

Περίδειγμα (Μηχανή με 2 καταστάσεις ή

M/M/1/1 ουρά)
 $\lambda \mu$

Μια μηχανή μπορεί να λειτουργεί ή να είναι χαλασμένη.

Αν λειτουργεί, θα χαλάσει μετά από χρόνο $\text{Exp}(\mu)$.

Ο χρόνος επιδιόρθωσης είναι $\text{Exp}(\lambda)$. Οι χρόνοι λειτουργίας

& επιδιόρθωσης είναι ανεξάρτητοι.

$$X(t) = \begin{cases} 0 & \text{αν δεν λειτουργεί τη στιγμή } t \\ 1 & \text{αν λειτουργεί } \gg \gg \gg \end{cases}$$

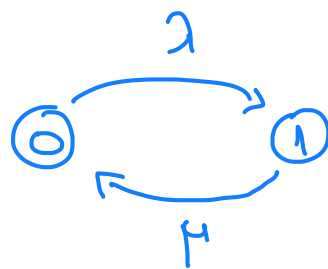
(i) Να βρεθεί ο Q

(ii) Να βρεθεί ο P(t).

Λύση

(i) $S = \{0, 1\}$

$$Q = \begin{matrix} & 0 & 1 \\ \begin{matrix} 0 \\ 1 \end{matrix} & \begin{bmatrix} -\lambda & \lambda \\ \mu & -\mu \end{bmatrix} \end{matrix}$$



(ii) $P'(t) = P(t) Q \Rightarrow$

$$\begin{bmatrix} p_{00}'(t) & p_{01}'(t) \\ p_{10}'(t) & p_{11}'(t) \end{bmatrix} = \begin{bmatrix} p_{00}(t) & p_{01}(t) \\ p_{10}(t) & p_{11}(t) \end{bmatrix} \begin{bmatrix} -\lambda & \lambda \\ \mu & -\mu \end{bmatrix}$$

$$p_{00}'(t) = -\lambda p_{00}(t) + \mu p_{01}(t) \quad (1)$$

$$p_{00}(0) = 1 \quad (7)$$

$$p_{01}'(t) = \lambda p_{00}(t) - \mu p_{01}(t) \quad (2)$$

$$p_{01}(0) = 0 \quad (8)$$

$$p_{10}'(t) = -\lambda p_{10}(t) + \mu p_{11}(t) \quad (3)$$

$$p_{10}(0) = 0 \quad (9)$$

$$p_{11}'(t) = \lambda p_{10}(t) - \mu p_{11}(t) \quad (4)$$

$$p_{11}(0) = 1 \quad (10)$$

$$p_{10}(t) + p_{11}(t) = 1 \quad (5)$$

$$p_{00}(t) + p_{01}(t) = 1 \quad (6)$$

$$(1) \stackrel{(6)}{\Rightarrow} p_{00}'(t) = -\lambda p_{00}(t) + \mu (1 - p_{00}(t)) \Rightarrow$$

$$p_{00}'(t) = -(\lambda + \mu) p_{00}(t) + \mu \Rightarrow$$

$$p_{00}'(t) + (\lambda + \mu) p_{00}(t) = \mu \Rightarrow$$

$$e^{(\lambda + \mu)t} p_{00}'(t) + (\lambda + \mu) e^{(\lambda + \mu)t} p_{00}(t) = \mu e^{(\lambda + \mu)t} \Rightarrow$$

$$\left[e^{(\lambda + \mu)t} p_{00}(t) \right]' = \mu e^{(\lambda + \mu)t} \Rightarrow$$

$$e^{(\lambda + \mu)t} p_{00}(t) = \frac{\mu}{\lambda + \mu} e^{(\lambda + \mu)t} + c \Rightarrow$$

$$p_{00}(t) = \frac{\mu}{\lambda + \mu} + c e^{-(\lambda + \mu)t} \quad (11)$$

$$(7): p_{00}(0) = 1 \stackrel{(11)}{\Rightarrow} \frac{\mu}{\lambda + \mu} + c = 1 \Rightarrow c = 1 - \frac{\mu}{\lambda + \mu} \Rightarrow$$

$$c = \frac{\lambda}{\lambda + \mu}$$

Άρα

$$p_{00}(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t}$$

$$(6) \Rightarrow p_{01}(t) = 1 - p_{00}(t) \Rightarrow$$

$$p_{01}(t) = 1 - \frac{\mu}{\lambda + \mu} - \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t} \Rightarrow$$

$$p_{01}(t) = \frac{\lambda}{\lambda + \mu} - \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t}$$

Από τις (3), (5) και (9) προκύπτει ότι

$$p_{10}(t) = \frac{\mu}{\lambda + \mu} - \frac{\mu}{\lambda + \mu} e^{-(\lambda + \mu)t}$$

και

$$p_{11}(t) = \frac{\lambda}{\lambda + \mu} + \frac{\mu}{\lambda + \mu} e^{-(\lambda + \mu)t}$$

Οπότε

$$P(t) = \begin{bmatrix} \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t} + \frac{\mu}{\lambda + \mu} & \frac{\lambda}{\lambda + \mu} (1 - e^{-(\lambda + \mu)t}) \\ \frac{\mu}{\lambda + \mu} (1 - e^{-(\lambda + \mu)t}) & \frac{\lambda}{\lambda + \mu} + \frac{\mu}{\lambda + \mu} e^{-(\lambda + \mu)t} \end{bmatrix}$$