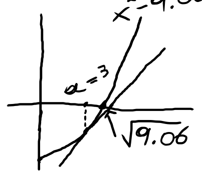
Επιλέγουμε για $a = 9$

$f(a) = \sqrt{9} = 3$, $f'(a) = \frac{1}{2\sqrt{9}} = \frac{1}{6}$

$f(9.06) = \sqrt{9} + (9.06 - 9) \cdot \frac{1}{6} \Leftrightarrow$

$f(9.06) = 3.01$

Μέθοδος N-R



$F(x) = x^2 - 9.06$

Θεωρώ $x_0 = 3$

$F'(x) = 2x$

$x_1 = x_0 - \frac{F(x_0)}{F'(x_0)} =$

$= 3 - \frac{3^2 - 9.06}{2 \cdot 3} =$

$= 3 - \frac{(-0.06)}{6} =$

$= 3 + 0.01 = 3.01$

$3.01^2 = 9.0601$

$x_1 = 3.01$ $x_2 = x_1 - \frac{F(x_1)}{F'(x_1)} =$

$= 3.01 - \frac{3.01^2 - 9.06}{6.02} =$

$= 3.01 - \frac{9.0601 - 9.06}{6.02} =$

$= 3.01 - \frac{0.0001}{6.02}$

Error

$x_2^2 - 9.06 = 0.00000001$

Παράδειγμα 2:

Βρείτε την τιμή $e^{0.01}$

$f(x) = e^x$ $f'(x) = e^x$

Επιλέγουμε για $a=0$

$$e^{0,01} = e^0 + (0.01-0)\cdot e^0$$

$$e^{0.01} = 1 + 0,01$$

$$e^{0.01} = 1,01$$

$$e^x = 1 + (x-0) = \underline{\underline{1+x}}$$

Το ανάπτυγμα του e^x

$$\underline{\underline{1+x}} + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \dots$$

Περικομένη Poisson

$$f(x) = \frac{\lambda^x}{x!\,(e^\lambda - 1)}, \quad x = 1,2,\dots, \quad \lambda > 0$$

$$L(\lambda) = \prod_{i=1}^{n} f(x_i) = \prod_{i=1}^{n} \frac{\lambda^{x_i}}{x_i!\,(e^\lambda - 1)}$$

$$l(\lambda) = \log L(\lambda) = \log \prod_{i=1}^{n} \frac{\lambda^{x_i}}{x_i!\,(e^\lambda - 1)} =$$

$$= \sum_{i=1}^{n} \log \frac{\lambda^{x_i}}{x_i!\,(e^\lambda - 1)} =$$

$$= \sum_{i=1}^{n} x_i \log\lambda - \sum_{i=1}^{n} \log x_i! - \sum_{i=1}^{n} \log(e^\lambda - 1)$$

$$= \log\lambda \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \log x_i! - n \log(e^\lambda - 1)$$

$$\frac{\partial l(\lambda)}{\partial \lambda} = \frac{1}{\lambda} \cdot \sum_{i=1}^{n} x_i - n \cdot \frac{e^\lambda}{e^\lambda - 1}$$

Θέτω για να βρω το λ

$$\frac{\partial l(\lambda)}{\partial \lambda} = 0 \iff \frac{1}{\lambda} \sum_{i=1}^{n} x_i - n \cdot \frac{e^\lambda}{e^\lambda - 1} = 0$$

$\nexists$ κλειστού τύπου εξίσωση
για το λ
και καταφεύγουμε
σε N-R.

Παράδειγμα 2: Κατανομή Poisson:

$$f(x_i/\lambda) = \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}$$

$$L(\lambda) = \prod_{i=1}^{n} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}$$

$$l(\lambda) = \sum_{i=1}^{n} \log\left(\frac{e^{-\lambda}\lambda^{x_i}}{x!}\right) =$$

$$= -n\lambda + \log\lambda \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \log x_i!$$

$$\frac{\partial l(\lambda)}{\partial(\lambda)} = -n + \frac{1}{\lambda}\sum_{i=1}^{n} x_i$$

$$\frac{\partial l(\lambda)}{\partial(\lambda)} = 0 \Leftrightarrow -n + \frac{1}{\lambda}\sum_{i=1}^{n} x_i = 0 \Leftrightarrow$$

$$\boxed{\lambda = \bar{x}}$$

Κατανομή Weibull:

$$f(x_i|\lambda,\theta) = \frac{\lambda \cdot x_i^{\lambda-1}}{\theta^{\lambda}} \cdot e^{-\left(\frac{x_i}{\theta}\right)^{\lambda}}, \quad x_i > 0$$

(Για το παράδειγμα θα θεωρήσουμε ότι $\lambda=2$. Μας ενδιαφέρει να βρούμε το θ. Στο παράδειγμα $n=49$)

$$L(\theta) = \prod_{i=1}^{n} \frac{\lambda \cdot x_i^{\lambda-1}}{\theta^{\lambda}} \cdot e^{-\left(\frac{x_i}{\theta}\right)^{\lambda}}$$

$$l(\theta) = \log L(\theta) = \sum_{i=1}^{n} \left\{ \log\lambda + (\lambda-1)\cdot\log x_i - \lambda \log\theta - \left(\frac{x_i}{\theta}\right)^{\lambda} \right\}$$

$$\frac{\partial l(\theta)}{\partial\theta} = \sum_{i=1}^{n} \left\{ -\frac{\lambda}{\theta} + \frac{\lambda \cdot x_i^{\lambda}}{\theta^{\lambda+1}} \right\} =$$

$$= -\frac{\lambda \cdot n}{\theta} + \frac{\lambda \sum_{i=1}^{n} x_i^{\lambda}}{\theta^{\lambda+1}}$$

Επειδή $\lambda = 2$

$$-\frac{2n}{\theta} + \frac{2\cdot\sum_{i=1}^{n} x_i^2}{\theta^3} = 0 \iff$$

$$\theta = \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{n}}$$

Για τα δεδομένα των χρόνων επιβίωσης βρίσκουμε ότι

$$\hat{\theta} = 9892.177$$