Teaching Mathematics and Its Applications (2016) **35**, 172–186
doi:10.1093/teamat/hrw015

Mathematics, modelling and students in transition

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[Submitted January 2016; accepted May 2016]

This article is based on data from two major research projects that investigated students involved in mathematically demanding courses during their transition through college and into university. It explores the nature of modelling as a mathematical practice in this important transition phase for students. Theoretical considerations are informed by illustrative accounts of a college mathematical modelling lesson and engineering lecture exemplifying the complex nature of mathematical modelling in these two phases of post-compulsory education. This raises important issues concerning the teaching and learning of mathematical practices in relation to modelling and applications. The discussion presented here is seen through the lens of Cultural Historical Activity Theory that informed the project team's analysis of the case studies developed of both institutions and individuals. In this article, data and earlier findings are reinterpreted to better understand how we might support students as they move from learning mathematics to learning to use mathematics effectively in pursuit of their other studies. The accounts of classroom and lecture activity illustrate how 'doing mathematics' is mediated in different ways ensuring that students experience modelling and applications as mathematical practices very differently in each. This leads me to explain why, but also infer that students are likely to experience difficulties in transition both 'vertically' in progression from one activity system to another over time (college to higher education) and 'horizontally' between activity systems in which they participate concurrently (maths and engineering classes in university).

1. Introduction

In this article, I explore the changing nature of the mathematical practice of modelling as experienced by students in transition from college to university where they study mathematically demanding courses. The illustrative example of such a course in Higher Education used here is in structural engineering. However, much of what I write would be of relevance to those studying a wide range of courses, particularly those in science or technology. The data on which I draw are from the series of Transmaths projects that investigated students in transition in post-compulsory education in the UK (see Section 2). These projects sought to answer research questions that focussed on how teaching and learning experiences impacted on students' learning outcomes in terms of their developing identities due to their changing relationship with mathematics. The analysis here focuses on issues that arose relating to modelling and applications.

From the outset, I wish to problematize my use of the terms mathematics and mathematical modelling as it is dangerous to assume a common, generally agreed understanding of these concepts. As will become apparent, and I have already implied by my use of the term 'mathematical practices', I wish to extend the notion of mathematics to be more inclusive and wide ranging in our thinking about the discipline than is often the case.

Cardella (2008) identified five key aspects that seem a useful starting point when considering the learning of mathematics for engineering: knowledge base, problem solving strategies, mathematical practices, use of resources (including social resources, time and metacognitive processes such as planning and monitoring work) and beliefs and affects. Of particular importance in the discussion that this article initiates is that of mathematical practices: in particular those that relate to applying mathematics and mathematical modelling. There is a wide range of published material that describes mathematical modelling and its teaching and learning in educational settings: much of this arises from the International Community of Teachers of Modelling and Applications (ICTMA) group. For example, see the volumes that are published in the series 'International Perspectives on the Teaching and Learning of Mathematical Modelling' such as Stillman *et al.* (2015). In attempting to describe modelling as a mathematical practice or competency, many authors have broken this down to identify sub-competencies, which they suggest may be considered to be carried out as part of a cyclical process. This has led researchers to schematize the practice using a flow diagram such as that suggested by Blum & Leiß (2007); Blomhöj & Jensen (2003) and Maaß (2006) amongst others.

Fundamental to all different descriptions of the process of applied mathematical modelling is the connectivity between a problem set in a context outside of mathematics (such as that of business finance in the college classroom or that of structural engineering in the university lecture theatre) and the world of mathematics. Both context and mathematics are seen as brought together in the thinking of the student when they engage in:

- *mathematizing*—in which relevant mathematics that can lead to a solution, or sense-making, of the problem set in a particular context is identified.
- *interpreting*—making sense of their mathematical solution to the mathematical problem in terms of the context of the problem situation.

Such representations of modelling are often considered cyclical in that they allow that models might be refined, or completely rethought, to cater for the complexity of the situation they represent. As I have remarked elsewhere (Wake, 2015), it is at these key moments of connecting mathematics and a context outside of mathematics that students and others (such as workers) have difficulties.

Changes in university engineering students' relationships with mathematics are especially important as they become learners of how to *use* mathematics in pursuit of their problem solving in engineering as a discipline outside of mathematics (Harris *et al.*, 2015). This is fundamentally different to the pursuit of learning mathematics as a discipline in its own right. To add complexity to their learning of mathematics, engineering students not only need to learn to *use* mathematics that they already know but also need to learn and understand new mathematical rules, procedures and techniques which they will then be expected to go on to apply. Although their university course is likely to be structured around distinct sub-disciplines of engineering, it is likely that mathematics will form a separate course unit, especially in early stages of the course. Students will be expected to learn to become users of mathematics during almost all aspects of the course. This leads to a very different experience when compared to the experience of college where the study of mathematics will be very much self-contained and used only peripherally in other subjects: with the notable exception of Physics or vocational courses in engineering/technology. Through analysis of two classroom vignettes, this paper seeks to examine the teaching and learning of the application of mathematics and mathematical modelling in college and university engineering settings and explore the implications of this for students as they try to connect context and mathematics in their mathematical activity. The vignettes are chosen purposively to illustrate the complexity of similarities and differences that students may experience in mathematical activity when they are in transition from the study of college mathematics to a university course such as engineering. The intention is not to suggest that we might generalize findings from the vignettes to provide insight into learning College Mathematics and University Engineering rather their purpose is to provoke discussion about how we might better support transitions for students who need to engage with modelling and applications as mathematical practices.

2. Project methodologies

This article, in Section 4, presents two, necessarily brief, accounts of episodes from 'lessons' in which mathematical modelling and applications of mathematics are central. The data that underpin these accounts and the discussion that follows emanate from two research projects that looked at students' transition (i) through college (Mathematics learning, identity and educational practice: the transition into post-compulsory education, ESRC grant RES-000-22-2890) and, at a later stage, (ii) into university (Mathematics learning, identity and educational practice: the transition into Higher Education, ESRC grant RES-062-23-1213). These projects sought to add knowledge of mathematics and students in transition through what is typically the first three years of post-compulsory education in England, i.e. for students aged 16–19, when studying mathematics becomes a matter of choice. The projects were complex and called on a range of methods to gather, analyse and interpret both quantitative and qualitative data with a primary focus on the investigation of experiences and identities/dispositions of learners.

The project that investigated transition through college used a large-scale questionnaire survey that involved some 1800 students with the sample including many considered 'on the edge' between engagement and disengagement. The survey was administered three times with the same cohort of students as they progressed on their college programmes and included specially constructed instruments that were used to measure important learning outcomes. These included students' mathematical self-efficacy, their disposition to enter higher education and their dispositions towards studying mathematically demanding subjects in higher education (Pampaka *et al.*, 2012). Teaching practices were investigated using a self-report instrument that measured teachers' classroom practices in each of the classes they taught. Case studies of five colleges were developed based on in-depth interviews with staff and students together with lesson observations and analysis.

The second project investigated transitions into Higher Education and also drew on a variety of methods including a survey of students ($n > 1700$), case studies of 13 university courses (mostly in STEM), and longitudinal interviews with a number of students ($n > 50$) as they progressed through university. The case studies in both research projects were developed from mainly qualitative investigations and involved observations and video recordings of lessons/lectures and tutorials with interviews of students and teachers. Triangulation was supported by the collection of other institutional documents and data, and interviews with other stakeholders such as Heads of Departments.

This article draws on qualitative data from both projects.

3. Cultural Historical Activity Theory

To analyse students' mathematical activity we used Cultural Historical Activity Theory (CHAT) as a lens. CHAT has developed, particularly recently, from the fundamental thinking of Vygotsky (1978) which considered how learning takes place through engagement in joint activity mediated by 'instruments' such as artefacts (diagrammatic representations, texts, overhead projector, etc.) and discourse/ language (who speaks/listens, what/whose rules, etc.: Wertsch, 1991). This is represented by the upper triangle in Engström's well-known schema (Fig. 1). For instance, the representations, language and gestures that the teacher employs, as well as a range of other resources, including text books, assessment questions and so on are instruments that are essential in mediating the mathematics to be learnt. In the short timescale of a lesson, such as that of the vignette of learning mathematics in the college classroom that follows, the way the teacher structures the assessment question to introduce different mathematical thinking and modelling as a practice is important in prompting students to consider, more or less effectively, the relationship between (in this case the) financial context and mathematical model.

Vygotsky's notion of mediated action of the individual was later expanded by Leont'ev (1981) to include the community in which the activity is situated. This is represented by the lower triangles in Engström's schema which highlight additional mediating nodes that draw attention to how community, division of labour and rules (in the widest sense) mediate the activity of the subject in relation to the object of activity. The rules include both explicit rules such as the structuring of the day as imposed by institutional organizational constraints, the syllabus or specification of the course and more implicit rules that impact on how the members of the community operate. These include, for example, societal expectations in relation to teaching and learning as well as more locally derived expectations such as how students are expected to engage with lectures in light of other 'classes' that are provided, whether or how students are expected to take notes, and so on. Issues of division of labour are important: that is, issues of the relationships between teacher and learner. In classroom situations, for example, the teacher almost always has control of the knowledge and is seen as being 'in control'. This need not necessarily be the case, of course. We observed, at times, in a small number of classrooms, students seemingly exerting agency over the direction of their learning. However, even in such situations we noted that the teacher often had more of a controlling role than may at first have seemed apparent (Wake & Pampaka, 2008). It is clear that issues of division of labour are connected to how a sense of community develops, in terms of who has agency and control over deciding the direction of learning, etc.

For students in the university the situation is more complicated in relation to how we understand mathematical activity and mathematical practices. At times mathematics may be the object of study, for example, in lectures and tutorials which comprise specific mathematics units of a degree programme. On other occasions, it may be considered an instrument as students use mathematics to

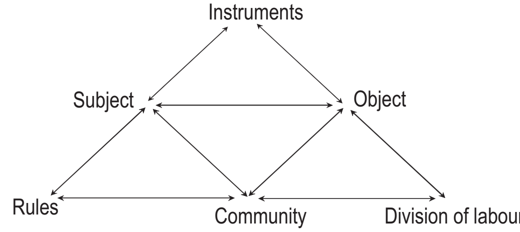

FIG. 1. Engström's CHAT model.

understand and make sense of engineering. Thus, we may consider mathematics as having a major, but different, role to play in two activity systems: one concerned primarily with, and consequently having as its object, the learning of mathematics and the other concerned primarily with, and consequently having as its object, the learning of engineering. In this latter activity system, mathematics plays a different role to that in the former, it shifts from its learning being the object to being a mediating instrument supporting the learning of engineering. In this sense, mathematics might be considered as a boundary object (Star & Griesemer, 1989), i.e. mathematics has different meaning in each of the different Activity Systems, while retaining its common essence across both.

4. In the 'classroom'

In colleges, we observed that by far the most common approach to teaching was heavily influenced by preparation towards forthcoming timed-written assessment. The dominant teaching style, as observed, and also self-reported by teachers was teacher-centred (Wake & Pampaka, 2008) typically involving a lot of teacher exposition/input followed by students individually practising routines and procedures. In general, we found models of teaching and learning mathematics at university, both as a subject in its own right and in support of other subjects, also to be fairly restricted. Such models consisted of, perhaps iconically, lectures supported by 'problem-solving classes' or 'workshops' that provide opportunities to practise techniques, rules and procedures. Again almost all lecturing and tutoring in problem-solving classes involved a great deal of teacher exposition.

4.1 Vignette 1: college Use of Maths lesson

The first vignette provides an account of an episode from a 'lesson' in a college where the class were studying towards the AS qualification *Use of Mathematics*. At the time of the research this AS level qualification, designed as 'half an *A*-Level' in the English post-16 suite of qualifications, was available in a number of Colleges and schools. Most AS levels were mainly designed to be taken at the end of the first year of a two-year course of study leading to the full *A* Level. AS Use of Mathematics, however, was one of a handful of such qualifications designed specifically as a stand-alone qualification. It was designed to provide a course to support students with their other areas of study such as the sciences, business/economics and the social sciences. Neither this course nor the full *A* Level in Use of Mathematics was designed to support progression to university engineering courses. The vignette here is chosen due to the significant elements of mathematical modelling it contains. The Use of Mathematics courses make the mathematical modelling process explicit in many ways in the texts and assessment encouraging a modelling ethos and pedagogy—as becomes clear in classroom observations in our case studies and illustrated in this vignette.

The lesson starts with the teacher explaining that the class will work on an extended question (Fig. 2) that the teacher had designed as an example of the type of questions the students will eventually meet in their external assessment. He goes on to read out the stem of the question which gives background information. This is typically presented to students in advance of the examination in a 'data sheet' which allows them time to immerse themselves in contexts and terminology, here, e.g. 'revenue', 'unit price', etc., relevant to the questions to be asked. He asks the class to answer the first part of the question by referring to a version of the graph in Fig. 3 below. At this stage, the graph does not yet include the tangent to the curve that is parallel to the straight line.

A student initiates a brief period of interaction with the teacher by suggesting that 'if you sell no units then you will make no money' which the teacher confirms as being correct (although this

1 A local manufacturing firm makes a single product. The revenue (money coming in) from selling this product depends on the price they charge according to the following equation:

$$y = 160x - 8x^2$$

where x is the price in £ per unit, and y is the revue in thousands of £ per year.
A graph of this function is on the attached sheet.

(a) Explain briefly why it is reasonable for the curve for revenue to pass through the point (0,0).

(b) At what price will the firm maximise its revenue?

The revenue also depends on the number of products made, but so do the costs of making the product. The company has derived a simple model of costs against selling price £x as follows:

$$y = 32x + 384,$$

where y represents annual costs, in thousands of £.

(c) Briefly explain the meanings of the numbers 32 and 384 in this equation, in terms of the costs to the company.

The company will 'break even' when the yearly revenue is equal to the yearly costs.

(d) Explain briefly why the selling price £x in order to break even satisfies the equation

$$160x - 8x^2 = 32x + 384.$$

The yearly *profit* is the difference between the revenue and the costs. This is maximised when costs and revenue are both increasing at the same *rate*.

(e) By drawing a suitable tangent to the revenue curve, estimate the selling price which will maximise the yearly profit for the company.

(f) What is the maximum yearly profit.

FIG. 2. Sample AS Use of Mathematics examination-style question used in college lesson.

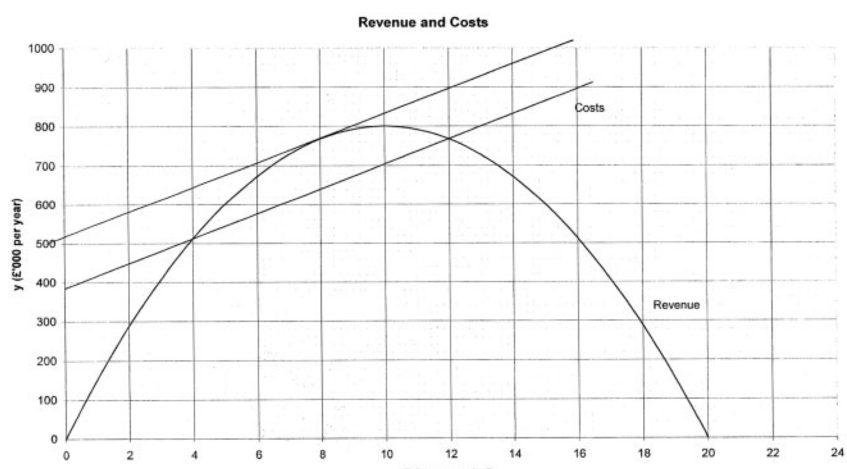

FIG. 3. Graph relating to example examination question used in college AS Use of Mathematics lesson.

interpretation is not quite correct as the graph plots price per unit against revenue). This is quickly passed over and the teacher asks the students why they think the curve 'starts to come down?' displaying a copy of the curve in Fig. 3. Throughout the lesson this pattern of the teacher carefully directing the class with questions that focus them on thinking of how mathematics relates to the context of finance continues. For example, following this early exchange a student contributes that, 'as you increase the price then you start making a total amount of money (revenue) that falls.' The teacher encourages such discourse by adding a further explanation: ‘So there is a certain point at which the firm is going to maximise it’s revenue.’ A student goes on to contribute that this is at ‘10’, with the teacher adding context again, suggesting that this is when the price is £10 per unit, but that this is not necessarily where the profit is maximized. Thus the teacher explicitly and repeatedly adds contextual detail, moving the discourse to the financial concepts of the ‘firm’, its ‘revenue’ and ‘profit’, and so forth.

The teacher then draws attention to the model, $y = 32x + 384$. He explains that this was suggested by someone in the company to show how costs increase with unit selling price. He asks students the next part of the written question, i.e. to explain the meanings of the numbers 32 and 384 in the ‘simple model’. A student contributes that the 32 is the ‘gradient’. The teacher confirms that this is ‘correct - in terms of the line’, explicitly drawing attention to the mathematical domain of the term, with the student then adding that the 384 gives the intercept (on the vertical axis, although this is not made explicit). The teacher then asks why the line goes through 384 rather than the origin. Another student identifies this as the £384, 000 of ‘starting up costs’. Attention is now focused on the meaning of ‘32’, with a student suggesting that this gives the ‘rate of change’ (expanding on the notion of gradient).

The students clearly have difficulty with interpreting this and the teacher suggests that the way to interpret such graphs is to consider what happens if x increases by one pound of unit-price, ‘So, for every £1 increase in the price of the unit it is going to cost you another £32,000 per year to make.’ He emphasizes what each of the variables represent together with their associated units by writing this information on the whiteboard at the front of the classroom.

The teacher goes on to point out that there are two points where the ‘costs line’ crosses the revenue curve and that these are ‘the break even points’ where the costs match the revenue coming in. This allows him to explain that this is where the two equations are equal: he writes the equation given in the question on the whiteboard and emphasizes for students why the two equations for costs and revenue are equal at the points where their graphs intersect. At this point the teacher realizes that when devising the question he had not asked students to solve this equation so he introduces a new sub-part of part (d) into the question, ‘Find the solutions of this equation using the graph.’ The students suggest that the ‘answers’ are $x = 4$ and $x = 12$.

The teacher then moves the class on to the next part of the question that considers profit as the difference between the revenue and the costs and asks at what price the company would maximize their profit. He goes on to suggest that the way to find this is to find where the costs and revenue are both going up at the same rate: he gestures with his hands to indicate that this is where the line and the curve are parallel, ‘Because there comes a point after that where the revenue is not going up at the same rate and the costs are still going up at the same rate and so the difference starts to come down again.’ Having pointed out that at $x = 10$, although the revenue is maximized the difference between the two graphs is not maximum and this therefore needs to be found. A student suggests that this is in the region where $x = 8$. The teacher now suggests that the students draw a tangent to the curve that is as near as possible parallel to the straight line (Fig. 4). The students, immediately see that the answer to sub-part (e) of the question is where $x = 8$. The teacher suggests that sub-part (f) could be answered by counting squares on the graph but more accurately can be found by substituting $x = 8$ into the two which he proceeds to demonstrate.

The remainder of the lesson continues in much the same way with the teacher emphasizing that in the examination of the qualification the students will be expected to be able to use and interpret a graph in this way to solve questions such as this.

This vignette shows how the teacher seeks to ensure motivation of the mathematical ideas in the financial context of the question. This is explicitly attended to by the teacher ensuring that the mathematics is consciously constituted as ‘mathematical modelling’ for the learners, and not simply as an

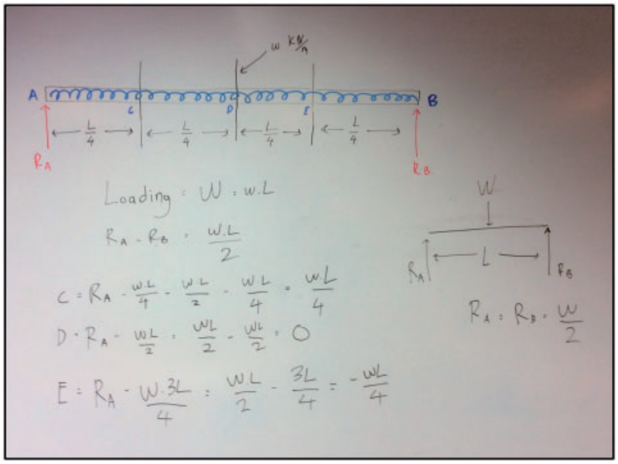

Fig. 4. Shearing force diagram for uniformly distributed load. This figure appears in colour in the online version of *Teaching Mathematics and its Applications*.

exercise in learning the mathematics of slope/gradient and rate of change for its own sake. I infer that this is necessary because this is a ‘maths class’ and the students might not otherwise choose to see their mathematics as a modelling activity with significance outside mathematics. This explicitness of mathematical modelling is not only evident in the dialogue of the lesson, but of course is explicitly named and codified in the Use of Mathematics syllabus/specification, assessment, and guidance for teachers; the whole design intends to make modelling explicit, in contrast to other mathematics courses where the presumption is that only ‘the mathematics itself’ counts.

4.2 Vignette 2: university structural engineering lecture

Vignette 2 presents a short episode from a structural engineering lecture in which ideas of structural analysis, important in civil engineering, rely heavily on mathematical modelling and applications. The choice of this episode from an engineering lecture rather than a lecture specifically aimed at the teaching and learning of mathematics might at first sight seem a little strange, but mathematics-specific lectures deal mainly with the pure and abstract rather than focussing on applications. It was in lectures such as the one partially described below that we found students engaged in meaningful activity that might be considered as mathematical modelling/applications under discussion here.

The lecturer starts by drawing attention to some conventions that had been introduced in previous lectures: that is (i) shearing forces at a section through a beam (represented as a point on the diagrams here draws, e.g. point C in Fig. 4) are considered positive upwards when on the left face and negative when downwards on the right-hand face, and (ii) bending moments are considered positive when a beam is sagging and negative when hogging. He introduces these ideas using overhead transparencies projected onto a screen at the front of the lecture theatre. Having done so, the lecturer works through a problem using a roll of blank overhead transparency film developing his solution freehand as he goes along. We saw many lecturers working in this way, demonstrating how they would tackle mathematical problems themselves. They either used an overhead projector or its modern equivalent, the visualizer connected to a computer together with a data projector, as they attempted to mimic the writing at the board that we observed in many college classrooms and university workshops or ‘problem-solving classes’ (Pepin, 2014).

The lecturer draws attention to the two types of force system that will be discussed naming them 'concentrated and uniformly distributed loading', before introducing a system of concentrated loading on a beam that is supported at both ends, A and B. He discusses this in abstract and mathematical terms arriving at algebraic expressions for the shearing forces in each section of the loaded beam. Having done so he then introduces some values that he uses to exemplify how these expressions might be applied to calculate shearing forces. The lecturer goes on to sketch a shearing force diagram. In a departure from the rest of the lecture where the lecturer usually works from the abstract to the concrete he then continues to develop a bending moment diagram for the situation by using the values of distances and forces which he introduced earlier.

He returns to developing abstract algebraic expressions when he goes on to introduce how to arrive at a shearing force diagram for uniformly distributed loading (w KN/m) on a beam supported at ends A and B a distance L metres apart (Fig. 4). However, he does give some possible suggestions for values of the evenly distributed load and the distance between the supports and emphasizes that the generalized working he goes through can be used for any values of w and L including those he specifically suggested.

At this point, the lecturer asks a question of the whole group: a rare event in the lecture theatre setting. After drawing attention to the uniform loading of w kN/m and the distance between the supports as L, he asks them to tell him the total loading on the whole beam. A student eventually responds 'wL'. The lecturer then attempts to get students to tell him the magnitude of the reactions at A and B. He is this time unsuccessful and has to tell them that due to the symmetry of the situation these forces will be $\frac{wL}{2}$ N, writing this down as he proceeds. In an attempt to explain how to determine the shearing forces at various points along the beam the lecturer introduces three new points C $\left(\frac{L}{4} fromA\right)$,$D$ $\left(\frac{L}{2} fromA\right)$ and E $\left(\frac{3L}{4} fromA\right)$, equally spaced between A and B. He proceeds to work out the shearing force at each of these points in terms of the generalized forces and dimensions he has introduced (Fig. 4).

The lecturer goes on to explain how although he calculated all shearing force values by considering forces to the left of each point, he could equally have used forces to the right and demonstrates the veracity of this by re-calculating the shearing force at D by considering forces to the right and arriving at the same result. By plotting these values at appropriate points along a diagram of the beam the lecturer arrives at a sketch of the shearing force diagram. Following this, a student draws attention (a relatively rare event in the lecture theatres observed as part of case-study work) to the expression 'uniformly distributed' and asks for clarification about the two types of loading that had been referred to in the lecture. The lecturer explains by asking students to consider (i) a car passing over a bridge which will give rise to concentrated loading and (ii) the loading due to the bridge itself (made from concrete) which leads to uniformly distributed loading in units of kilonewtons per metre. At last after some considerable time the lecturer is provoked into making the context of the problem clear to the students for the first time: of course, the lecturer will have understood the relationship between the mathematics that he had been discussing and its potential application in structural engineering contexts but his discourse focused on rules and procedures that kept this connectivity hidden from the students.

The lecture continues much in this vein with the lecturer carefully introducing rules and procedures in relation to developing shearing force and bending moment diagrams, but without making explicit appropriate engineering contexts that are modelled in this way. In contrast to the previous vignette, there seems to be no emphasis on the connection of the mathematics to its context being important per se: it seems to be taken for granted. In this case I infer that, at least for the lecturer, it is a given that mathematics is a tool for doing engineering (calculations): no significant questioning of this is needed. In terms of the CHAT analysis, this use of mathematics is implicit in the lecturer's practice and pedagogy. This is in contrast to the previous vignette where the teacher brings the use of mathematics to make sense of a situation explicit through his questioning of students as the lesson proceeds.

5. Comparative analysis and discussion

The two vignettes presented here highlight a significant feature of how students engage in applying their mathematics as a practice in the two different settings. Essentially: the most stark contrast is the amount of attention directed by the teachers, and therefore, experienced by the students, towards the connectivity between context and mathematics. In the college lesson, although the students were often inclined to focus on mathematical features such as the gradient and intercept of the straight line, the teacher is keen for them to make sense of this in terms of the financial situation it represents. Although the written questions he had designed are concerned with, and require proficiency in, the use of procedures and techniques, his oral questioning ensures that students need to consider how these mathematical features connect with the financial context. The example assessment question is used by the teacher to ensure students have the opportunity to focus their attention on the connection between model and context.

In contrast, in the engineering lecture, in which you might have expected the engineering lecturer to have emphasized such connectivity, the emphasis is much more on the efficient and accurate implementation of rules and procedures. This is most clearly brought to our attention when well into the lecture a student asks for the concept of a uniformly distributed load to be clarified. It was only at this point that the lecturer injected any sense of context (a car driving across a bridge) into the abstract mathematical models that had been his focus until this point. Here, although the mathematics has the potential (as an instrument) to connect consciously with the engineering context of the loading of a bridge, this opportunity is only taken by their lecturer in response to a student's question.

To provide further insight into how mathematical practices come to be shaped by the activity systems in which they are situated, Table 1 provides a very brief summary of some of the key features of the two activity systems of college Use of Maths and university engineering 'classrooms' in general summarizing our observations across case studies. There is not space here to explain the details of this fully but the key contrast in mediating factors that I draw out in this article are emboldened.

Table 1 illustrates important factors that are important to our understanding of our case studies. These factors have significant impact on how the collective or community view of mathematics, and mathematical modelling and application as a practice is socially experienced and constructed. (In the university, this collective view derives from experiences when learning both mathematics and engineering.) In this respect, it can be considered that different student groups share and produce different cultural models of mathematics (Gee, 1999) depending on the different experiences they have in the different activity systems in which they learn mathematics. The purposively chosen vignettes presented here together with the analysis of the two activity systems illustrate the complexity that underlies how different cultural models may develop and be influenced.

There is no doubt that the different teachers' narratives in respect to mathematical modelling and applications have considerable impact on the way in which students come to see, and develop, their own mathematical practices. In general in classrooms, in both colleges and universities, we rarely saw evidence of discussion of mathematical modelling and application as a practice. Any learning of how to apply mathematics was almost entirely tacit with little overt support in teaching. The main difference we detect in the vignettes here is the college teacher's focus on, and insistence that students engage in making sense of, connections between context and mathematics. This was undoubtedly due to the assessment requirements of the AS Use of Mathematics course. In parallel classrooms where students were following the 'traditional' *A* Level in mathematics, we saw no evidence of such

TABLE 1. CHAT analysis of college Use of Mathematics lessons and University engineering lectures

College: AS Use of Mathematics lesson	University: Structural engineering lecture
Object	
Learning mathematics	*Learning engineering (the role of mathematics)*
Mathematics as an individual discipline rarely connected with other discipline. Teachers reflect this compartmentalization (Morgan, 2011) with their teaching almost always restricted to the subject of mathematics.	Unit courses in engineering strongly bounded for students but lecturers have weakly boundaries between disciplines in their own engineering practice particularly in their own connectivity between mathematics and engineering.
Outcome	*Outcome*
Qualification—set and awarded externally.	Qualification (unit pass)—set and awarded internally.
Individual goal	*Individual goal*
Predominantly focused on gaining a qualification to move on to further study or work. Very few students focused on learning as support for learning other disciplines or for interest.	Predominantly to gain course unit credits but also important in supporting engineering problem solving.
System motivation	*System motivation*
High level of performativity – aggregated student grades in external assessment leading to college league table positions are extremely important to managers and affect all aspects of system including at classroom level. Courses developed in response to external specifications.	The importance of competence in applying mathematics effectively in engineering is widely recognized and sought by lecturers. Courses controlled internally but with requirements for graduates to meet externally controlled chartered engineering standards.
Instruments	
Resources	*Resources*
Heavily informed by external assessment with text books and teacher notes aligned to this. Low use of technology: mainly scientific calculator. Informal student use of technology as learning resource such as web-based revision resources.	Mainly developed internally, bespoke to courses. Internally controlled and developed. Increasing use of computer technology both as a mathematical tool and increasingly as a learning resource (including providing bespoke e-learning and assessment).
Pedagogy	*Pedagogy*
Characteristically long periods of exposition with procedural approaches highlighted, followed by practise of these. AS Use of Maths assessment design supports pedagogies that focus on connecting a range of context situated problems and mathematics.	Focused on learning rules and procedures. Problem solving classes/workshops provide models of problem solving practices. Connectivity between context and mathematics not necessarily highlighted.

(Continued)

TABLE 1. Continued.

College: AS Use of Mathematics lesson	University: Structural engineering lecture
Rules	
At an institutional level aggregated scores of students' performance in national assessment dominate. These permeate all aspects of college *and* classroom activity with maximizing grades seen as beneficial to students *and* the institution (Wake, 2013).	At an institutional level student performance is considered as mainly a function of the student rather than teaching. Even though students are monitored and support mechanisms are provided whether students engage or not is left to the student.
Attendance and performance continuously monitored with targeted intervention.	Assessment is internally controlled and reflects courses developed internally by lecturers.
Assessment is externally controlled and dominates teaching and learning. AS Use of Maths focuses on modelling and applications. Traditional Maths on pure mathematics.	
Division of labour	
Clearly defined roles for teachers and students with teachers mediators of static body of knowledge.	Clearly defined roles for teachers and students with teachers mediators of static body of knowledge.
Community	
Students taught in relatively small groups often with a strong sense of sociality.	Students taught in both large and small groups often with a weak sense of sociality.
Teachers and students aligned together in adversity against the examination system.	Lecturers and students seemingly opposed with lecturers in control of assessment of students.
Teachers supportive of students to maximize performance which is attributed to their teaching.	Student performance attributed mainly to them rather than as a function of teaching.
The wider role of college community often influential.	

connectivity: the course leading to this *A* Level has no similar requirement to engage with modelling and applications.

6. Conclusion

Analysis of the learning of mathematics in post-compulsory education for progression to engineering leads to the conclusion that students have diverse experiences of, and presumably come to a wide range of understandings of, mathematical modelling and applications as a mathematical practice. CHAT points to how this is the case because of variations in the mediating factors that come to define the social and cultural relations which affect the learning of mathematics as an activity.

Significant in this regard, and pertinent to the two carefully selected vignettes in this article, is the role that the assessment design of AS Use of Mathematics plays in providing rules that promote explicit engagement with modelling and applications.

Of particular importance is how teachers and students come to understand and experience the connections between context and the mathematics that they use to model this. The two vignettes illustrate quite different ways in which this can be constituted. In the engineering lecture, this coupling of context and mathematics was implicit as far as the lecturer was concerned and for him the mathematics did not need to be explicitly motivated by the engineering context. On the other hand, the college teacher made the coupling between the context of the financial problem and the mathematics explicit and encouraged students to consider how the structure of the situation and structure of the mathematics are connected. On numerous occasions the teacher highlighted how the abstract and general mathematical concepts that the students were using related to the applied problem under investigation.

It follows that to support students in transition we need to be cognizant of the issues raised here: that students have very different experiences of mathematical modelling and applications and these result from complex interactions of a large number of significant mediating factors. The vignettes are helpful in pointing to the kinds of factors that we need to be sensitive about: however, each particular activity system needs to be analysed to take account of its own specific conditions.

In general, our research leads me to conclude that if we wish to ensure students experience learning, in their classrooms, that prioritizes and emphasizes mathematical models and applications this needs to be designed into course specifications. This has to be achieved so as to inform all other mediating factors: instruments (including resources and pedagogies), rules (most importantly those relating to assessment), and the allied aspects of division of labour and community. Appropriate courses need to be implemented in ways that ensure students develop agency in enquiring into connectivity between context and mathematics.

In summary, to ensure that students are better prepared to use and apply mathematics it is important that teachers and learners:

- develop an expanded conceptualization of mathematics that goes beyond a well-defined knowledge base, to include problem solving strategies, mathematical practices, use of resources and pays attention as to how these come together to inculcate attitudes and beliefs;
- understand how societal, institutional and community rules, resources and ways of working impact on how they come to teach, learn and understand mathematics as a discipline and its potential relationship and connectivity with problems in other fields;
- ensure that mathematical modelling and applications whenever, and wherever, being used, are made explicit (even when they are usually tacit in the teacher's day-to-day practice and will eventually become so for the students);
- prioritize modelling, applications and problem solving strategies in their mathematical practices in ways that allows them to focus on how mathematics connects with the world outside of mathematics and how they can use such connectivity to gain insight into situations and solve problems.

7. Post-script

In addition to the important aspects of the AS Use of Mathematics programme reported here, the Transmaths research found that it performs more effectively than the AS Traditional programme in developing students' self-efficacy in mathematics and dispositions towards study of mathematics. Over the period of the course, it was also effective in retaining students with lower entry grades in the study of mathematics and the students were motivated in their learning by engaging in modelling (Hernandez-Martinez *et al.*, 2011). An overview of the impact of the qualification on student outcomes is reported in detail elsewhere: for a summary, see Wake (2011)).

Despite these positive aspects, the qualification was terminated in 2015. A full *A* Level in Use of Mathematics that was developed after the research period was introduced, but this will recruit for the last time in 2016. New qualifications in 'Core Mathematics' have been recently developed that share some of the underlying principles and philosophy of the Use of Mathematics qualifications, are similar in size to an AS Level qualification and are currently in early stages of implementation.

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