

MSc in Statistics and Operational Research
Bayesian Inference Project: First Question
Bayesian Test for Statistical Significance

1. For the data set `RegressionData` consider the simple linear regression model

$$y_i = \alpha + \beta x_i + \epsilon_i, \quad \epsilon_i \sim \text{Normal}(0, \tau^{-1}), \quad i = 1, \dots, 100.$$

Test the hypotheses

$$H_0 : \beta = 0$$

$$H_1 : \beta \neq 0$$

Consider equal prior probabilities for the two hypotheses and assume the following prior distributions for the model parameters.

$$\alpha | \tau \sim \text{Normal}(0, (c_1 \tau)^{-1})$$

$$\beta | \tau \sim \text{Normal}(0, (c_2 \tau)^{-1})$$

$$\tau \sim \text{Gamma}(p, q),$$

where $c_1 = c_2 = 0.5$ and $p = q = 0.01$.

2. Consider Bayesian Inference for the simple linear regression model in Part 1. Let $\theta = (\alpha, \beta)'$ and obtain the conjugate joint prior distribution of θ, τ . Then, using a conjugate prior distribution for the model parameters, calculate the joint posterior distribution of the model parameters, the marginal posterior distribution of τ and the conditional posterior distribution of $\theta | \tau$.