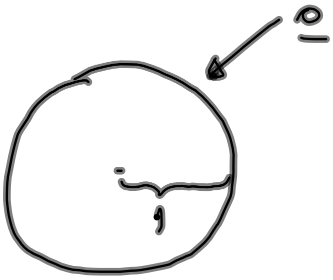
Ρίψη βέλους σε κυκλ στόχο

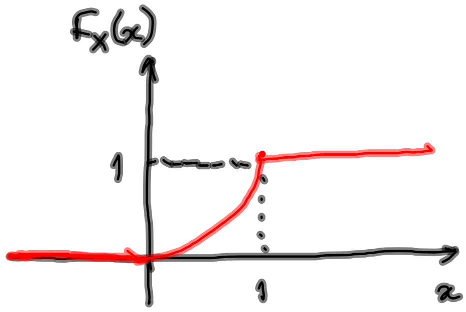
X = Αποστ. σημείου από κέντρο



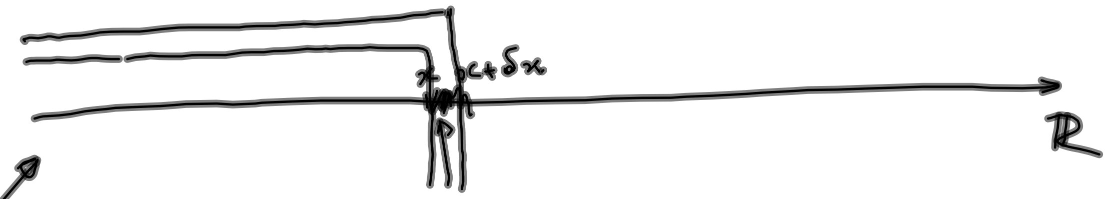
$$F_X(x) = P(X \le x), \quad x \in \mathbb{R}$$

$$= \frac{\text{Εμβ. } \{X \le x\}}{\underbrace{\text{Εμβ. } \underline{\circ}}_{\pi \cdot 1^2 = \pi}}$$

$$= \begin{cases} 0, & x < 0 \\ \frac{\pi x^2}{\pi} = x^2, & 0 < x < 1 \\ 1, & x \ge 1 \end{cases}$$



Πυκνότητα Πιθανότητας



Τιμές της X

$$f_X(x) = \lim_{\delta x \to 0^+} \frac{P(x < X \leq x + \delta x)}{\delta x}$$

πυκν. στο x

$$= \lim_{\delta x \to 0^+} \frac{P(X \leq x + \delta x) - P(X \leq x)}{\delta x}$$

$$\cancel{P_X(x) = P(X = x)} \qquad = \lim_{\delta x \to 0^+} \frac{F_X(x+\delta x) - F_X(x)}{\delta x} = \frac{dF_X(x)}{dx}$$

Διαφορές - Ομοιότητες σμπ και σππ
σπ

X διακρ	X συνεχής
$p_X(x) = P(X=x)$ χρησ.	άχρηστη $p_X(x) = P(X=x) = 0$
$F_X(x) = P(X \le x)$ ολ	ολ $F_X(x) = P(X \le x)$
	$f_X(x) = \dfrac{dF_X(x)}{dx}$
$p_X(x) \ge 0$, $\sum_x p_X(x) = 1$	$f_X(x) \ge 0$, $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$
$p_X(x) \le 1$	$f_X(x)$ μπορεί > 1
$P(X \in B) = \sum_{x \in B} p_X(x)$	$P(X \in B) = \int_B f_X(x)\,dx$

$p_X(x) \leftrightarrow f_X(x)$

$\Sigma \leftrightarrow \int$

Έστω X Bernoulli

$P(X=0)=1-p$

$P(X=1)=p$

$P(0<X<1)=0$

$P(0\leq X<1)=P(X=0)=1-p$

$P(0<X\leq 1)=P(X=1)=p$

$P(0\leq X\leq 1)=1$

X συνεχής

Τότε

$P(a<X<b)$

$=P(a\leq X<b)$

$=P(a<X\leq b)$

$=P(a\leq X\leq b)$

$=\int_a^b f_X(x)\,dx$

Δεκ 2-9:48 πμ

$$E[X] = \sum_x x \, p_X(x)$$

$$Var[X] = E\left((X - E[X])^2\right)$$

$$E[g(X)] = \sum_x g(x) \, p_X(x)$$

$$E[X] = \int_{-\infty}^{\infty} x \, f_X(x) \, dx$$

$$Var[X] = E\left((X - E[X])^2\right)$$

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) \, f_X(x) \, dx$$

$$E[aX + b] = a\,E[X] + b$$

$$Var[aX + b] = a^2\, Var[X]$$

$$Var[X] = E[X^2] - (E[X])^2$$

Δεκ 2-9:53 πμ

Ομοιόμορφη κατανομή στο $[a,b]$

$X \sim \text{Uniform}([a,b])$

$$f_X(x) = \begin{cases} c, & x \in [a,b] \\ 0, & \text{διαφορ.} \end{cases}$$

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1 \Rightarrow \cancel{\int_{-\infty}^{a} 0\,dx} + \int_a^b c\,dx + \cancel{\int_b^{\infty} 0\,dx} = 1$$

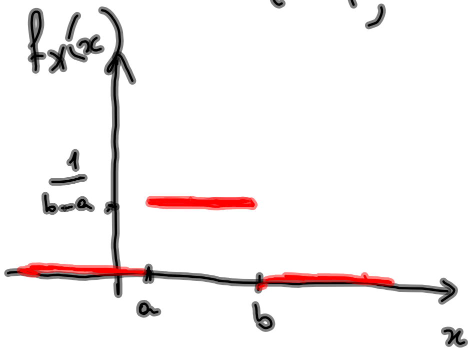
$$\Rightarrow c(b-a) = 1 \Rightarrow c = \frac{1}{b-a}.$$

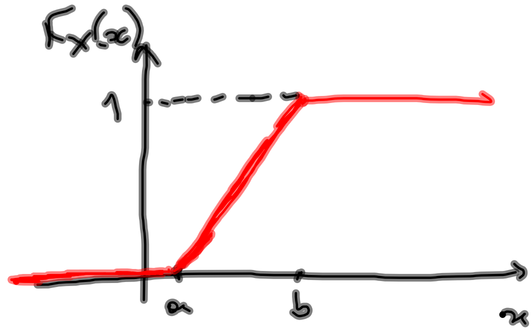
$$f_X(x) = \begin{cases} \frac{1}{b-a}, & x \in [a,b] \\ 0, & \text{διαφορ.} \end{cases}$$

$$F_X(x) = P(X \leq x) = P(X \in (-\infty, x])$$

$$= \int_{-\infty}^{x} f_X(t)\,dt$$

$$= \begin{cases} 0, & x < a \\ \int_a^x \frac{1}{b-a}\,dt = \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x > b \end{cases}$$





$$E(X) = \int_{-\infty}^{\infty} x\, f_X(x)\, dx$$

$$= \int_a^b x \, \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a} \int_a^b x\, dx$$

$$= \frac{1}{b-a} \left. \frac{x^2}{2} \right|_{x=a}^{b}$$

$$= \frac{b^2-a^2}{2(b-a)}$$

$$= \frac{a+b}{2}$$

$$Var(X) = E(X^2) - (E(X))^2$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx$$

$$= \int_a^b x^2 \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a} \int_a^b x^2 dx$$

$$= \frac{1}{b-a} \cdot \frac{b^3-a^3}{3}$$

$$= \frac{a^2+ab+b^2}{3}$$

$$Var(X)$$

$$= \frac{a^2+ab+b^2}{3} - \left(\frac{a+b}{2}\right)^2$$

$$= \frac{a^2+ab+b^2}{3} - \frac{a^2+2ab+b^2}{4}$$

$$= \frac{a^2-2ab+b^2}{12}$$

$$= \frac{(b-a)^2}{12}$$

Δεκ 2-10:33 πμ

Η Εκθετική Κατανομή $Exp(\lambda)$

X = Χρόνος ζωής εξαρτήμ. που δεν γηράσκει

Έχει την αμνήμονη ιδιότητα

$$P(X>s+t \mid X>s) = P(X>t)$$

Η μοναδική τ.μ. συνεχής που έχει αυτή την ιδιοτ. είναι με σ.π.π.

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x>0 \\ 0, & \text{διαφ.} \end{cases}$$ για κάποιο λ

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & x \le 0 \end{cases}$$

$$F_X(x) = \int_{-\infty}^{x} f_X(t)\,dt = \begin{cases} 1 - e^{-\lambda x}, & x > 0 \\ 0, & x \le 0 \end{cases}$$

οπότε για $s, t > 0$

$$P(X > s+t \mid X > s) = \frac{P(X > s+t,\ X > s)}{P(X > s)} = \frac{\overbrace{P(X > s+t)}^{1 - F_X(s+t)}}{\underbrace{P(X > s)}_{1 - F_X(s)}}$$

$$= \frac{e^{-\lambda(s+t)}}{e^{-\lambda s}} = e^{-\lambda t} = P(X > t)$$

$$E[X] = \int_{-\infty}^{\infty} x f_X(x)\,dx$$

$$= \int_0^{\infty} x \lambda e^{-\lambda x}\,dx$$

$$= \lambda \int_0^{\infty} x e^{-\lambda x}\,dx$$

$$= \lambda \int_0^{\infty} x \left(\frac{e^{-\lambda x}}{-\lambda}\right)' dx$$

ολοκληρ. κατά παράγοντες

$$= \lambda \left(\left[\frac{x e^{-\lambda x}}{-\lambda}\right]_0^{\infty} - \int_0^{\infty} \frac{e^{-\lambda x}}{-\lambda}\,dx \right)$$

$$= \dots = 1/\lambda.$$

$$\boxed{Var[X] = \frac{1}{\lambda^2}}$$