Διαλεξη 5

Λυσεις Ασκησεων 1.4, 1.5, 1.6

1.4 Duffing: $\ddot{x} + x + \varepsilon x^3 = 0$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 - \varepsilon x_1^3 \end{cases}$$

$$f_1 = 0, \quad f_2(x_1, x_2) = -x_1^3$$

$$\dot{E}(x_1, x_2) = x_1 f_1(x_1, x_2) + x_2 f_2(x_1, x_2) = -x_2 x_1^3$$

$$F(A) = \int_0^{2\pi} \dot{E}(A\cos t, -A\sin t)\, dt$$

$$= \int_0^{2\pi} +A\sin t \, A^3 \cos^3 t \, dt$$

$$= -A^4 \int_0^{2\pi} \cos^3 t \, d(\cos t) = -A^4 \left. \frac{\cos^4 t}{4} \right|_0^{2\pi} = 0.$$

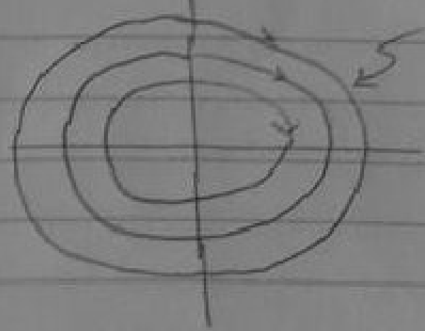
Σχλ. Συντηρητικο Συστημα - Συντηρητικη Διαταραχη

$$H_\varepsilon(x, \dot{x}) = \underbrace{\frac{\dot{x}^2}{2}}_{E_K} + \underbrace{\left(\frac{x^2}{2} + \varepsilon \frac{x^4}{4}\right)}_{E_\Delta}$$

$$\frac{d}{dt} H_\varepsilon(x_1(t), x_2(t)) = \dot{x}\,(\ddot{x} + x + \varepsilon x^3) = 0$$

$$\dot{x}_1 = \frac{\partial H_\varepsilon}{\partial x_2}$$

$$\dot{x}_2 = -\frac{\partial H_\varepsilon}{\partial x_1}$$



$H_\varepsilon = C$ (καμπυλες σταθμ

ΟΛΕΣ ΟΙ ΤΡΟΧΙ

ΠΕΡΙΟΔΙΚΕΣ.

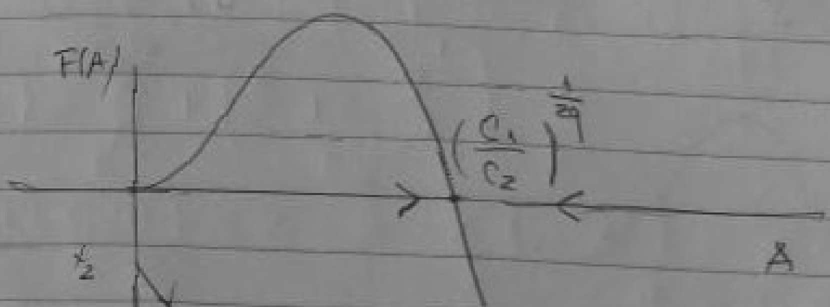
1.5 $\ddot{x} = -x + \varepsilon |\dot{x}|^{p} \dot{x} (1 - x^{2q}) = 0$, $p \geq 0$, $q > 0$.

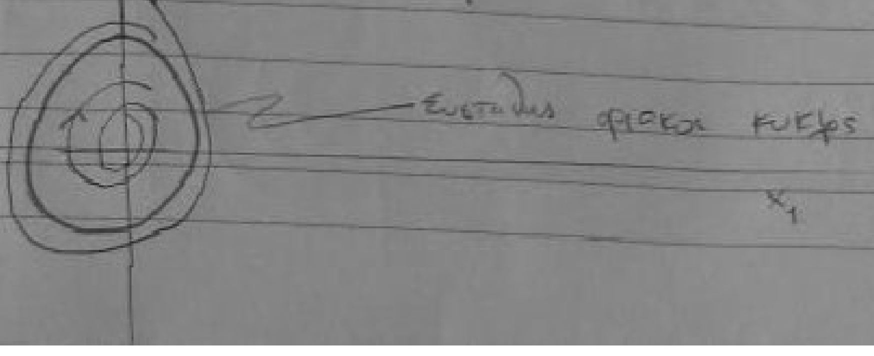
Van der Pol: $p = 0$, $q = 1$.

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_1 + \varepsilon |x_2|^{p} x_2 (1 - x_1^{2q})$$

$$\dot{E}(x_1, x_2) = x_1 f_1(x_1, x_2) + x_2 f_2(x_1, x_2)$$
$$= x_1 x_2 + x_2 (-x_1 + x_2 |x_2|^{p} (1 - x_1^{2q}))$$
$$= x_2^{2} |x_2|^{p} (1 - x_1^{2q})$$

$$F(A) = \int_0^{2\pi} \dot{E}(A\cos t, -A\sin t)\, dt$$
$$= \int_0^{2\pi} A^{p+2} |\sin t|^{p} (\sin t)^{2} (1 - A^{2q} \cos^{2q} t)$$
$$= A^{p+2} \int_0^{2\pi} (|\sin t|^{p+2} - A^{2q} (\sin t)^{2} (\cos^{2q} t))\, dt$$
$$= A^{p+2} \left[C_1 - C_2 A^{2q} \right] \quad , C_1 > 0, C_2 > 0$$





1.6 $\ddot{x} = -x + \varepsilon f(\dot{x})(1-x^2)$ Τύπου Van der Pol

$$= -x + \varepsilon\left(\frac{f(\dot{x})}{\dot{x}}\right)\dot{x}\,(1-x^2)$$

f κατάλληλο πολυώνυμο ώστε $\exists!$ 3 περιοδικές για $|\varepsilon| << 1$.

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 + \varepsilon f(x_2)(1-x_1^2) = -x_1 + \varepsilon f_2 \end{cases}$$

$$E(x_1, x_2) = x_1 f_1(x_1,x_2) + x_2 f_2(x_1,x_2)$$

$$= x_2 f(x_2)(1-x_1^2) = x_2^2 \frac{f(x_2)}{x_2}(1-x_1^2) = x_2^2 g(x_2)(1-$$

$$F(A) = \int_0^{2\pi} E(A\cos t, -A\sin t)\,dt$$

$$= A^2\int_0^{2\pi} (\sin^2 t)\, g(A\sin t)(1-A^2\cos^2 t)\,dt.$$

Έστω $g(x_2) = a_0 + a_1 x^2 + a_2 x^4$

$$F(A) = A^2\int_0^{2\pi} (\sin^2 t)(a_0 + a_1 A^2\sin^2 t + a_2 A^4 \sin^4 t)(1 - A^2\cos^2 t)\,dt$$

$$= A^2\int_0^{2\pi} \sin^2 t\,\big[a_0 + A^2(a_1\sin^2 t - a_0\cos^2 t) + A^4(a_2\sin^4 t - a_1\sin^2 t\cos^2 t$$

$$- A^6 \sin^4 t\cos^2 t\big]\,dt$$

$$= A^2\left[\bar{a}_0 + \bar{a}_1 A^2 + \bar{a}_2 A^4 + \bar{a}_3 A^6\right] = A^2 P(A^2).$$

$P(z) = \bar{a}_0 + \bar{a}_1 z + \bar{a}_2 z^2 + \bar{a}_3 z^3$

$z^2 P(z^2)$