Microeconomic Theory I

Optimal choice and demand

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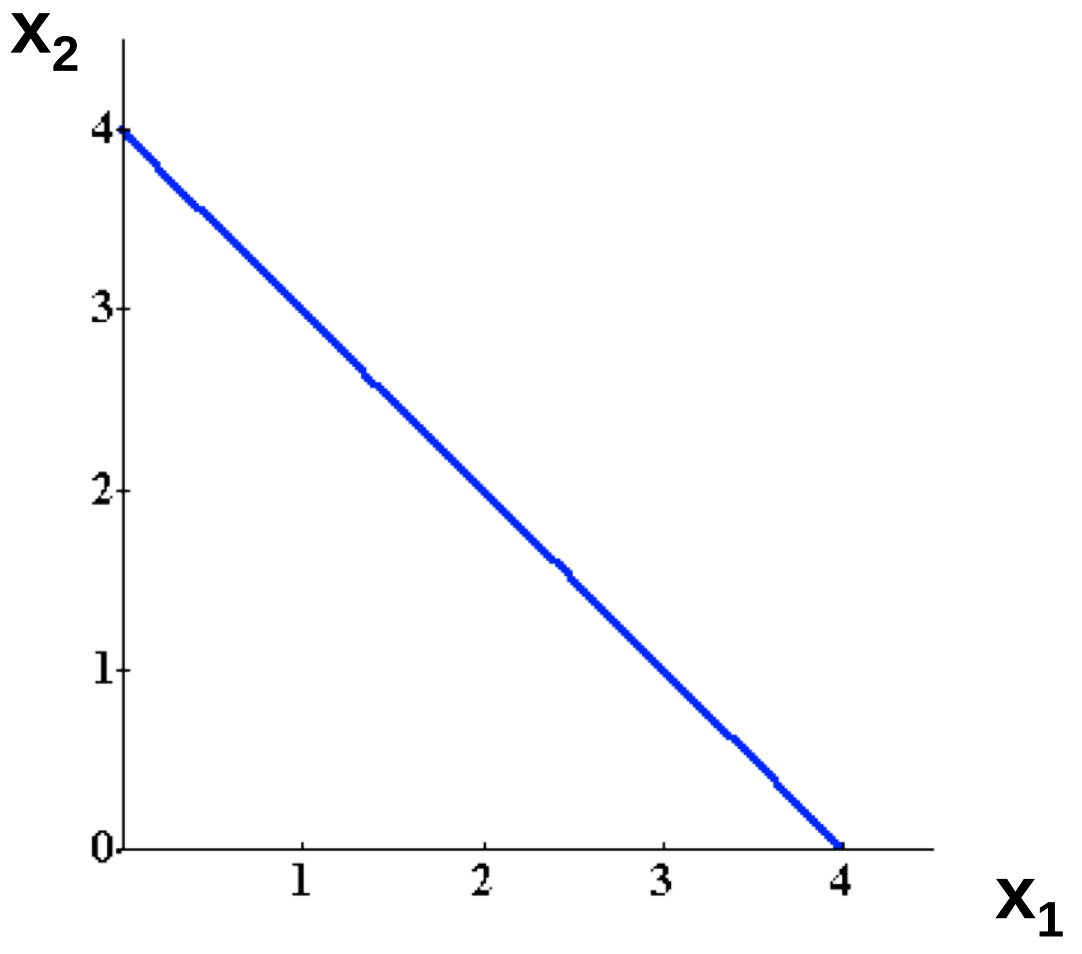
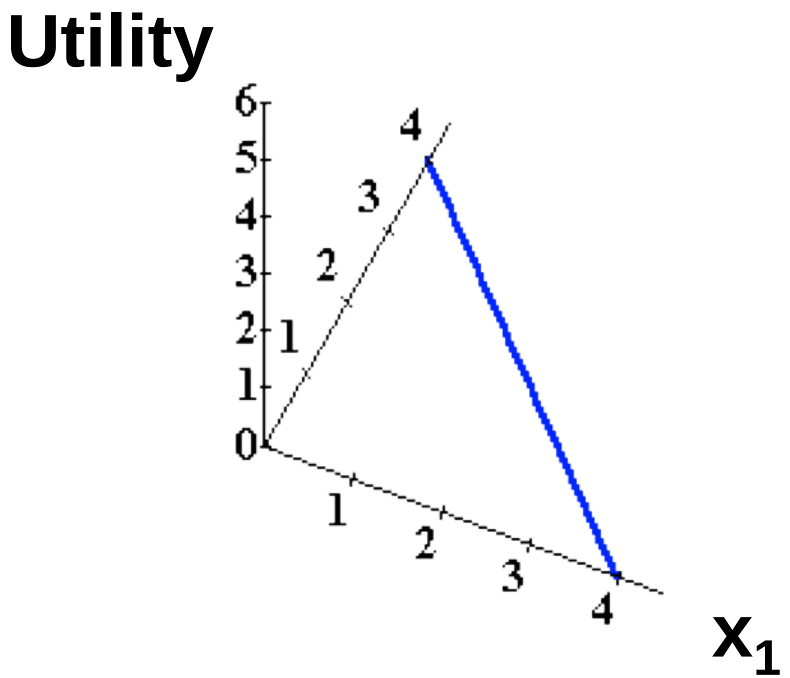
Economic Rationality

- **The principal behavioral postulate is that a decisionmaker chooses its most preferred alternative from those available to it.**
- **The available choices constitute the choice set.**
- **How is the most preferred bundle in the choice set located?**

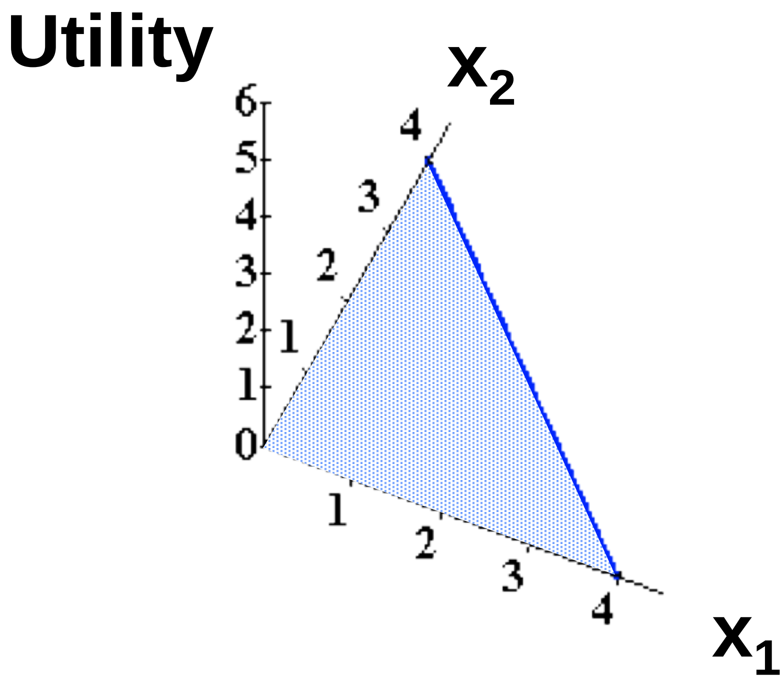
Rational Constrained Choice



Rational Constrained Choice



Rational Constrained Choice



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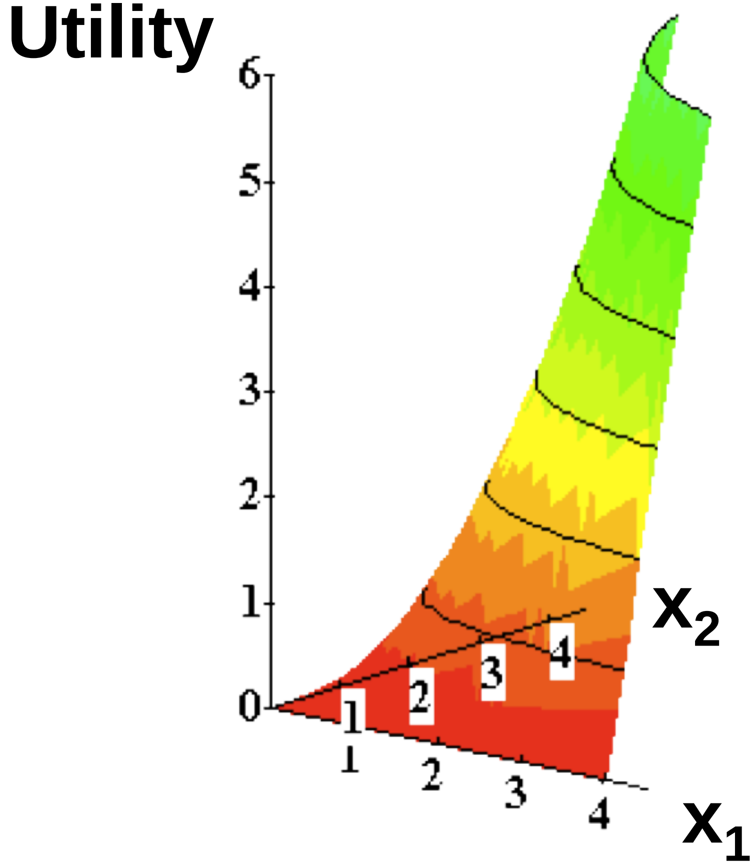

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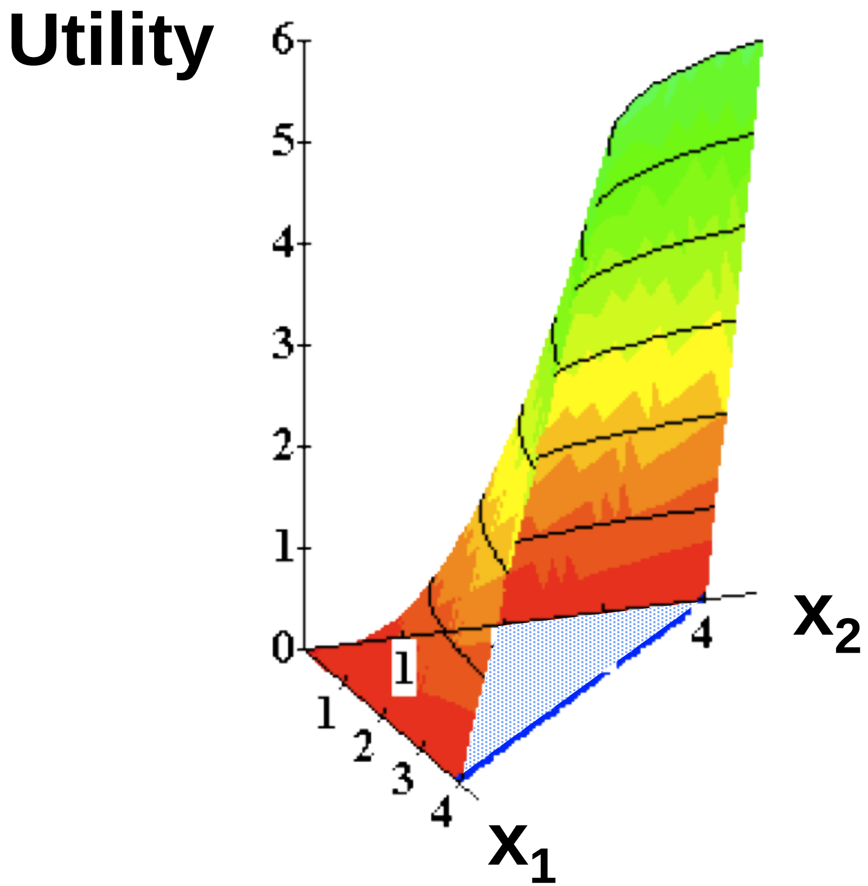

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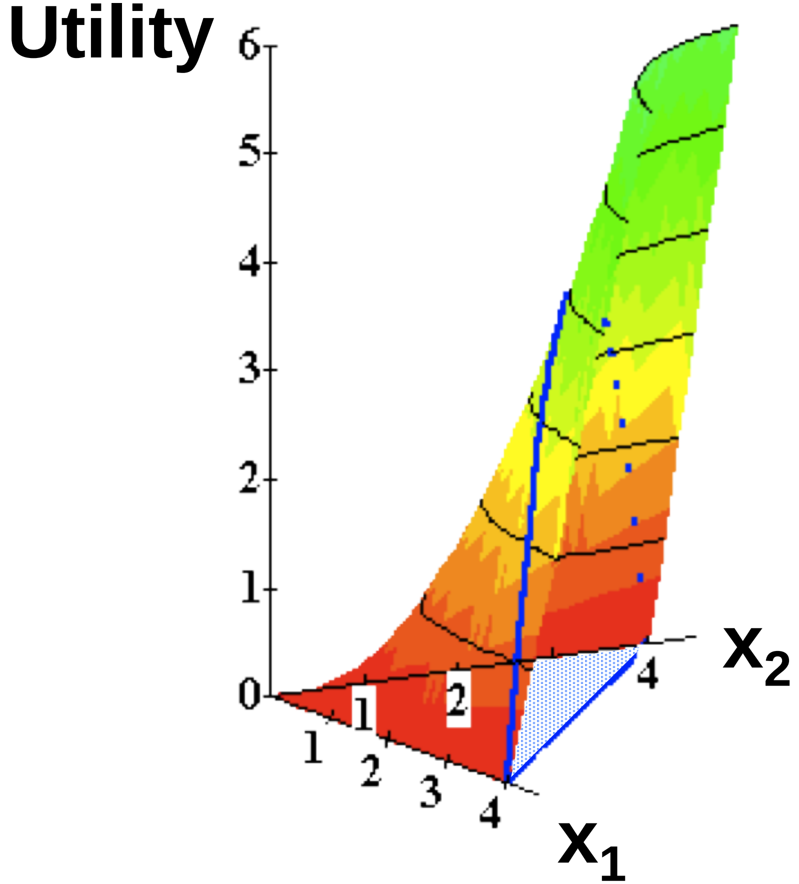

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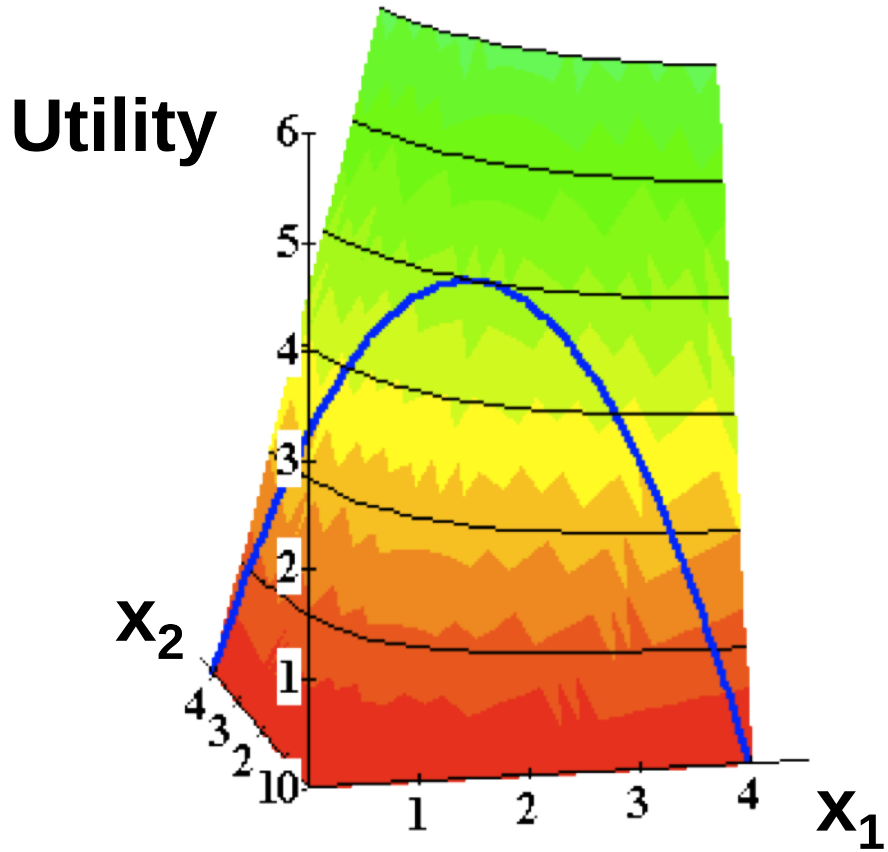

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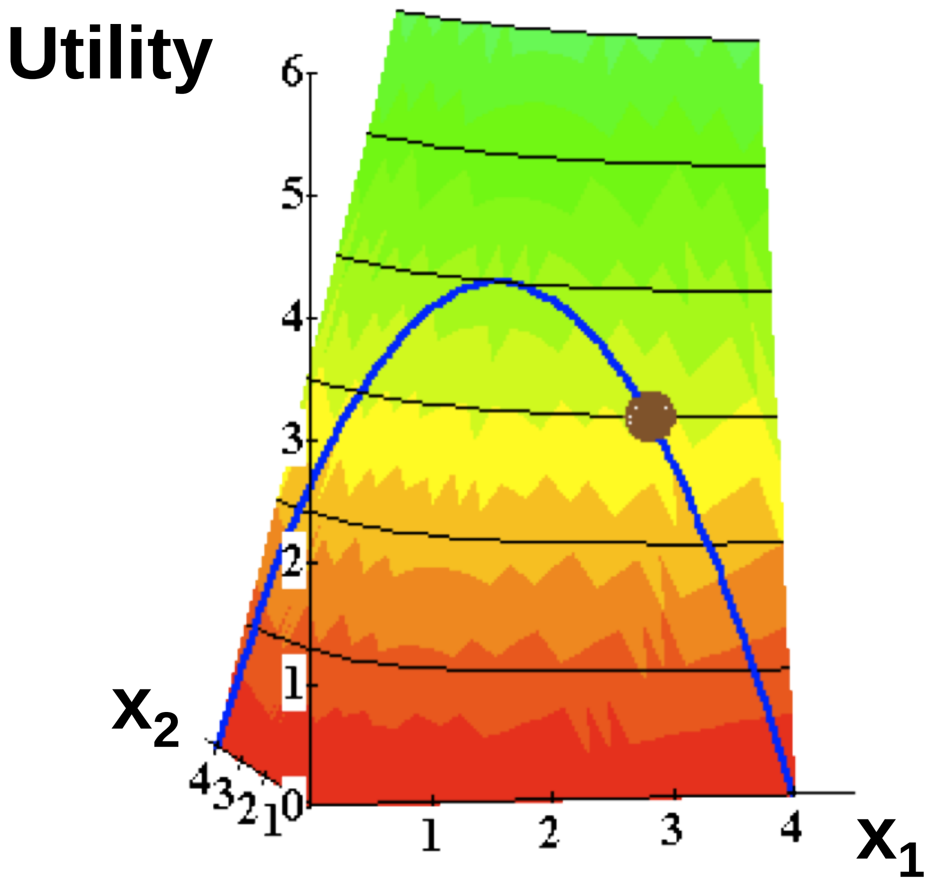

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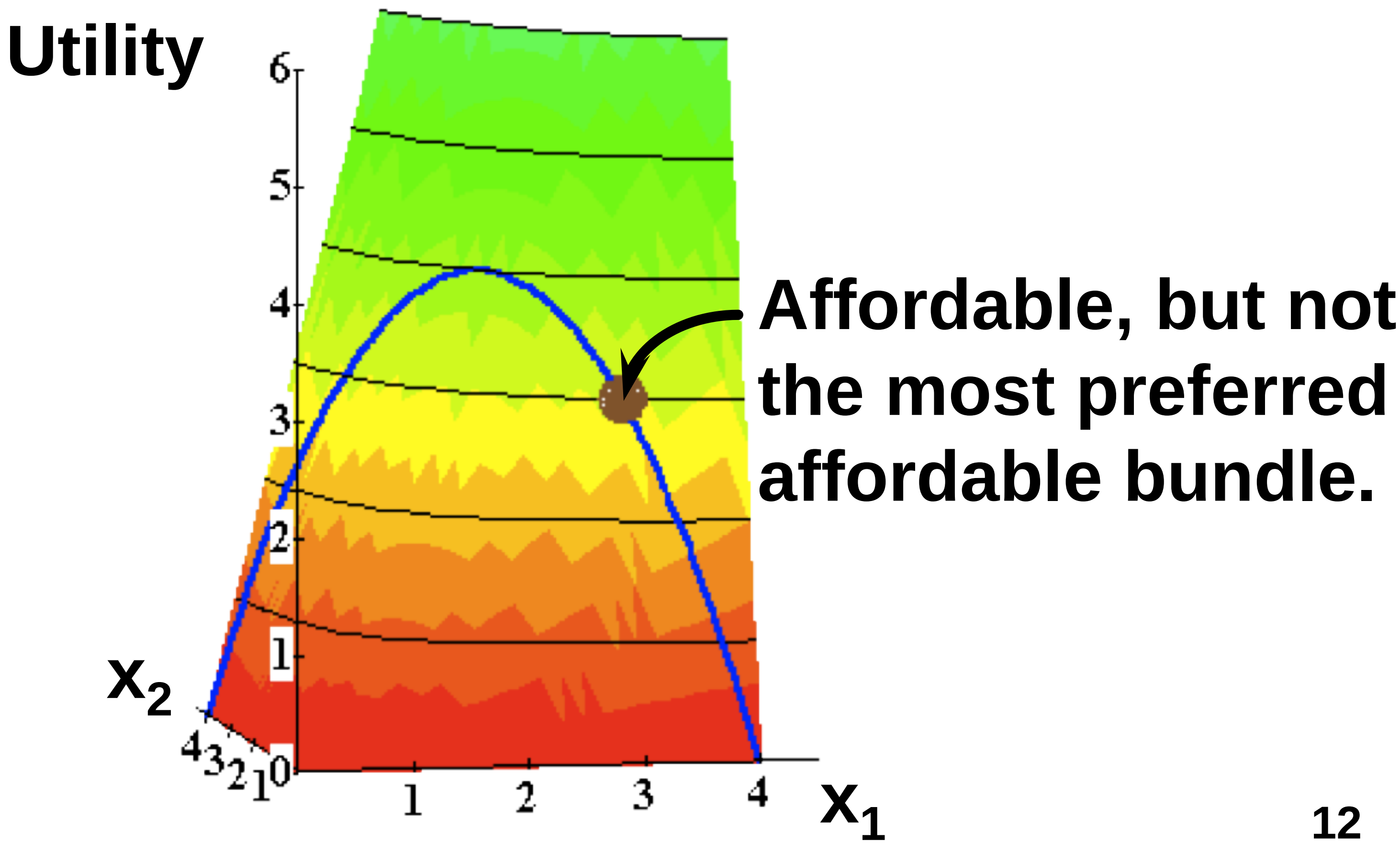
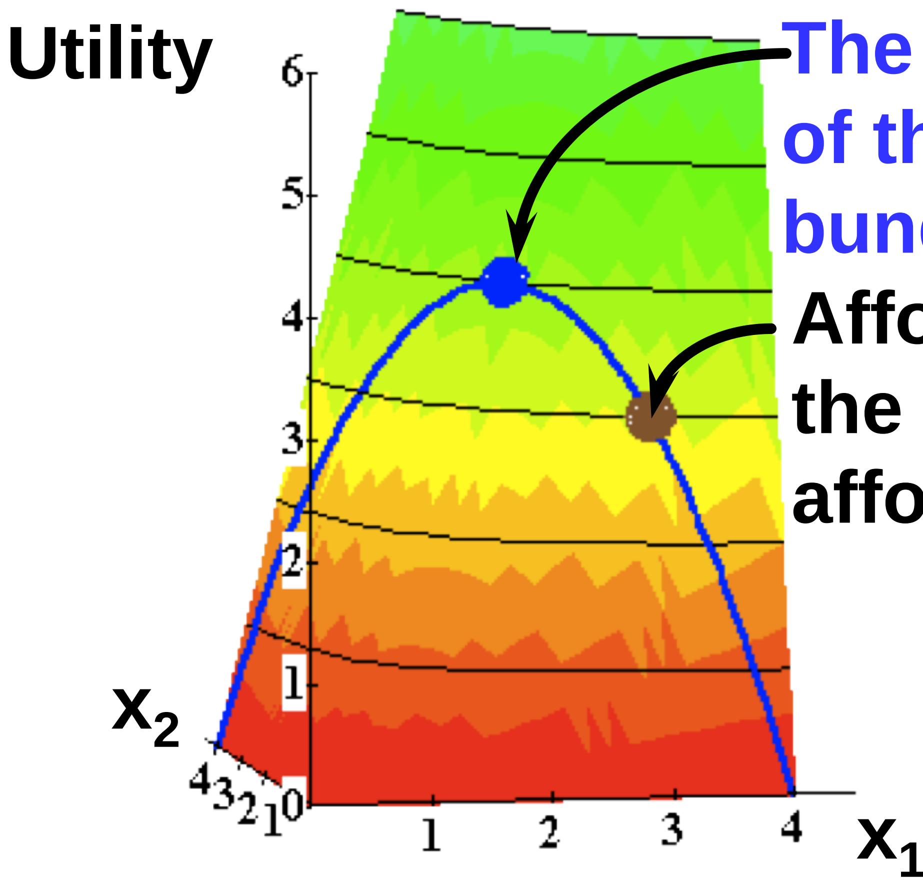


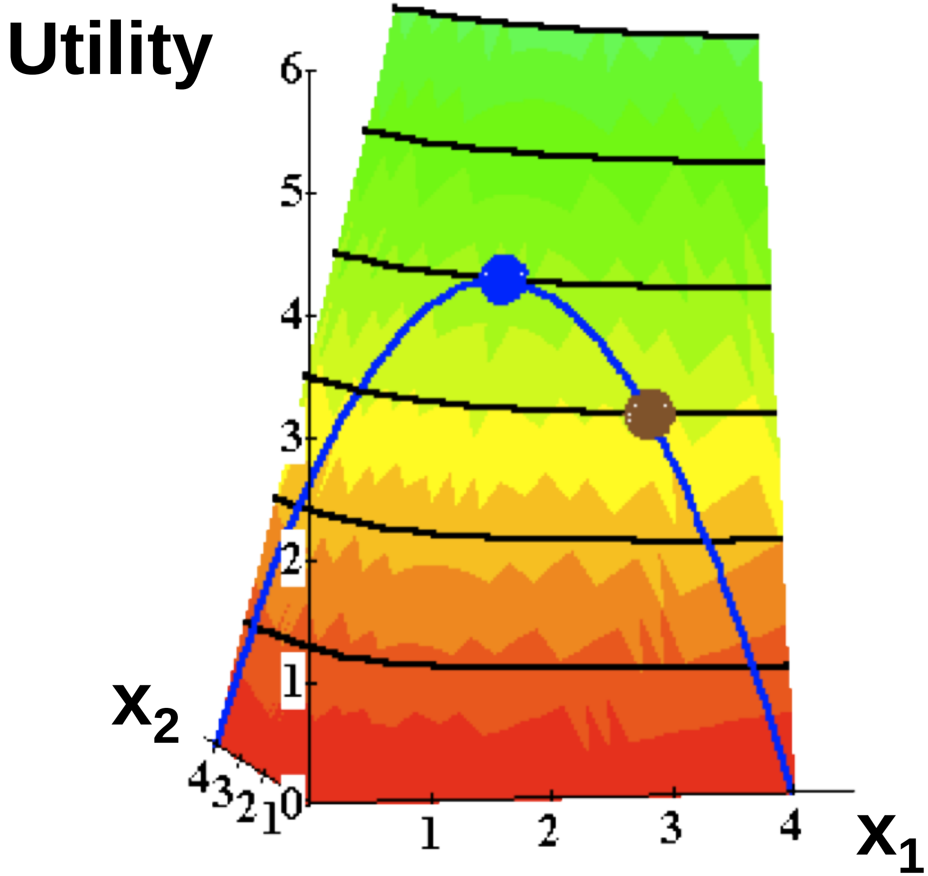
Rational Constrained Choice



The most preferred of the affordable bundles.

Affordable, but not the most preferred affordable bundle.

Rational Constrained Choice



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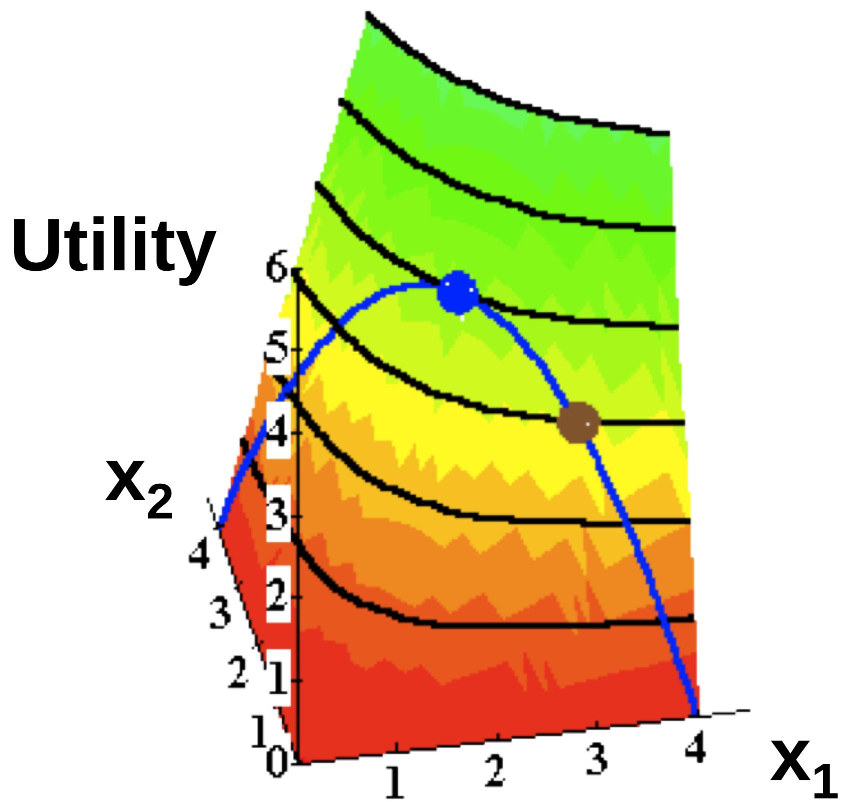

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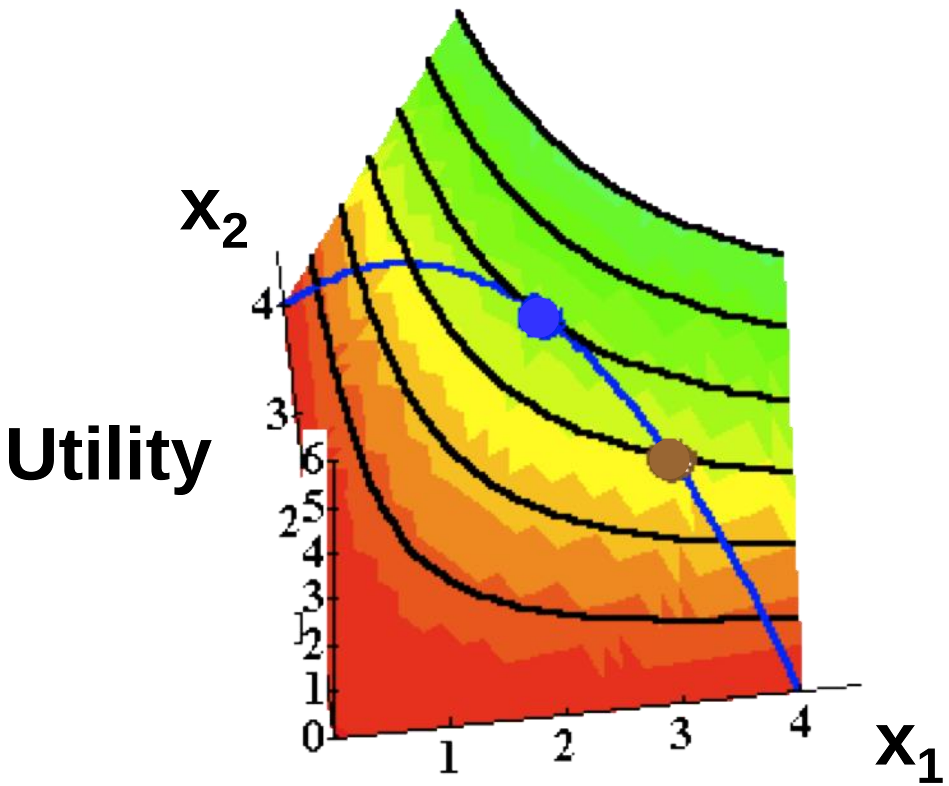

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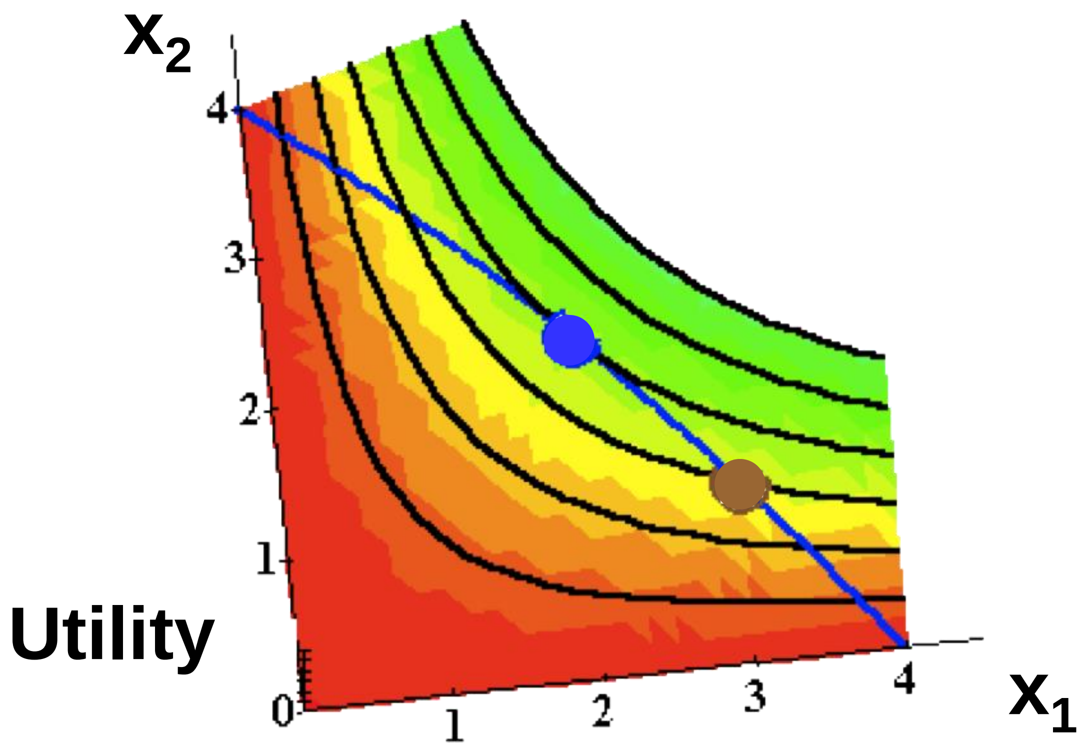

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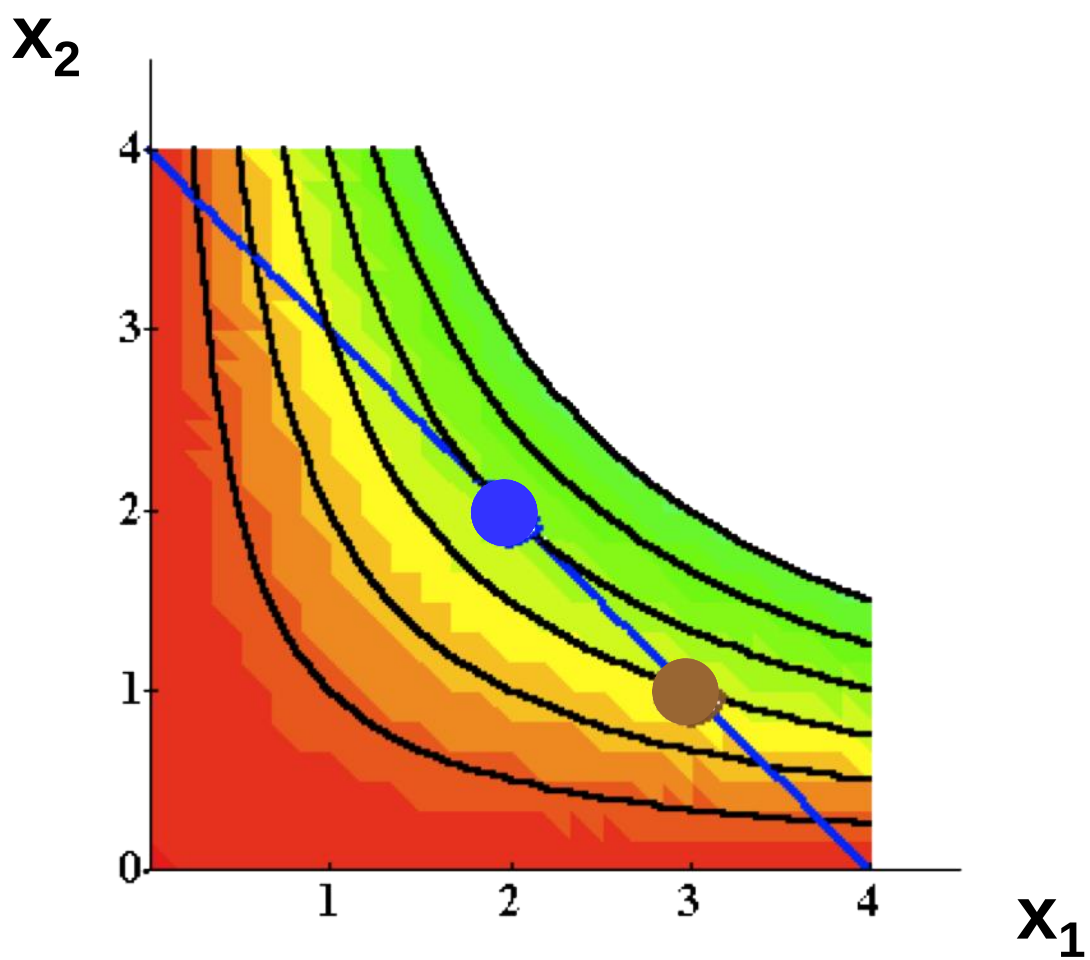

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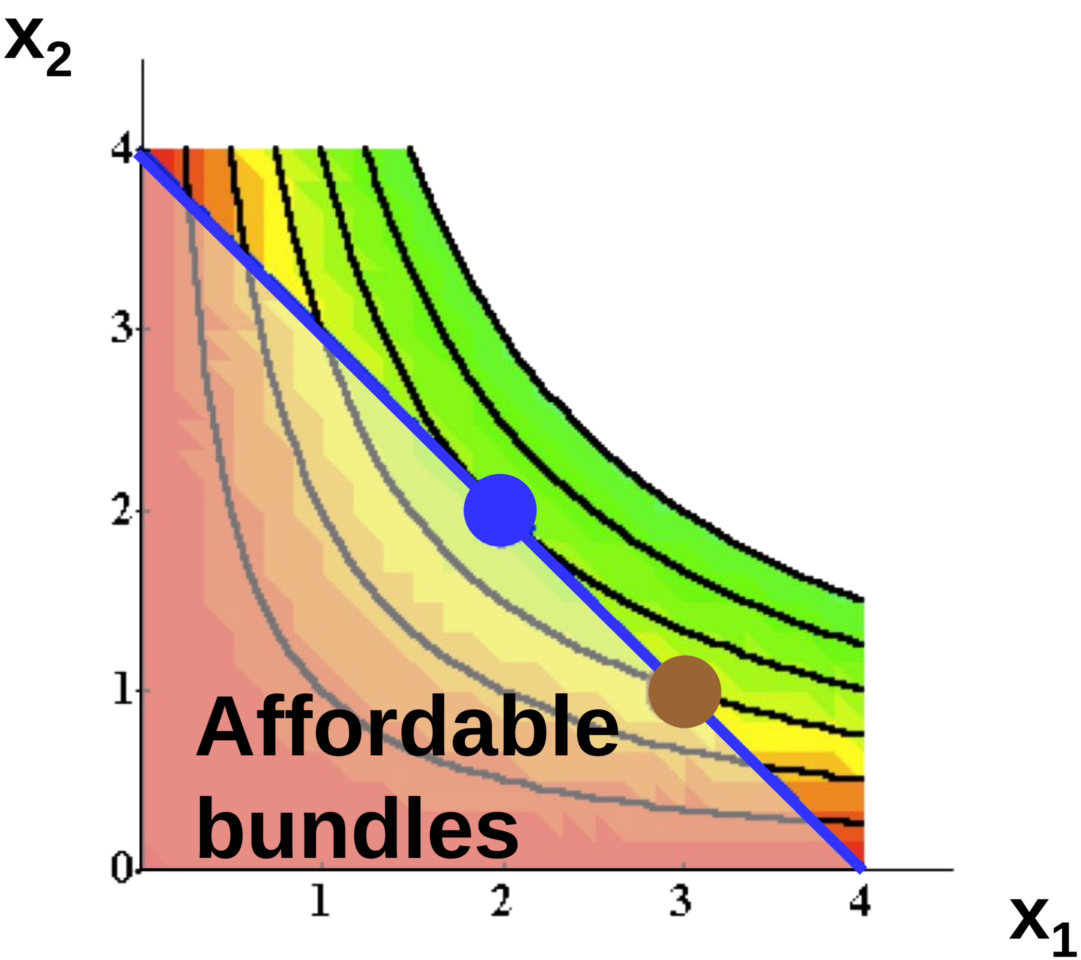

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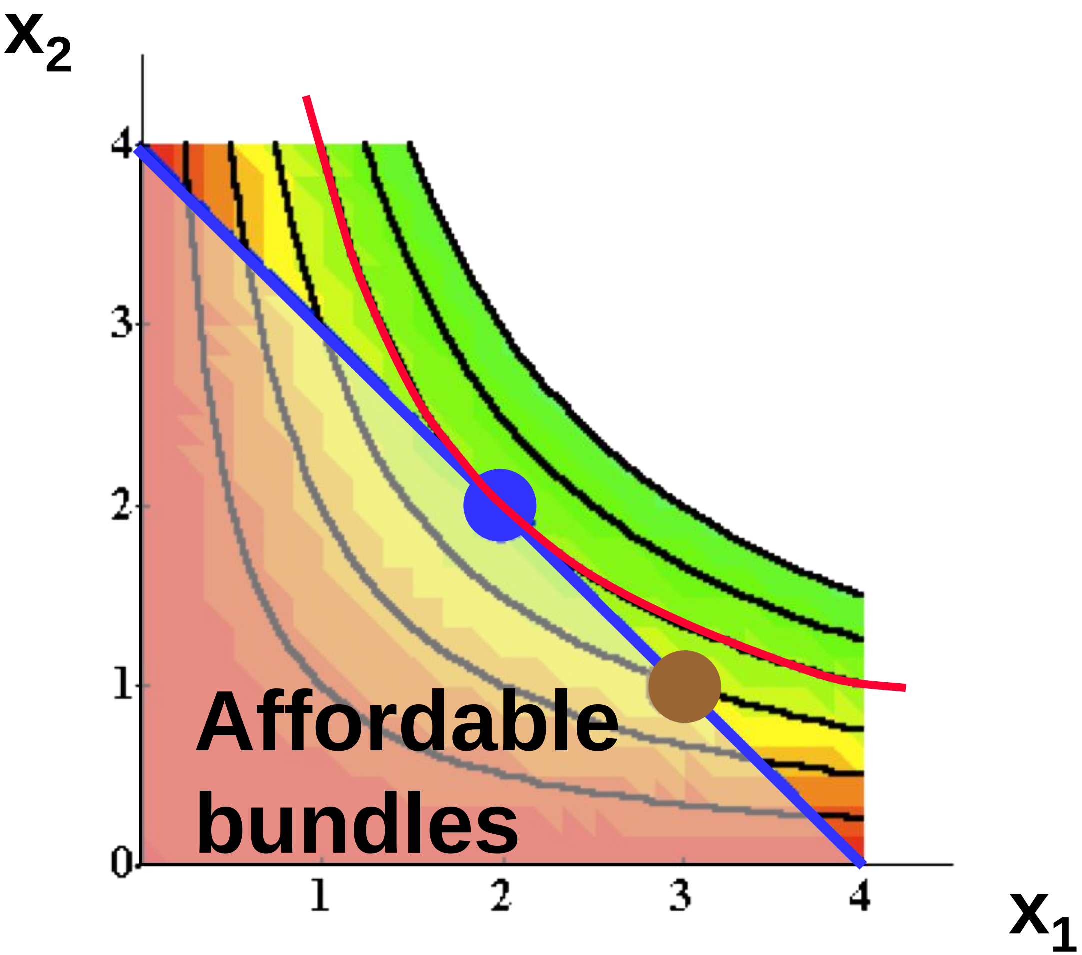

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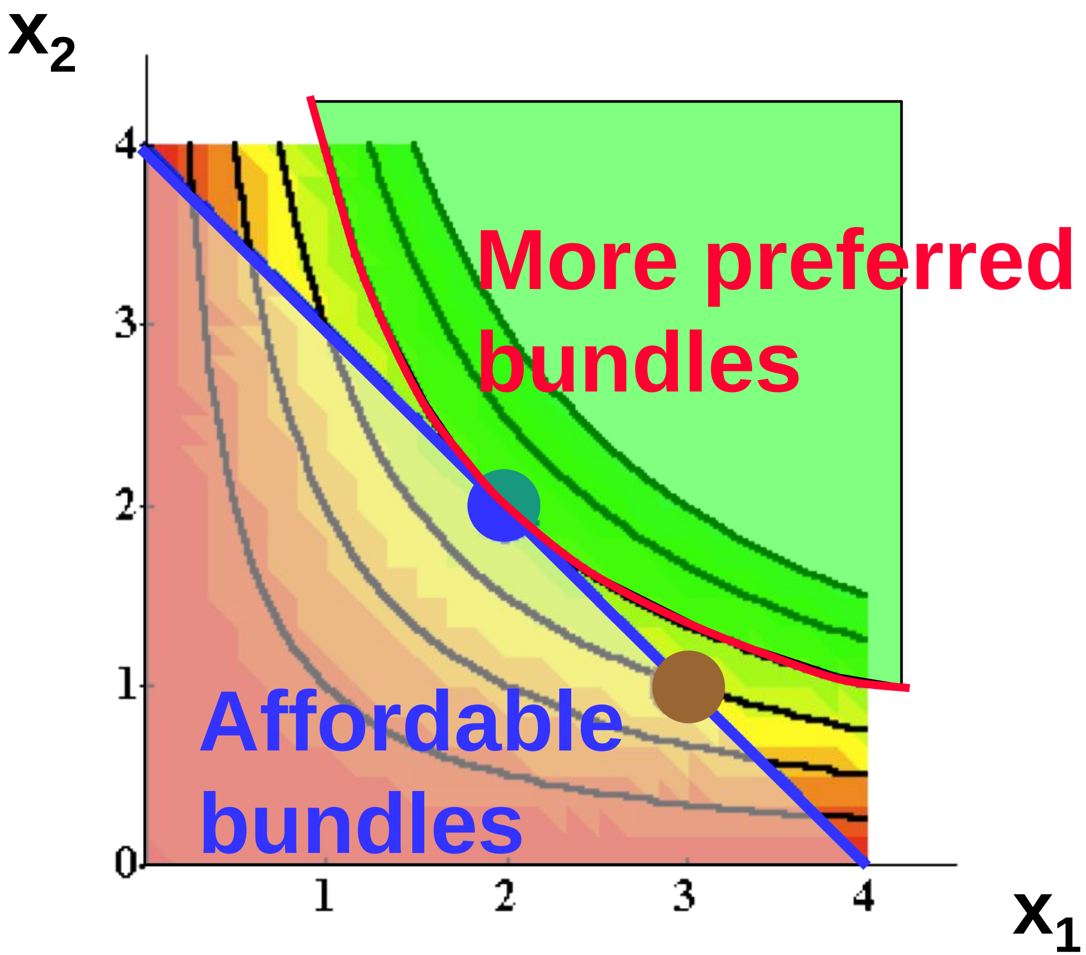

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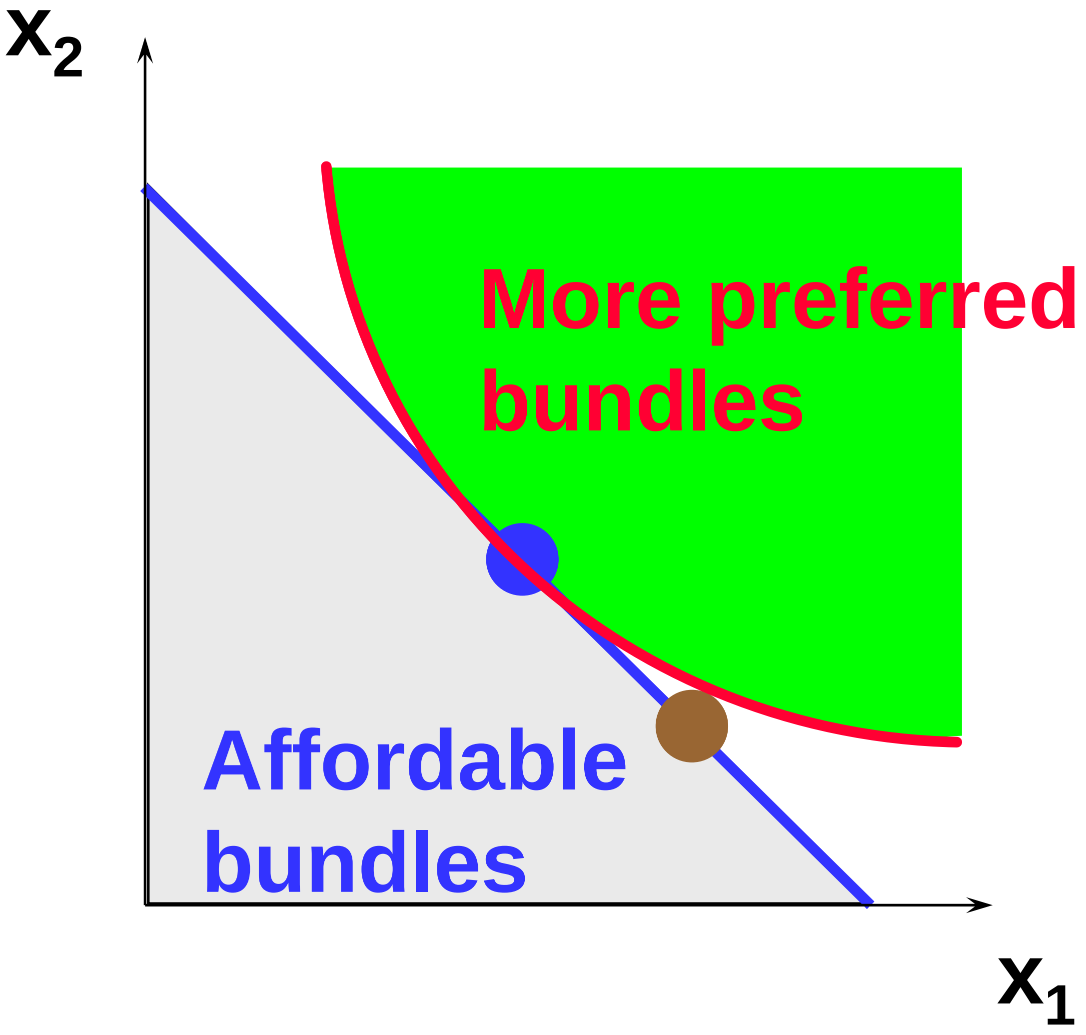

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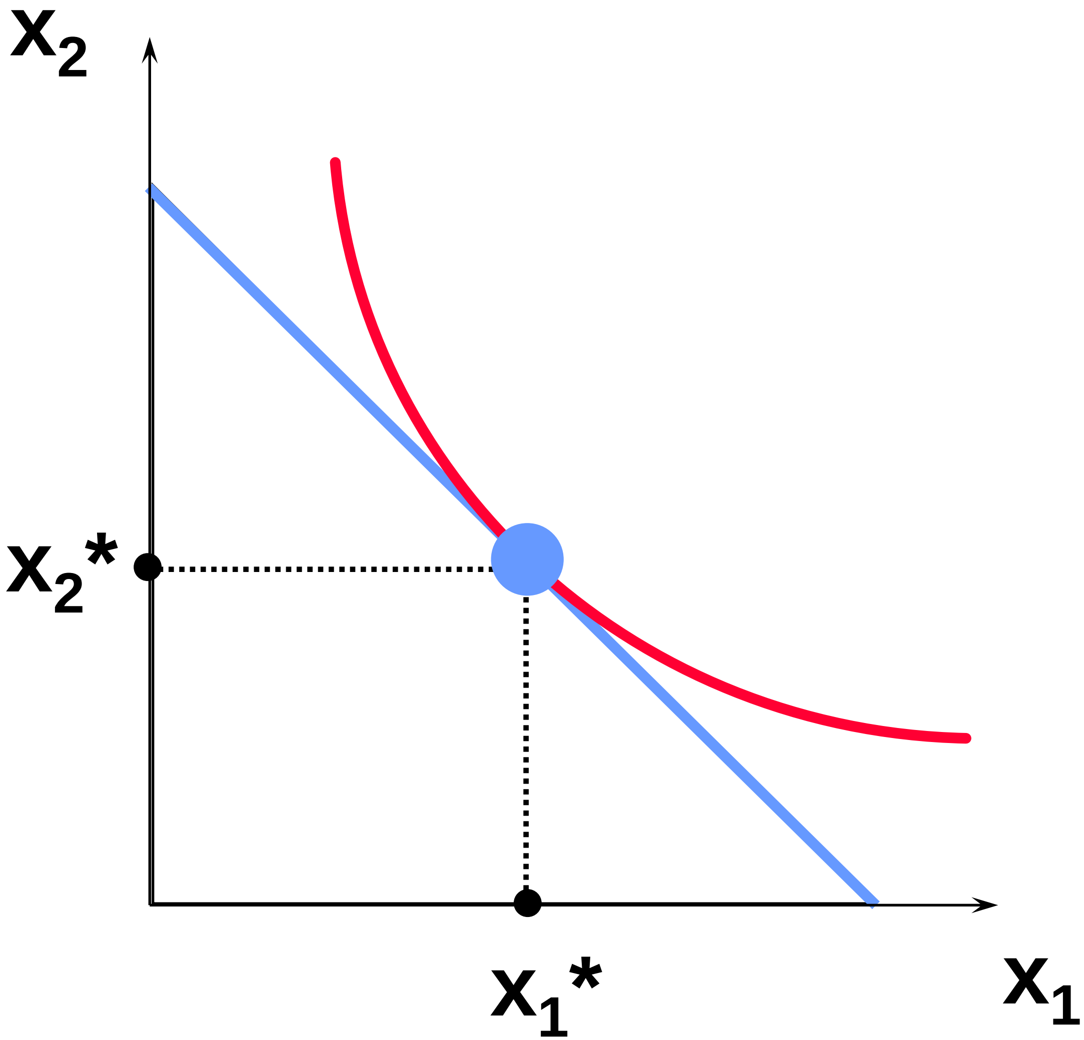
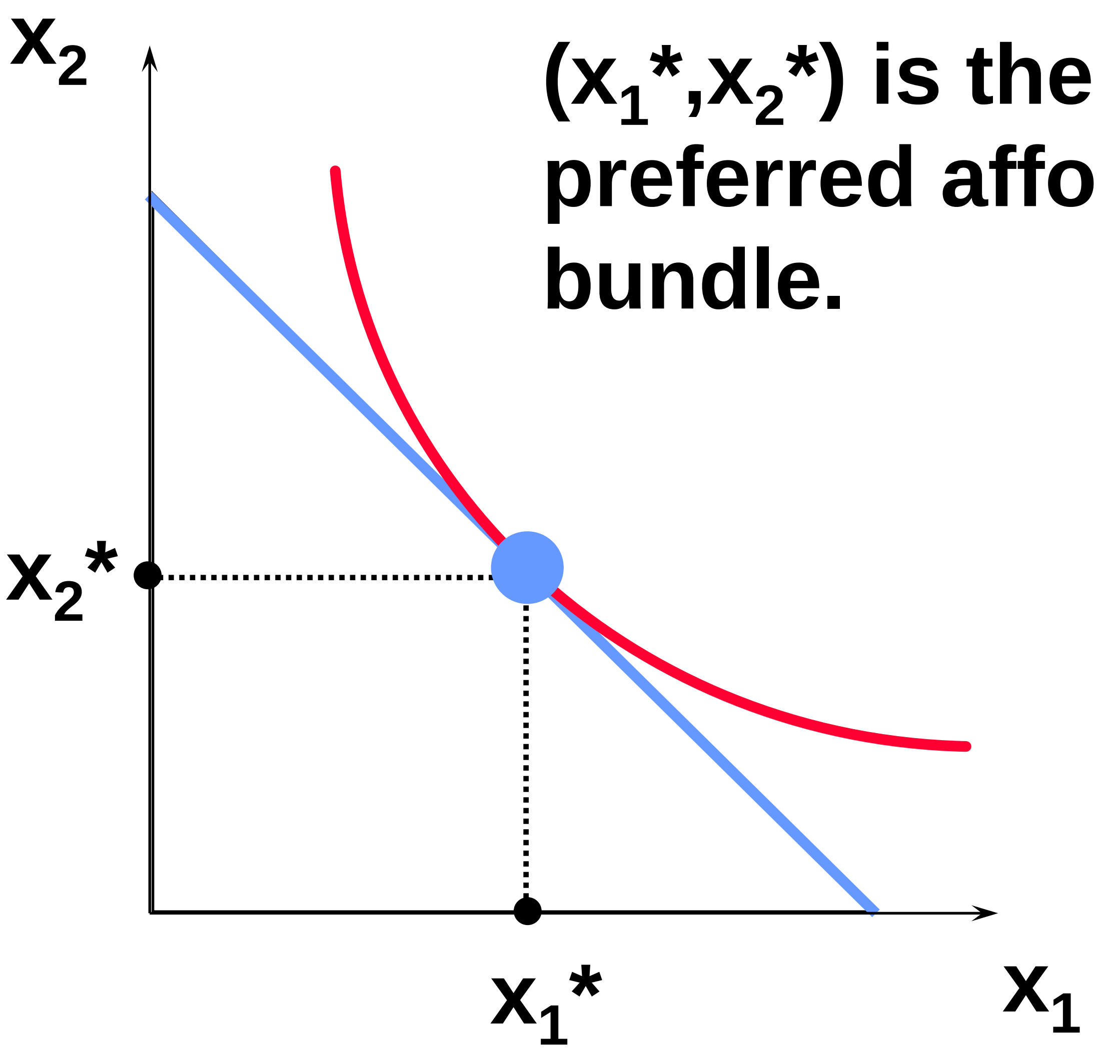


Rational Constrained Choice



(x_1^*, x_2^*) is the most preferred affordable bundle.

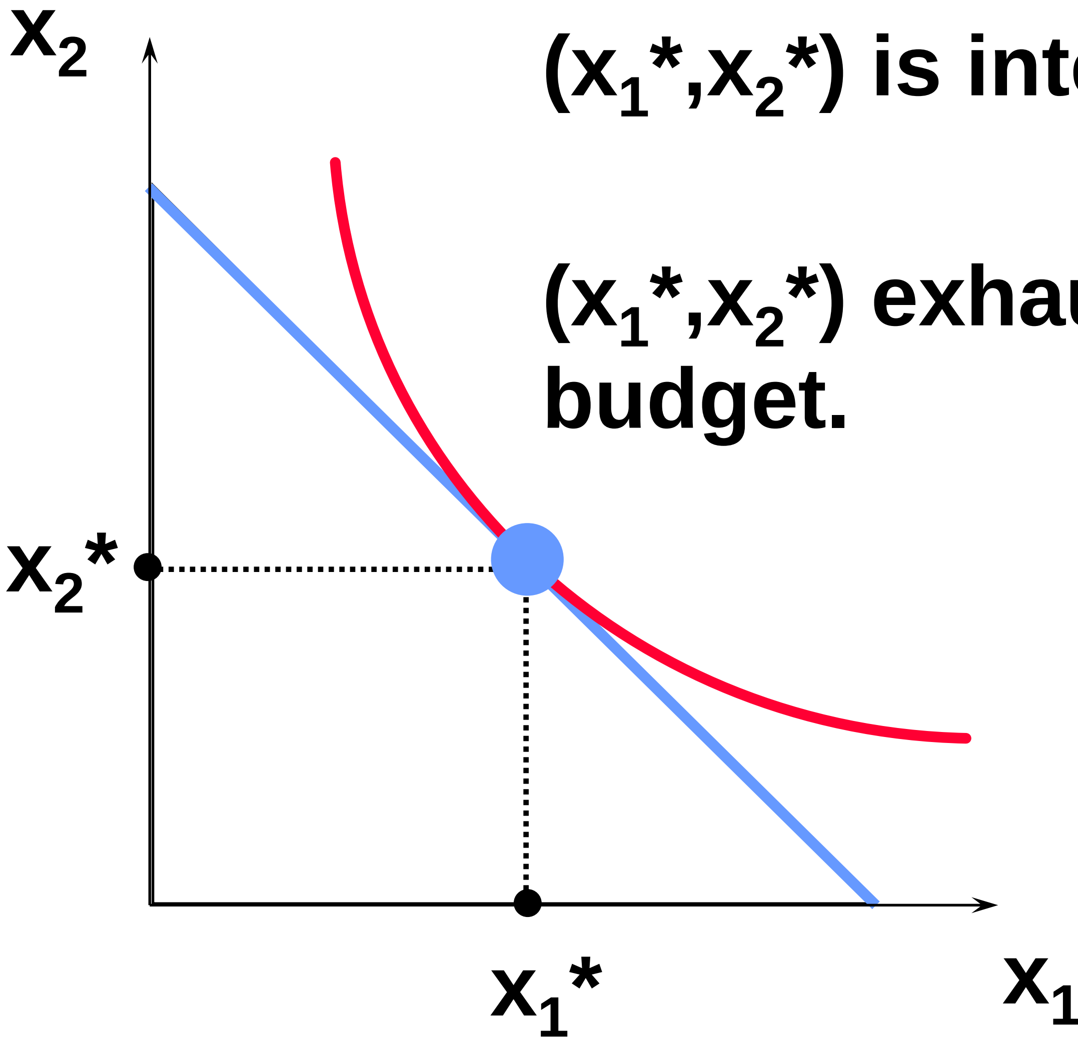
Rational Constrained Choice

- **The most preferred affordable bundle is called the consumer's ORDINARY DEMAND at the given prices and budget.**
- **Ordinary demands will be denoted by $x_1*(p_1,p_2,m)$ and $x_2*(p_1,p_2,m)$.**

Rational Constrained Choice

- **When $x_1^* > 0$ and $x_2^* > 0$ the demanded bundle is INTERIOR.**
- **If buying (x_1^*, x_2^*) costs \$m then the budget is exhausted.**

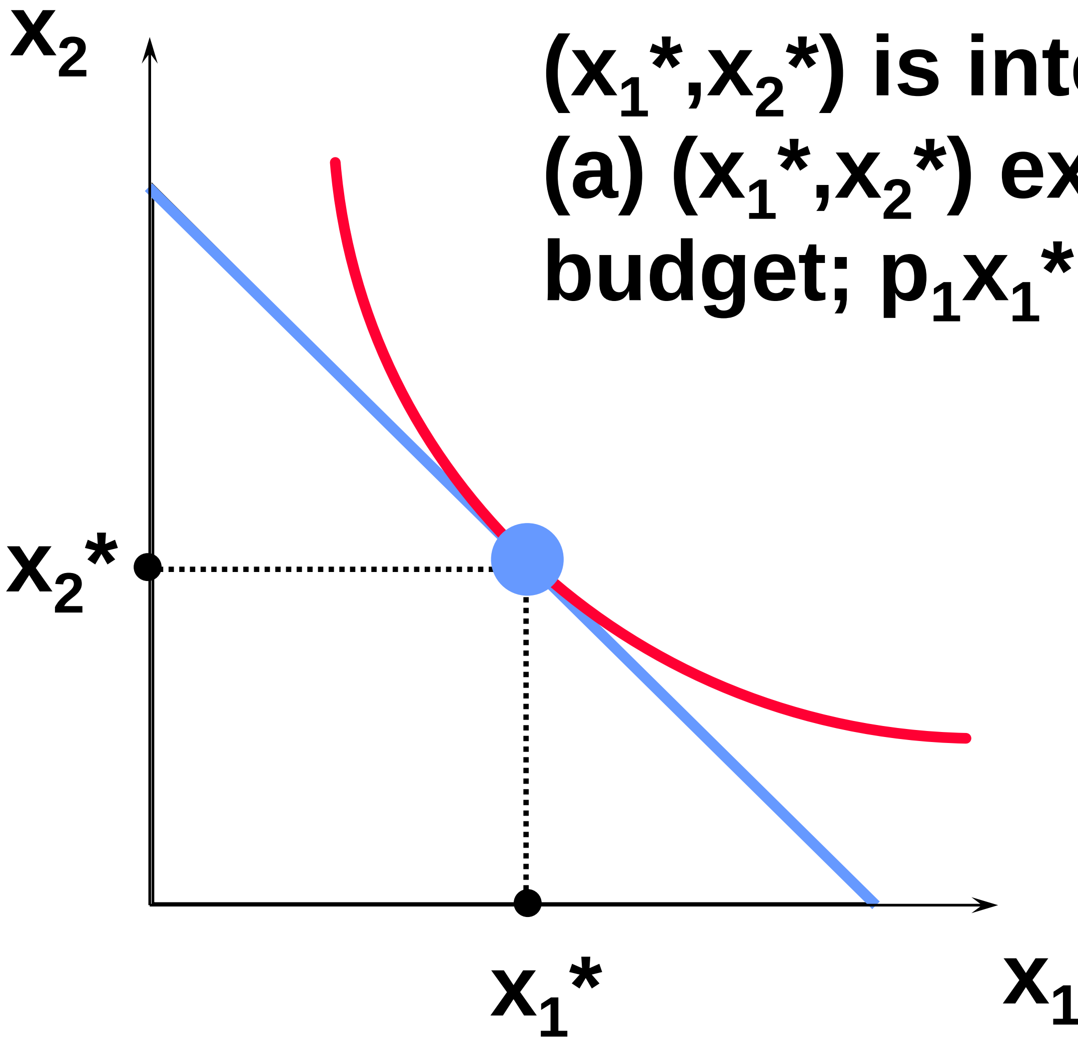
Rational Constrained Choice



(x_1^*, x_2^*) is interior.

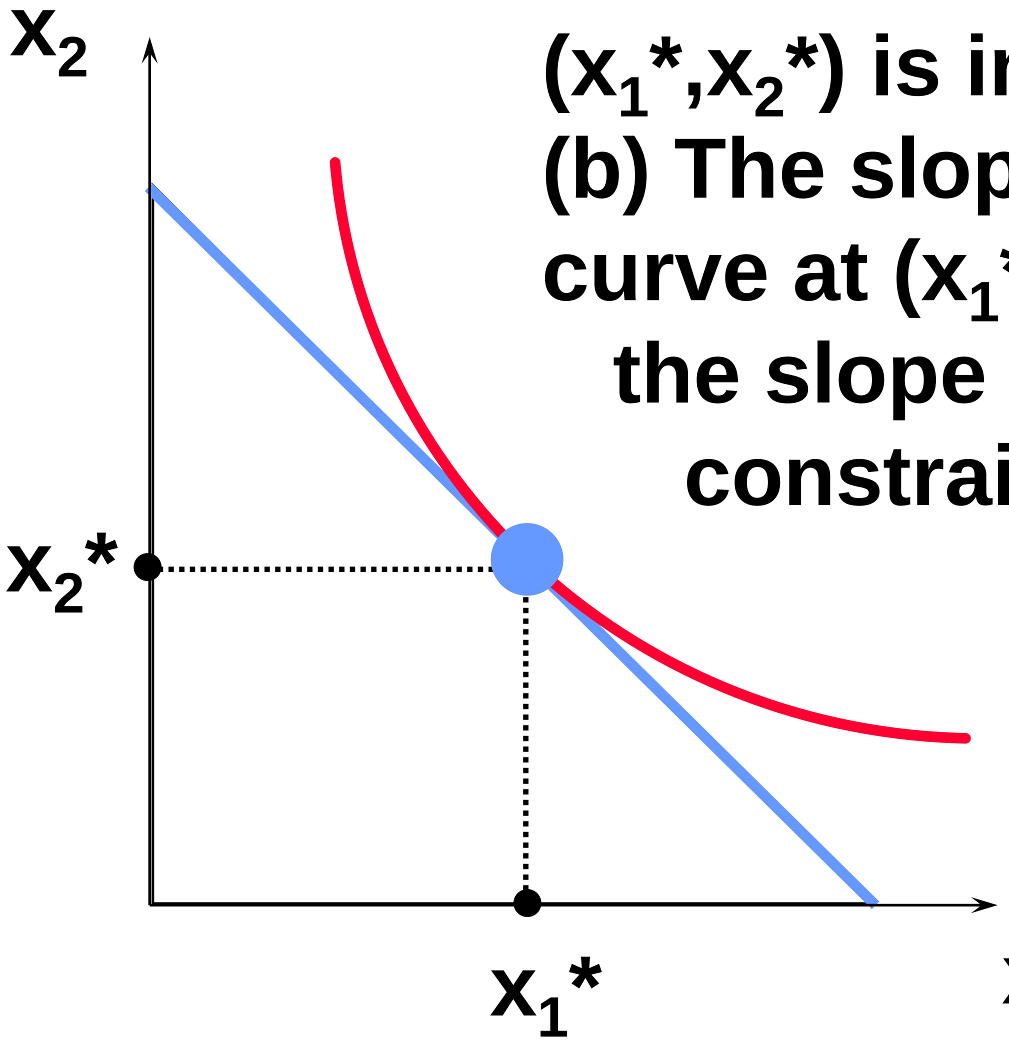
(x_1^*, x_2^*) exhausts the budget.

Rational Constrained Choice



(x_1^*,x_2^*) is interior.
(a) (x_1^*,x_2^*) exhausts the budget; $p_1x_1^* + p_2x_2^* = m$.

Rational Constrained Choice



(x_1^*, x_2^*) is interior .
(b) The slope of the indiff. curve at (x_1^*, x_2^*) equals the slope of the budget constraint.

Rational Constrained Choice

- **(x_1^*,x_2^*) satisfies two conditions:**
- **(a) the budget is exhausted;**

  $$p_1x_1^* + p_2x_2^* = m$$

- **(b) the slope of the budget constraint, $-p_1/p_2$, and the slope of the indifference curve containing (x_1^*,x_2^*) are equal at (x_1^*,x_2^*).**

Computing Ordinary Demands

- **How can this information be used to locate (x_1*,x_2*) for given p_1, p_2 and m?**

Computing Ordinary Demands - a Cobb-Douglas Example.

- **Suppose that the consumer has Cobb-Douglas preferences.**

$$U(x_1, x_2) = x_1^a x_2^b$$

Computing Ordinary Demands - a Cobb-Douglas Example.

- **Suppose that the consumer has Cobb-Douglas preferences.**

$$U(x_1, x_2) = x_1^a x_2^b$$

- **Then** $$MU_1 = \frac{\partial U}{\partial x_1} = a x_1^{a-1} x_2^b$$

$$MU_2 = \frac{\partial U}{\partial x_2} = b x_1^a x_2^{b-1}$$

Computing Ordinary Demands - a Cobb-Douglas Example.

- So the MRS is

$$\mathrm{MRS} = \frac{dx_2}{dx_1} = -\frac{\partial U/\partial x_1}{\partial U/\partial x_2} = -\frac{a x_1^{a-1} x_2^{b}}{b x_1^{a} x_2^{b-1}} = -\frac{a x_2}{b x_1}.$$

Computing Ordinary Demands - a Cobb-Douglas Example.

- So the MRS is

$$MRS = \frac{dx_2}{dx_1} = -\frac{\partial U/\partial x_1}{\partial U/\partial x_2} = -\frac{ax_1^{a-1}x_2^b}{bx_1^a x_2^{b-1}} = -\frac{ax_2}{bx_1}.$$

- At (x_1^*, x_2^*), MRS = $-p_1/p_2$ so

Computing Ordinary Demands - a Cobb-Douglas Example.

- **So the MRS is**

$$\mathrm{MRS} = \frac{dx_2}{dx_1} = -\frac{\partial U/\partial x_1}{\partial U/\partial x_2} = -\frac{a x_1^{a-1} x_2^{b}}{b x_1^{a} x_2^{b-1}} = -\frac{a x_2}{b x_1}.$$

- **At (x_1^*, x_2^*), MRS = $-p_1/p_2$ so**

$$-\frac{a x_2^*}{b x_1^*} = -\frac{p_1}{p_2} \quad \Rightarrow \quad x_2^* = \frac{b p_1}{a p_2} x_1^*. \qquad \textbf{(A)}$$

Computing Ordinary Demands - a Cobb-Douglas Example.

- (x_1^*, x_2^*) also exhausts the budget so

$$p_1 x_1^* + p_2 x_2^* = m. \qquad \textbf{(B)}$$

Computing Ordinary Demands - a Cobb-Douglas Example.

- **So now we know that**

$$x_2^* = \frac{bp_1}{ap_2}x_1^* \qquad \textbf{(A)}$$

$$p_1x_1^* + p_2x_2^* = m. \qquad \textbf{(B)}$$

Computing Ordinary Demands - a Cobb-Douglas Example.

- **So now we know that**

$$x_2^* = \frac{bp_1}{ap_2} x_1^* \qquad \textbf{(A)}$$

Substitute

$$p_1 x_1^* + p_2 x_2^* = m. \qquad \textbf{(B)}$$

Computing Ordinary Demands - a Cobb-Douglas Example.

- **So now we know that**

$$x_2^* = \frac{bp_1}{ap_2} x_1^* \qquad \textbf{(A)}$$

Substitute

$$p_1 x_1^* + p_2 x_2^* = m. \qquad \textbf{(B)}$$

and get

$$p_1 x_1^* + p_2 \frac{bp_1}{ap_2} x_1^* = m.$$

This simplifies to ….

Computing Ordinary Demands - a Cobb-Douglas Example.

$$x_1^* = \frac{am}{(a+b)p_1}.$$

Computing Ordinary Demands - a Cobb-Douglas Example.

$$x_1^* = \frac{am}{(a+b)p_1}.$$

Substituting for x_1* in

$$p_1x_1^* + p_2x_2^* = m$$

then gives

$$x_2^* = \frac{bm}{(a+b)p_2}.$$

Computing Ordinary Demands - a Cobb-Douglas Example.

So we have discovered that the most preferred affordable bundle for a consumer with Cobb-Douglas preferences

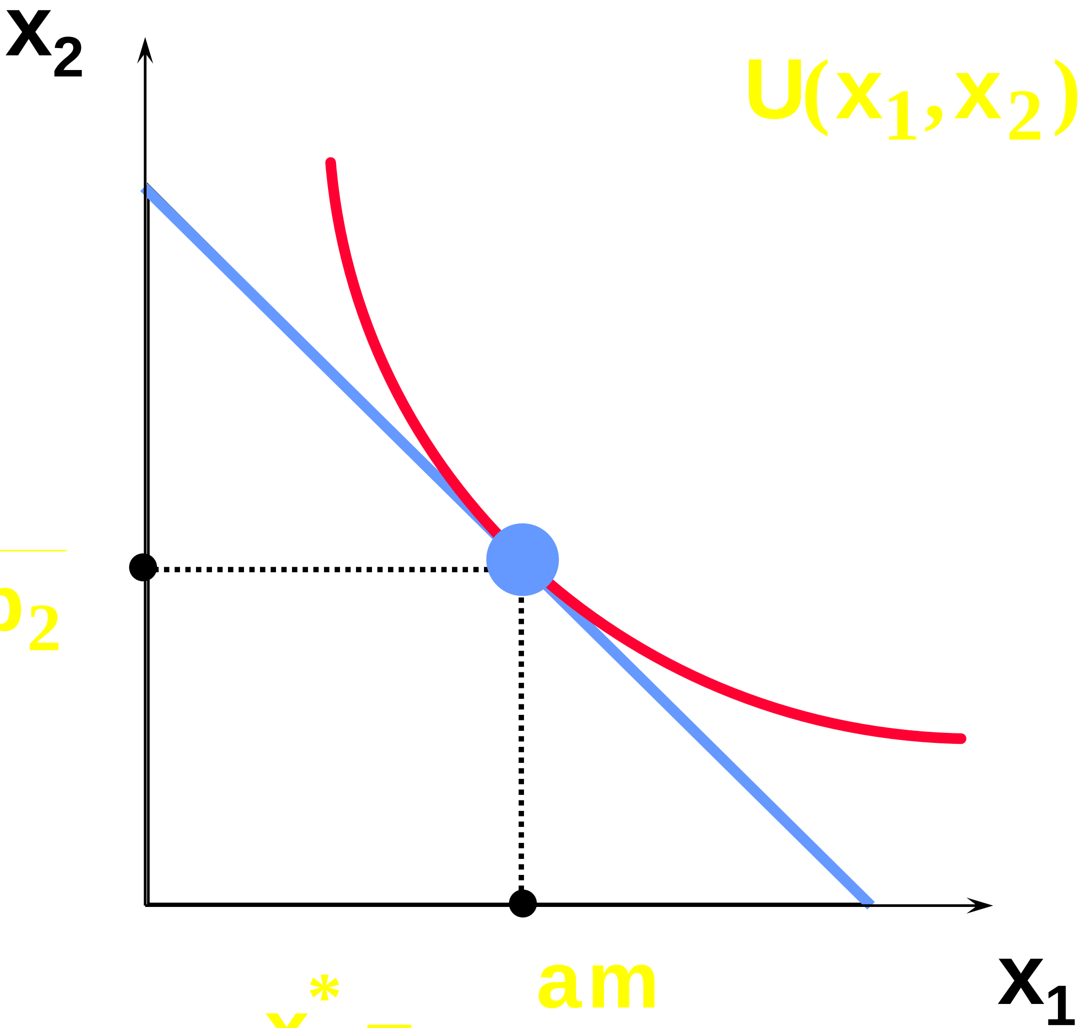
$$U(x_1, x_2) = x_1^a x_2^b$$

is

$$(x_1^*, x_2^*) = \left(\frac{am}{(a+b)p_1}, \frac{bm}{(a+b)p_2} \right).$$

Computing Ordinary Demands - a Cobb-Douglas Example.

$$U(x_1, x_2) = x_1^a x_2^b$$



$$x_2^* = \frac{bm}{(a+b)p_2}$$

$$x_1^* = \frac{am}{(a+b)p_1}$$

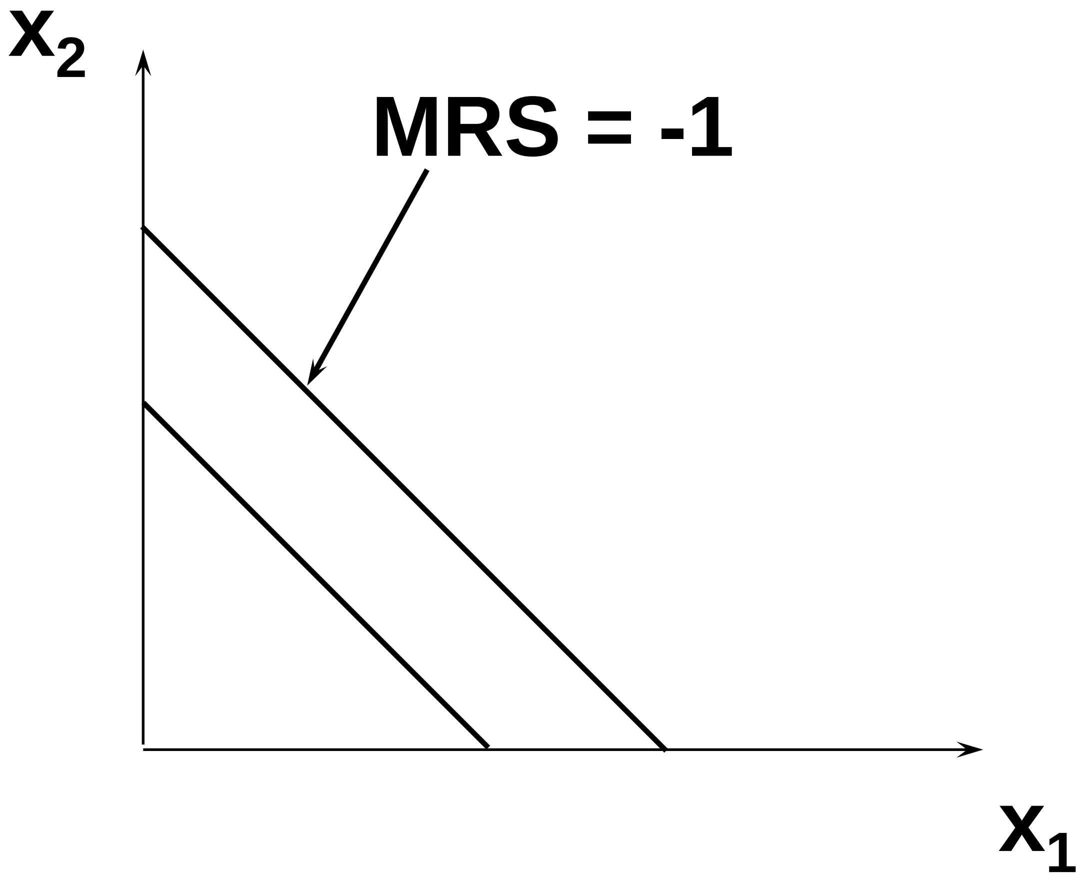
Rational Constrained Choice

- **When $x_1^* > 0$ and $x_2^* > 0$**
  and (x_1^*, x_2^*) exhausts the budget,
  and indifference curves have no 'kinks', the ordinary demands are obtained by solving:
- **(a) $p_1x_1^* + p_2x_2^* = y$**
- **(b) the slopes of the budget constraint, $-p_1/p_2$, and of the indifference curve containing (x_1^*, x_2^*) are equal at (x_1^*, x_2^*).**

Rational Constrained Choice

- **But what if $x_1^* = 0$?**
- **Or if $x_2^* = 0$?**
- **If either $x_1^* = 0$ or $x_2^* = 0$ then the ordinary demand (x_1^*, x_2^*) is at a corner solution to the problem of maximizing utility subject to a budget constraint.**

Examples of Corner Solutions -- the Perfect Substitutes Case



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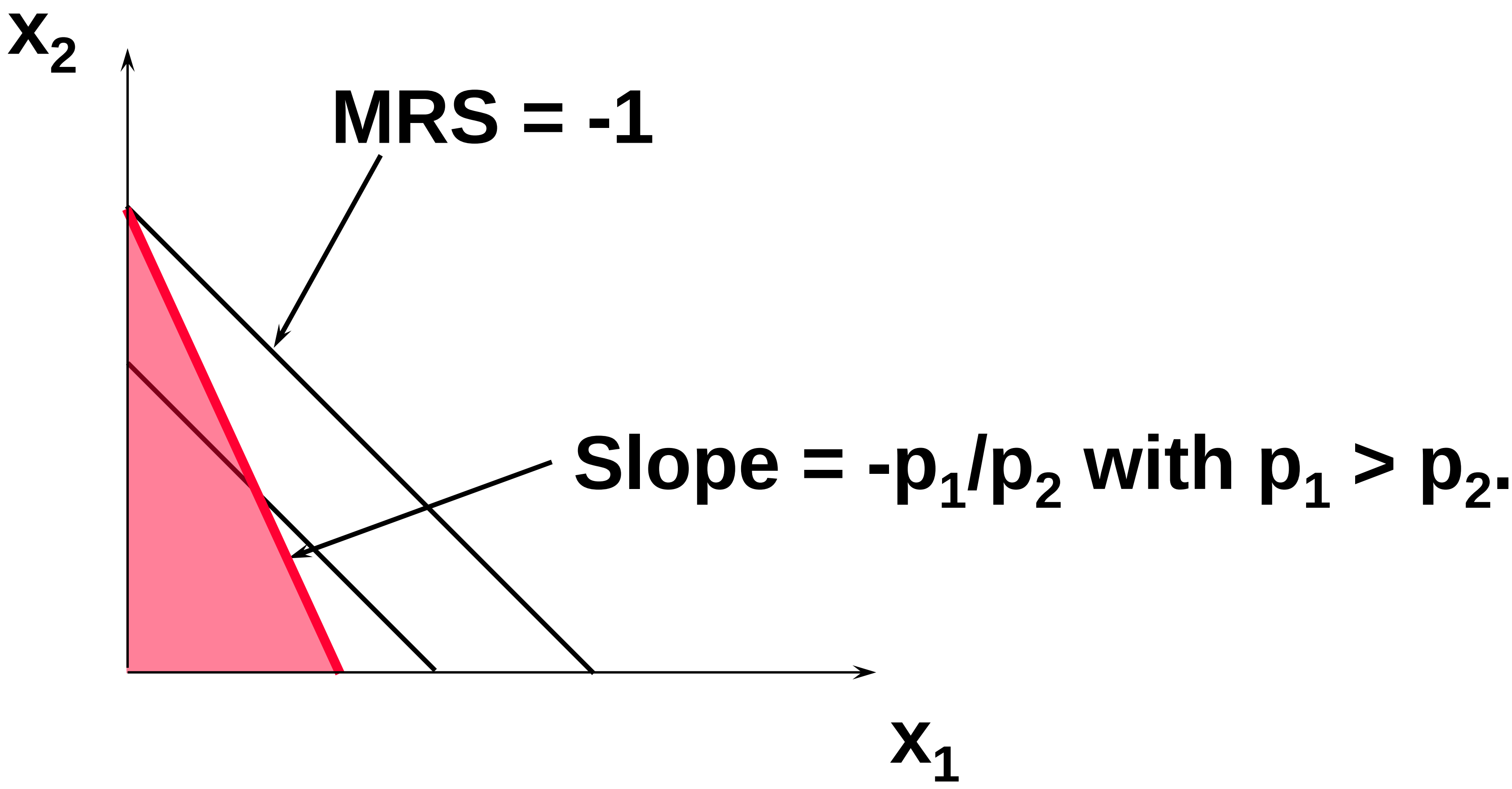

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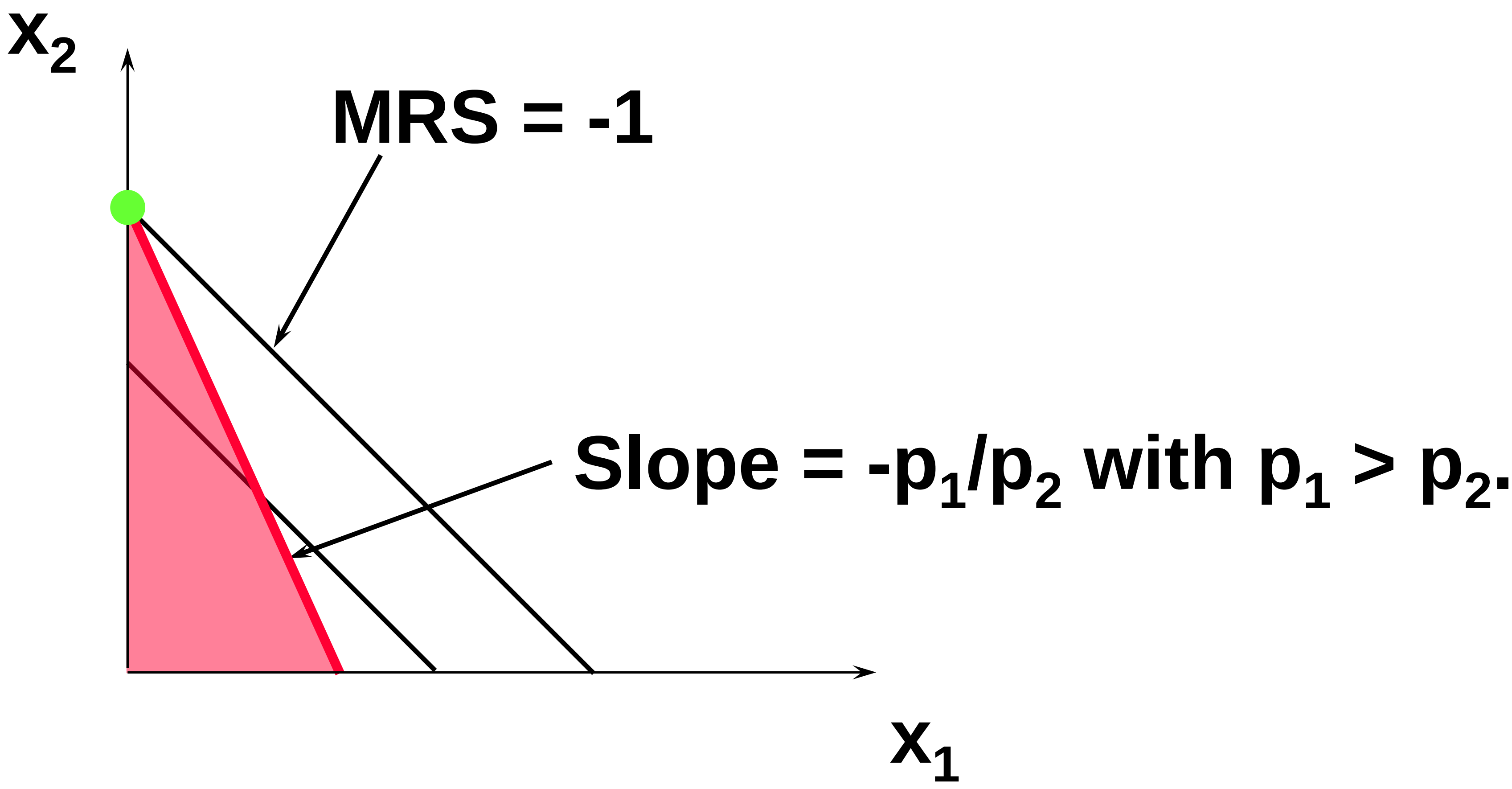

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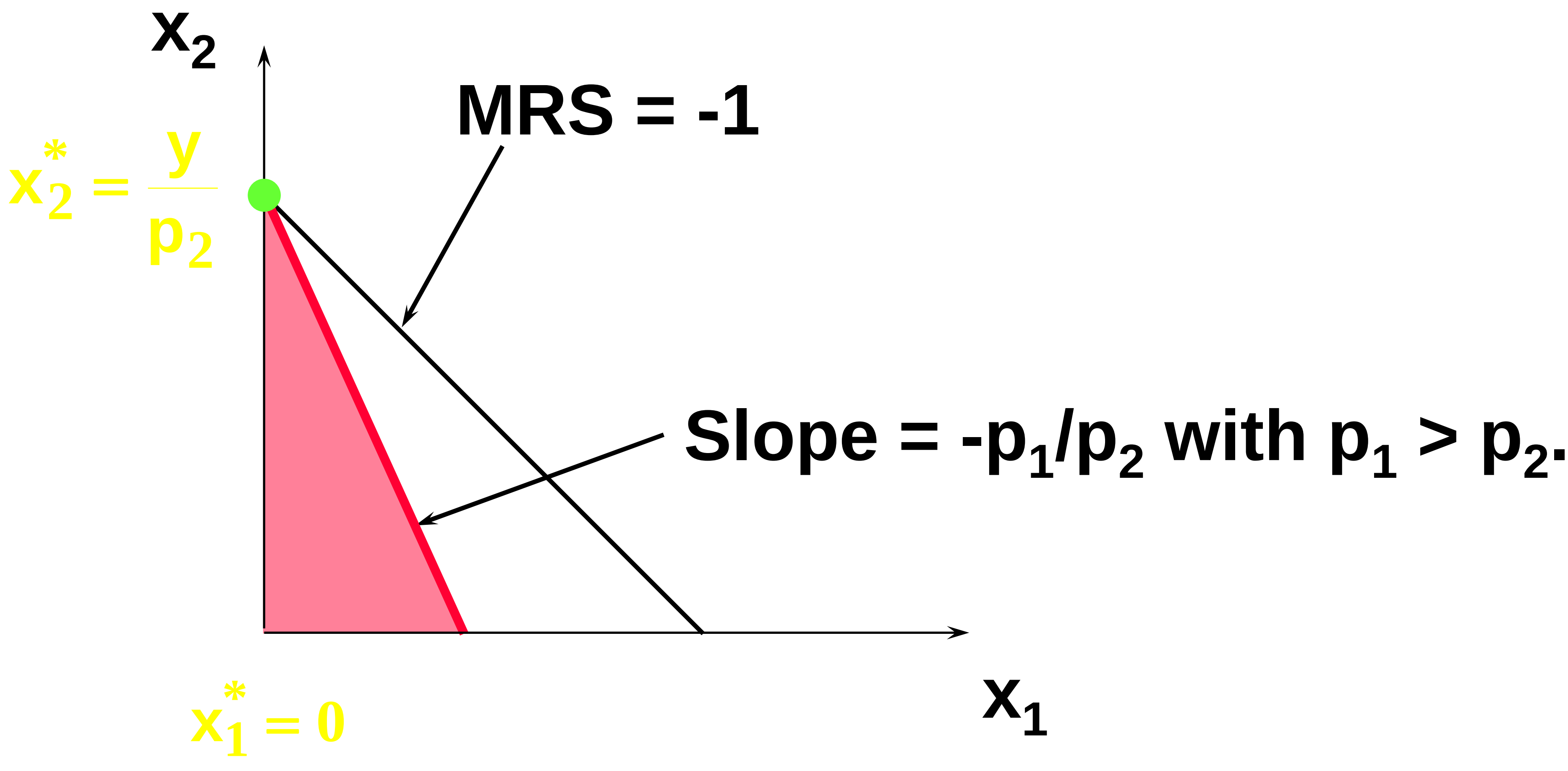
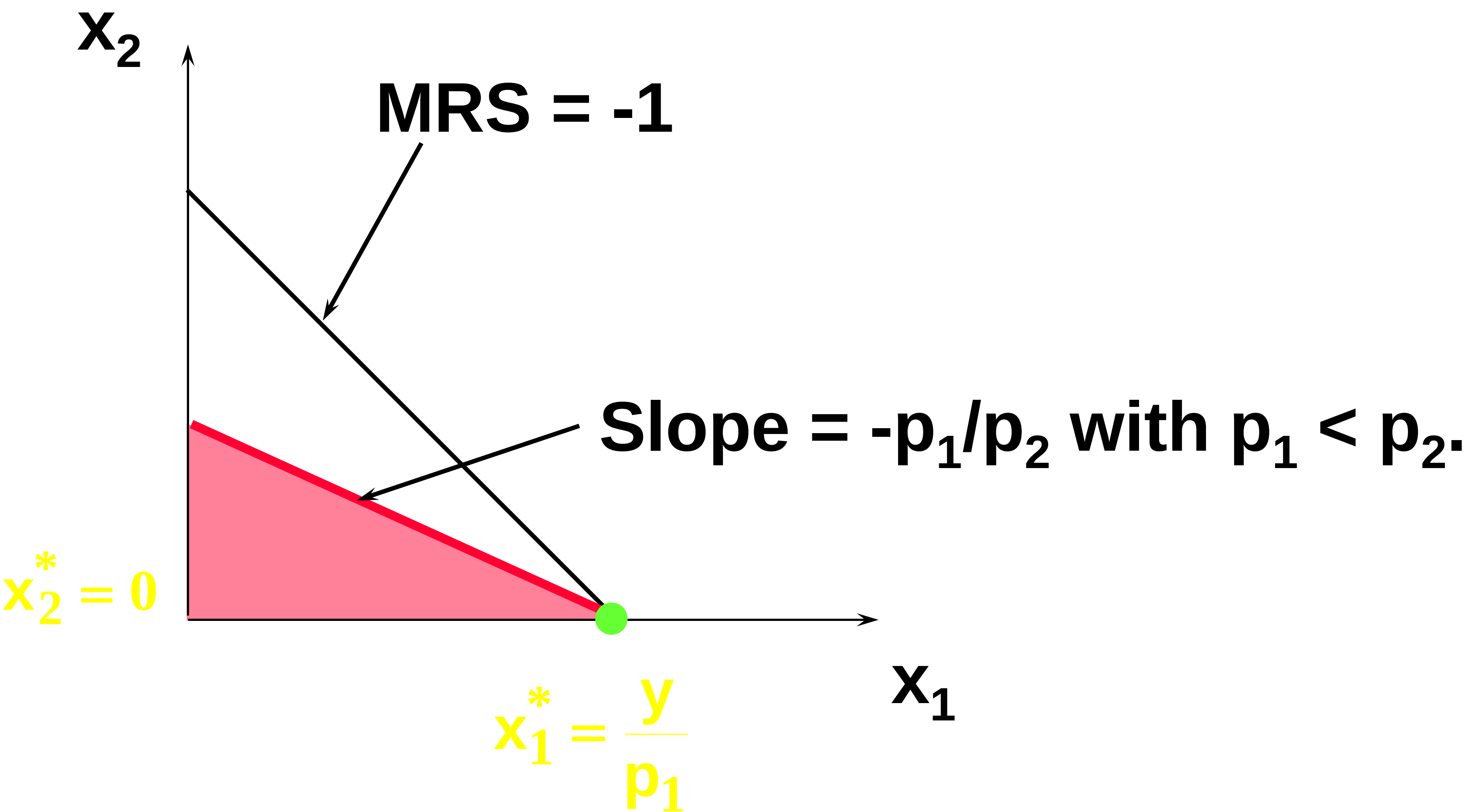


Examples of Corner Solutions -- the Perfect Substitutes Case



Examples of Corner Solutions -- the Perfect Substitutes Case

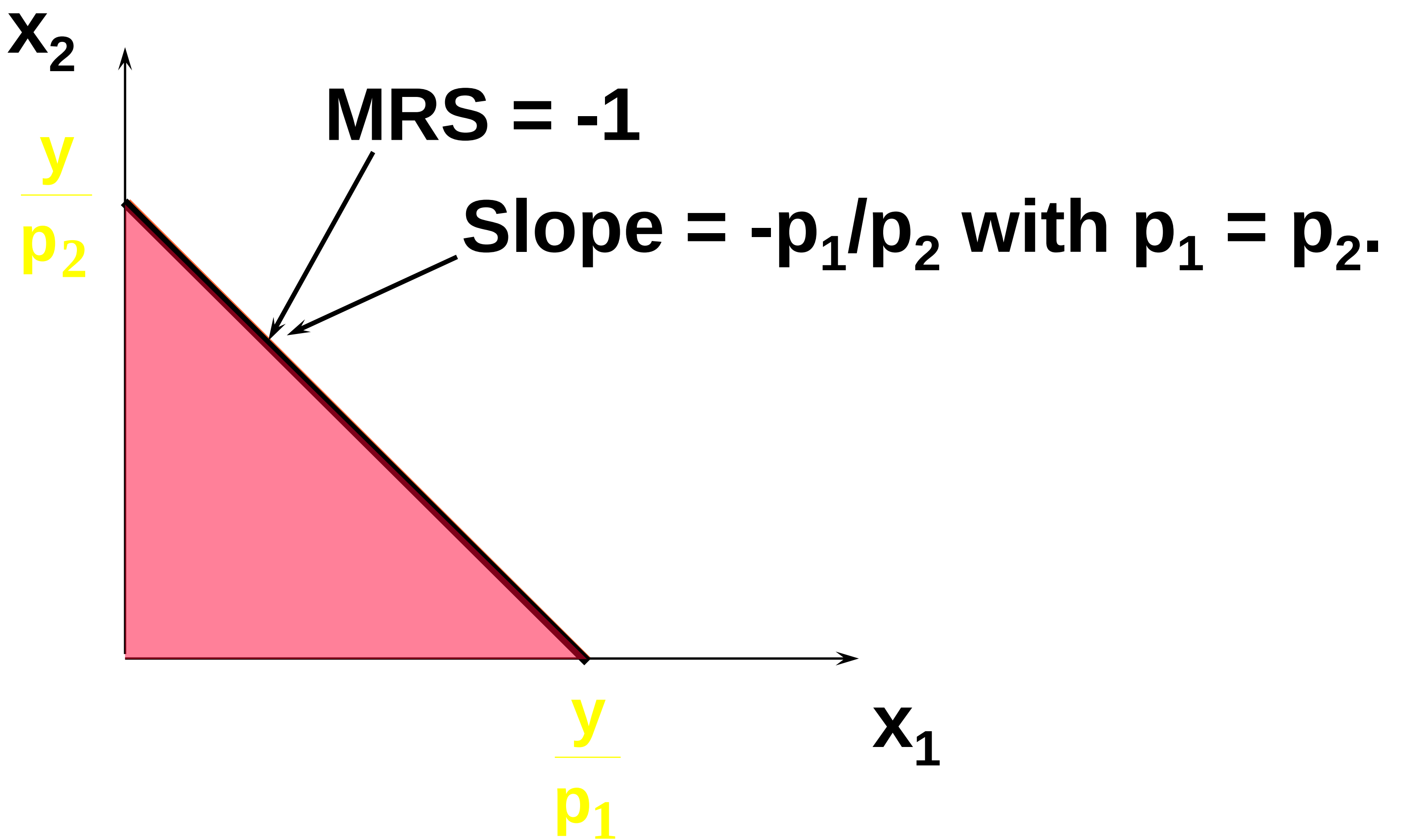
So when $U(x_1,x_2) = x_1 + x_2$, the most preferred affordable bundle is (x_1^*,x_2^*) where

$$(x_1^*,x_2^*) = \left(\frac{y}{p_1},0\right) \quad \text{if } p_1 < p_2$$

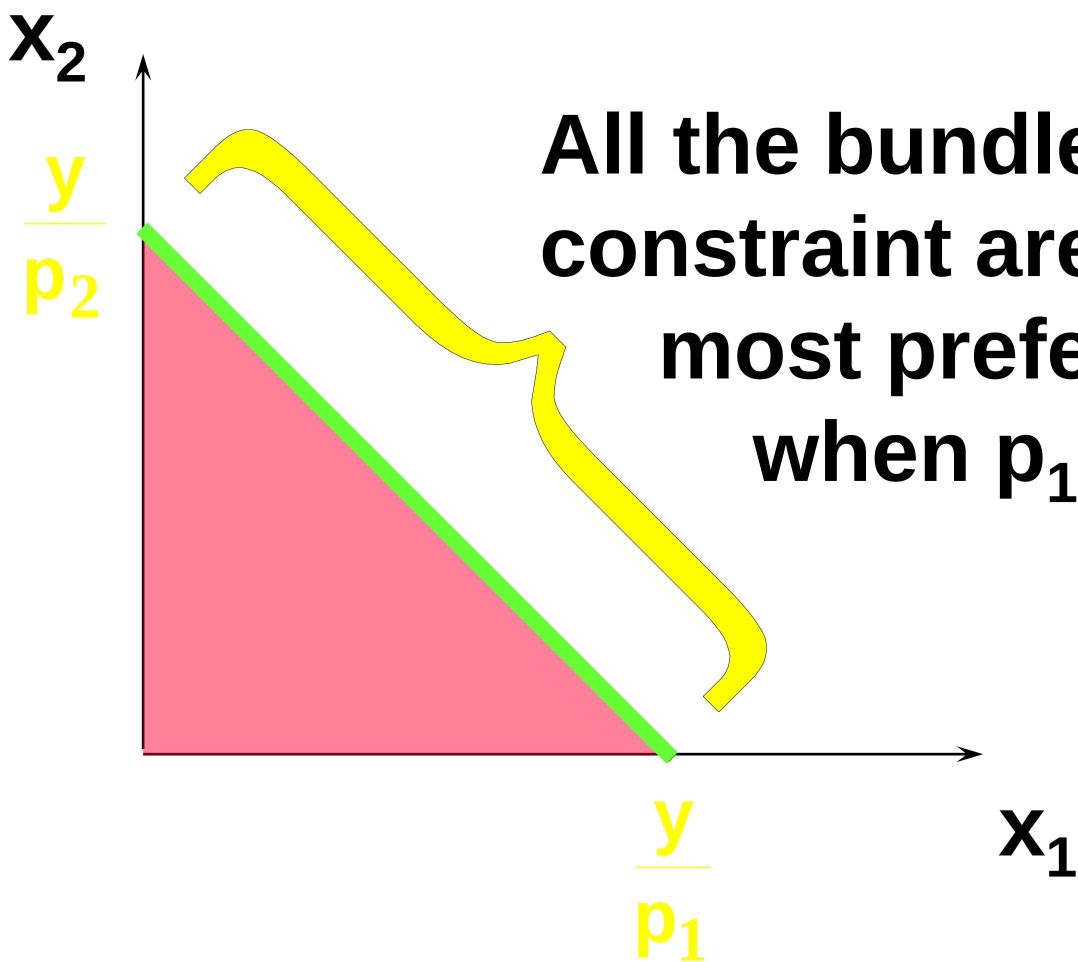
and

$$(x_1^*,x_2^*) = \left(0,\frac{y}{p_2}\right) \quad \text{if } p_1 > p_2.$$

Examples of Corner Solutions -- the Perfect Substitutes Case

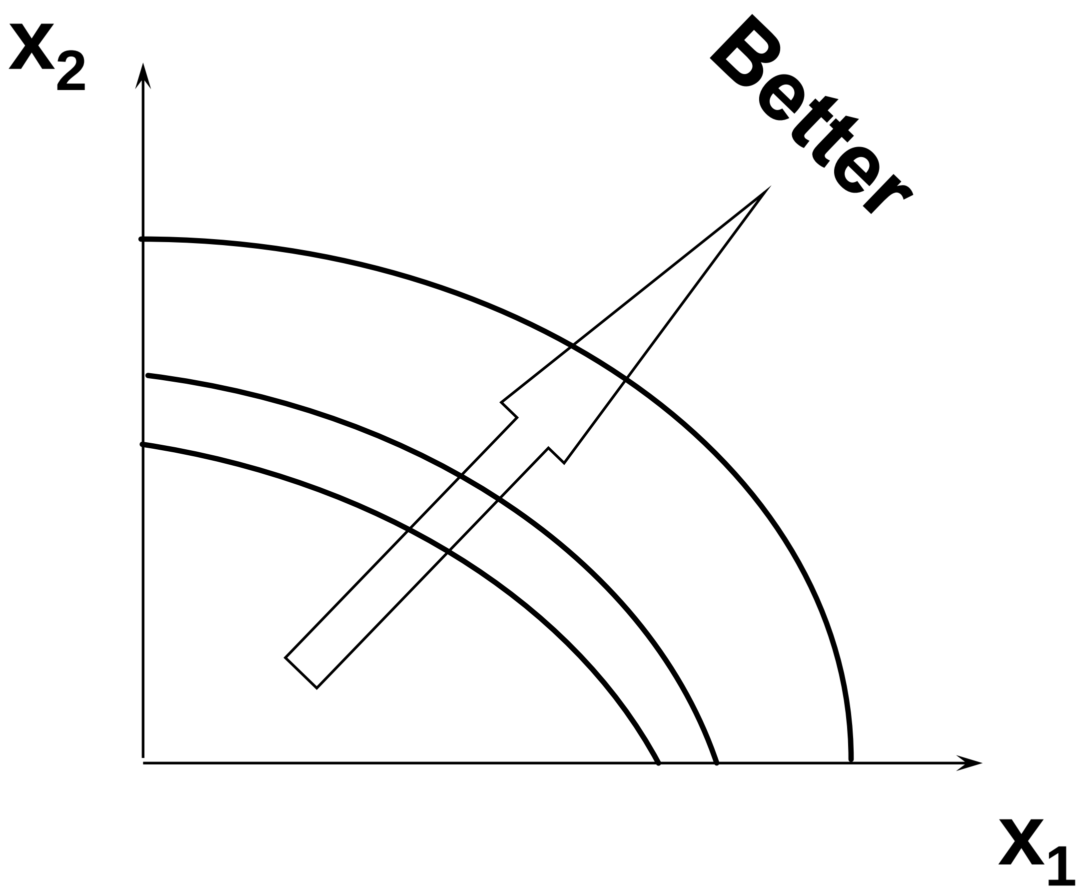


Examples of Corner Solutions -- the Perfect Substitutes Case



All the bundles in the constraint are equally the most preferred affordable when $p_1 = p_2$.

Examples of Corner Solutions -- the Non-Convex Preferences Case



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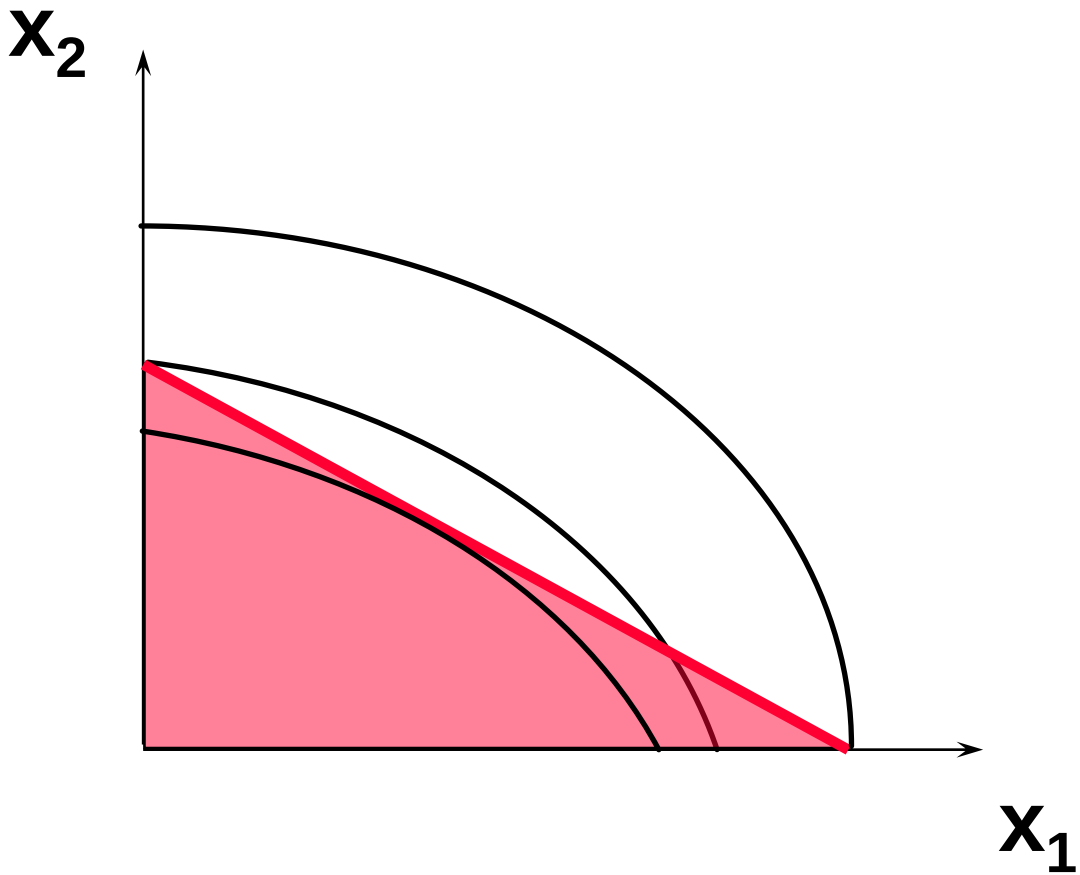

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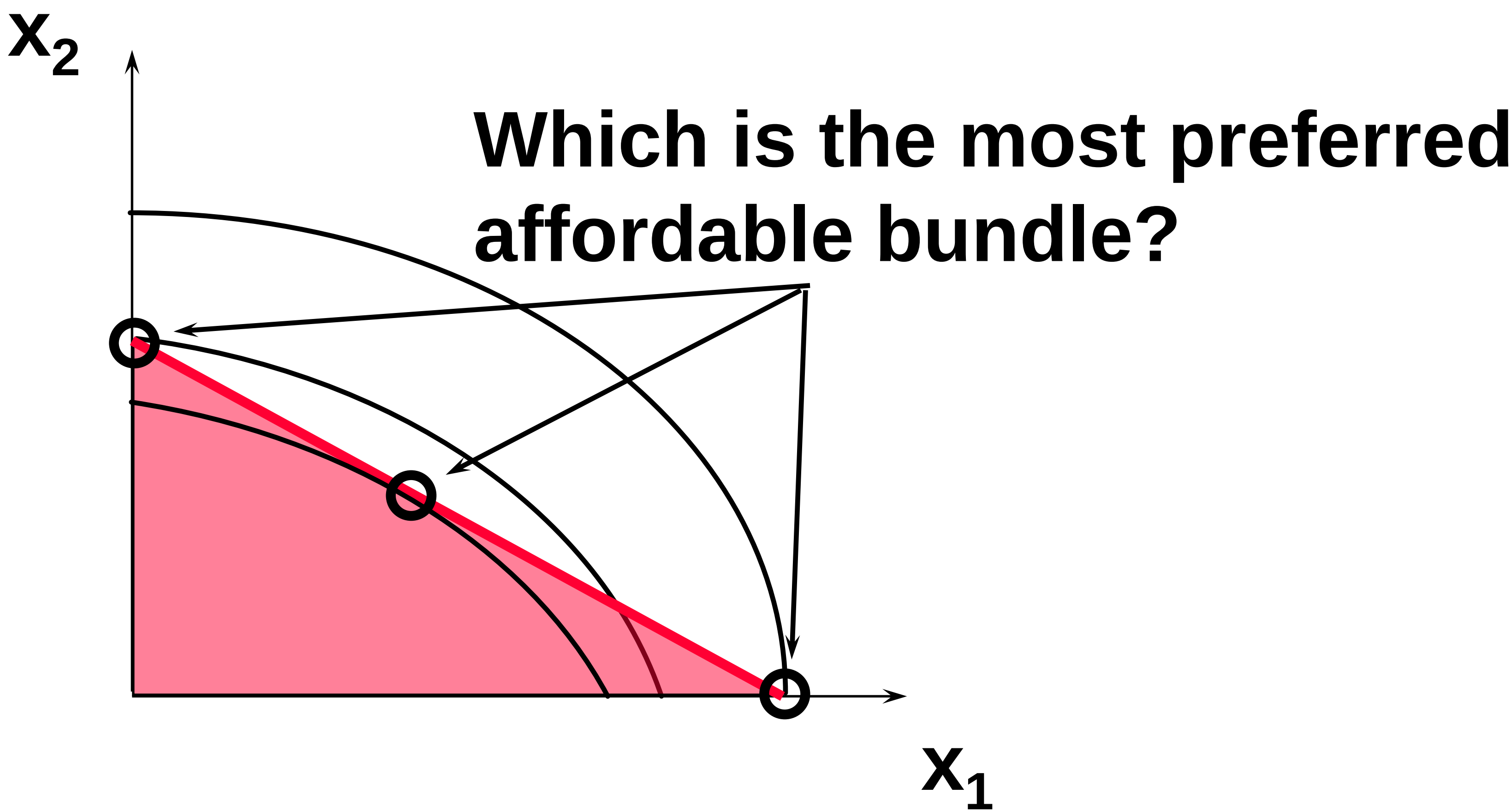

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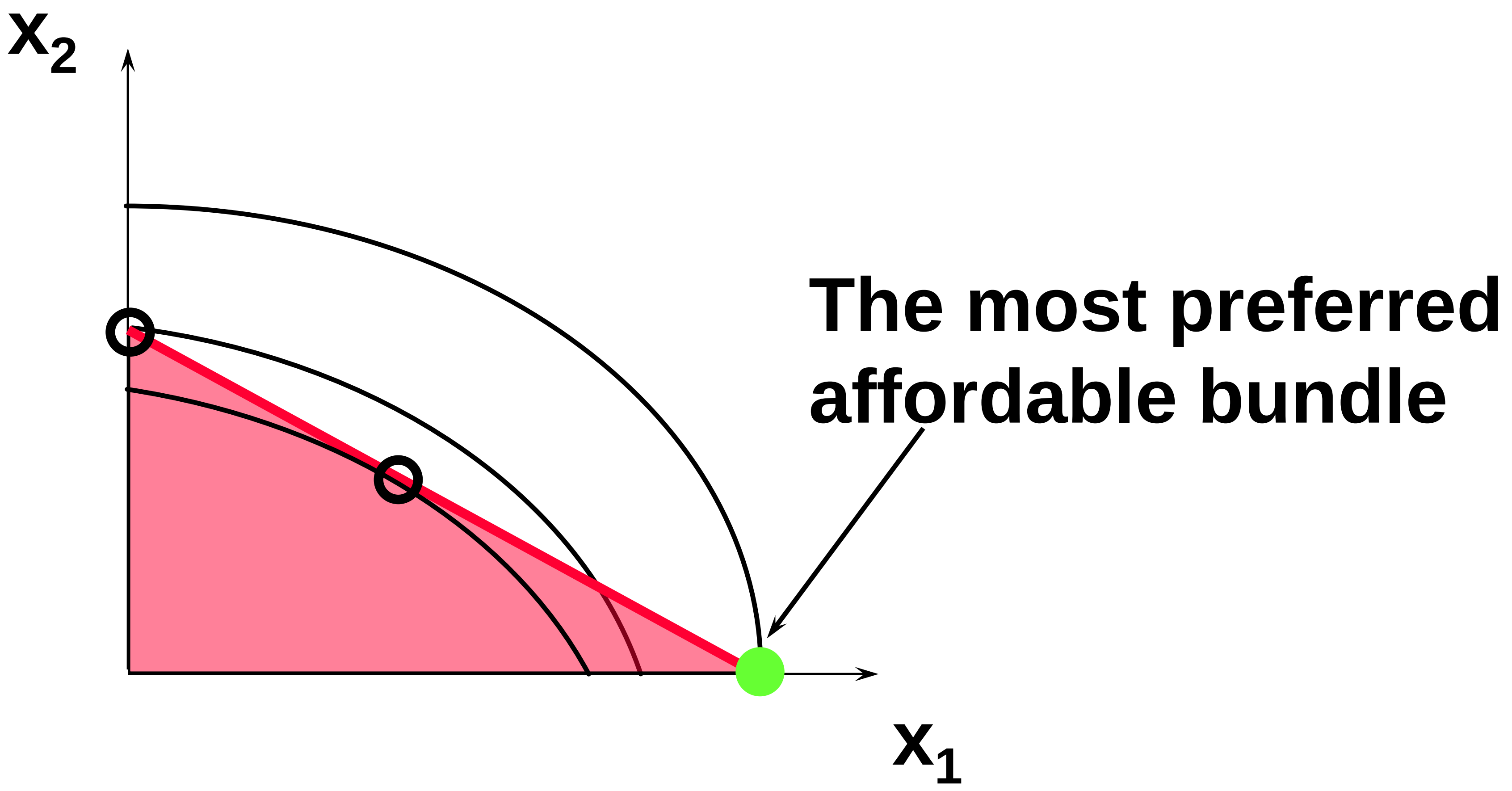
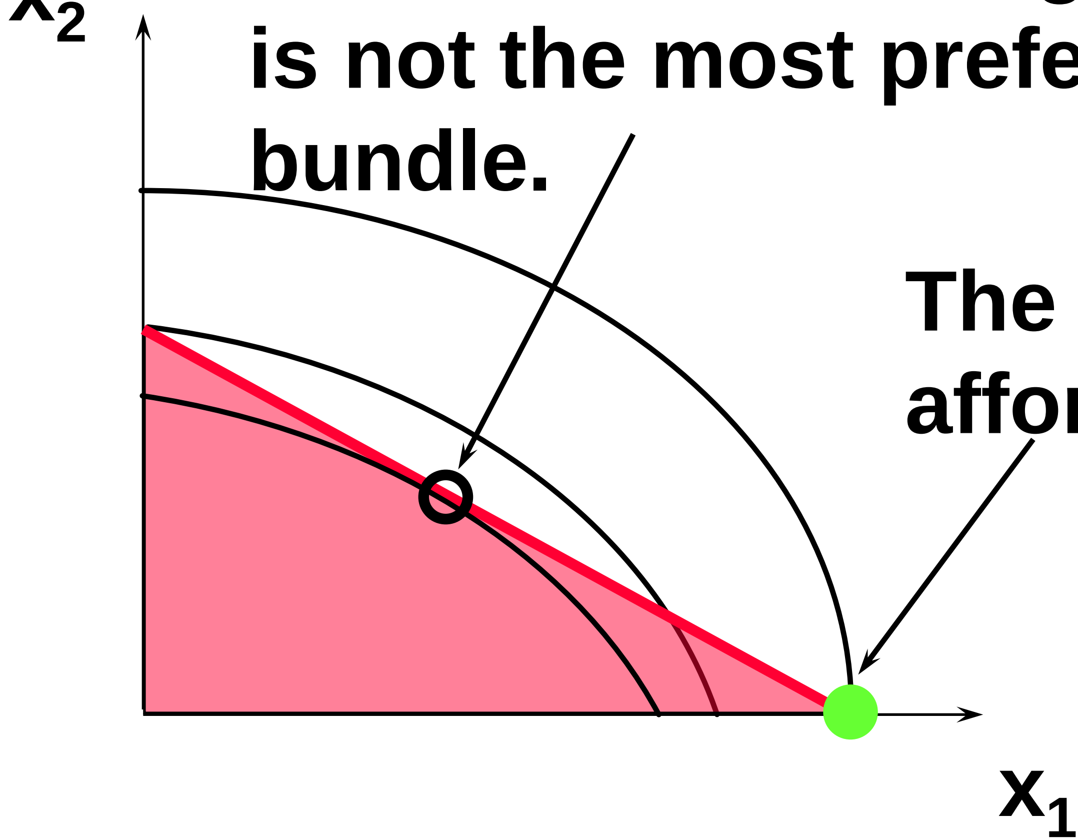


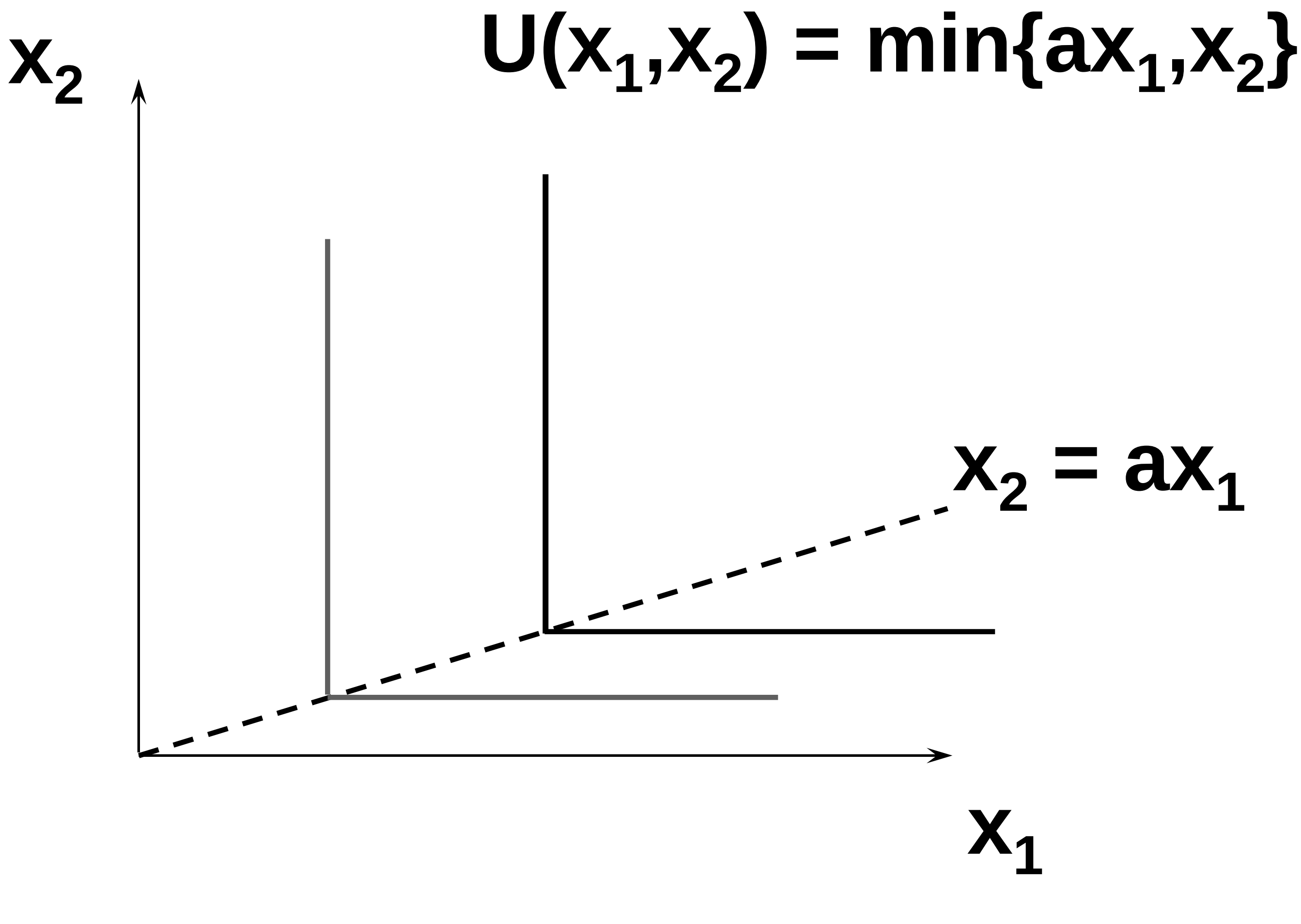
Examples of Corner Solutions -- the Non-Convex Preferences Case

Notice that the "tangency solution" is not the most preferred affordable bundle.



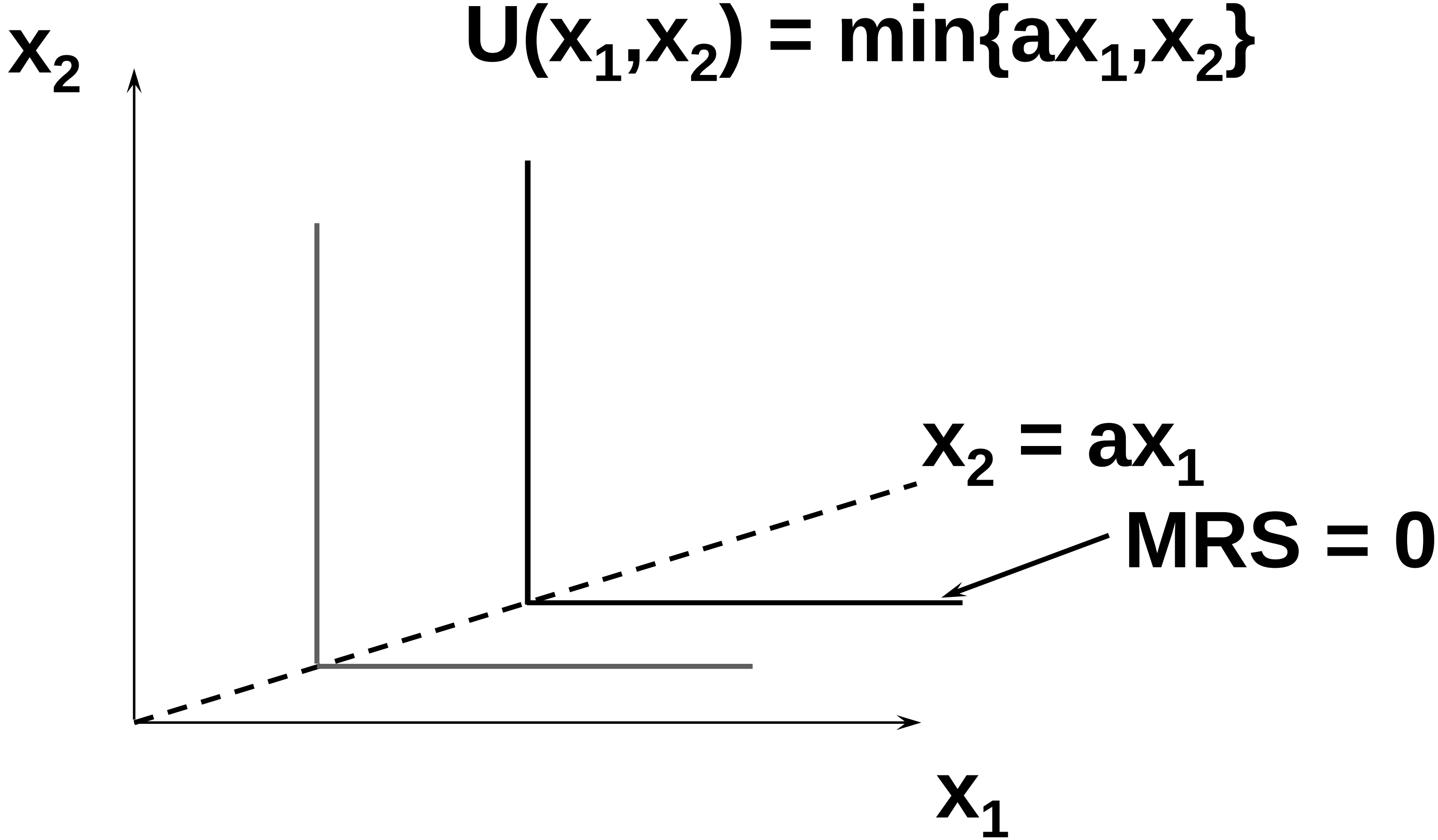
The most preferred affordable bundle

Examples of ‘Kinky’ Solutions -- the Perfect Complements Case



Examples of ‘Kinky’ Solutions -- the Perfect Complements Case

$U(x_1,x_2) = \min\{ax_1,x_2\}$



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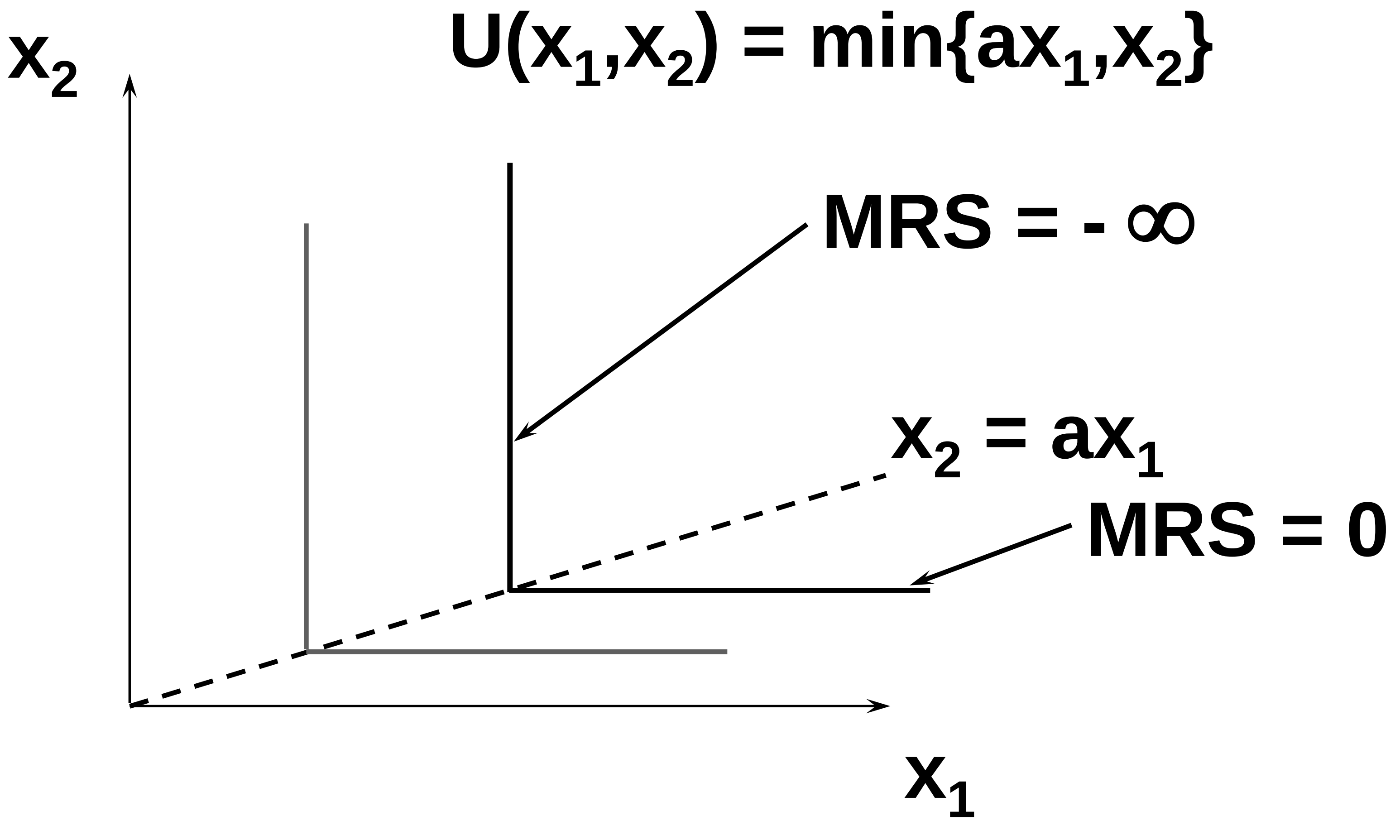

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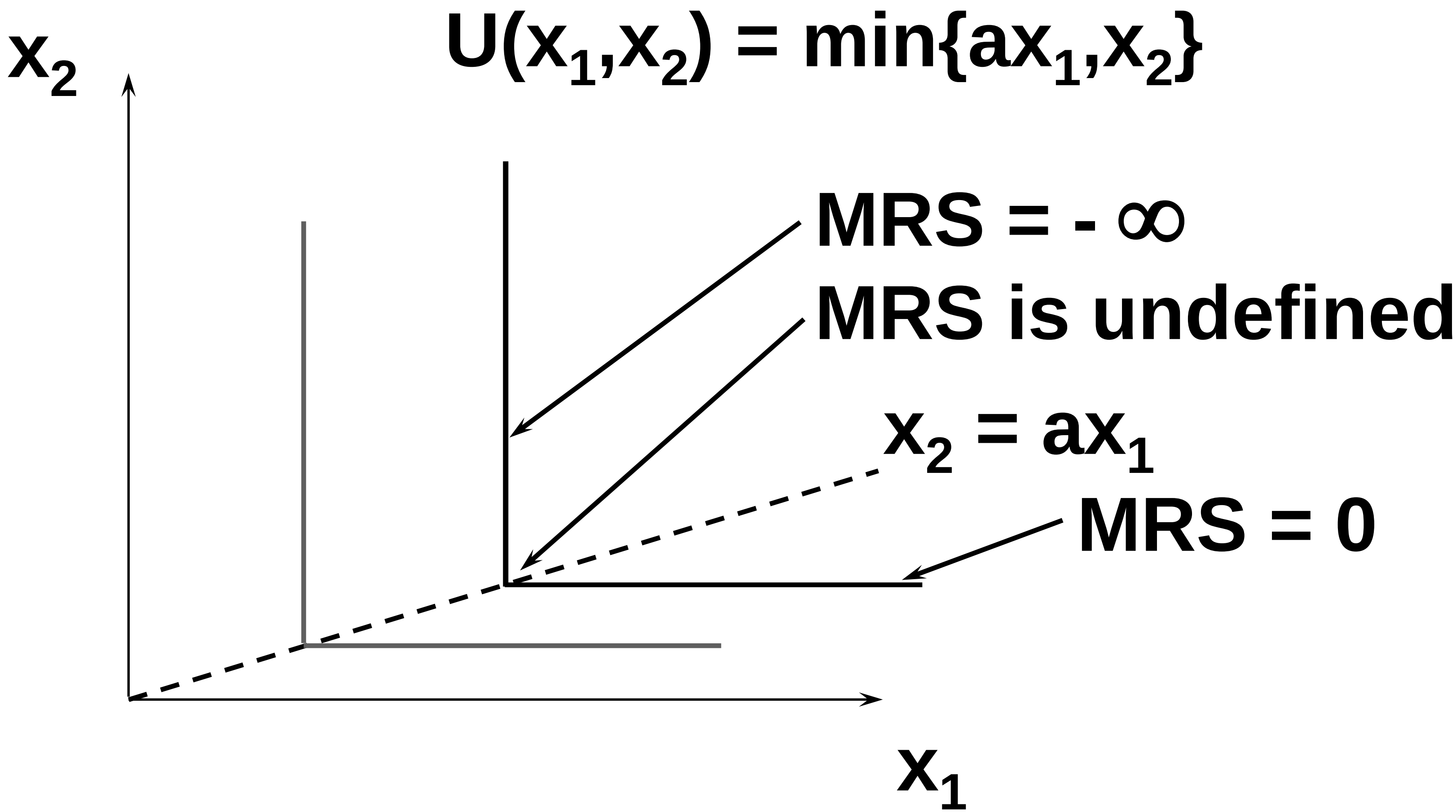

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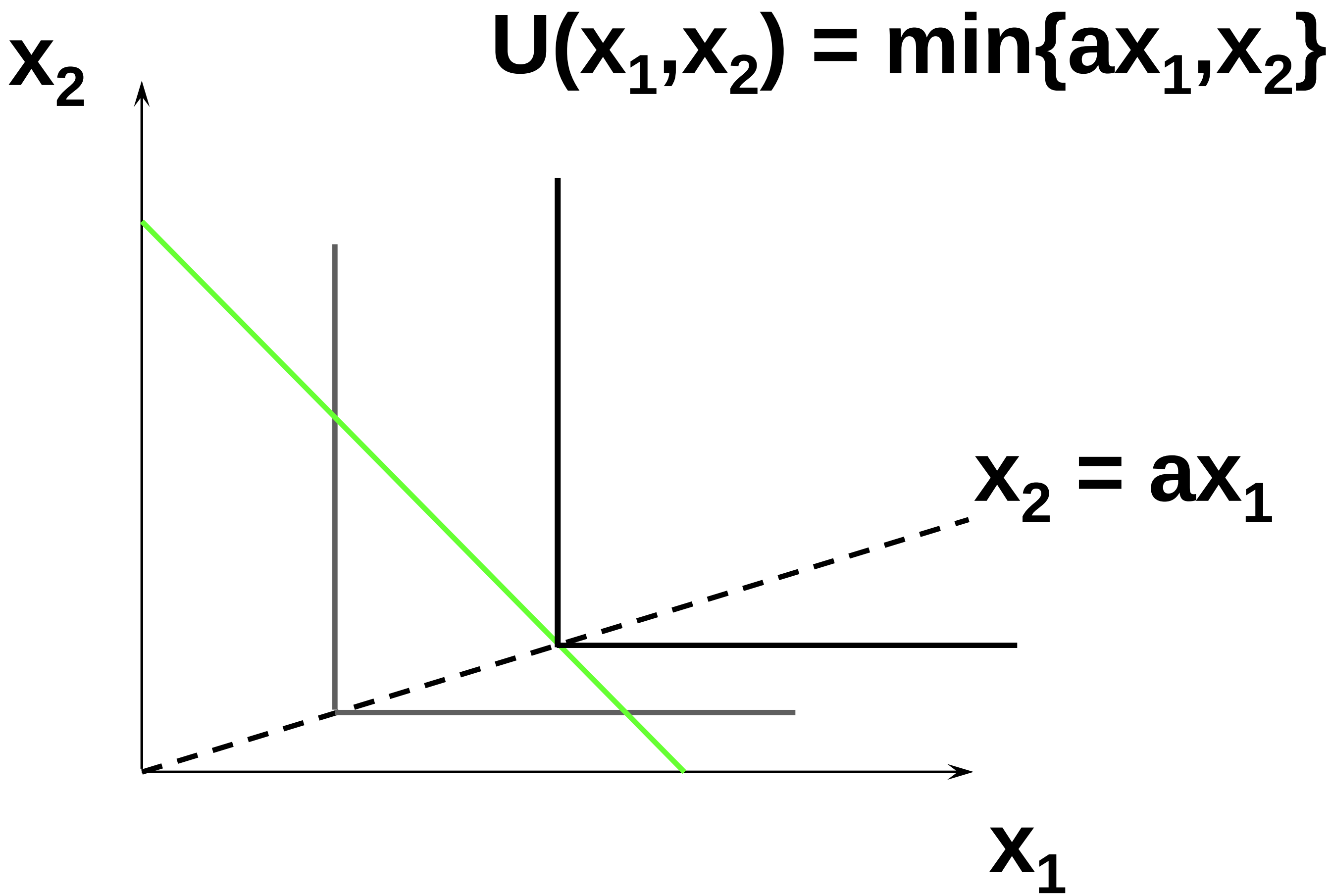

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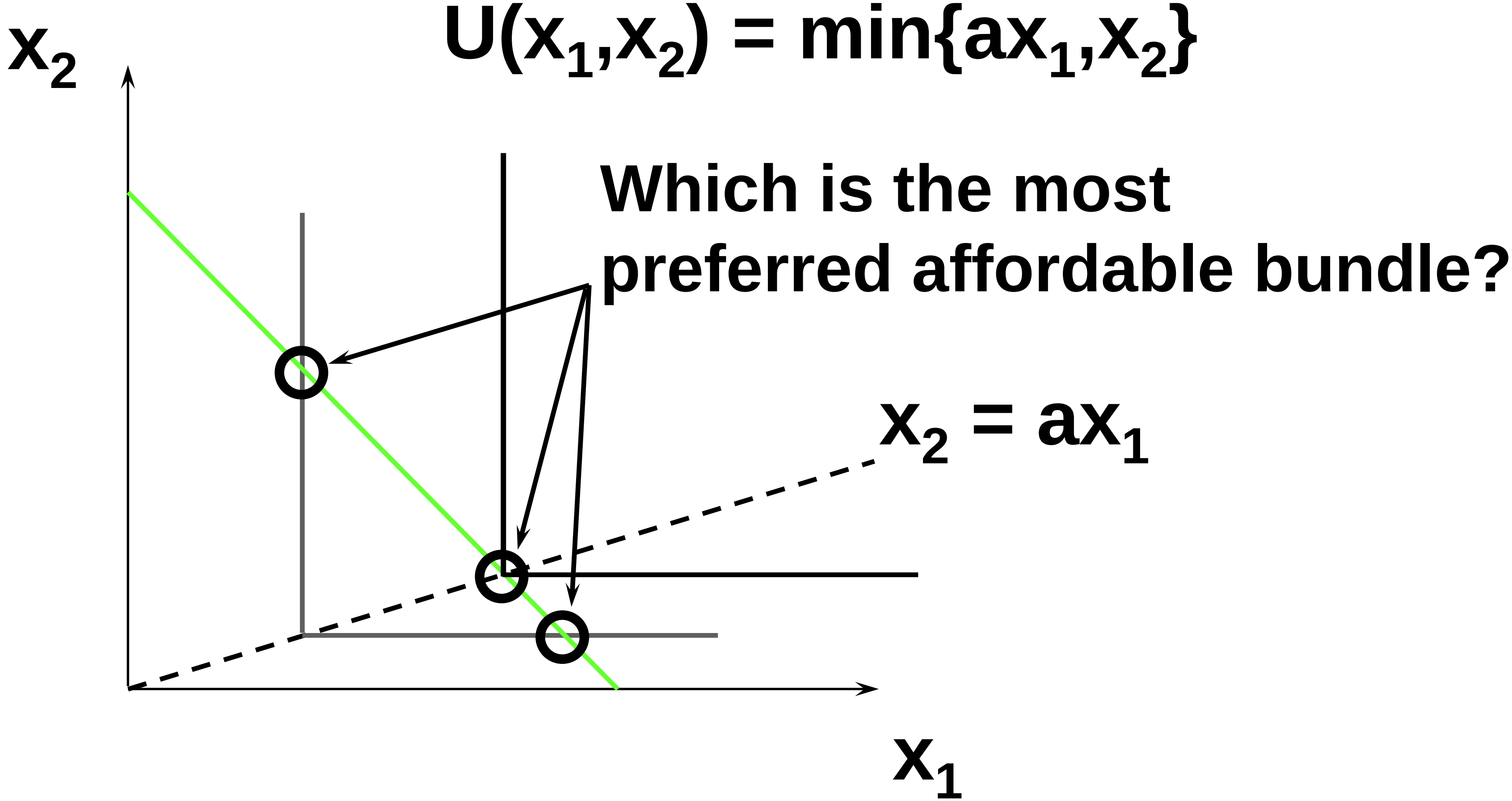
$U(x_1,x_2) = \min\{ax_1,x_2\}$



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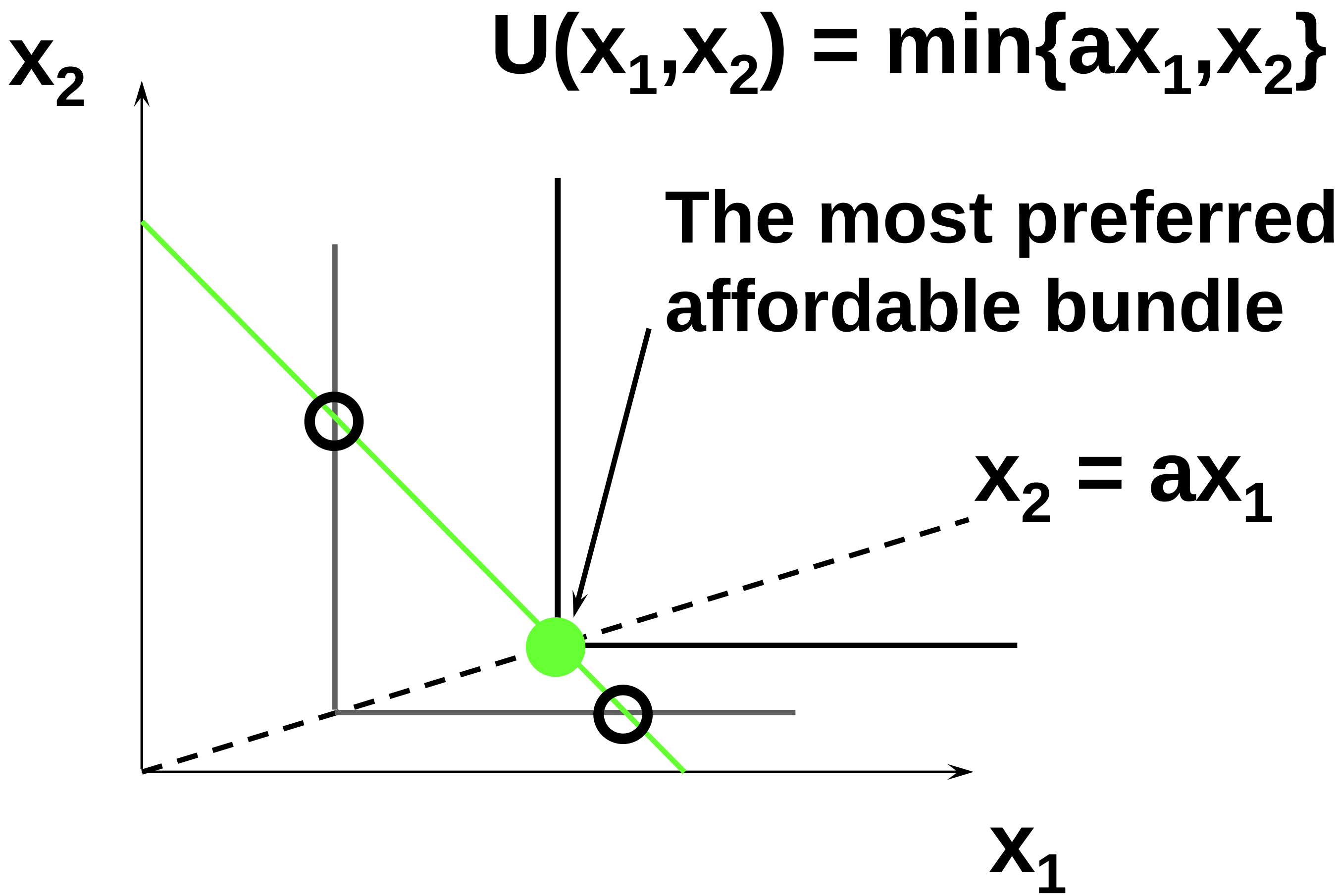

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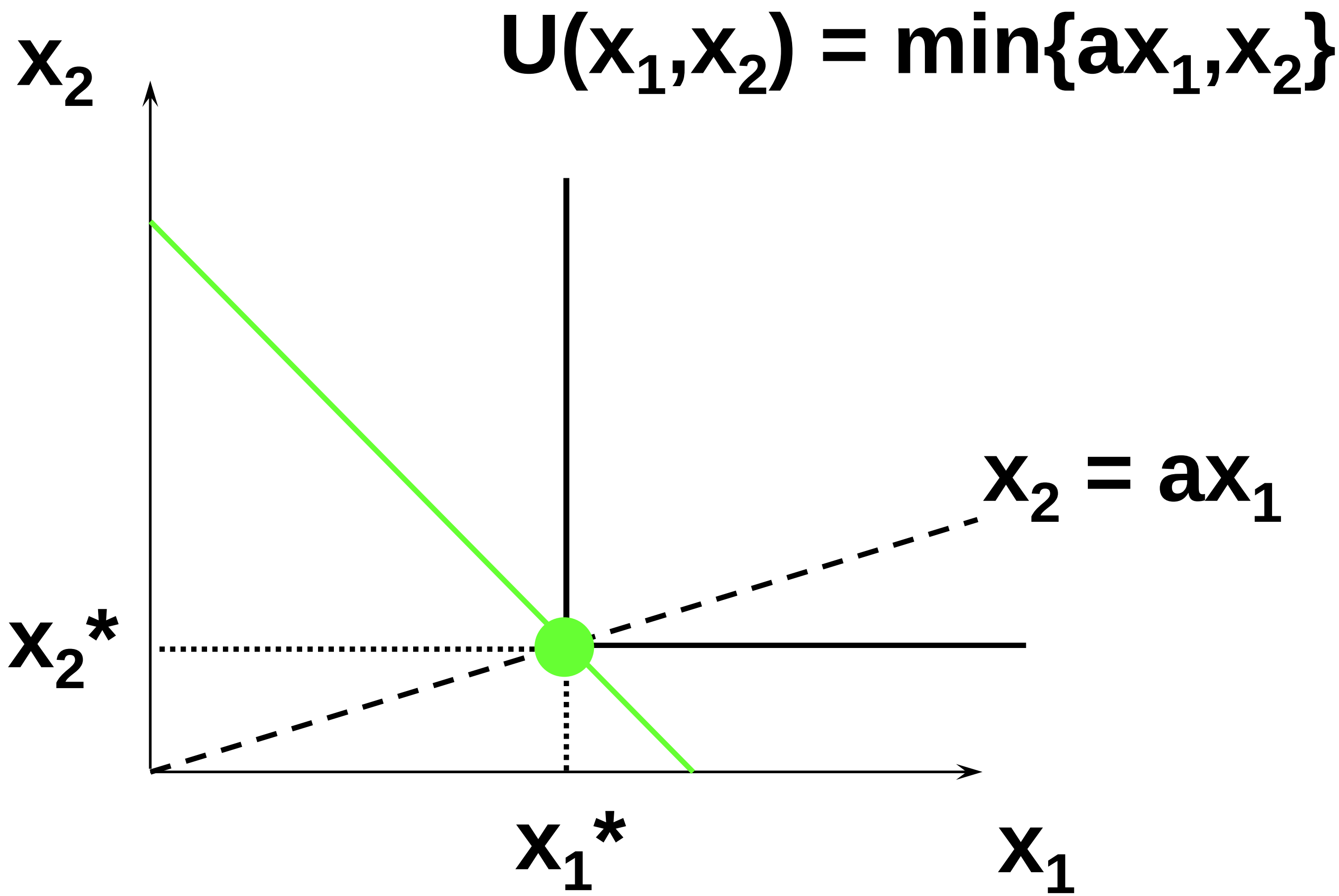
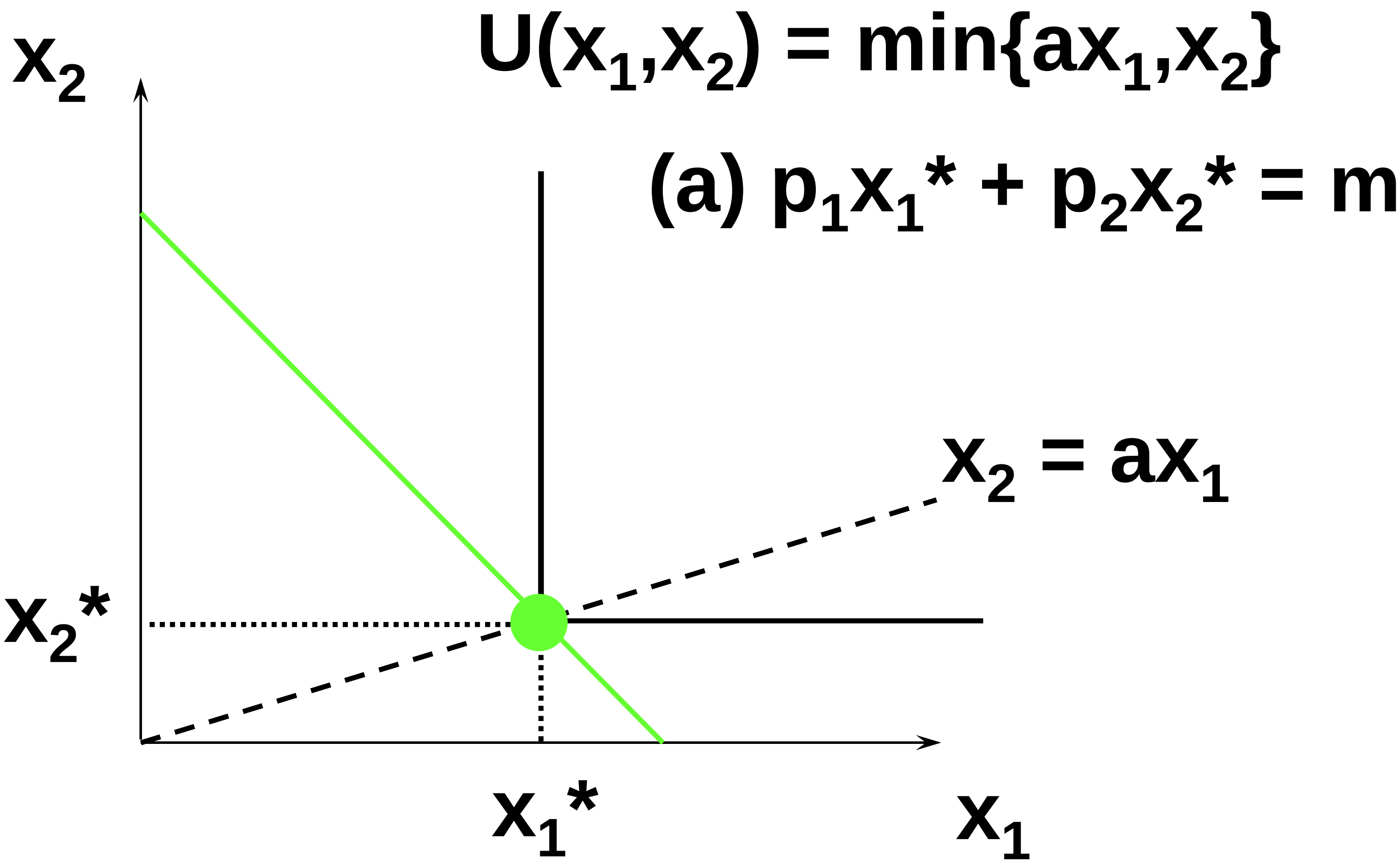


Examples of ‘Kinky’ Solutions -- the Perfect Complements Case

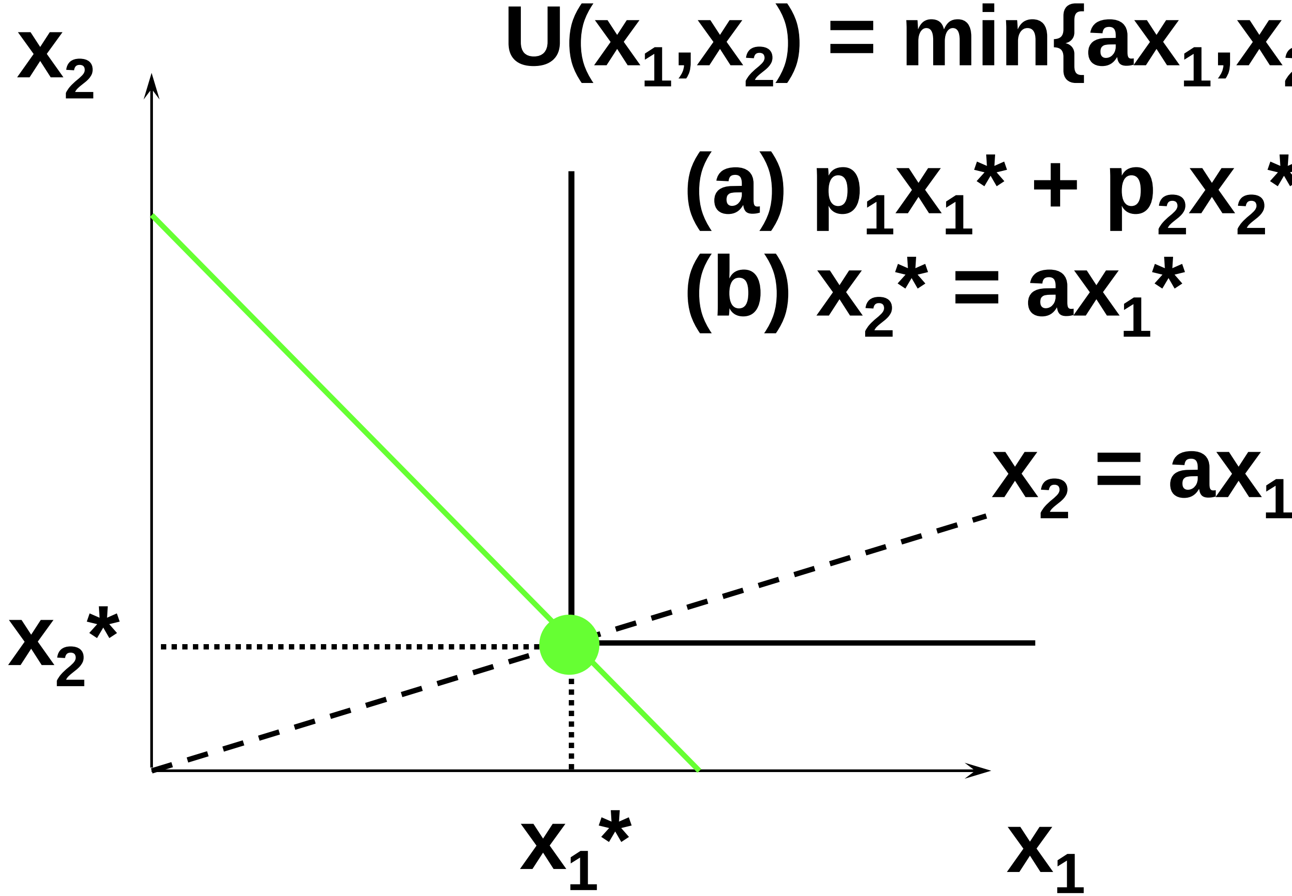


Examples of 'Kinky' Solutions -- the Perfect Complements Case

$U(x_1,x_2) = \min\{ax_1,x_2\}$

(a) $p_1x_1^* + p_2x_2^* = m$

(b) $x_2^* = ax_1^*$



Examples of ‘Kinky’ Solutions -- the Perfect Complements Case

(a) $p_1x_1^* + p_2x_2^* = m$; (b) $x_2^* = ax_1^*$.

Examples of 'Kinky' Solutions -- the Perfect Complements Case

(a) $p_1x_1^* + p_2x_2^* = m$; (b) $x_2^* = ax_1^*$.

Substitution from (b) for x_2^* in (a) gives $p_1x_1^* + p_2ax_1^* = m$

Examples of 'Kinky' Solutions -- the Perfect Complements Case

(a) $p_1x_1^* + p_2x_2^* = m$; (b) $x_2^* = ax_1^*$.

Substitution from (b) for x_2^* in (a) gives $p_1x_1^* + p_2ax_1^* = m$ which gives

$$x_1^* = \frac{m}{p_1 + ap_2}$$

Examples of 'Kinky' Solutions -- the Perfect Complements Case

(a) $p_1x_1^* + p_2x_2^* = m$; (b) $x_2^* = ax_1^*$.

Substitution from (b) for x_2^* in (a) gives $p_1x_1^* + p_2ax_1^* = m$ which gives

$$x_1^* = \frac{m}{p_1 + ap_2};\ x_2^* = \frac{am}{p_1 + ap_2}.$$

Examples of 'Kinky' Solutions -- the Perfect Complements Case

(a) $p_1x_1^* + p_2x_2^* = m$; (b) $x_2^* = ax_1^*$.

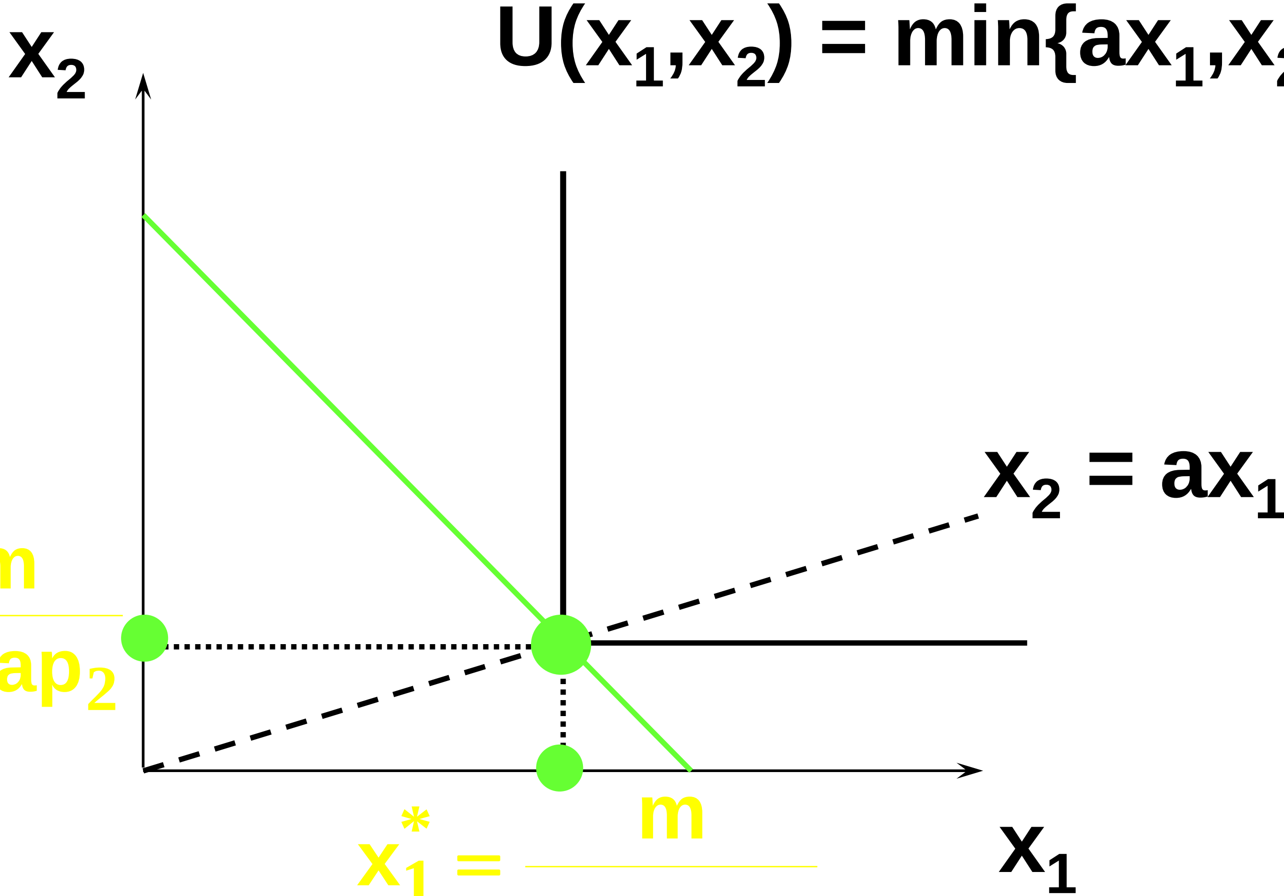
Substitution from (b) for x_2^* in (a) gives $p_1x_1^* + p_2ax_1^* = m$ which gives

$$x_1^* = \frac{m}{p_1 + ap_2}; x_2^* = \frac{am}{p_1 + ap_2}.$$

A bundle of 1 commodity 1 unit and *a* commodity 2 units costs $p_1 + ap_2$; $m/(p_1 + ap_2)$ such bundles are affordable.

Examples of ‘Kinky’ Solutions -- the Perfect Complements Case

$U(x_1,x_2) = \min\{ax_1,x_2\}$



$$x_2^* = \frac{am}{p_1 + ap_2}$$

$$x_1^* = \frac{m}{p_1 + ap_2}$$

Properties of Demand Functions

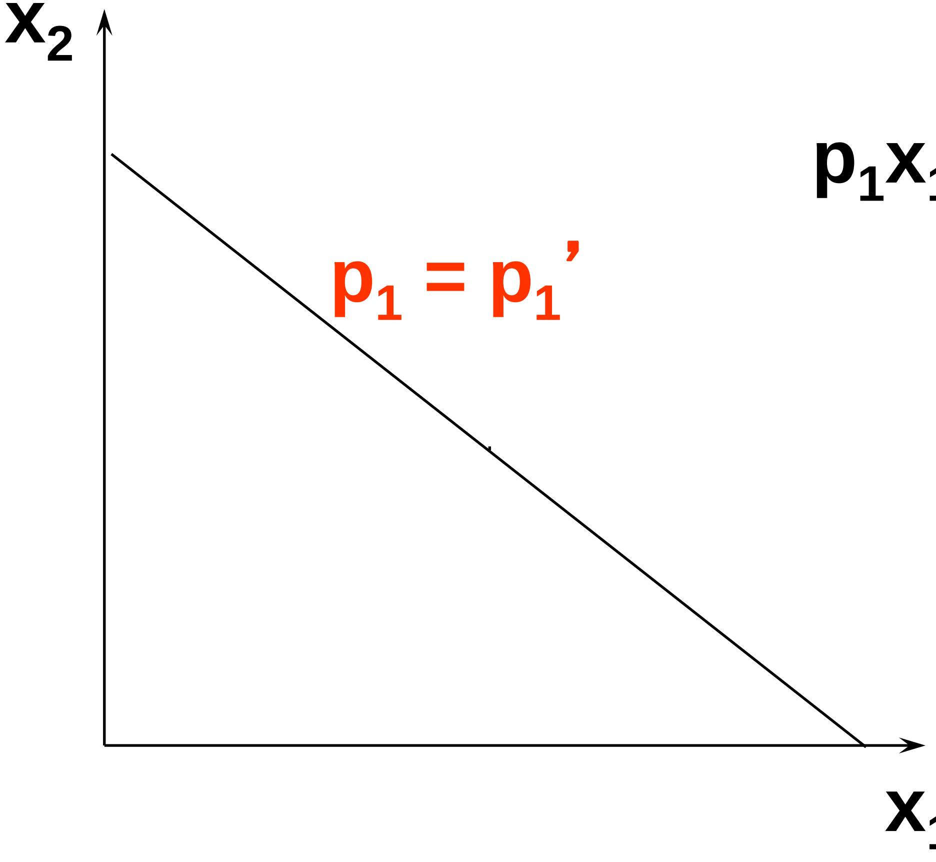
- **Comparative statics analysis of ordinary demand functions -- the study of how ordinary demands $x_1^*(p_1,p_2,y)$ and $x_2^*(p_1,p_2,y)$ change as prices p_1, p_2 and income y change.**

Own-Price Changes

- **How does $x_1^*(p_1,p_2,y)$ change as p_1 changes, holding p_2 and y constant?**
- **Suppose only p_1 increases, from p_1' to p_1'' and then to p_1'''.**

Own-Price Changes

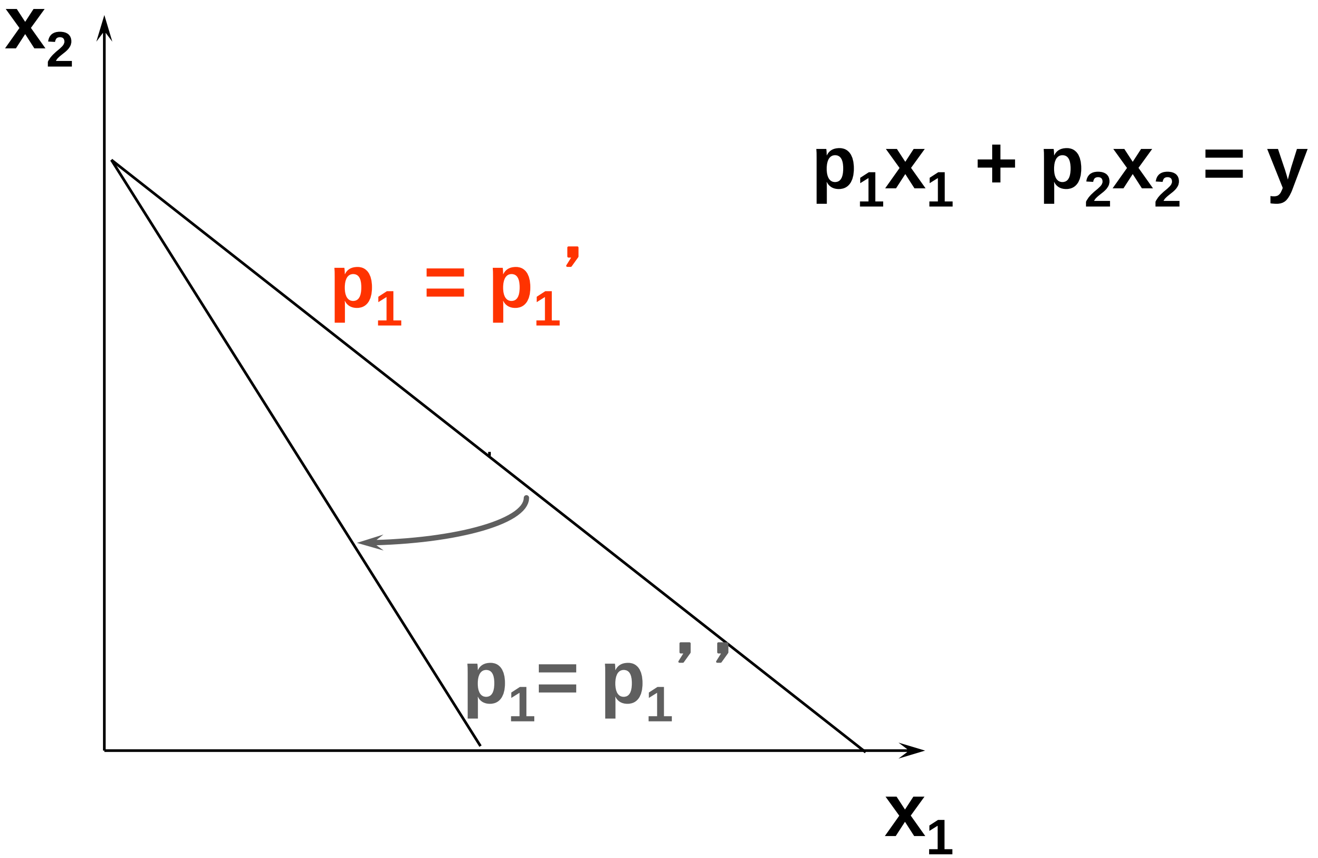
Fixed p_2 and y.



$p_1x_1 + p_2x_2 = y$

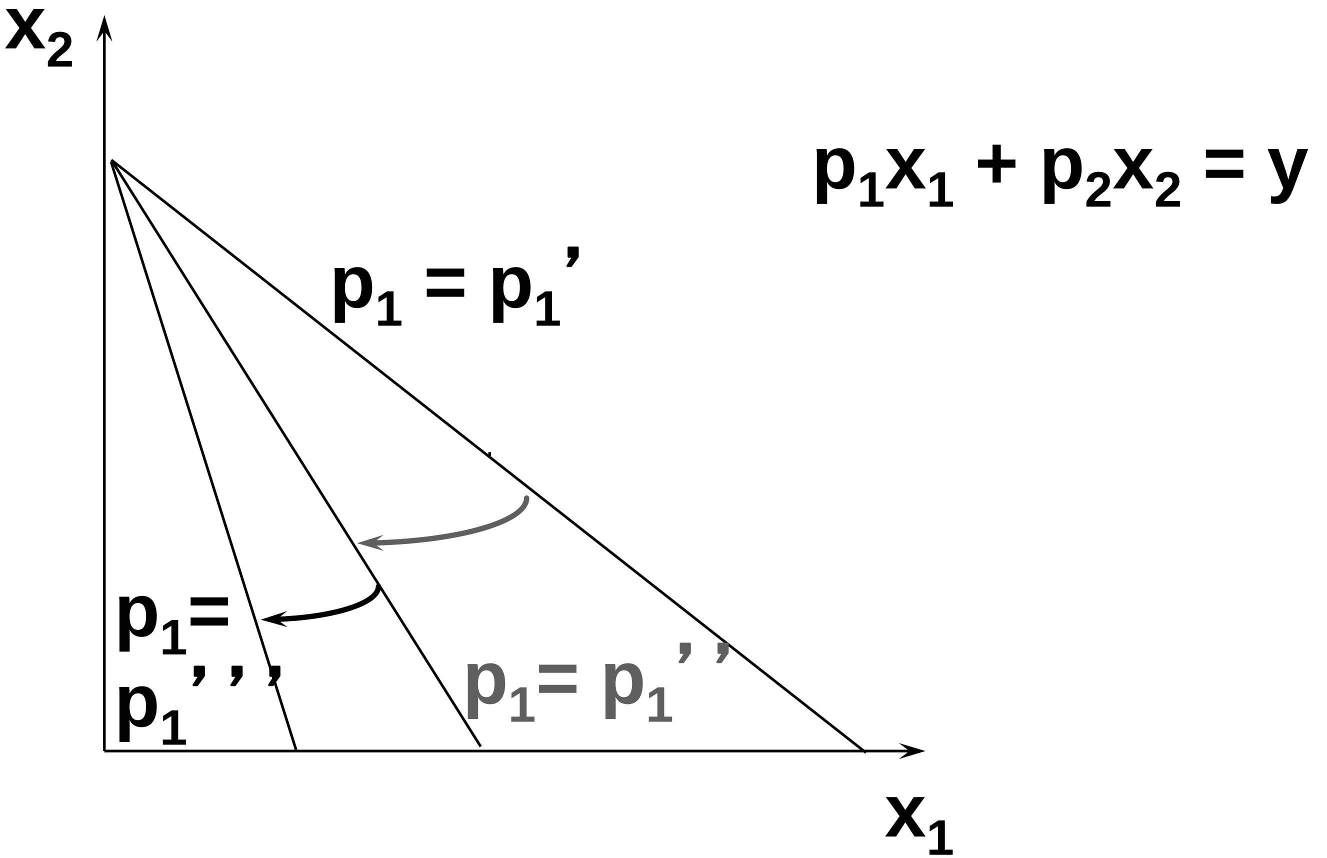
Own-Price Changes

Fixed p_2 and y.



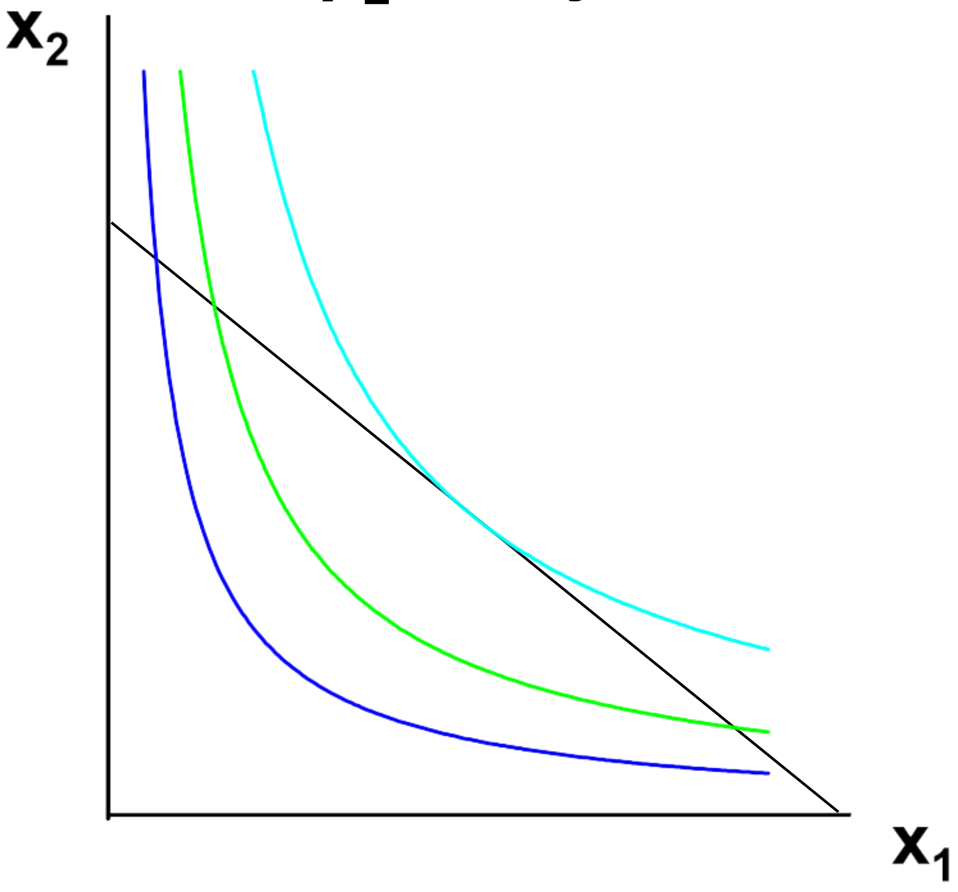
Own-Price Changes

Fixed p_2 and y.



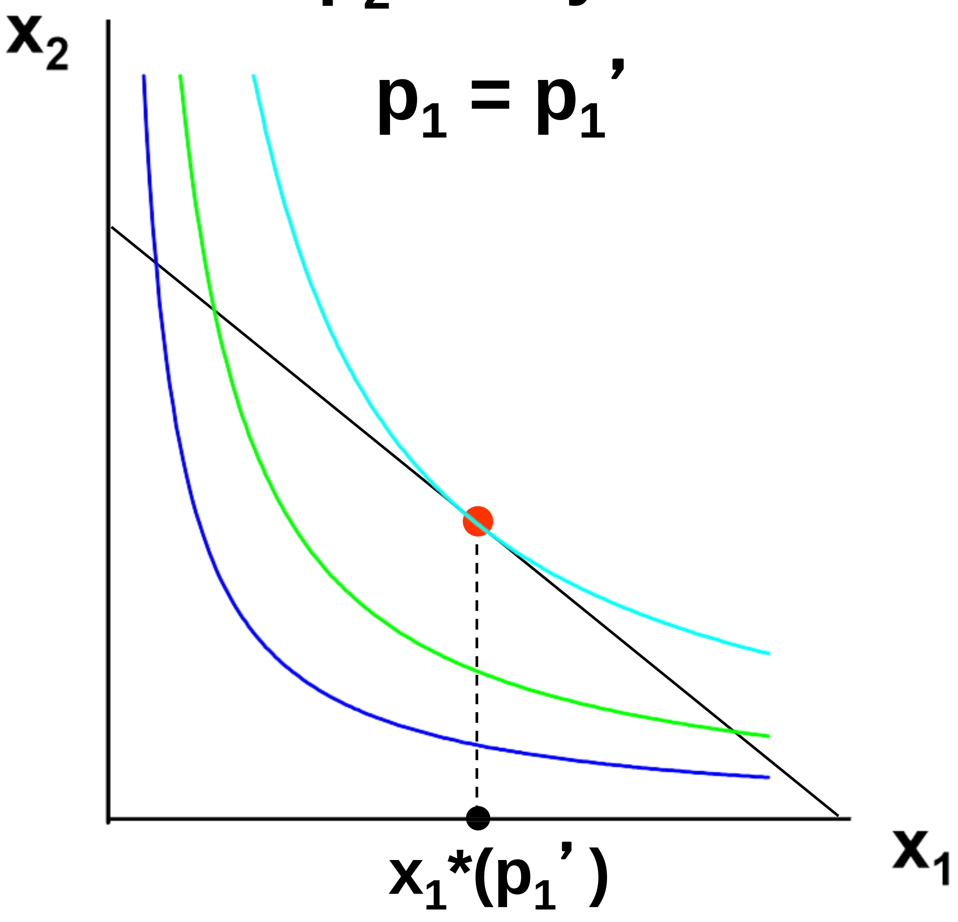
Own-Price Changes

Fixed p_2 and y.

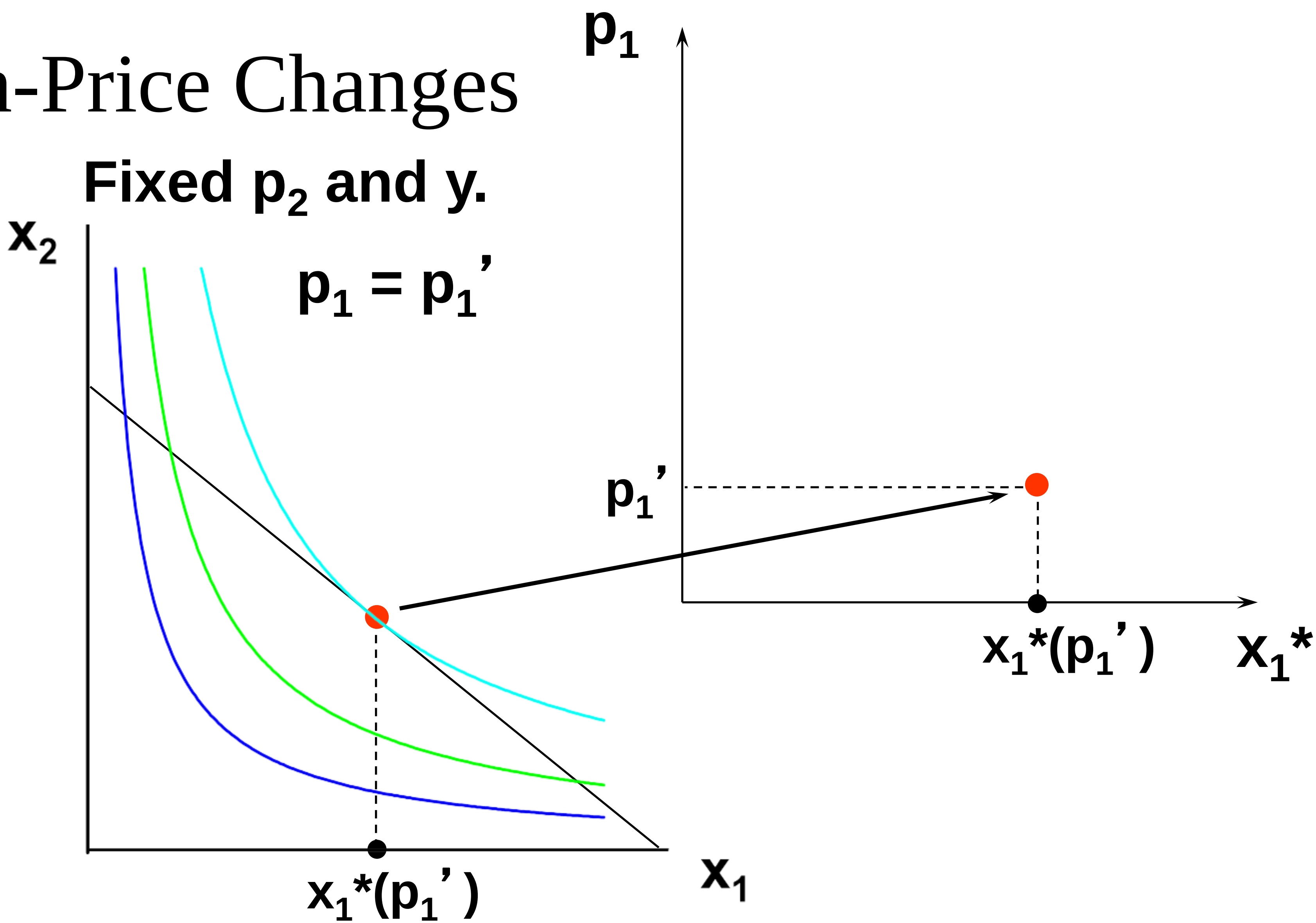


Own-Price Changes

Fixed p_2 and y.

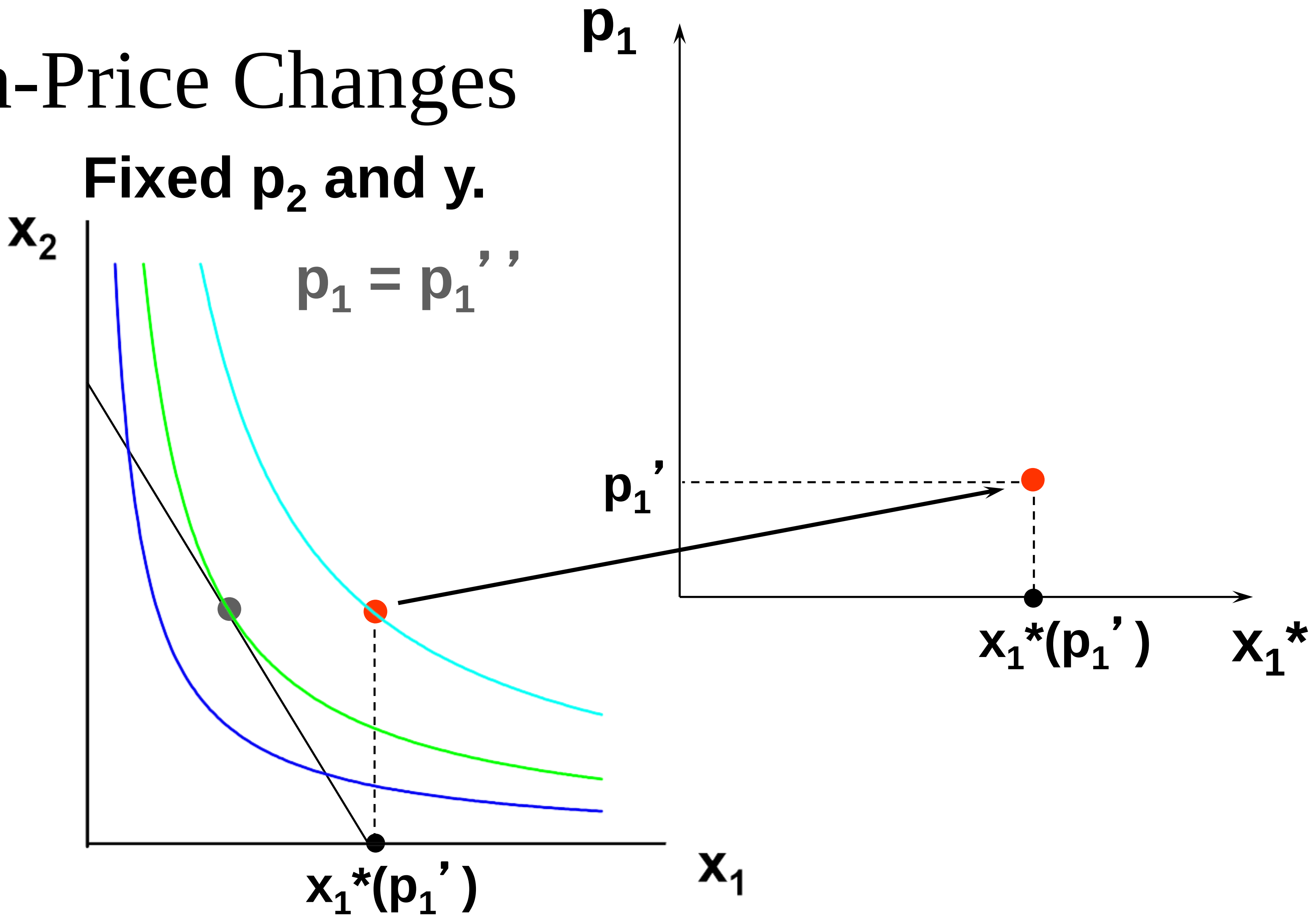


Own-Price Changes

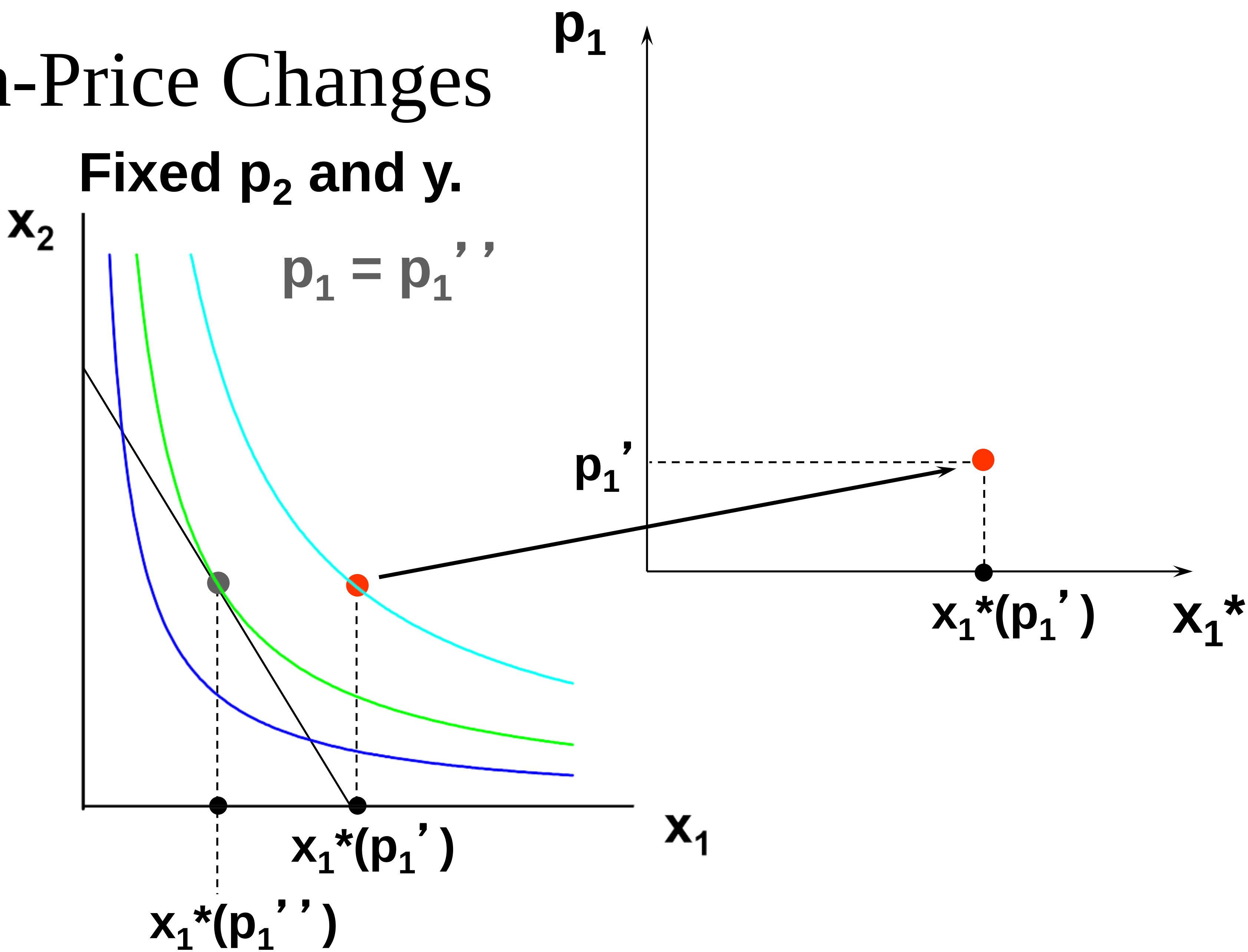


Own-Price Changes

Fixed p_2 and y.



Own-Price Changes



Own-Price Changes

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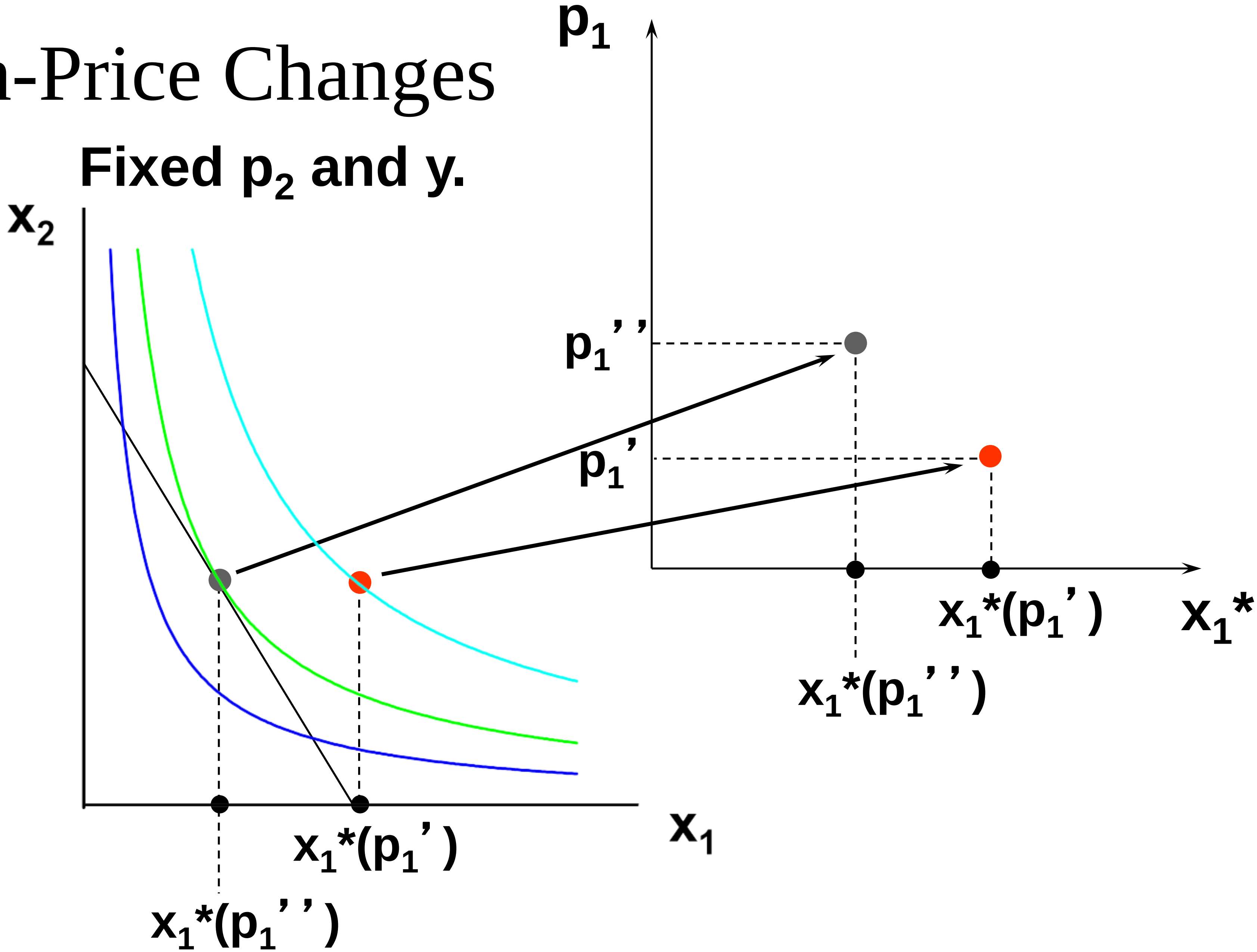
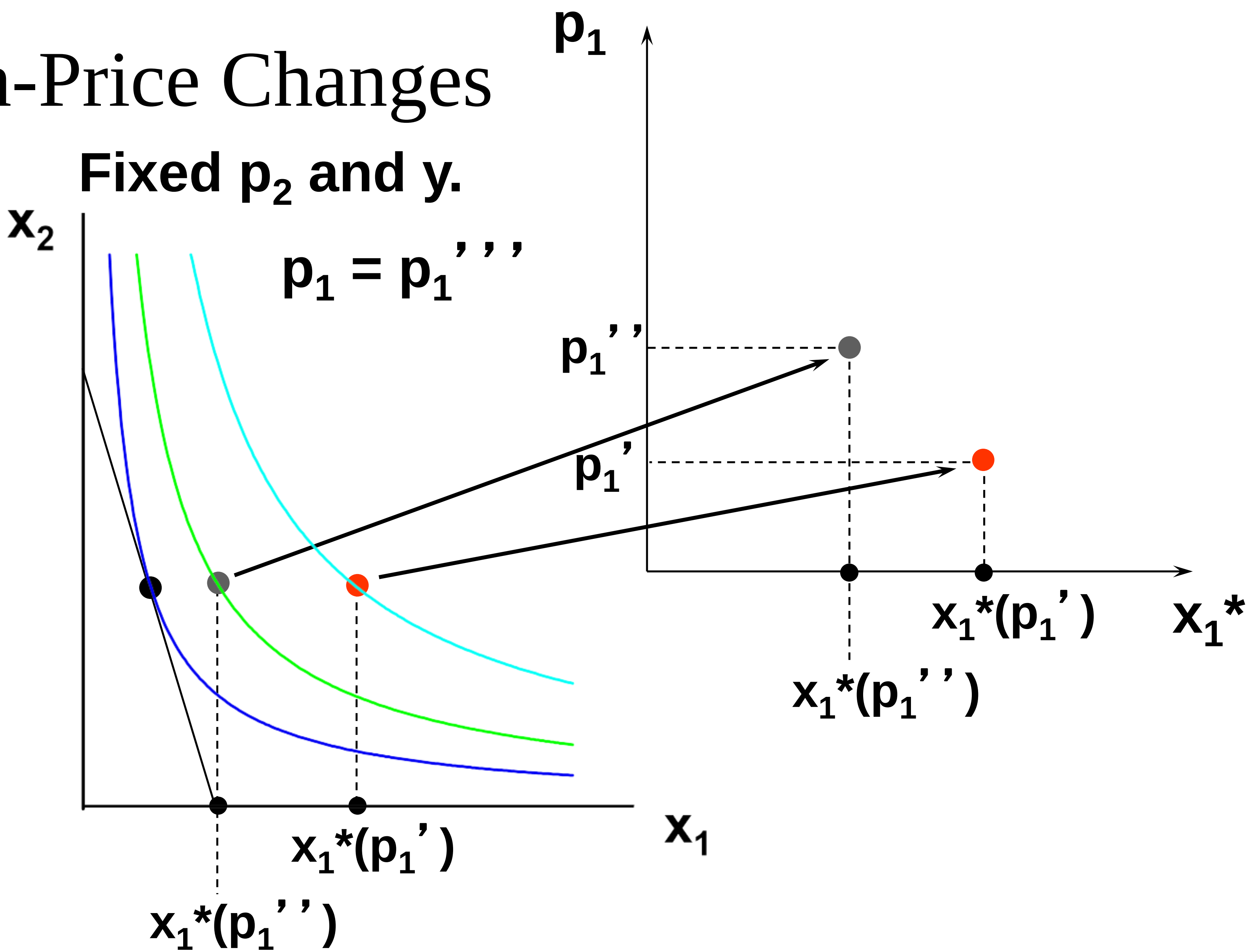


Own-Price Changes



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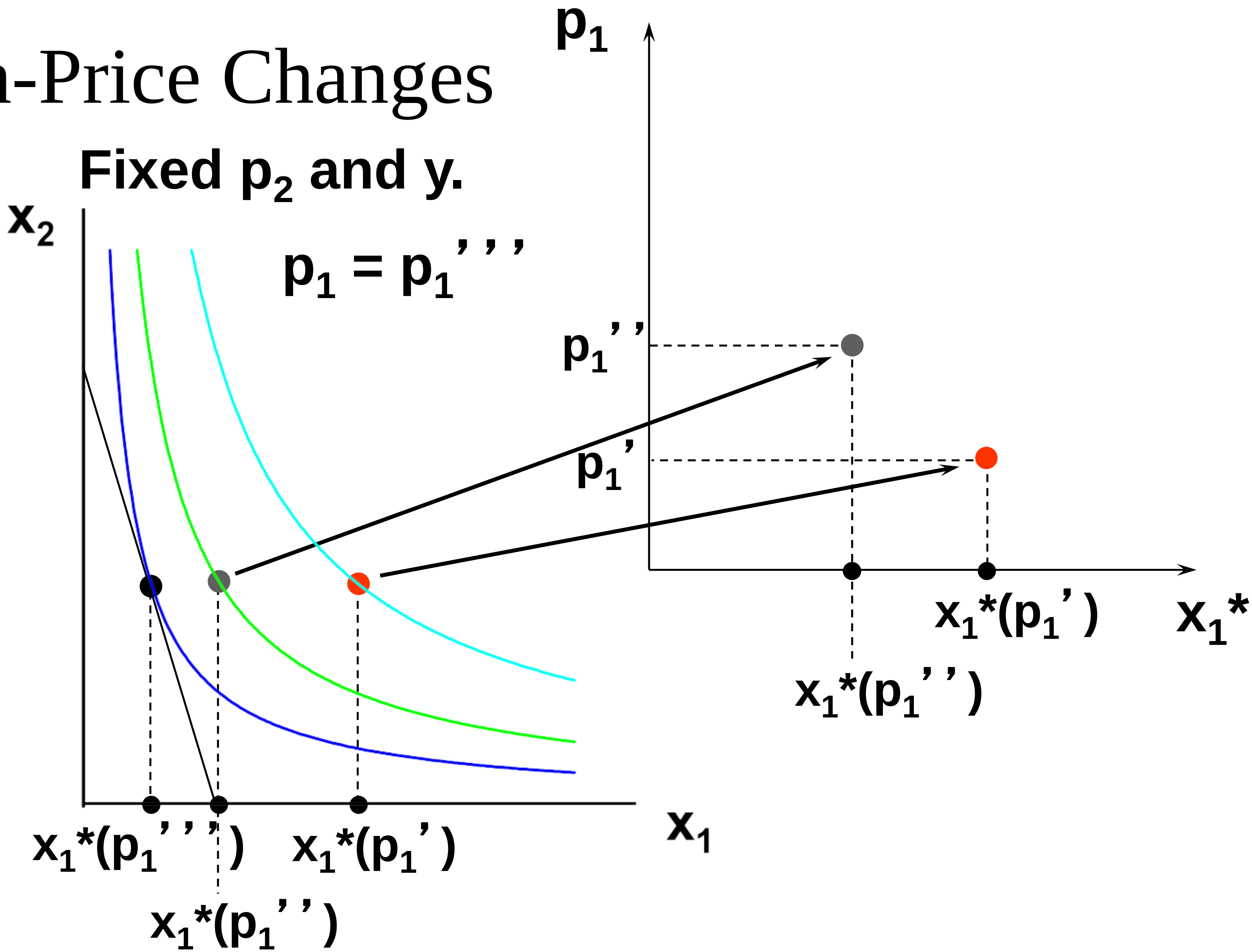

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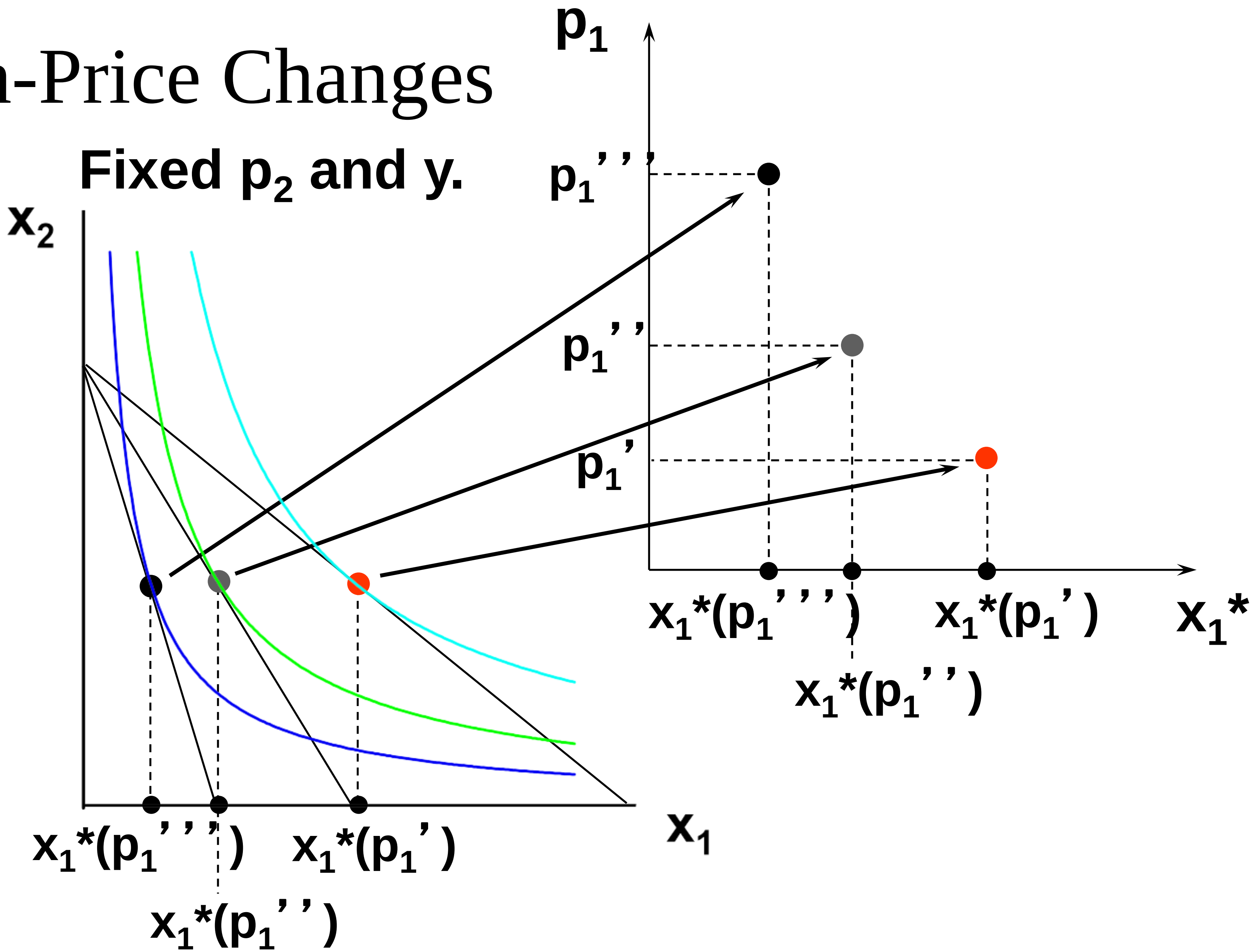

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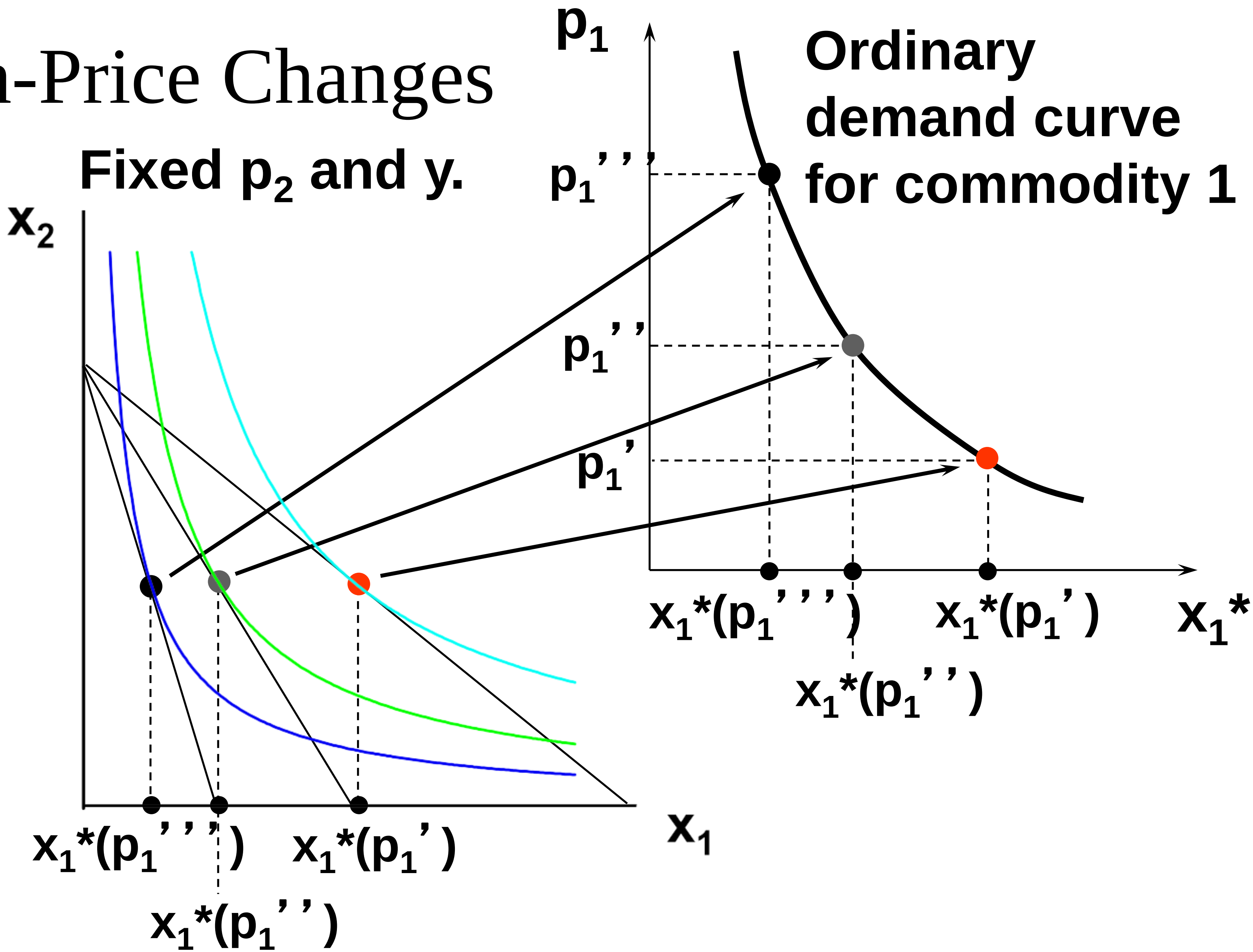
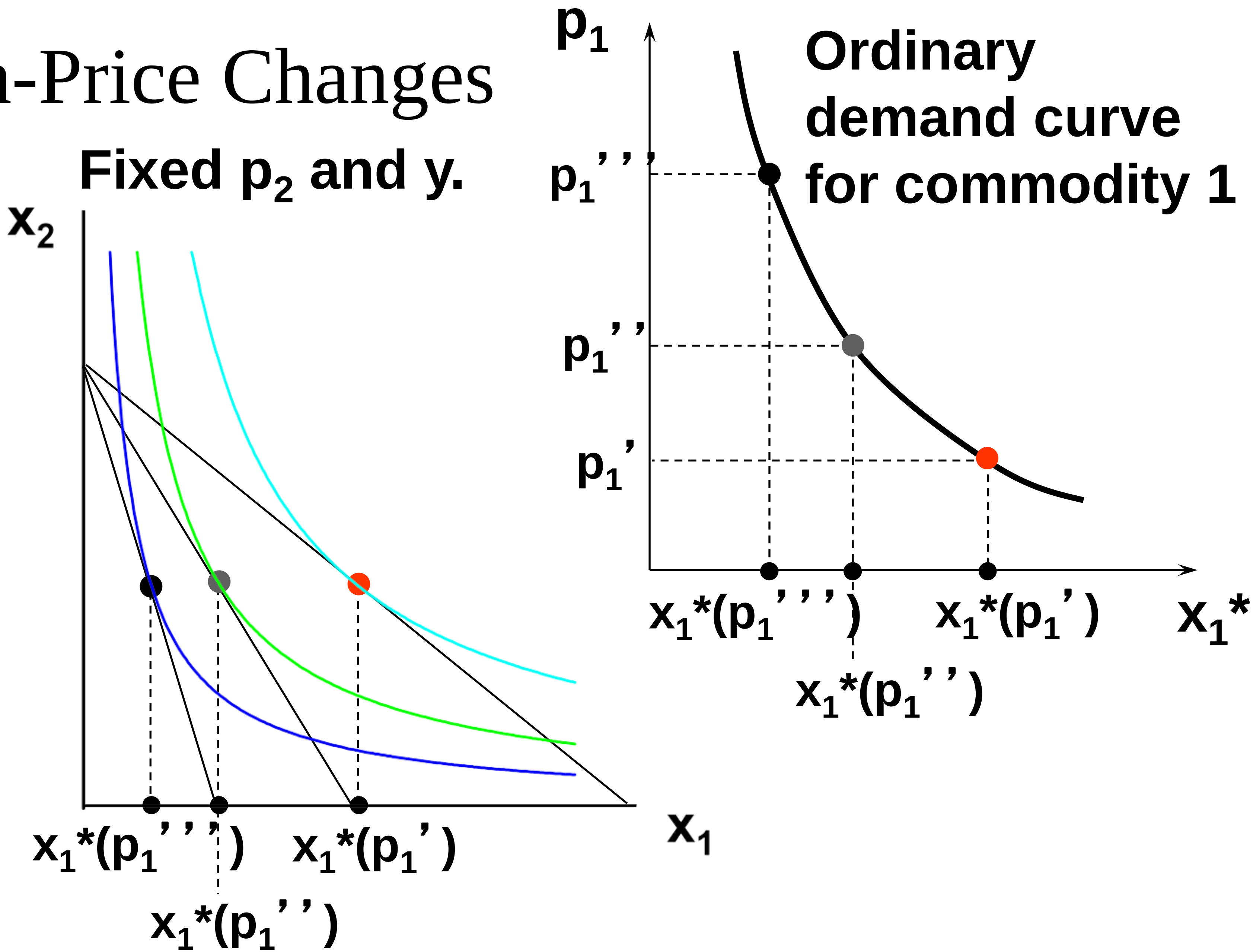
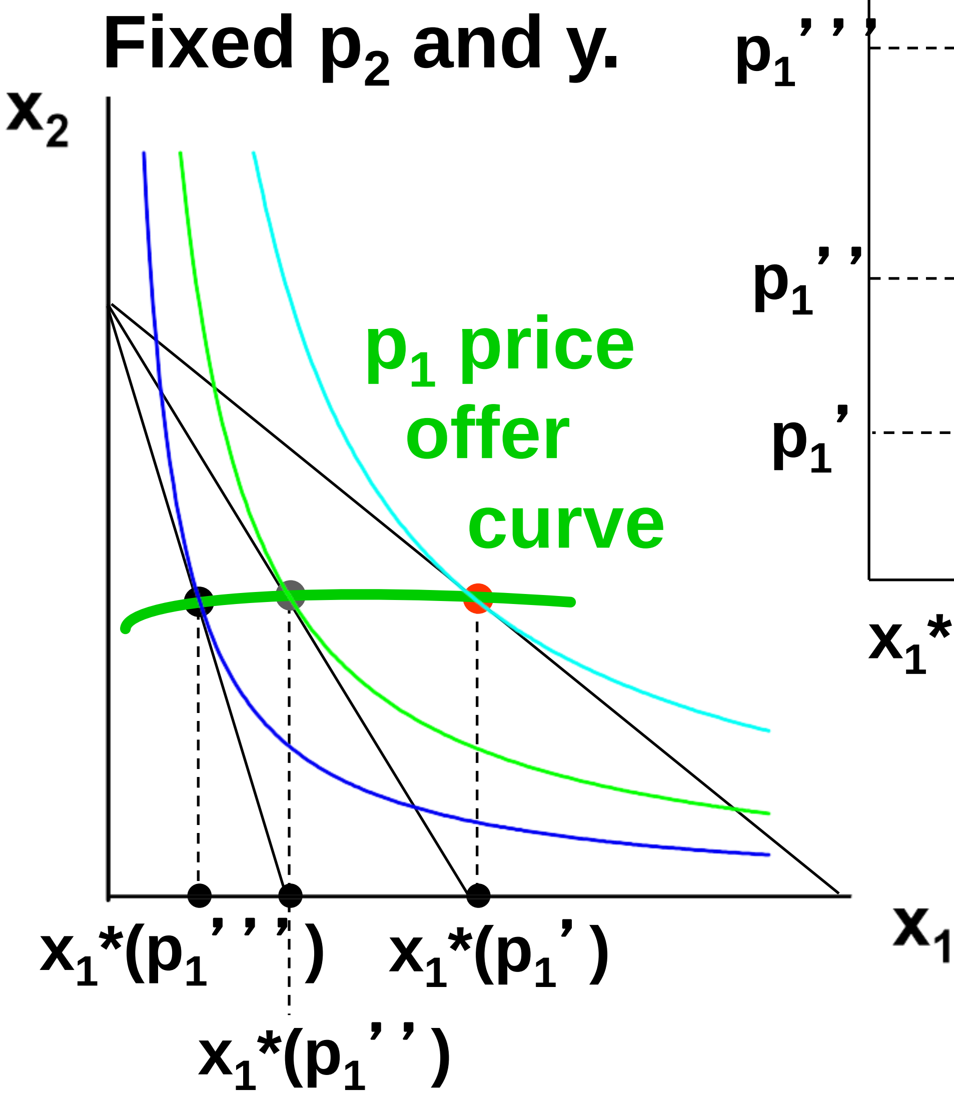
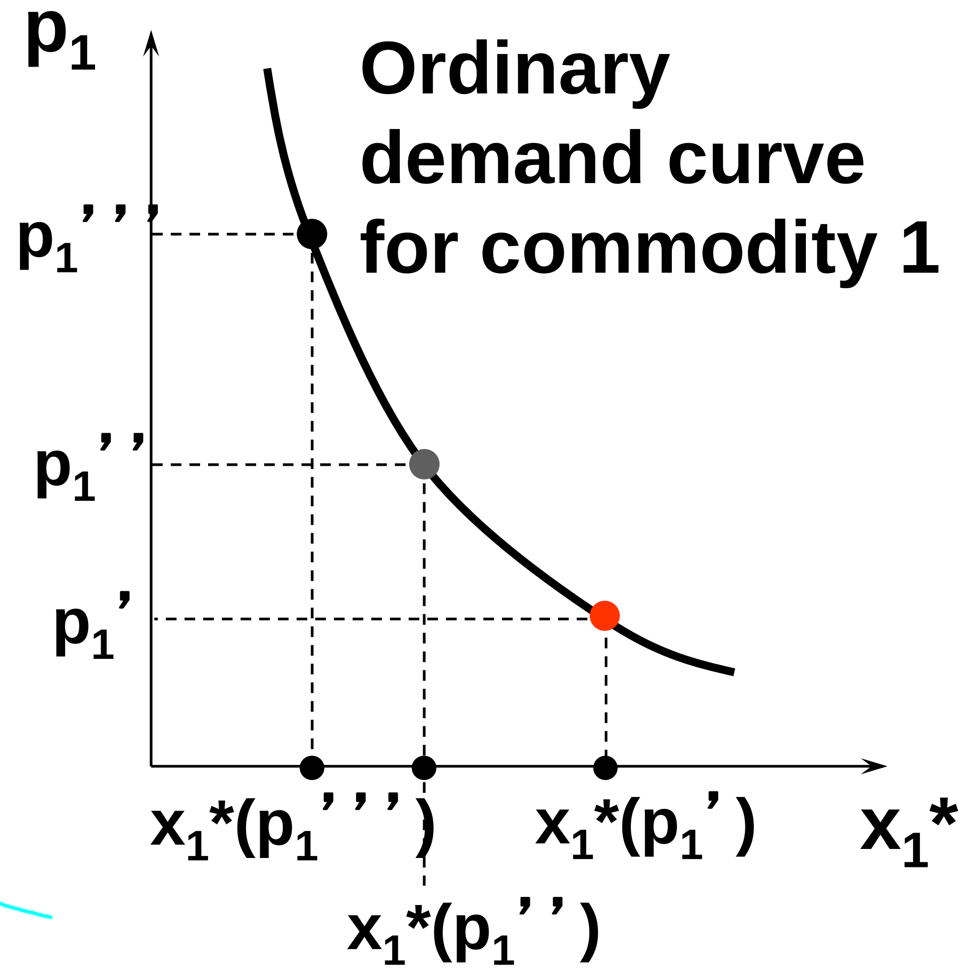


Own-Price Changes



Own-Price Changes





Own-Price Changes

- **The curve containing all the utility-maximizing bundles traced out as p_1 changes, with p_2 and y constant, is the p_1- price offer curve.**
- **The plot of the x_1-coordinate of the p_1- price offer curve against p_1 is the ordinary demand curve for commodity 1.**

Own-Price Changes

- What does a p_1 price-offer curve look like for a perfect-complements utility function?

Own-Price Changes

- What does a p_1 price-offer curve look like for a perfect-complements utility function?

$$U(x_1, x_2) = \min\{x_1, x_2\}.$$

Then the ordinary demand functions for commodities 1 and 2 are

Own-Price Changes

$$x_1^*(p_1,p_2,y) = x_2^*(p_1,p_2,y) = \frac{y}{p_1+p_2}.$$

Own-Price Changes

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With p_2 and y fixed, higher p_1 causes smaller x_1* and x_2*.

Own-Price Changes

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As $p_1 \rightarrow 0, \quad x_1^* = x_2^* \rightarrow \frac{y}{p_2}.$

Own-Price Changes

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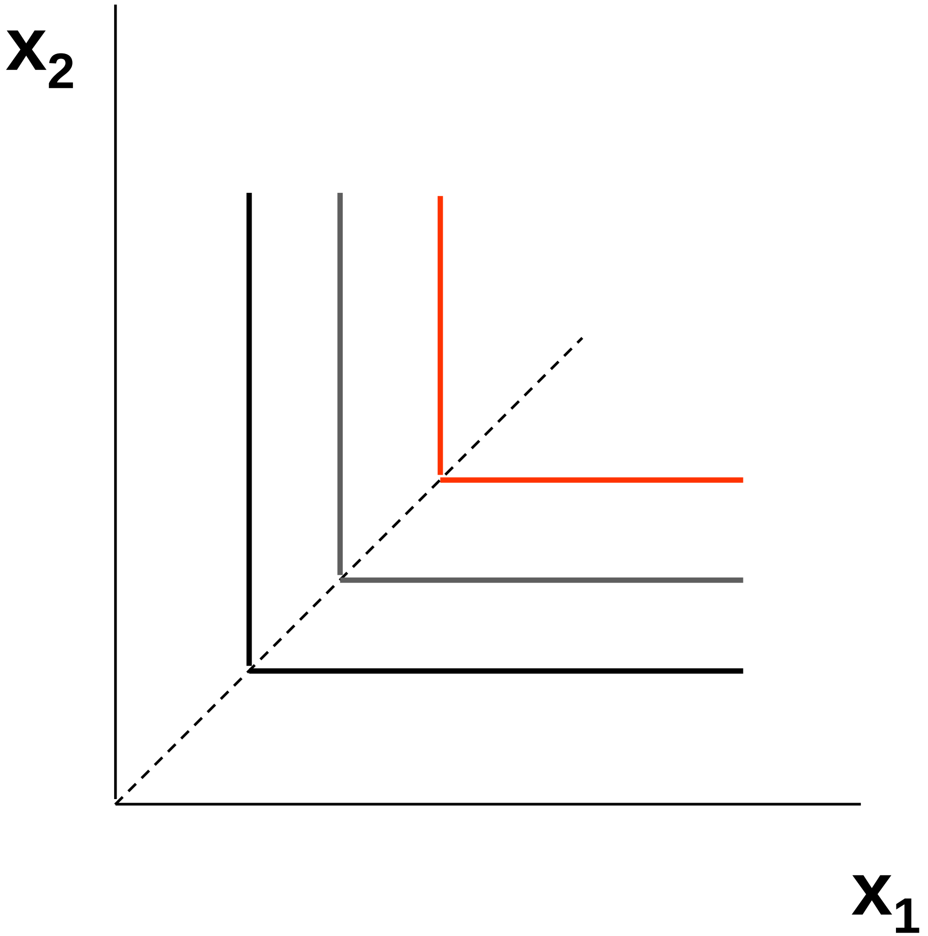
With p_2 and y fixed, higher p_1 causes smaller x_1* and x_2*.

As $p_1 \to 0, \quad x_1^* = x_2^* \to \dfrac{y}{p_2}.$

As $p_1 \to \infty, \quad x_1^* = x_2^* \to 0.$

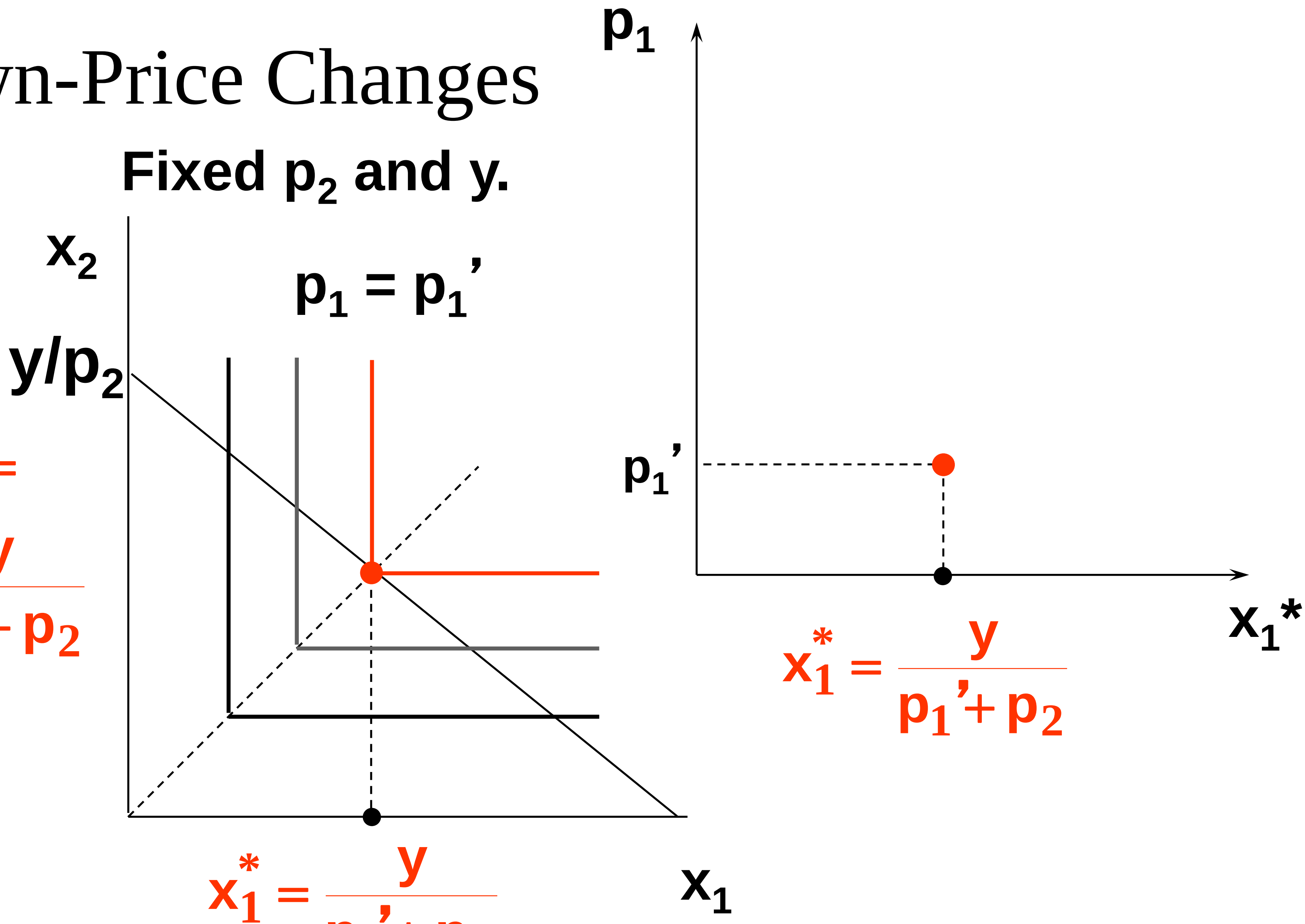
Own-Price Changes

Fixed p_2 and y.



Own-Price Changes

Fixed p_2 and y.



$$x_2^* = \frac{y}{p_1' + p_2}$$

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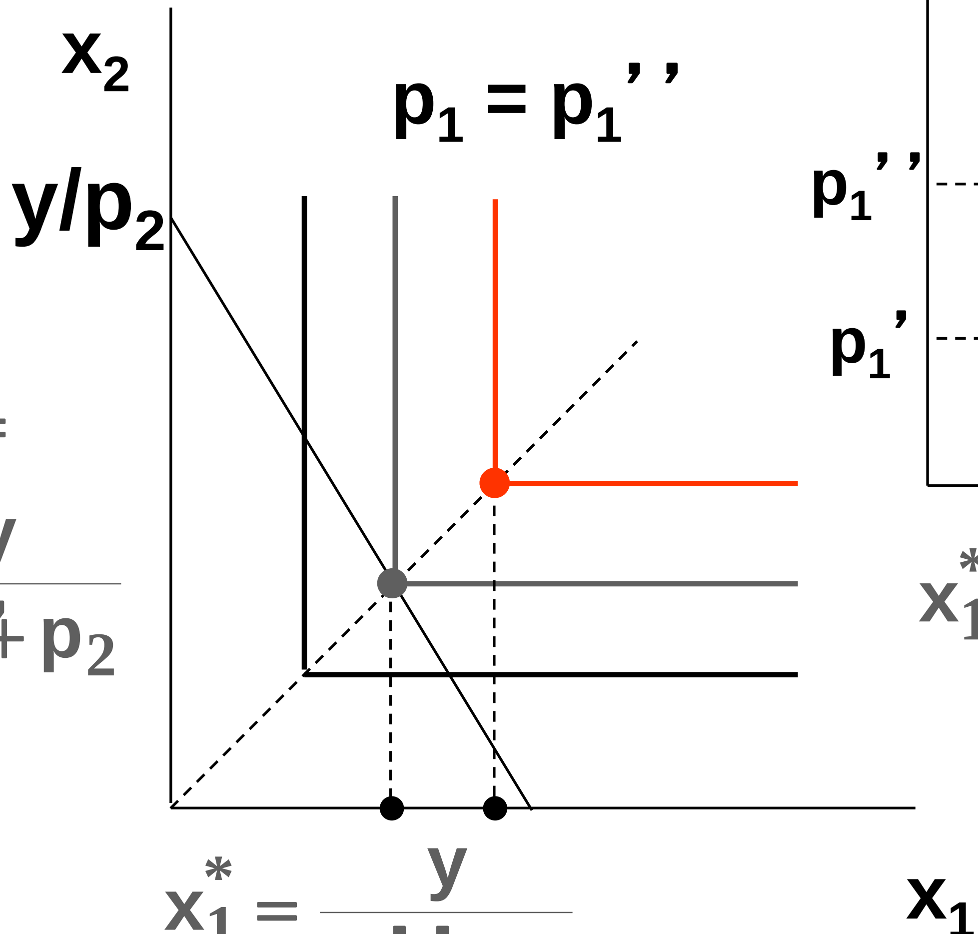
Own-Price Changes

Fixed p_2 and y.



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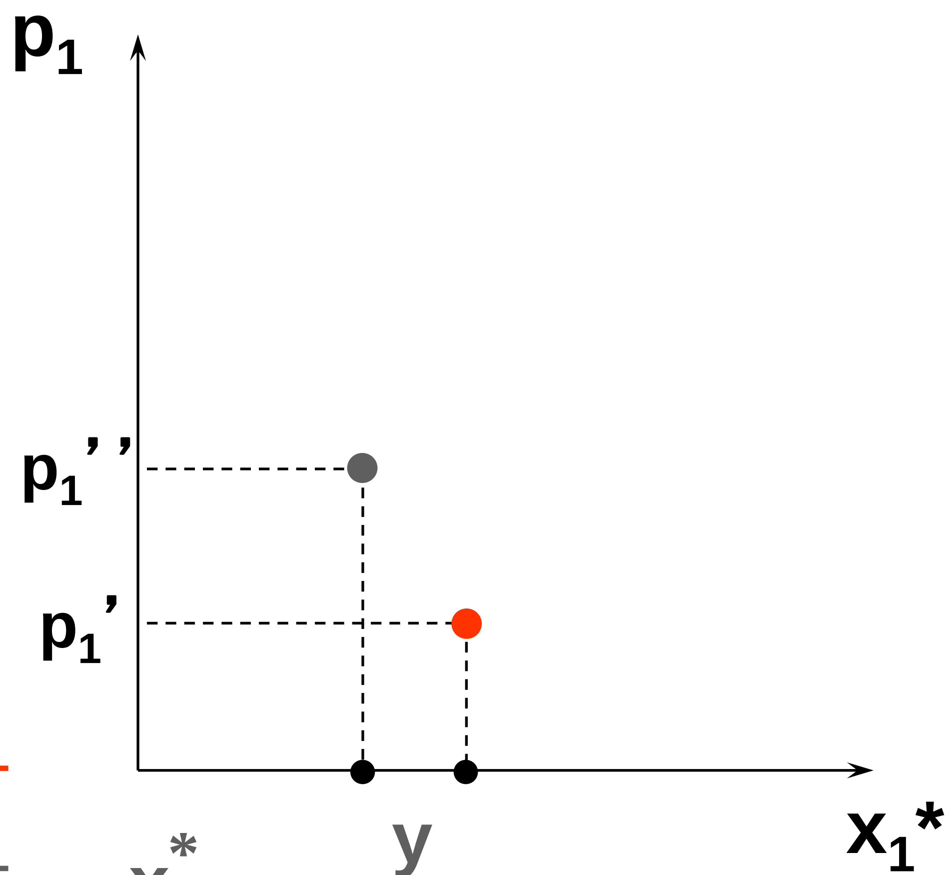

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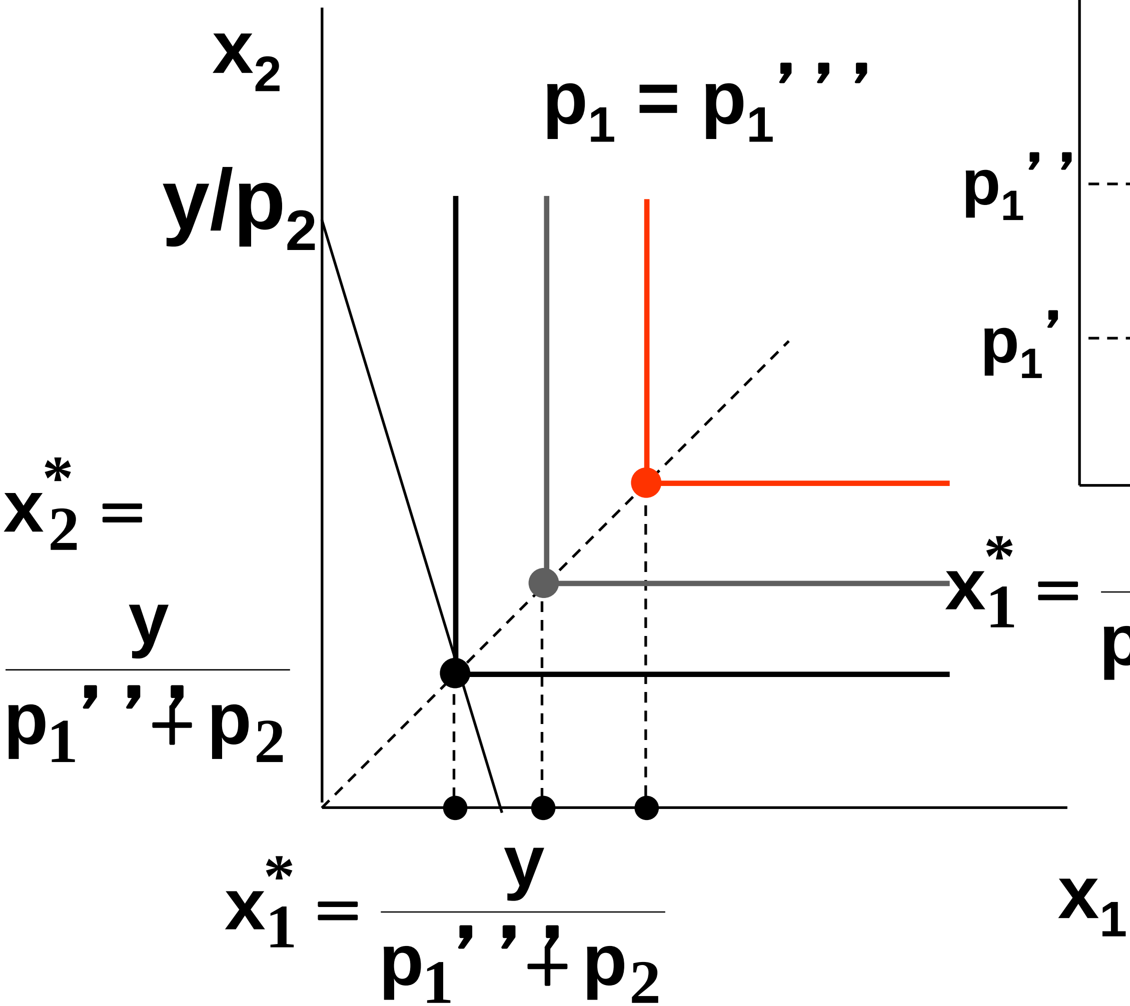
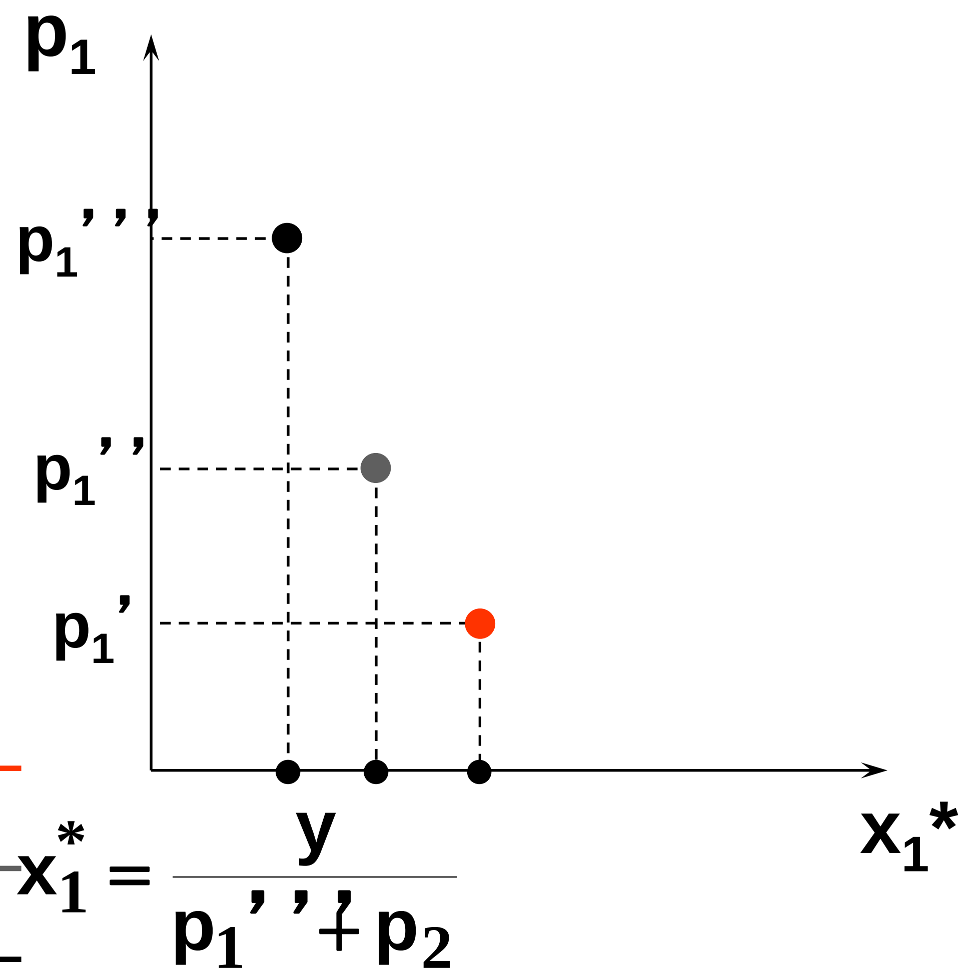
Own-Price Changes

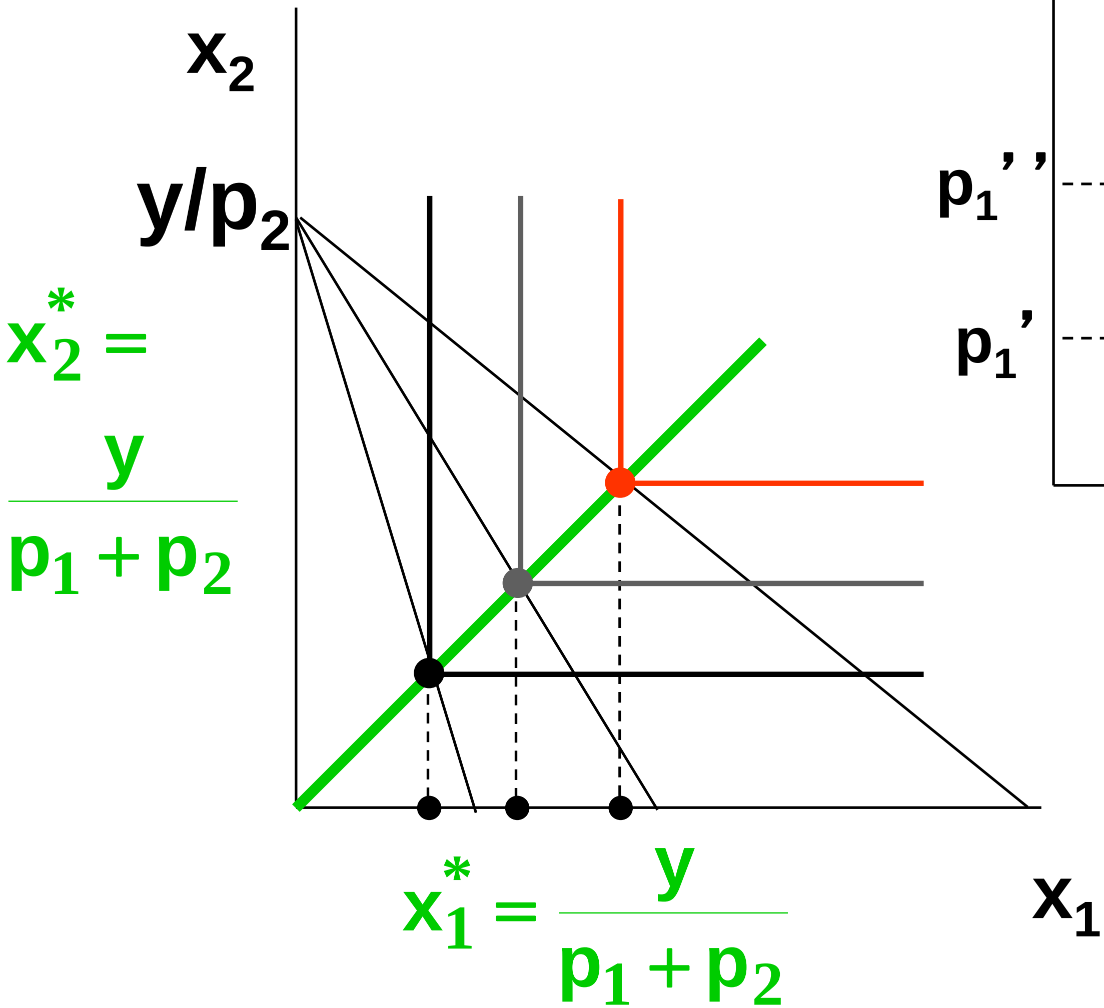
Fixed p_2 and y.

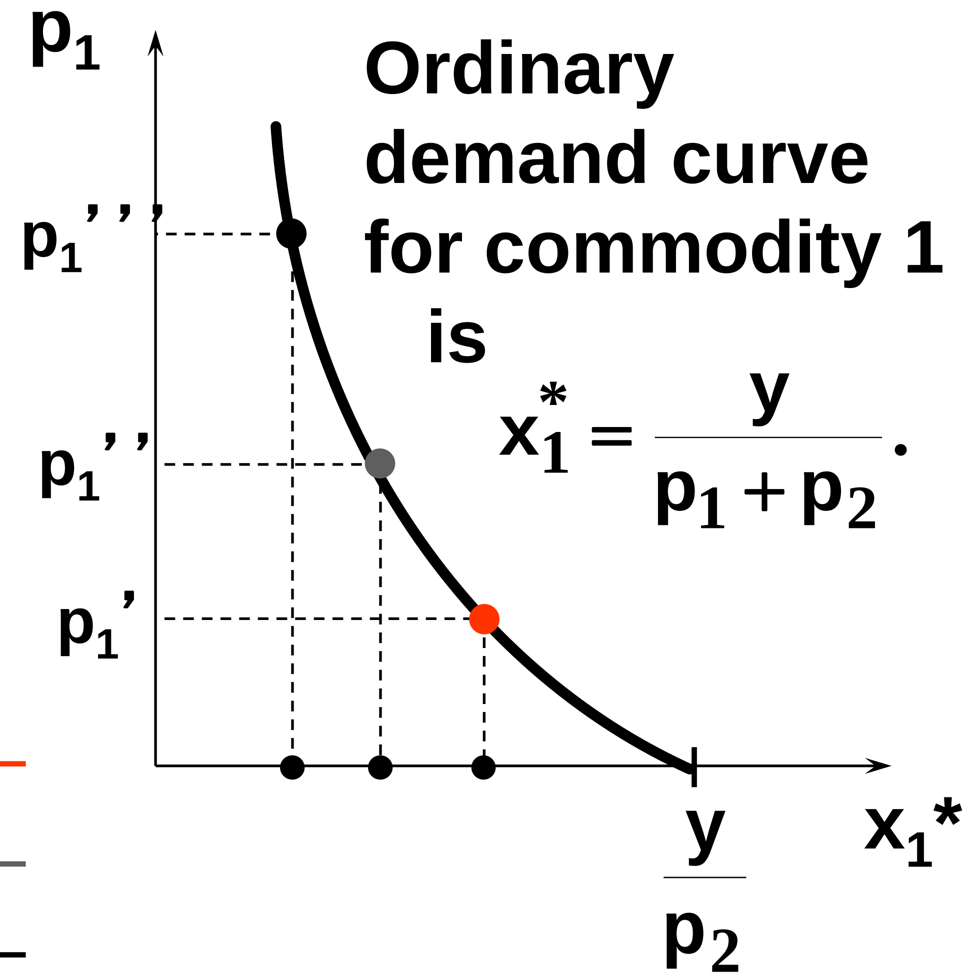




Own-Price Changes

Fixed p_2 and y.





Ordinary demand curve for commodity 1 is

$$x_1^* = \frac{y}{p_1 + p_2}.$$

Own-Price Changes

- What does a p_1 price-offer curve look like for a perfect-substitutes utility function?

$$U(x_1, x_2) = x_1 + x_2.$$

Then the ordinary demand functions for commodities 1 and 2 are

Own-Price Changes

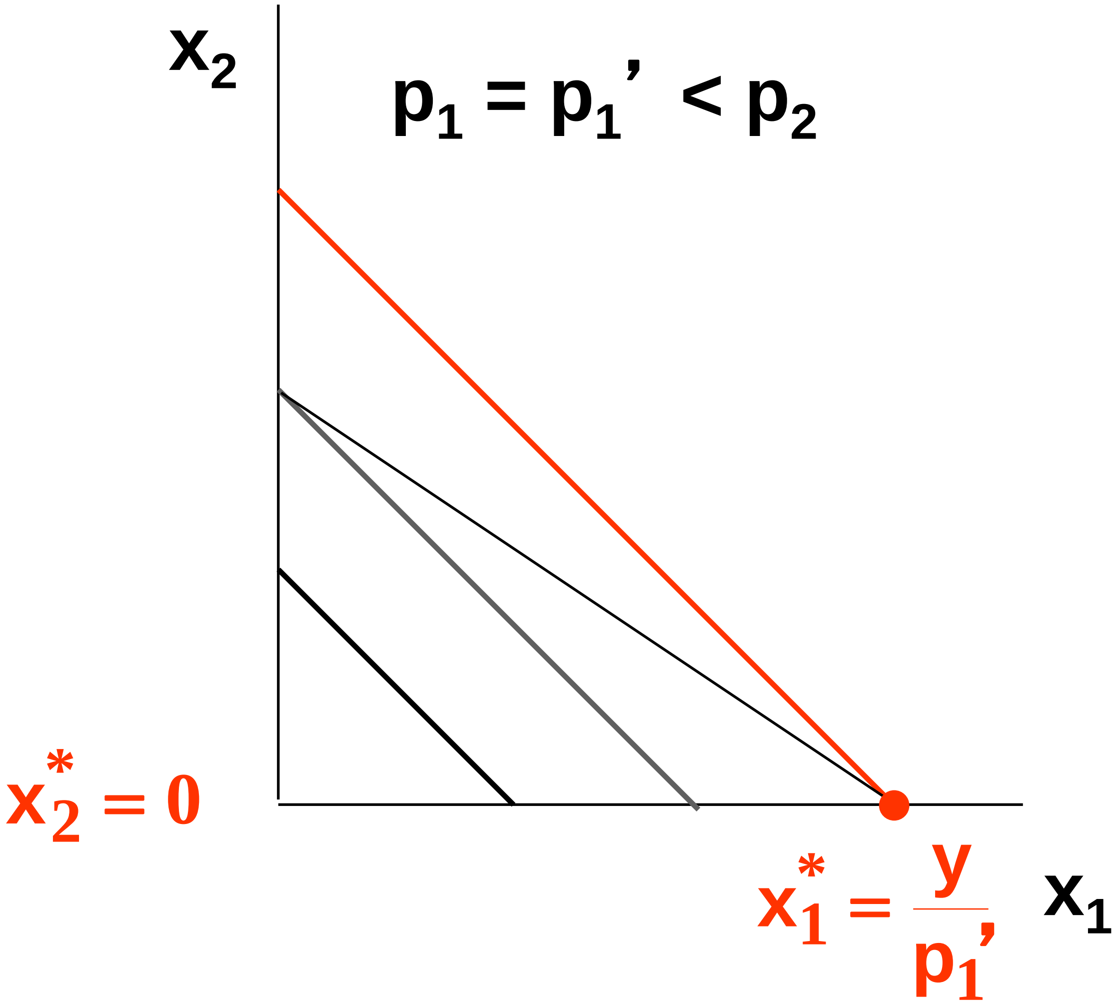
$$x_1^*(p_1, p_2, y) = \begin{cases} 0 & , \text{if } p_1 > p_2 \\ y/p_1 & , \text{if } p_1 < p_2 \end{cases}$$

and

$$x_2^*(p_1, p_2, y) = \begin{cases} 0 & , \text{if } p_1 < p_2 \\ y/p_2 & , \text{if } p_1 > p_2. \end{cases}$$

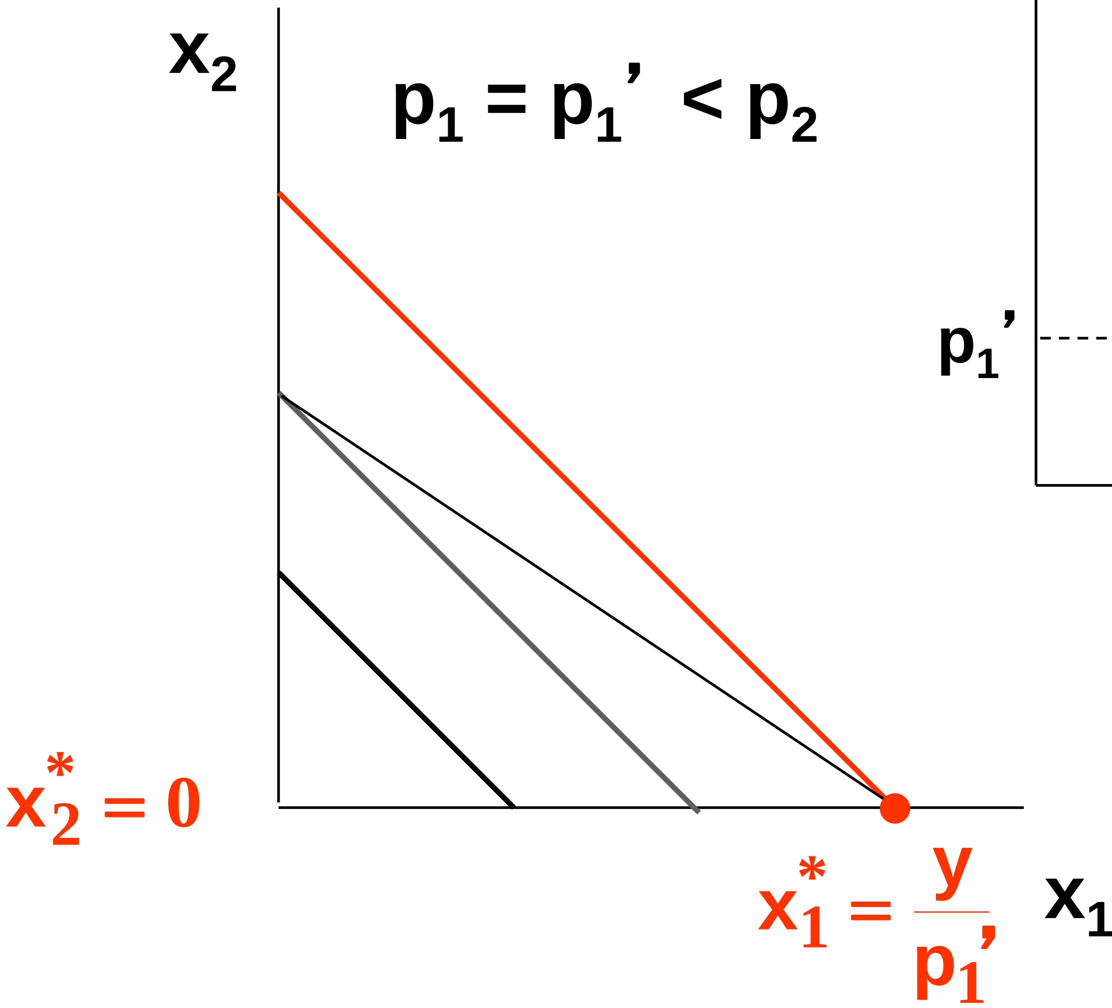
Own-Price Changes

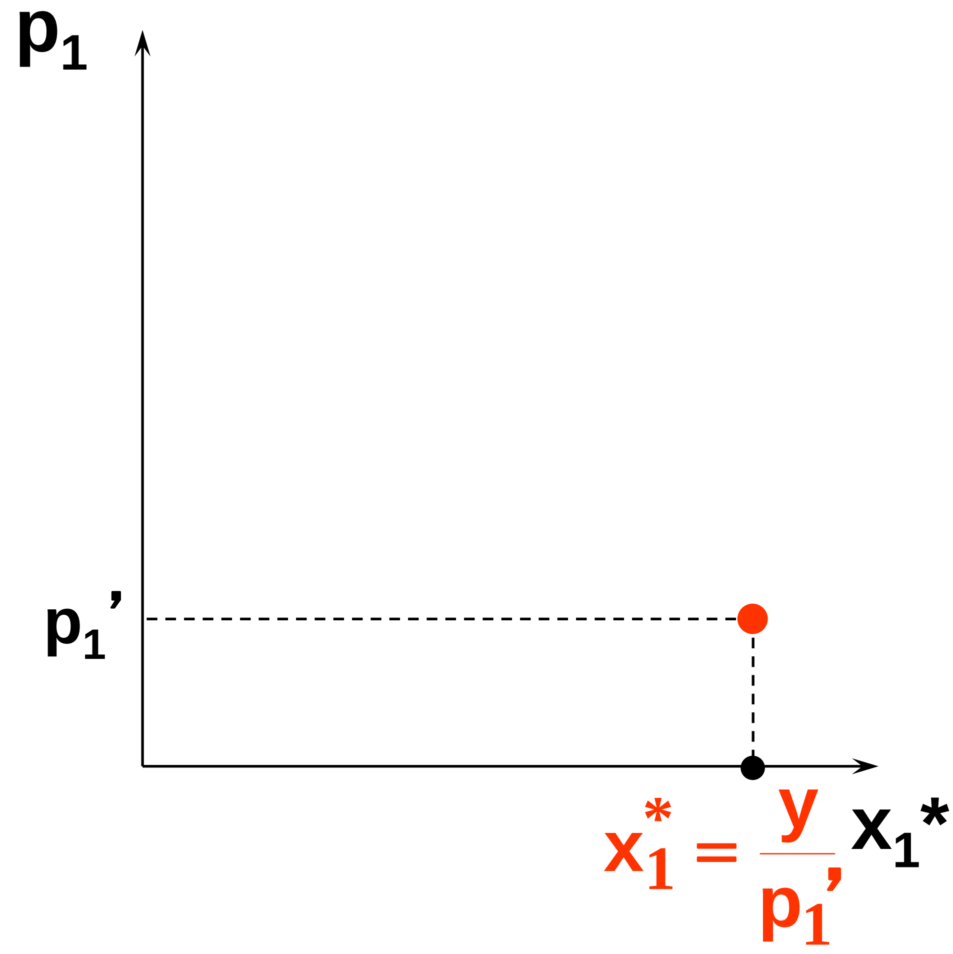
Fixed p_2 and y.



Own-Price Changes

Fixed p_2 and y.





Own-Price Changes

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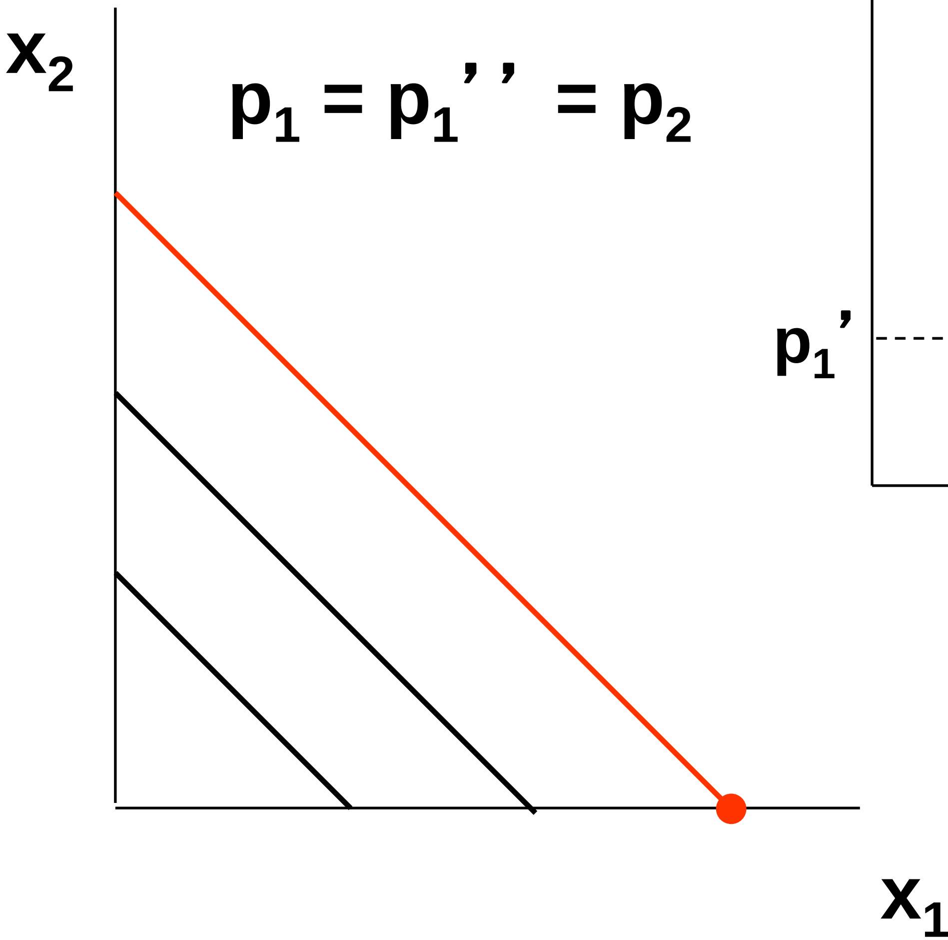
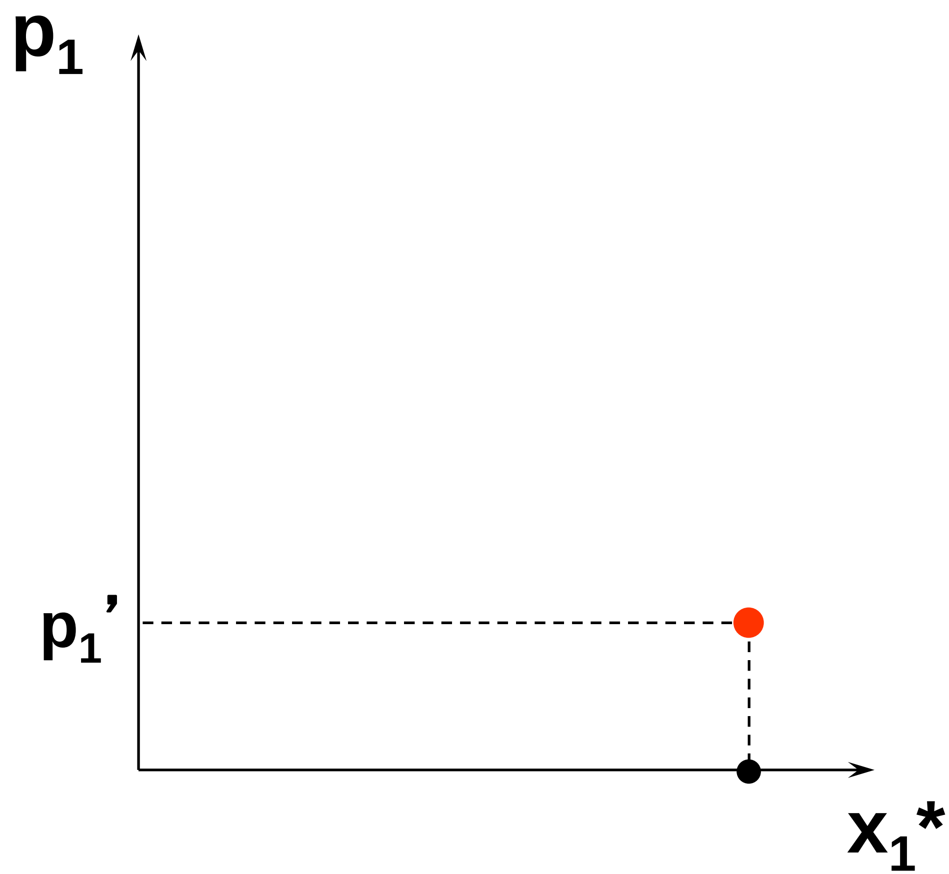
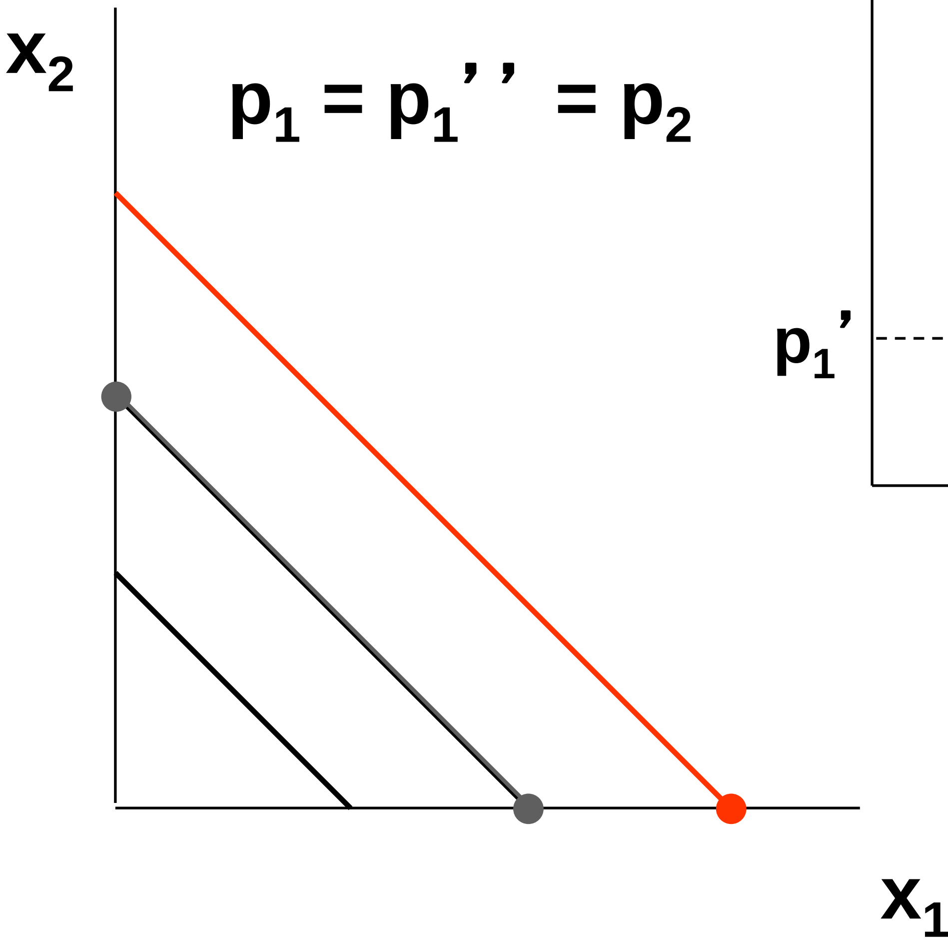
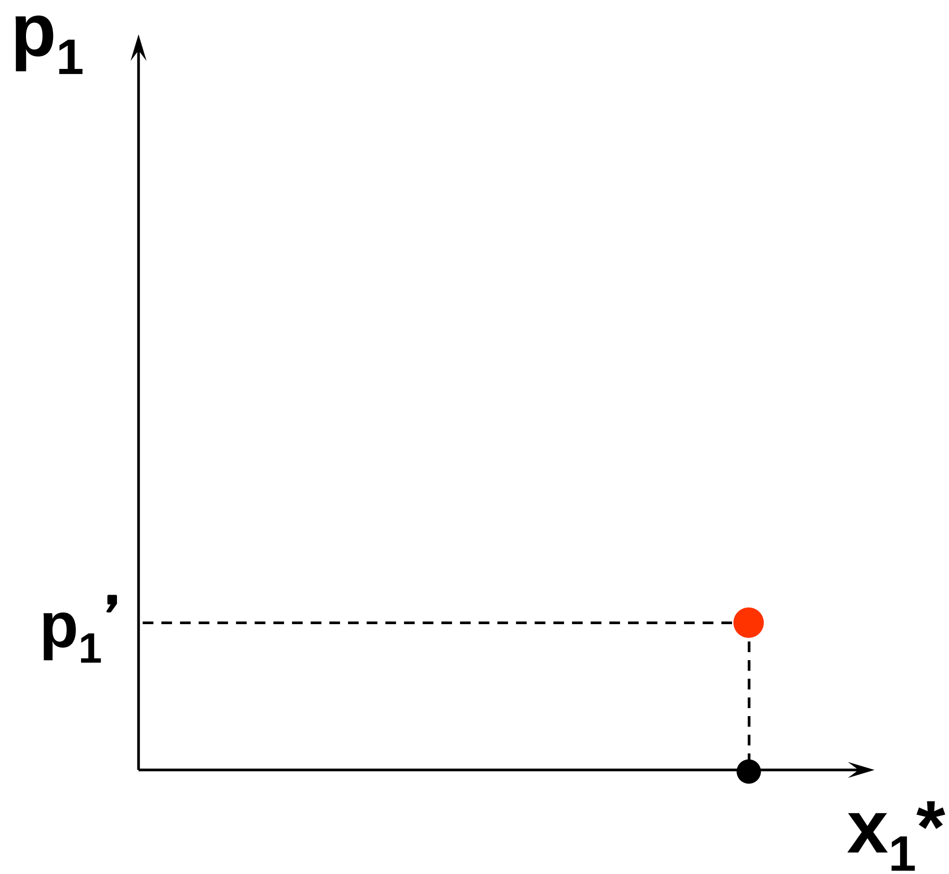




Own-Price Changes

Fixed p_2 and y.





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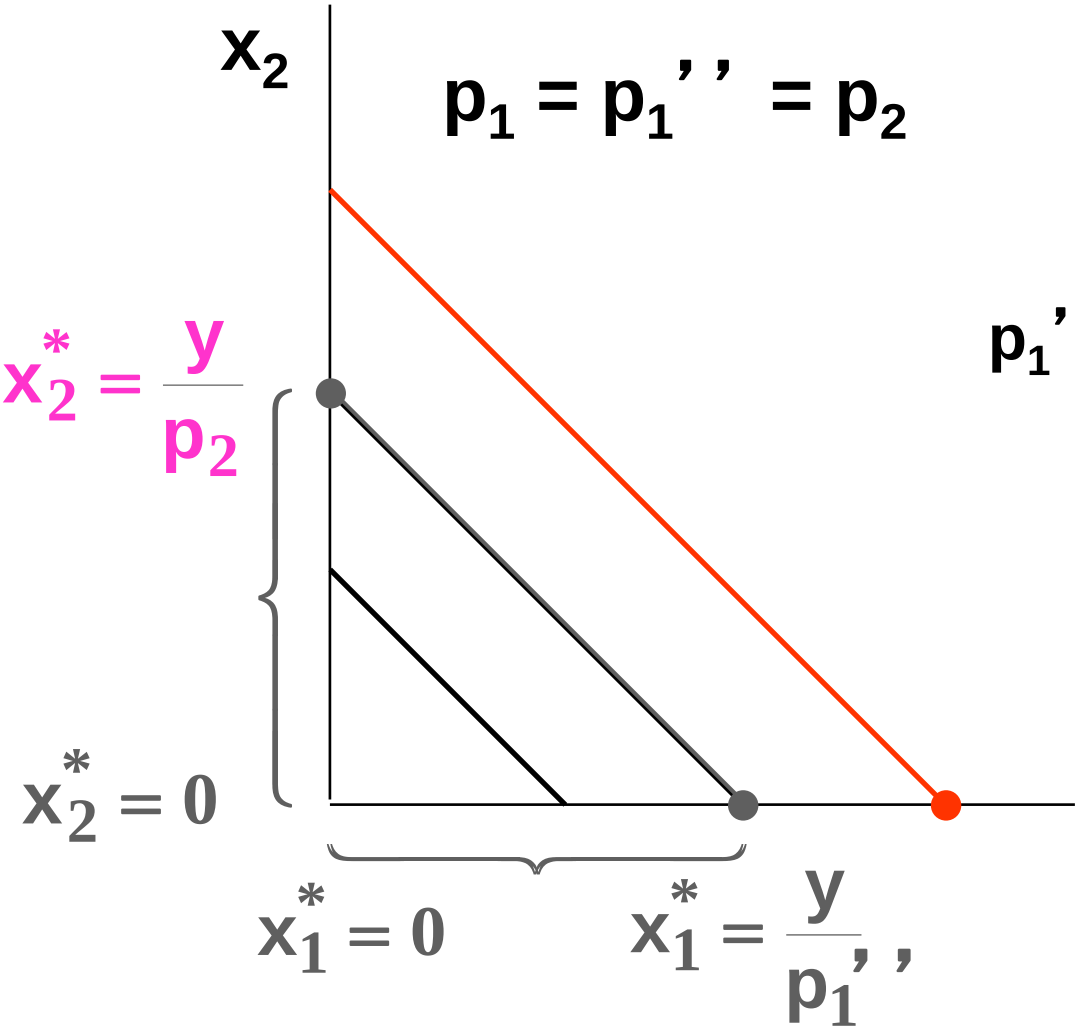

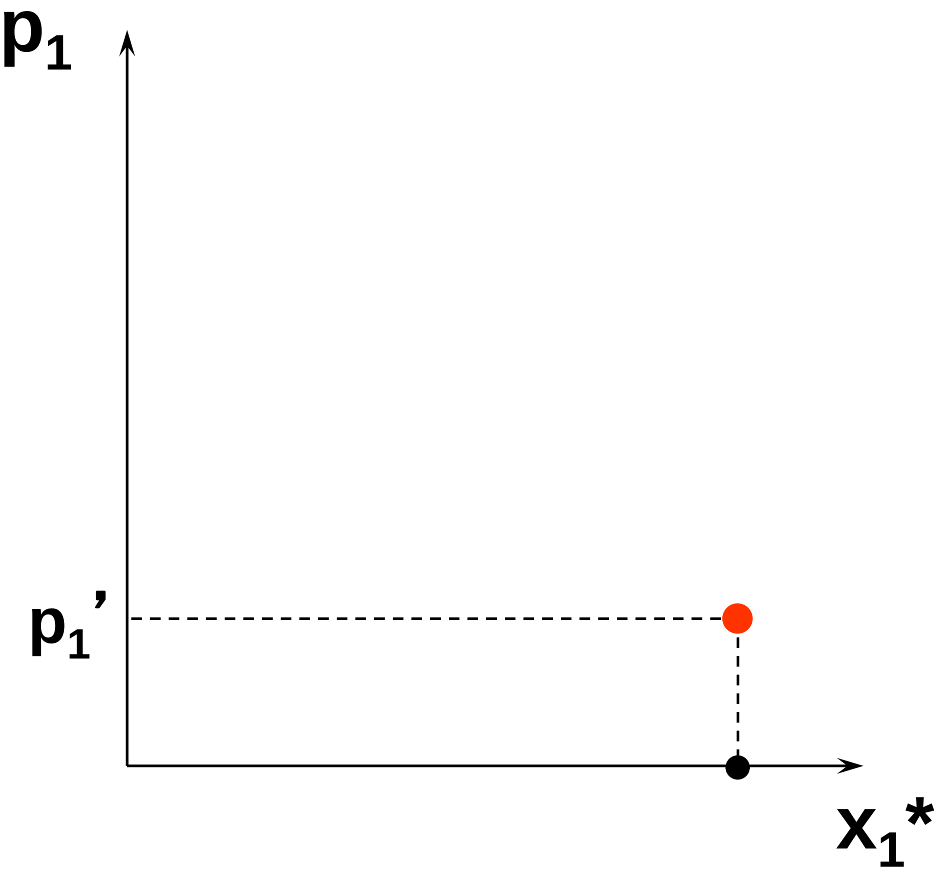

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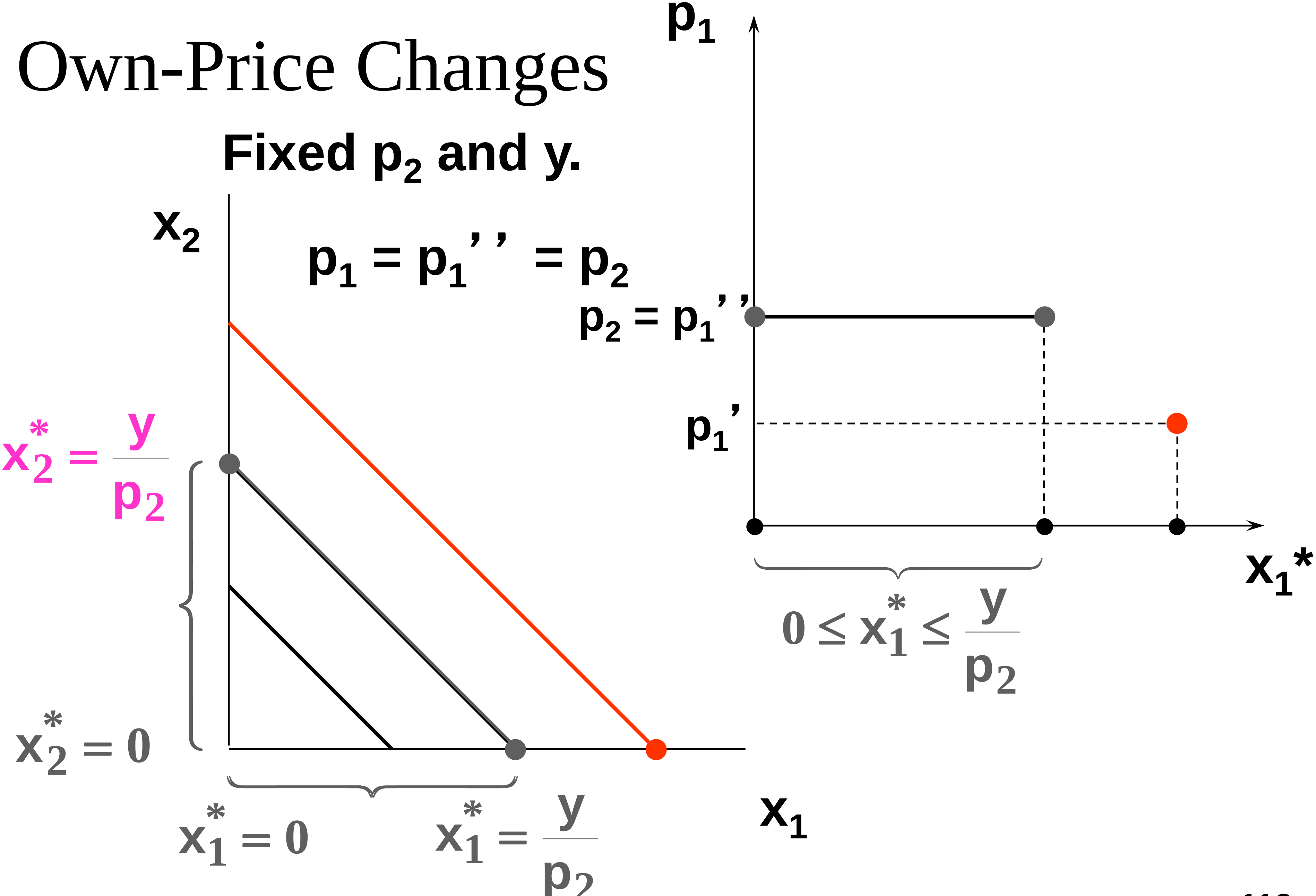
Fixed p_2 and y.



Own-Price Changes

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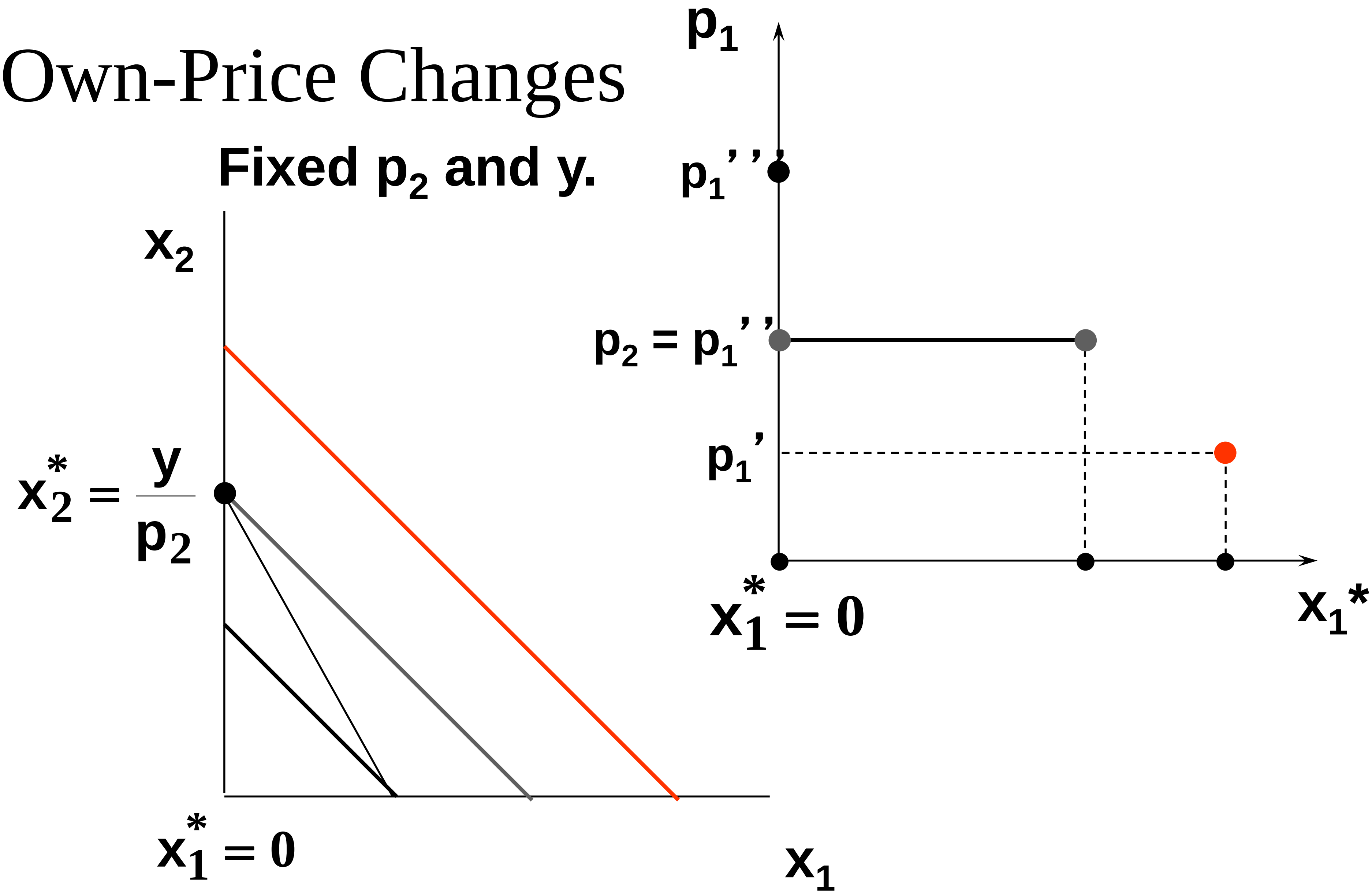
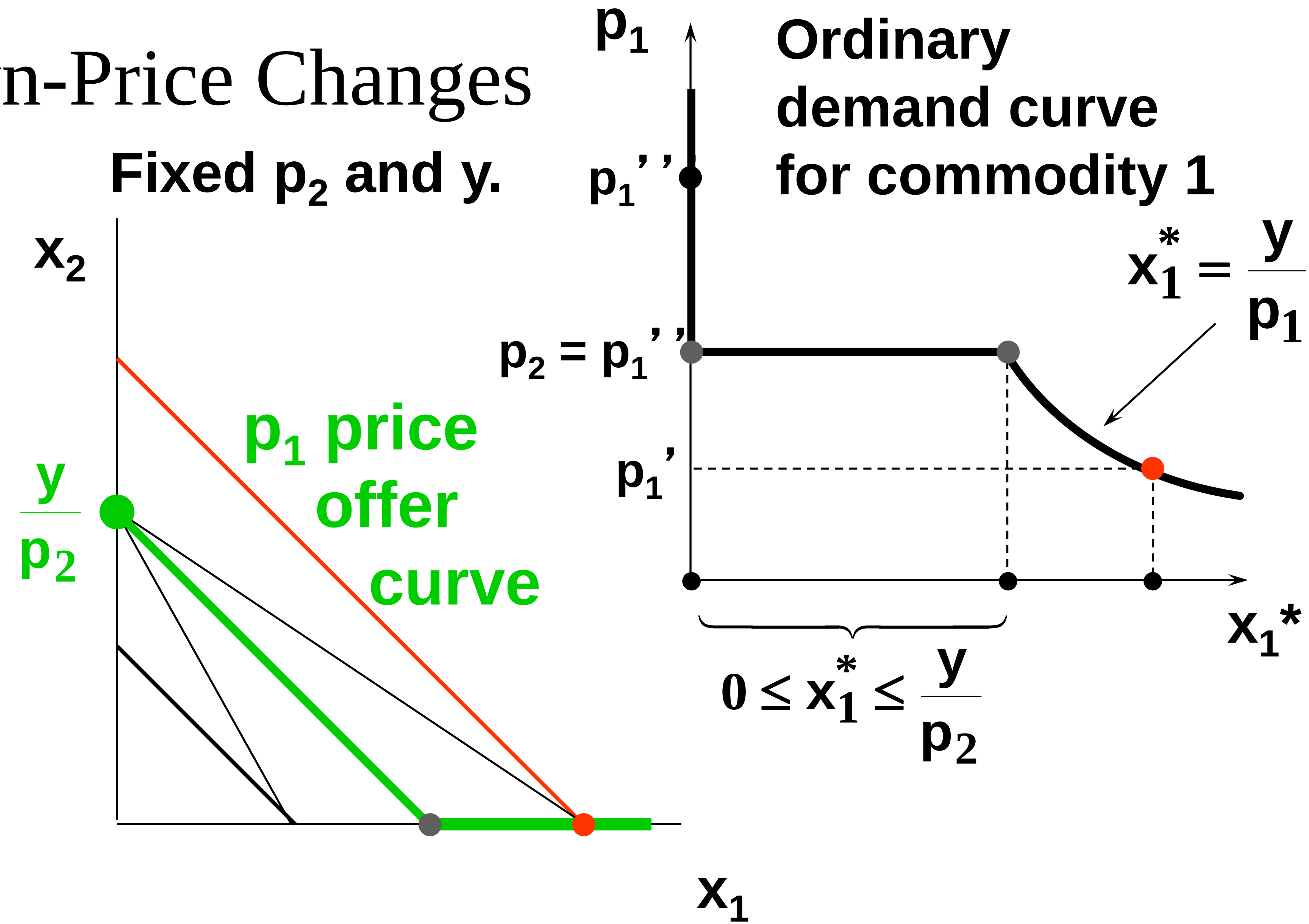


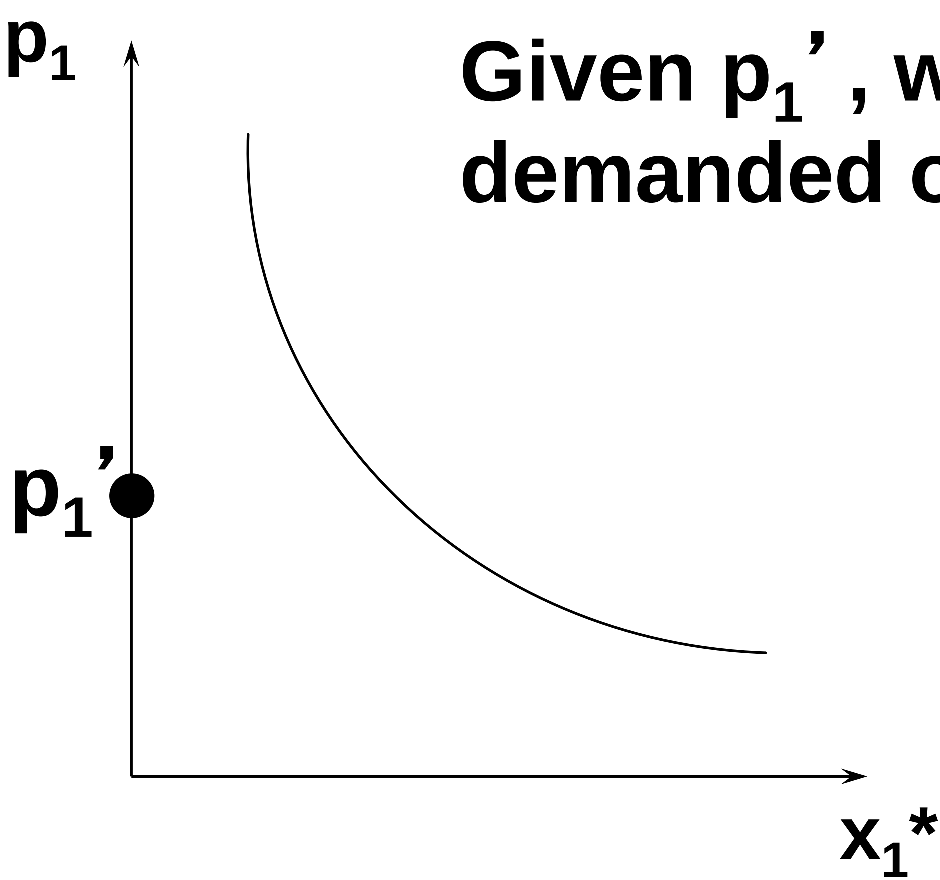
Own-Price Changes



Own-Price Changes

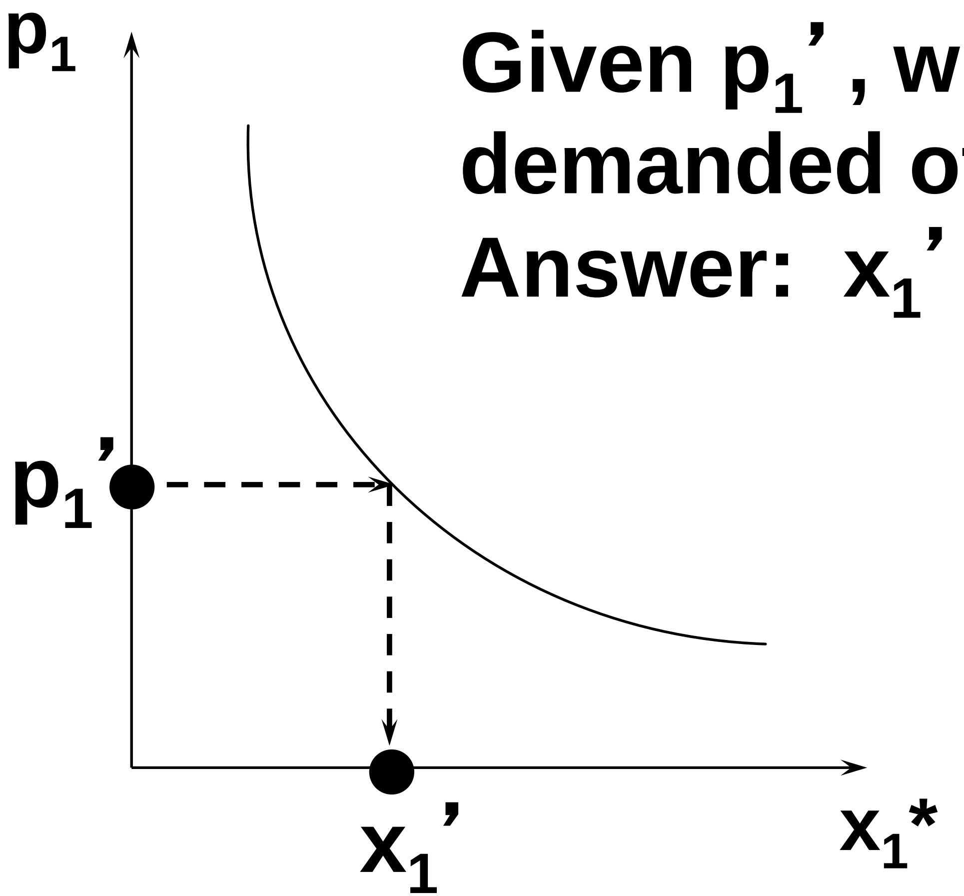
- Usually we ask “Given the price for commodity 1 what is the quantity demanded of commodity 1?”
- But we could also ask the inverse question “At what price for commodity 1 would a given quantity of commodity 1 be demanded?”

Own-Price Changes



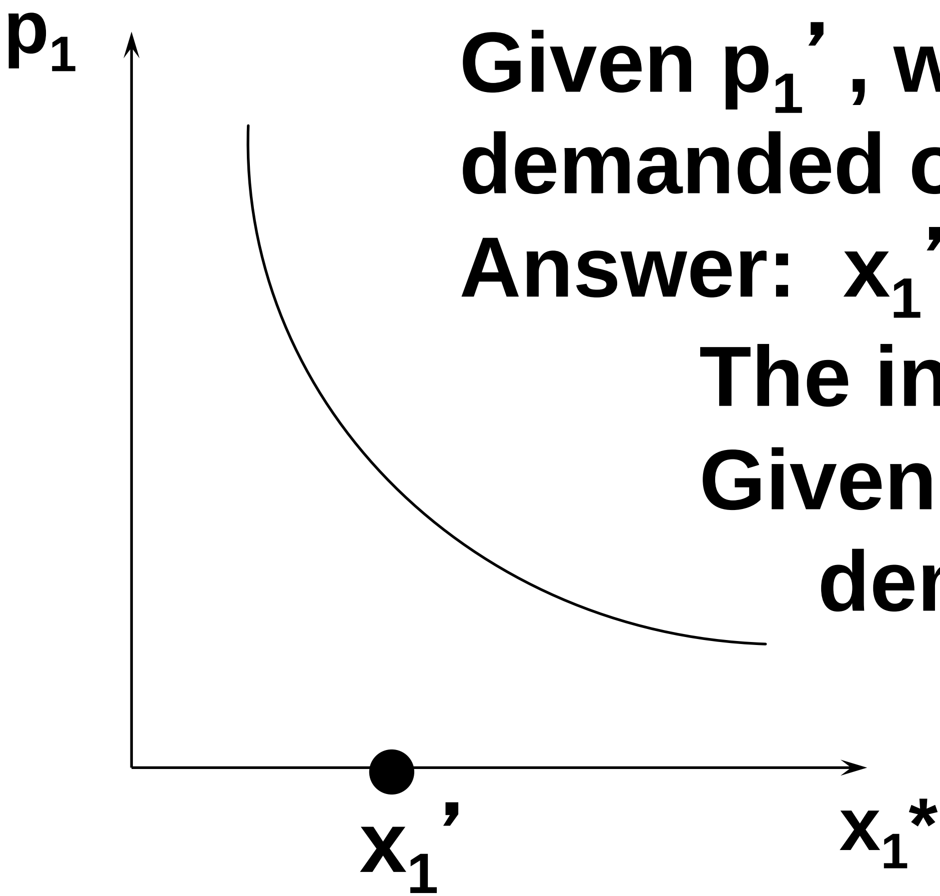
Given p_1', what quantity is demanded of commodity 1?

Own-Price Changes



Given p_1', what quantity is demanded of commodity 1? Answer: x_1' units.

Own-Price Changes

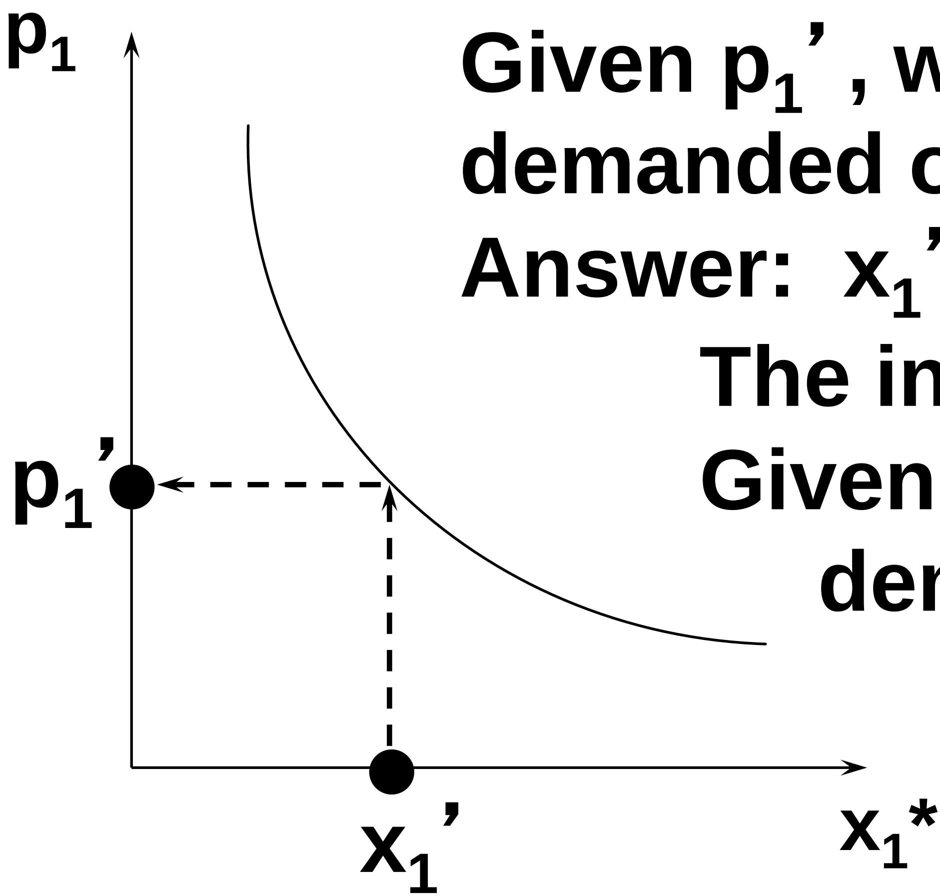


Given p_1', what quantity is demanded of commodity 1?

Answer: x_1' units.

The inverse question is: Given x_1' units are demanded, what is the price of commodity 1?

Own-Price Changes



Given p_1', what quantity is demanded of commodity 1? Answer: x_1' units.

The inverse question is: Given x_1' units are demanded, what is the price of commodity 1? Answer: p_1'

Own-Price Changes

- **Taking quantity demanded as given and then asking what must be price describes the inverse demand function of a commodity.**

Own-Price Changes

A Cobb-Douglas example:

$$x_1^* = \frac{ay}{(a+b)p_1}$$

is the ordinary demand function and

$$p_1 = \frac{ay}{(a+b)x_1^*}$$

is the inverse demand function.

Own-Price Changes

A perfect-complements example:

$$x_1^* = \frac{y}{p_1 + p_2}$$

is the ordinary demand function and

$$p_1 = \frac{y}{x_1^*} - p_2$$

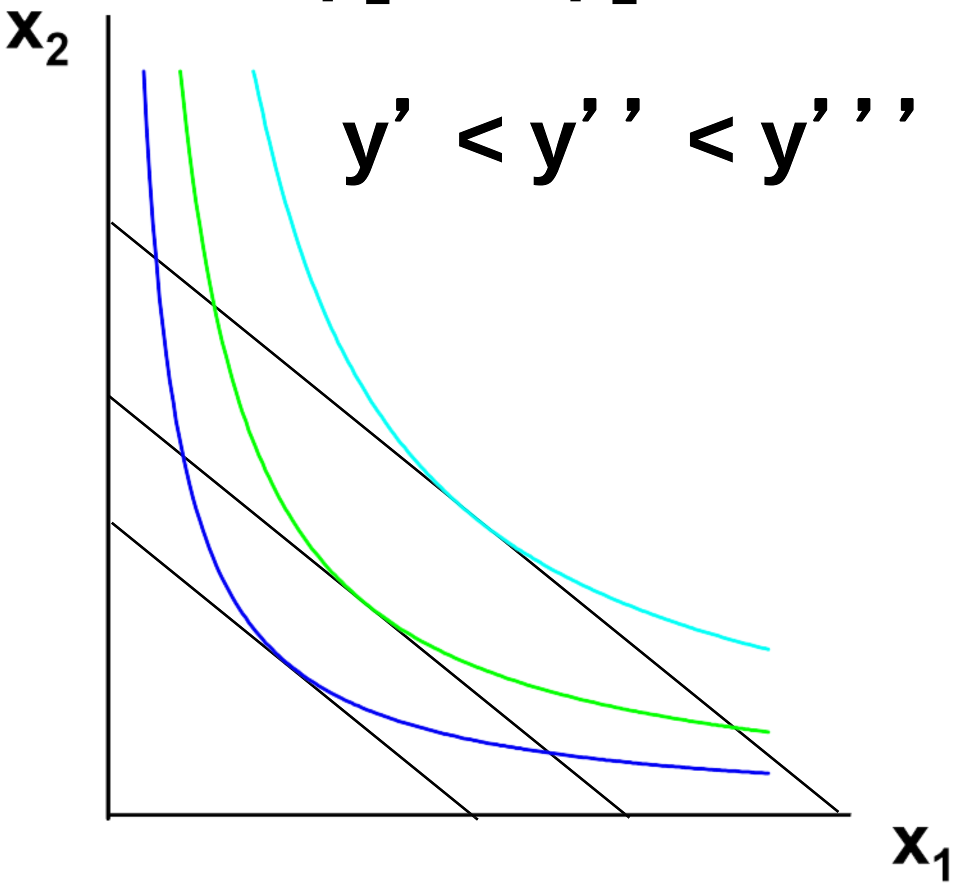
is the inverse demand function.

Income Changes

- **How does the value of $x_1^*(p_1,p_2,y)$ change as y changes, holding both p_1 and p_2 constant?**

Income Changes

Fixed p_1 and p_2.



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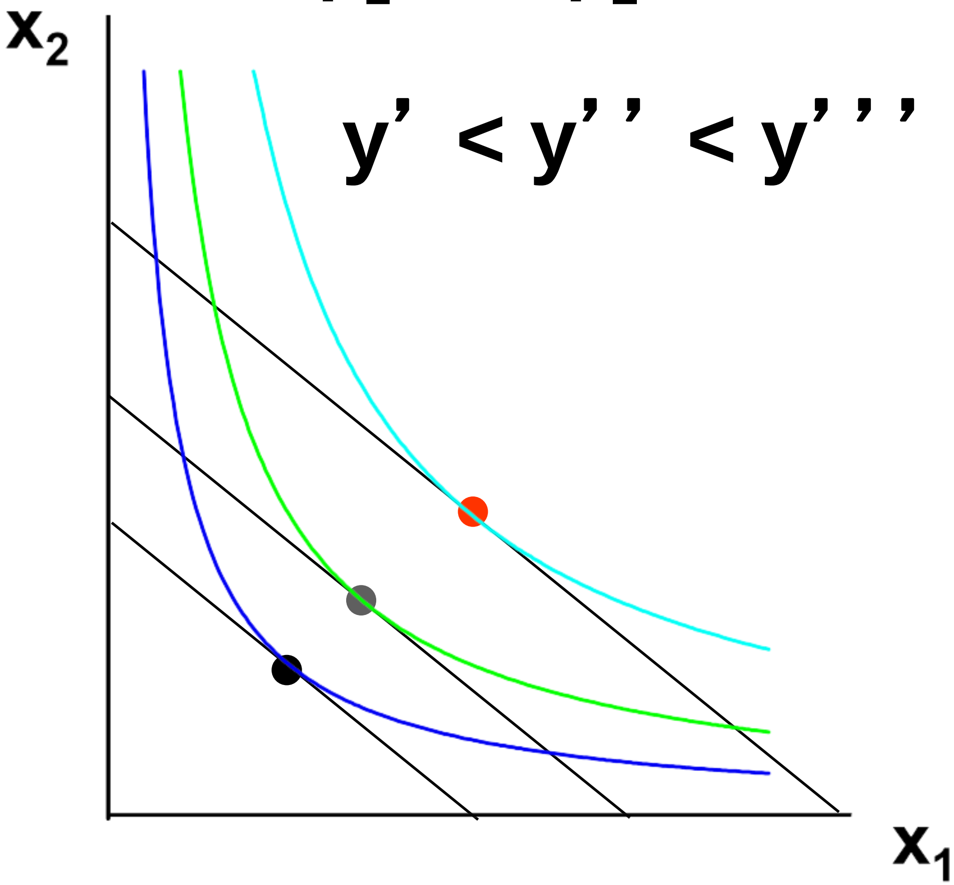

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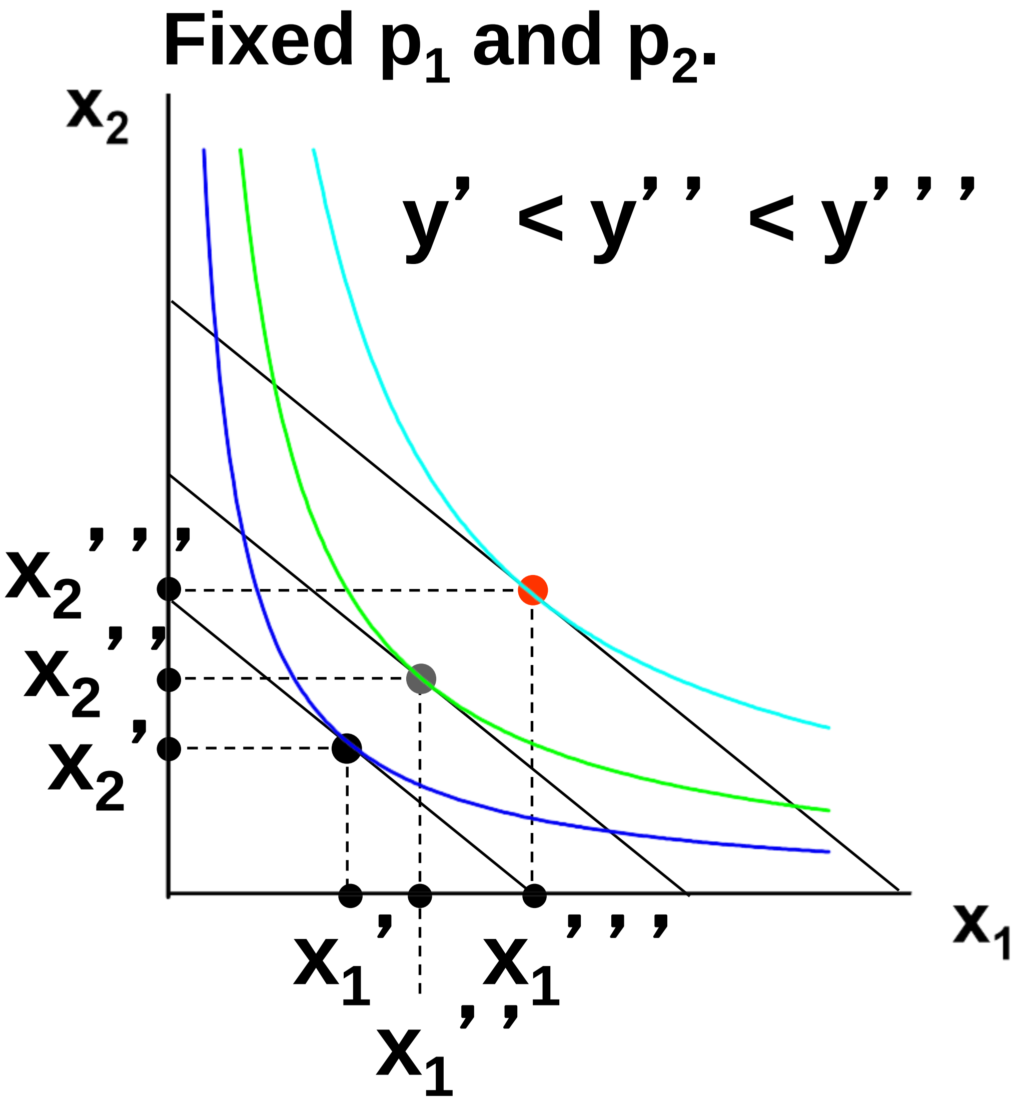

Income Changes

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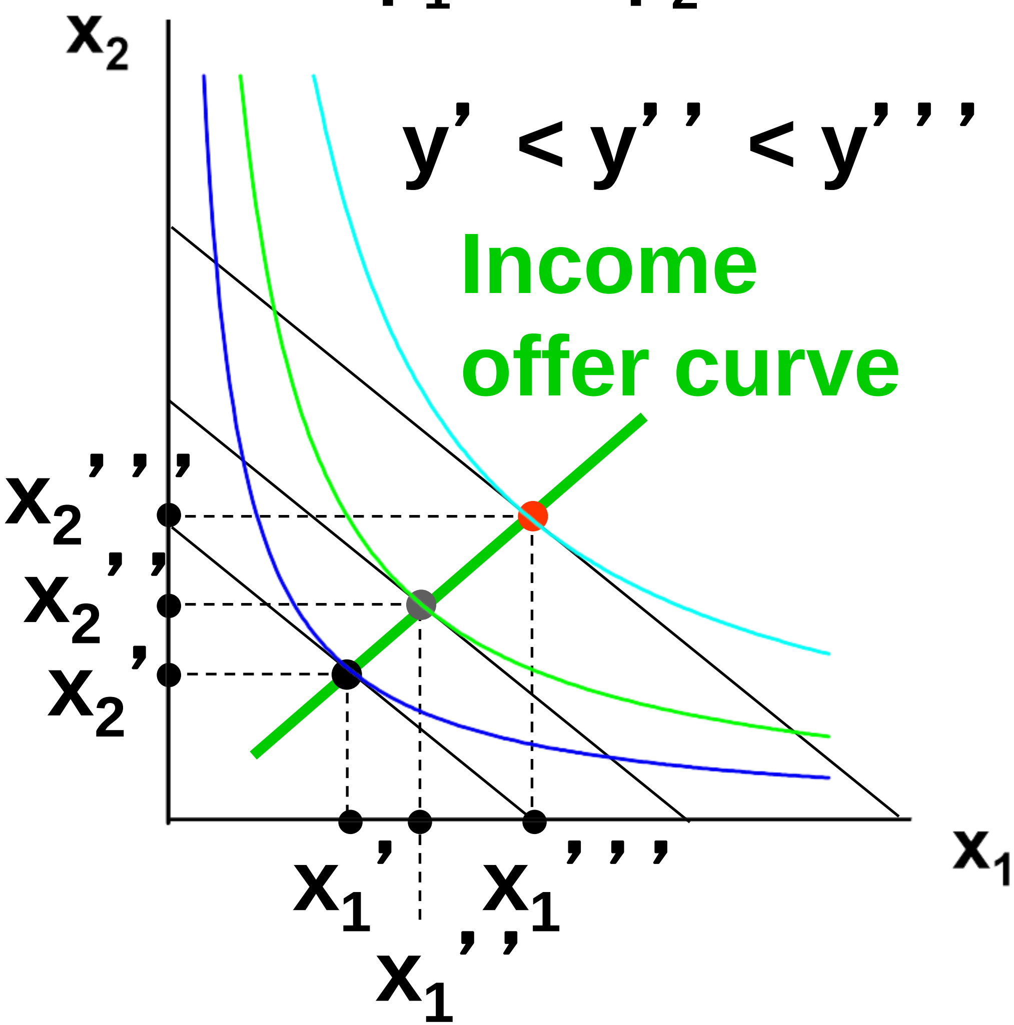
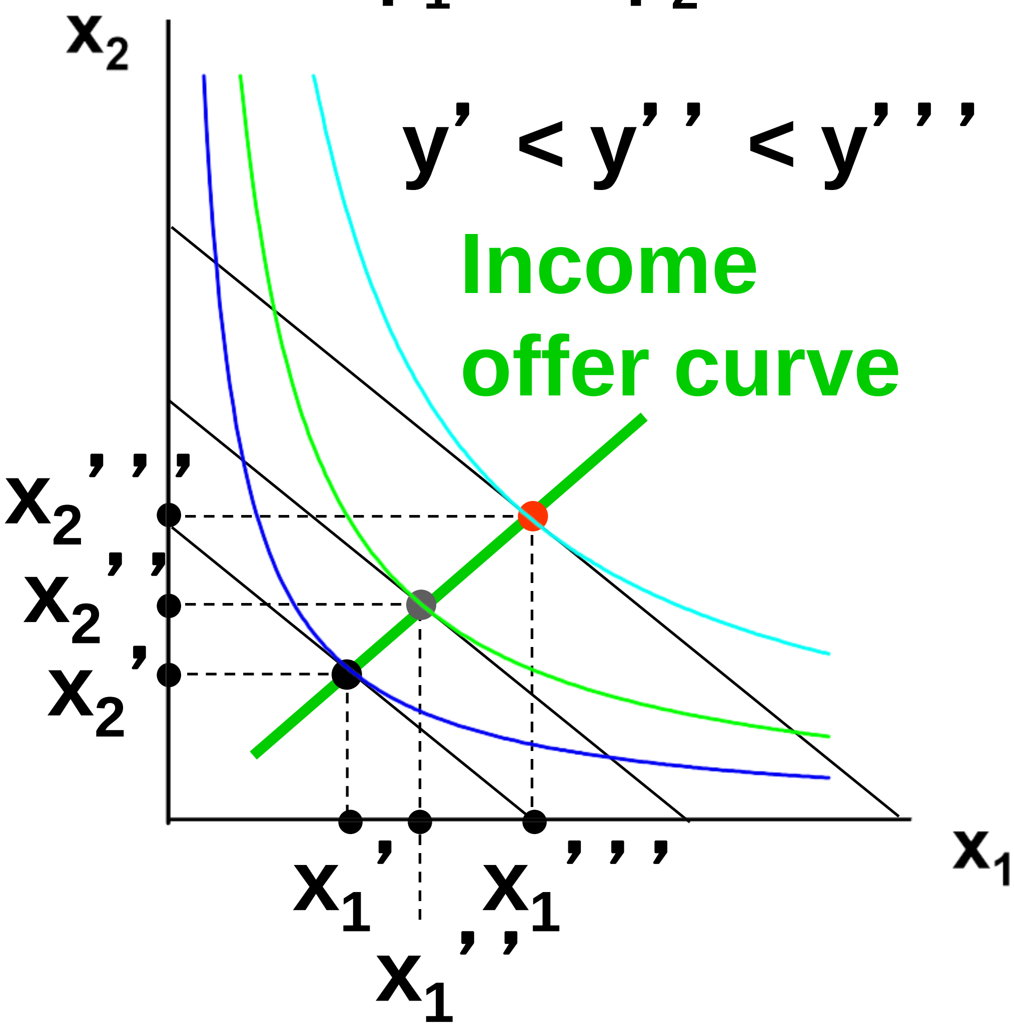


Income Changes

- **A plot of quantity demanded against income is called an Engel curve.**

Income Changes

Fixed p_1 and p_2.



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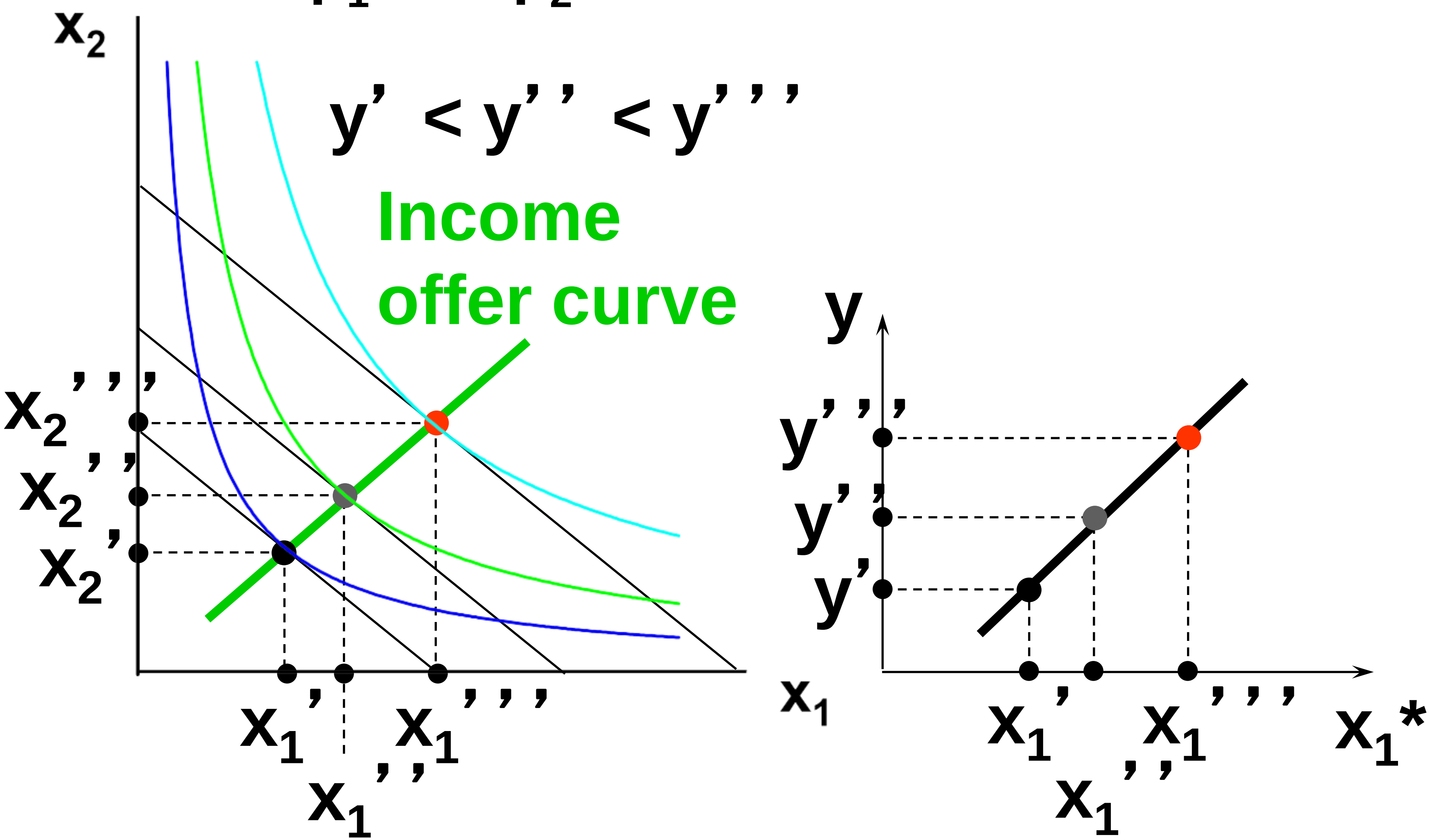

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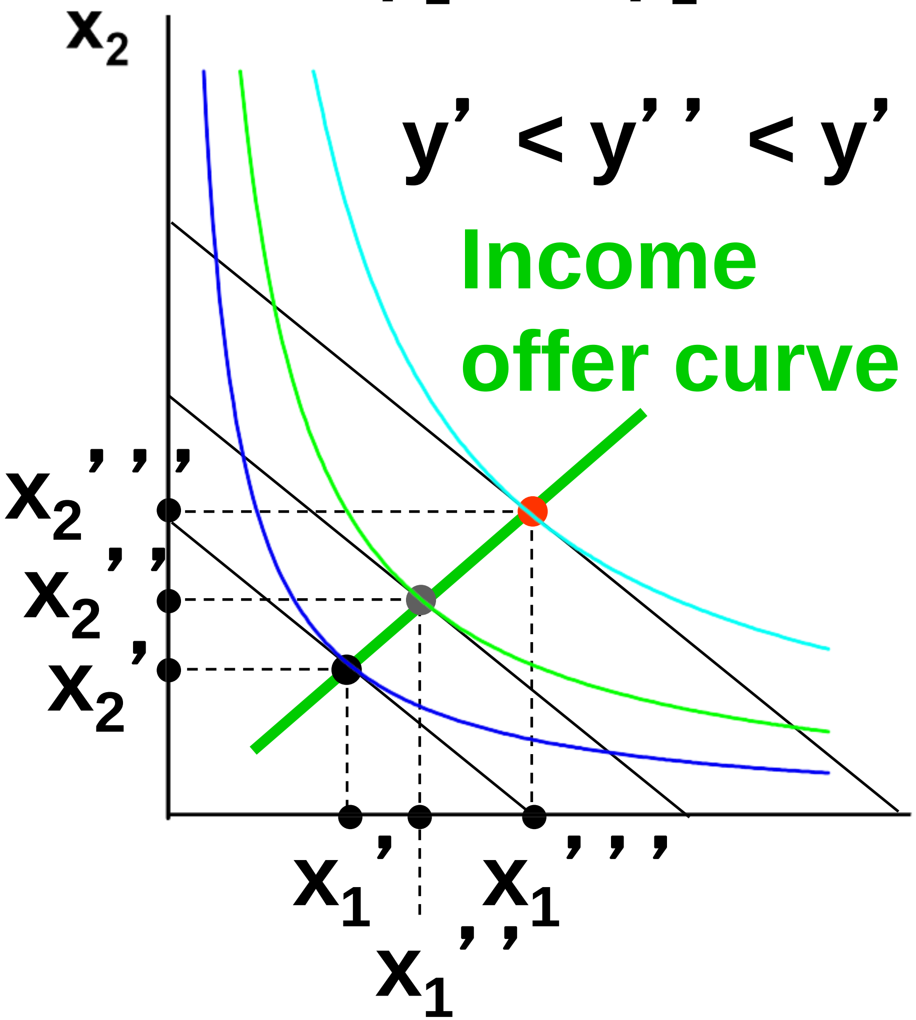

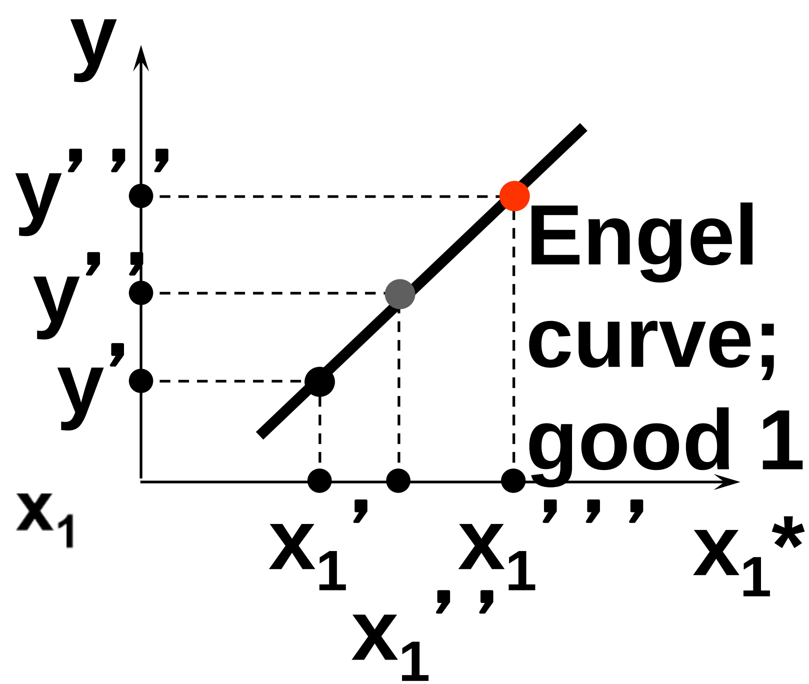

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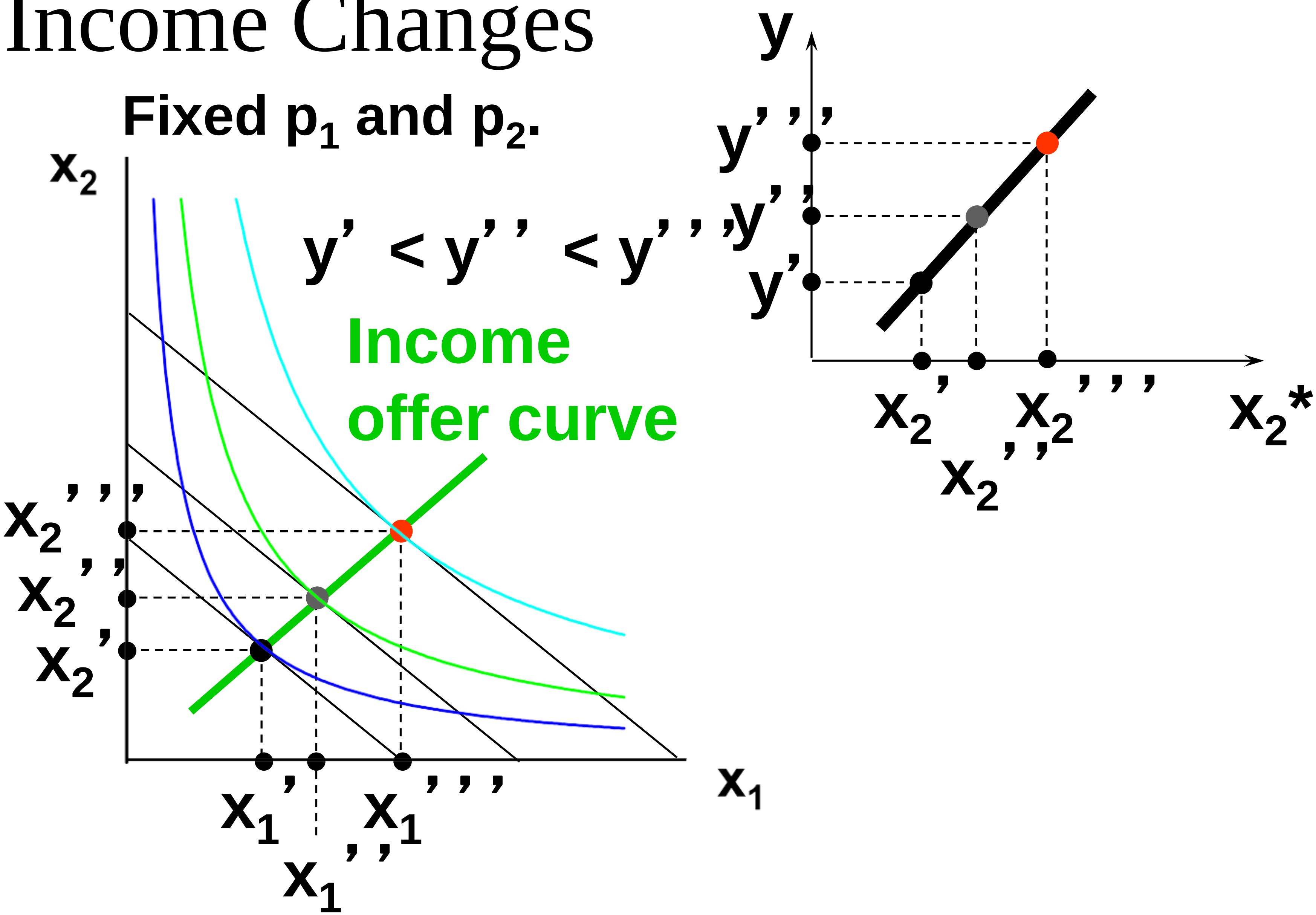

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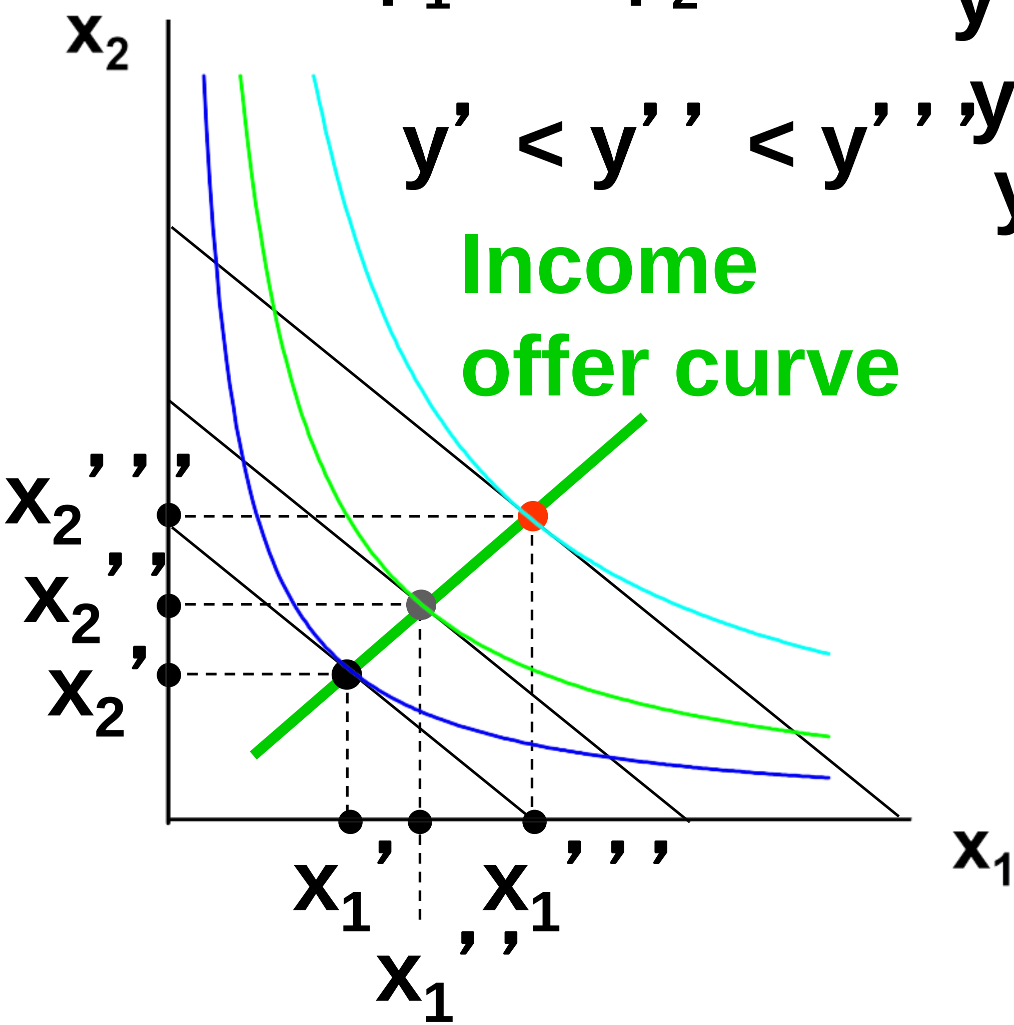

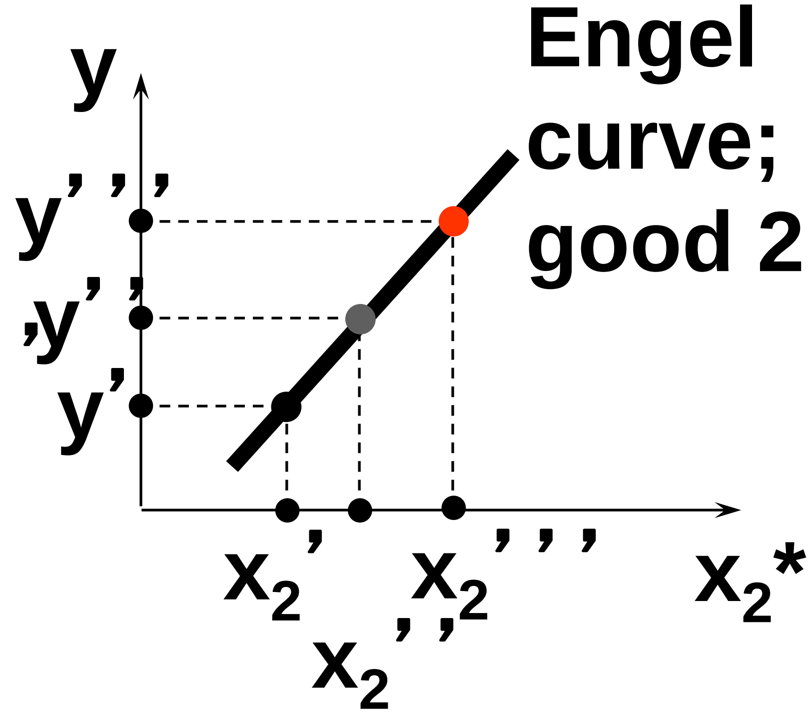

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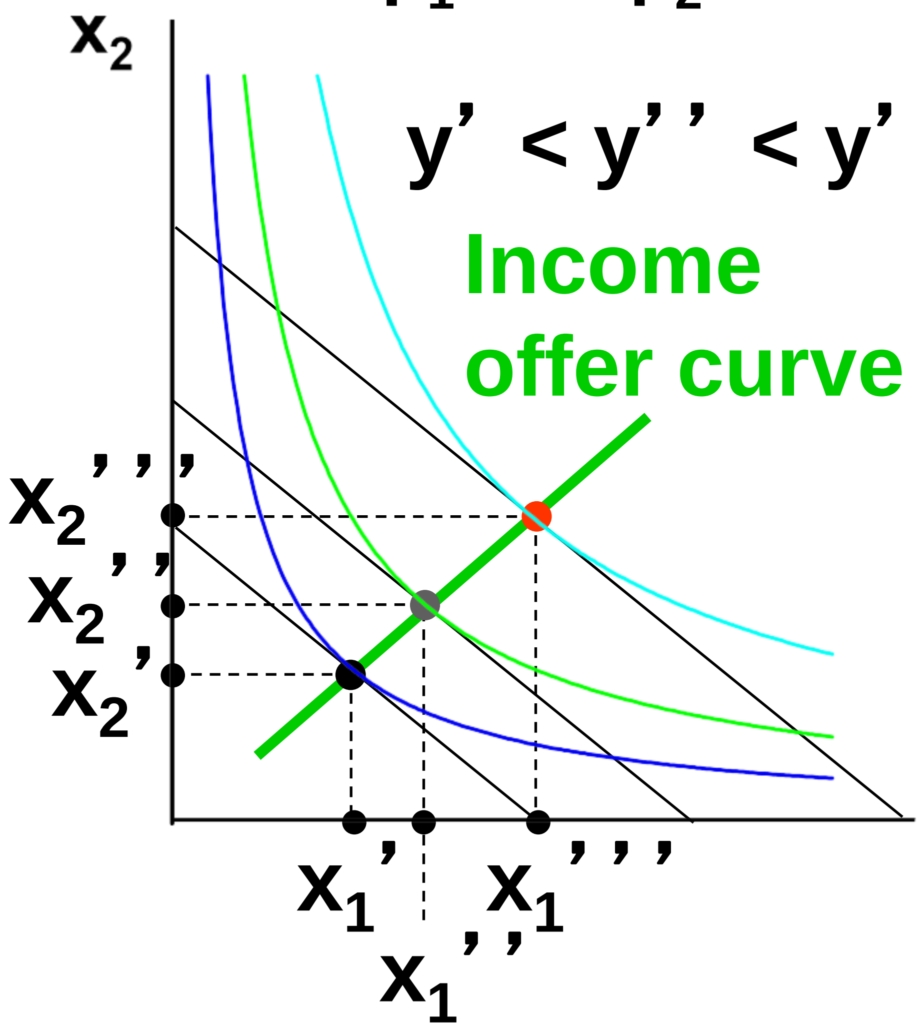
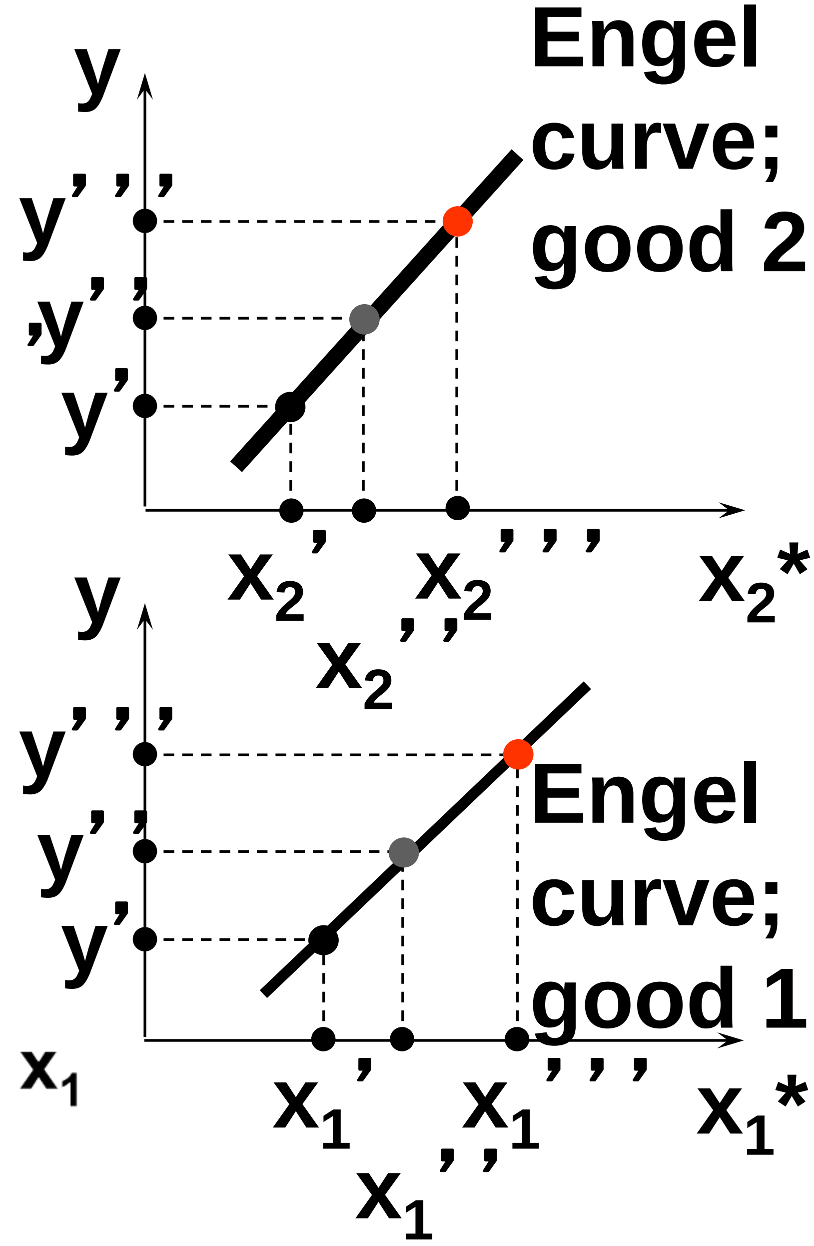




Income Changes and Cobb-Douglas Preferences

- **An example of computing the equations of Engel curves; the Cobb-Douglas case.**

$$U(x_1, x_2) = x_1^a x_2^b.$$

- **The ordinary demand equations are**

$$x_1^* = \frac{ay}{(a+b)p_1}; \quad x_2^* = \frac{by}{(a+b)p_2}.$$

Income Changes and Cobb-Douglas Preferences

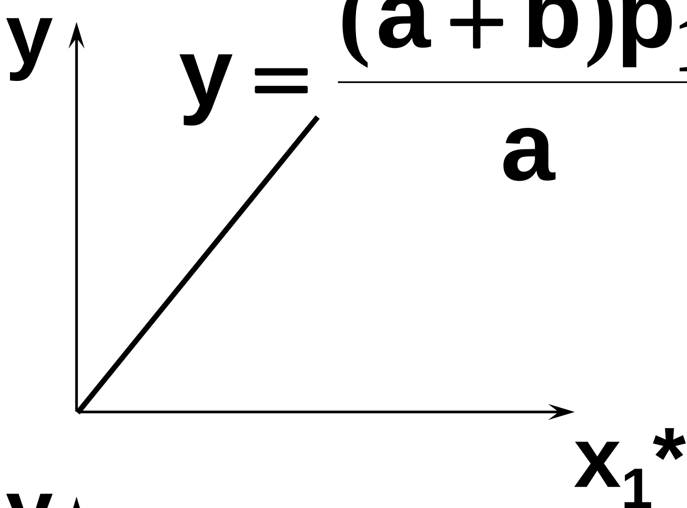
$$x_1^* = \frac{ay}{(a+b)p_1}; \quad x_2^* = \frac{by}{(a+b)p_2}.$$

Rearranged to isolate y, these are:

$$y = \frac{(a+b)p_1}{a} x_1^*$$ **Engel curve for good 1**

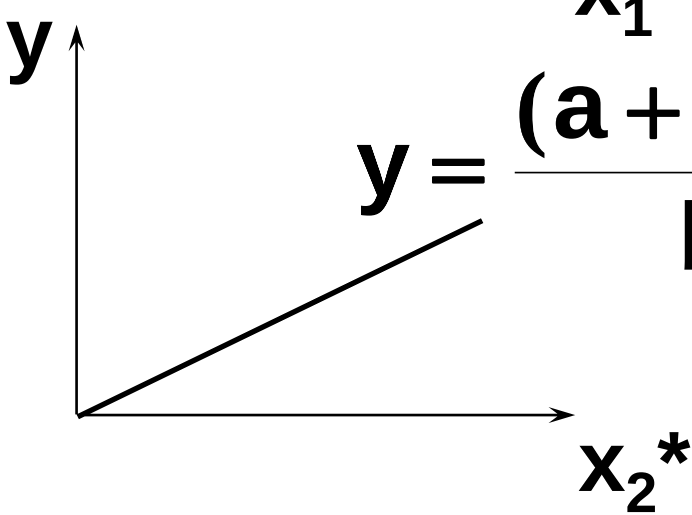
$$y = \frac{(a+b)p_2}{b} x_2^*$$ **Engel curve for good 2**

Income Changes and Cobb-Douglas Preferences



$$y = \frac{(a+b)p_1}{a} x_1^*$$

Engel curve for good 1



$$y = \frac{(a+b)p_2}{b} x_2^*$$

Engel curve for good 2

Income Changes and Perfectly-Complementary Preferences

- **Another example of computing the equations of Engel curves; the perfectly-complementary case.**

$$U(x_1, x_2) = \min\{x_1, x_2\}.$$

- **The ordinary demand equations are**

$$x_1^* = x_2^* = \frac{y}{p_1 + p_2}.$$

Income Changes and Perfectly-Complementary Preferences

$$x_1^* = x_2^* = \frac{y}{p_1 + p_2}.$$

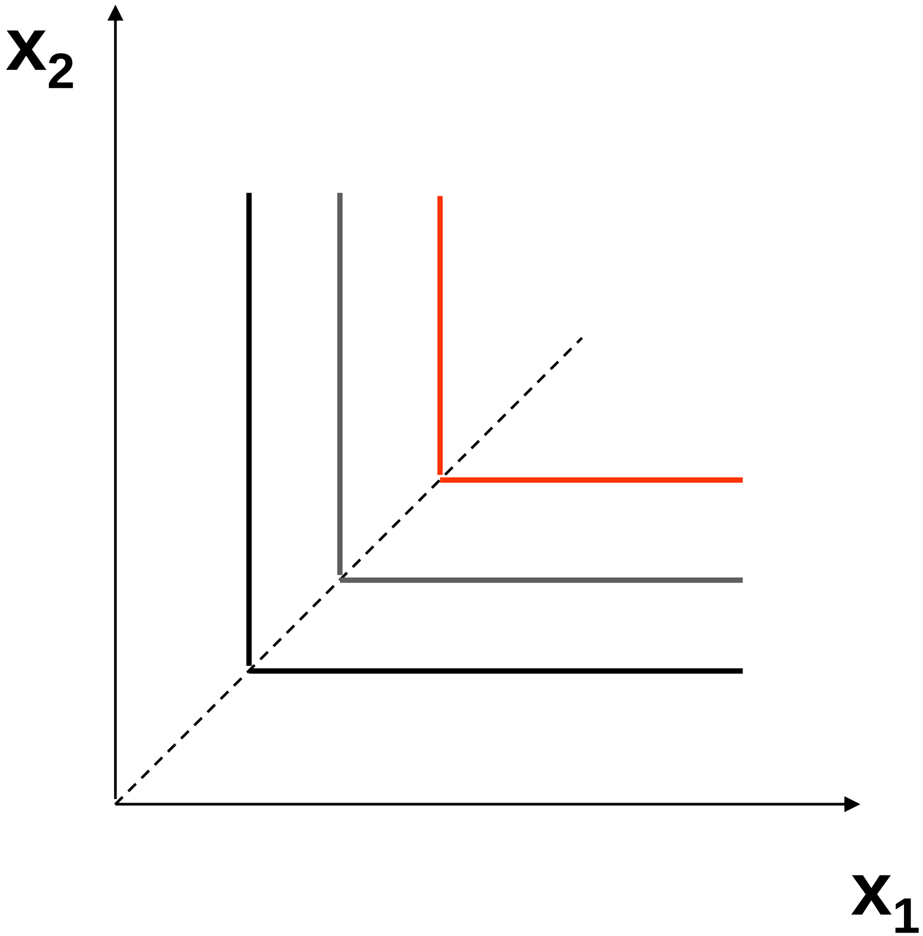
Rearranged to isolate y, these are:

$y = (p_1 + p_2)x_1^*$ **Engel curve for good 1**

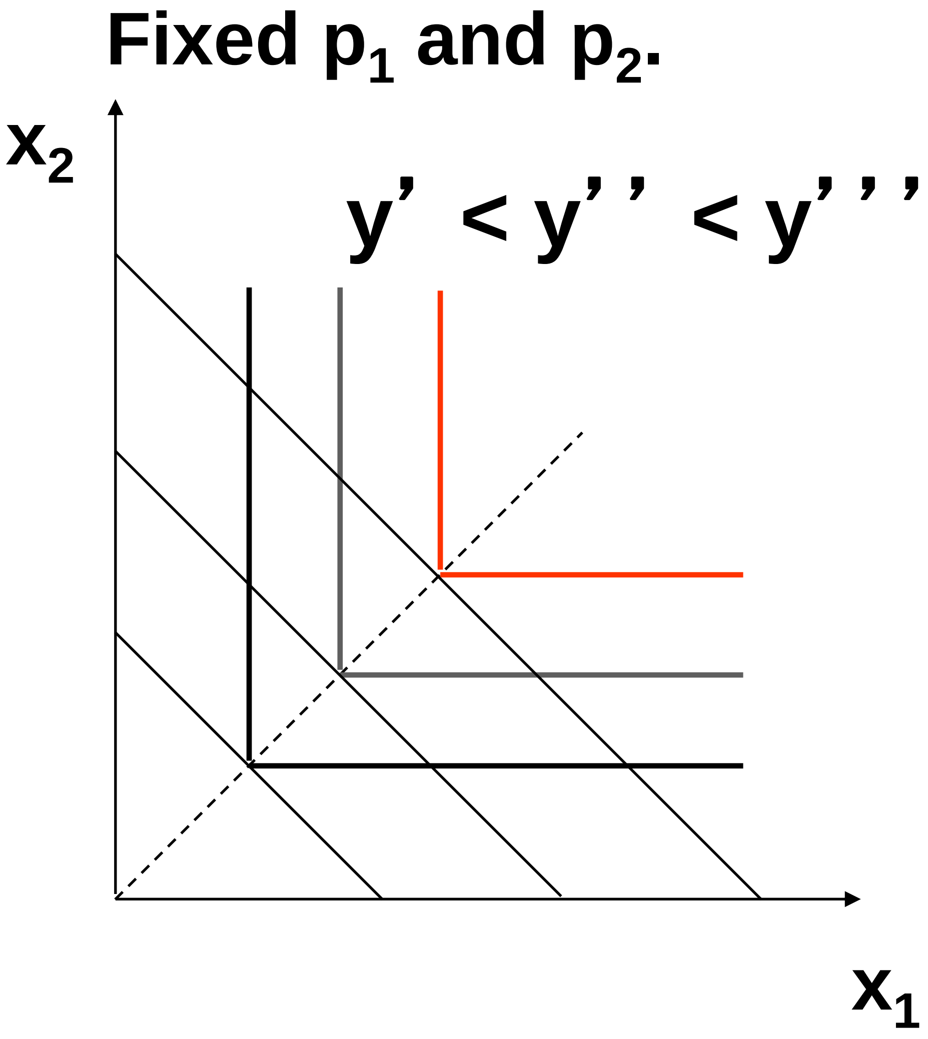
$y = (p_1 + p_2)x_2^*$ **Engel curve for good 2**

Income Changes

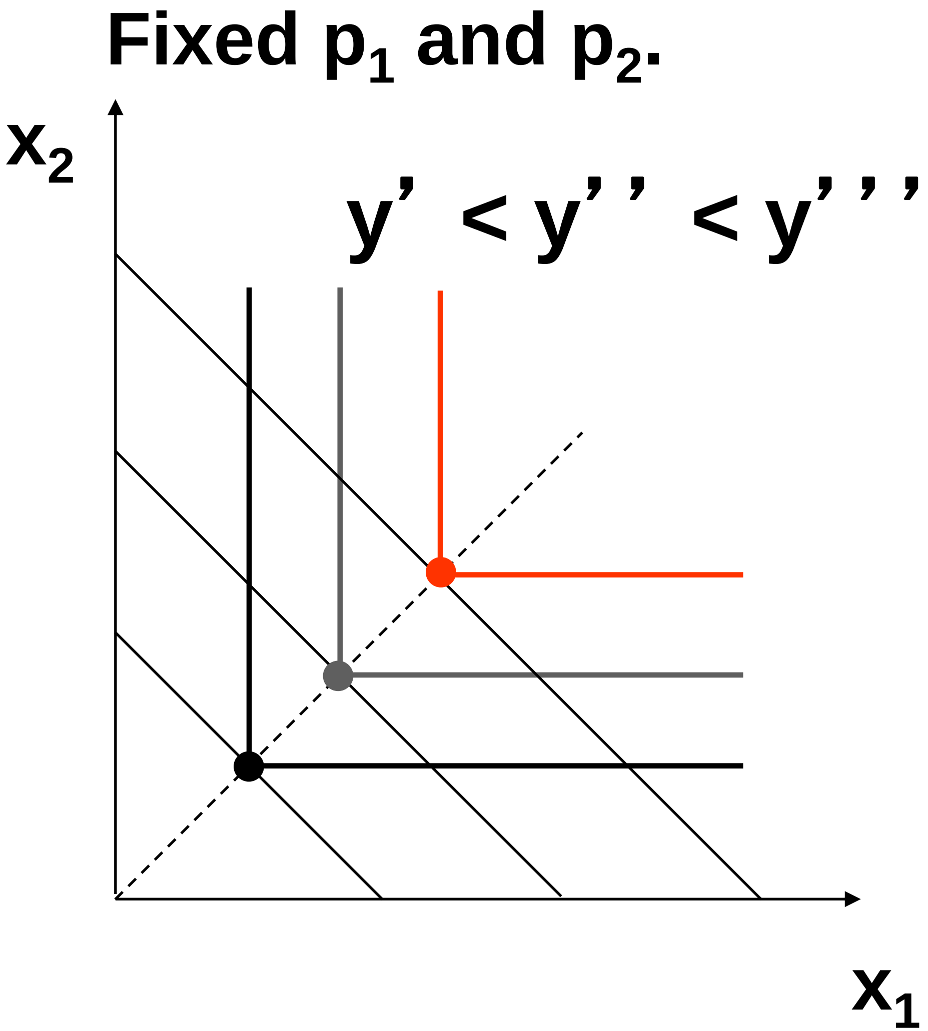
Fixed p_1 and p_2.



Income Changes

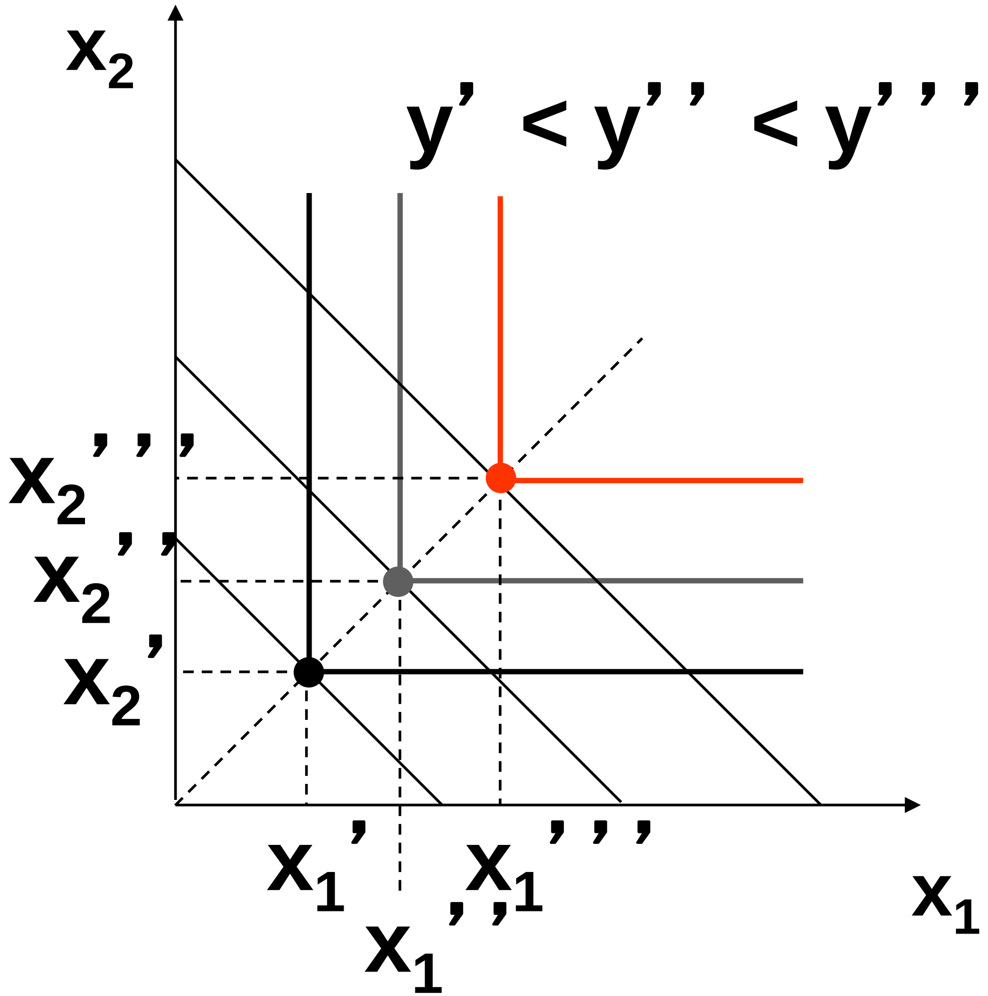


Income Changes



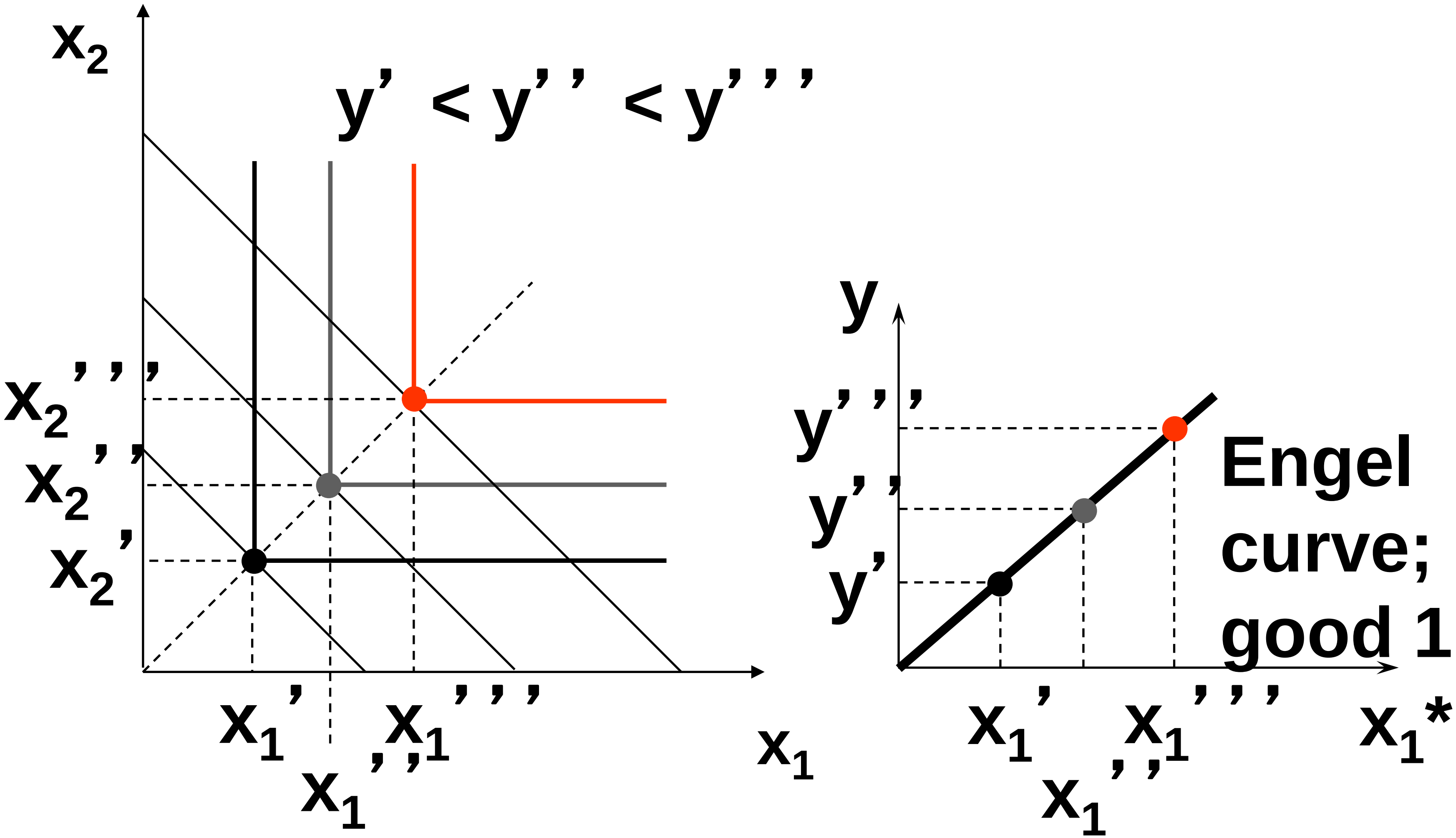
Income Changes

Fixed p_1 and p_2.



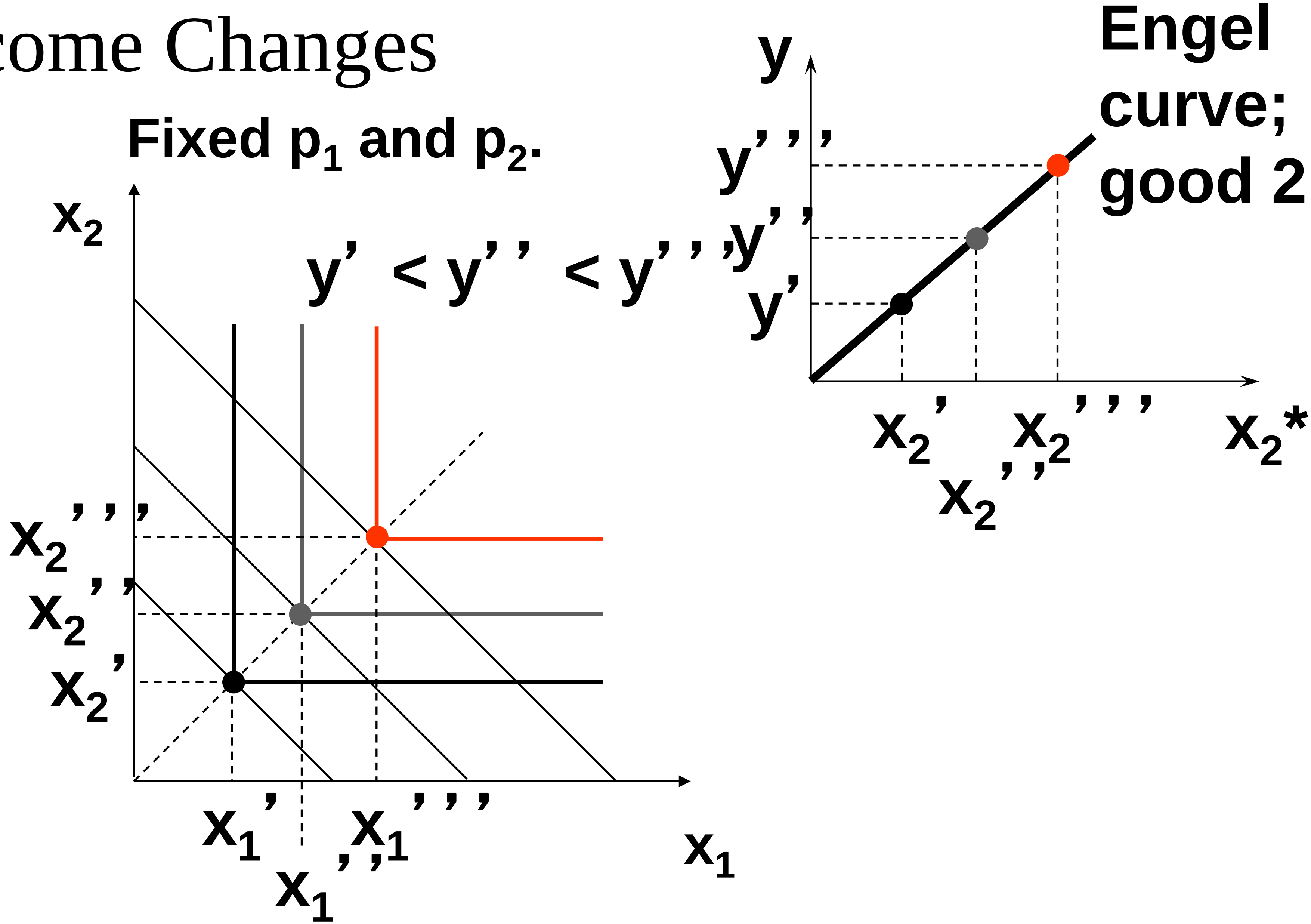
Income Changes

Fixed p_1 and p_2.



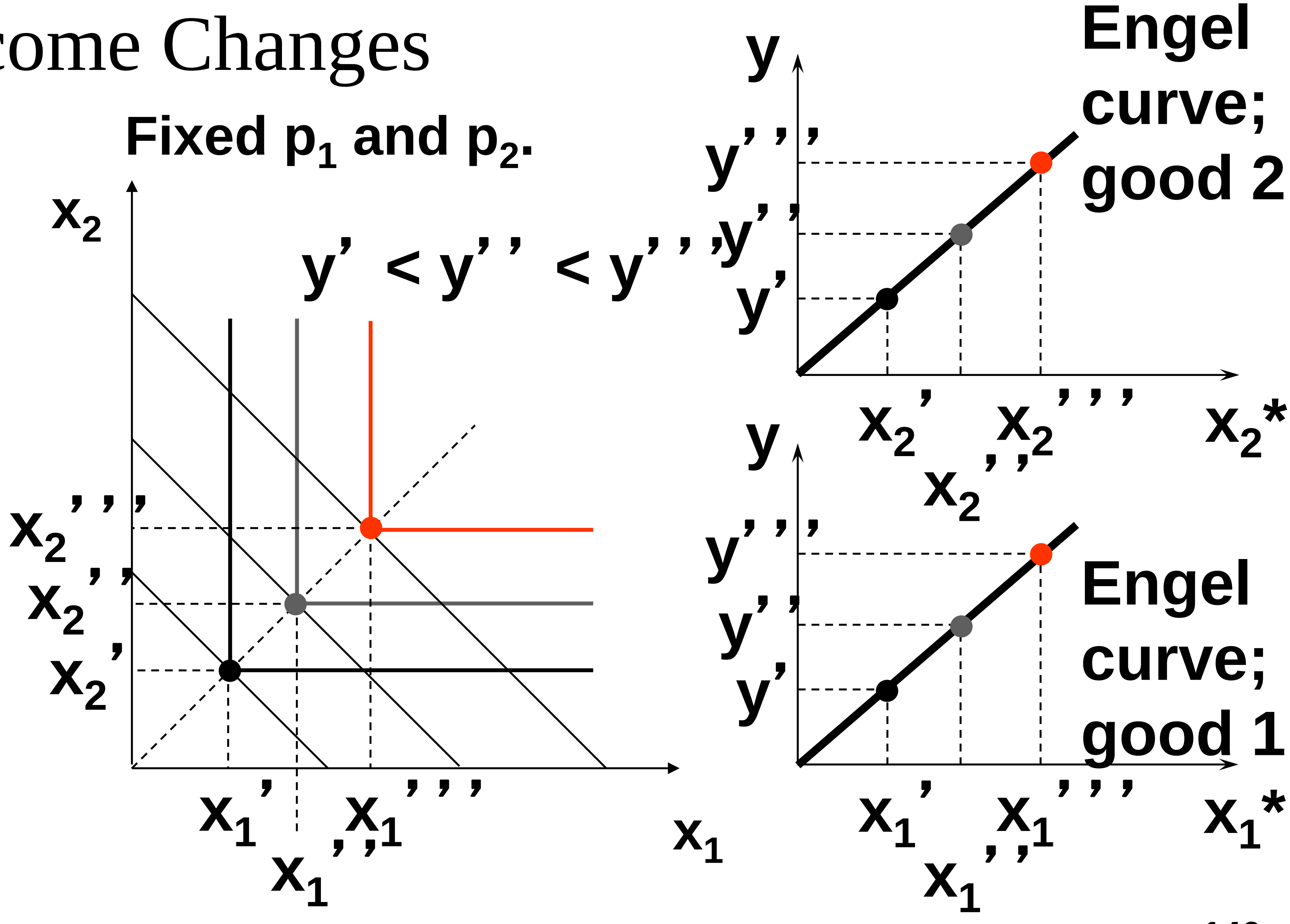
Income Changes

Fixed p_1 and p_2.



Income Changes

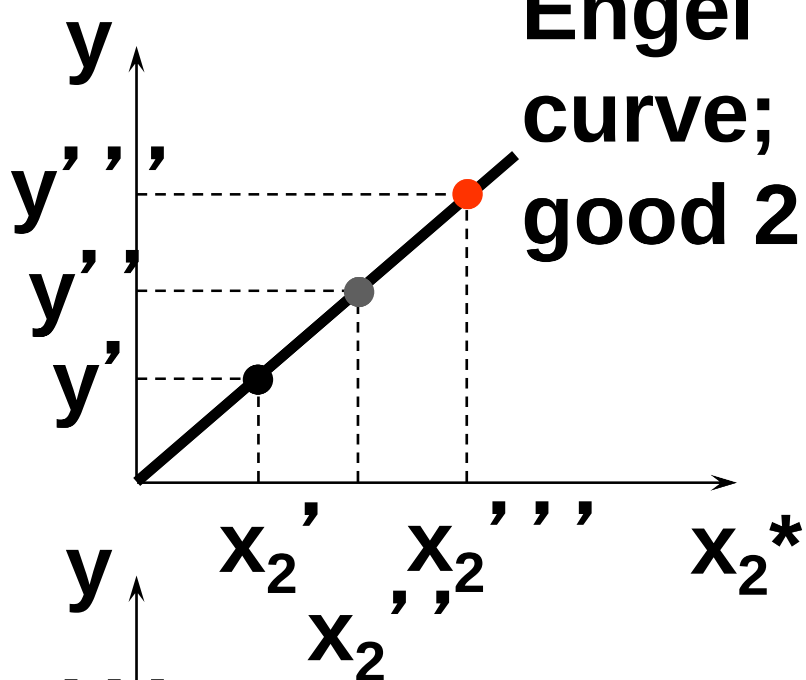
Fixed p_1 and p_2.



Income Changes

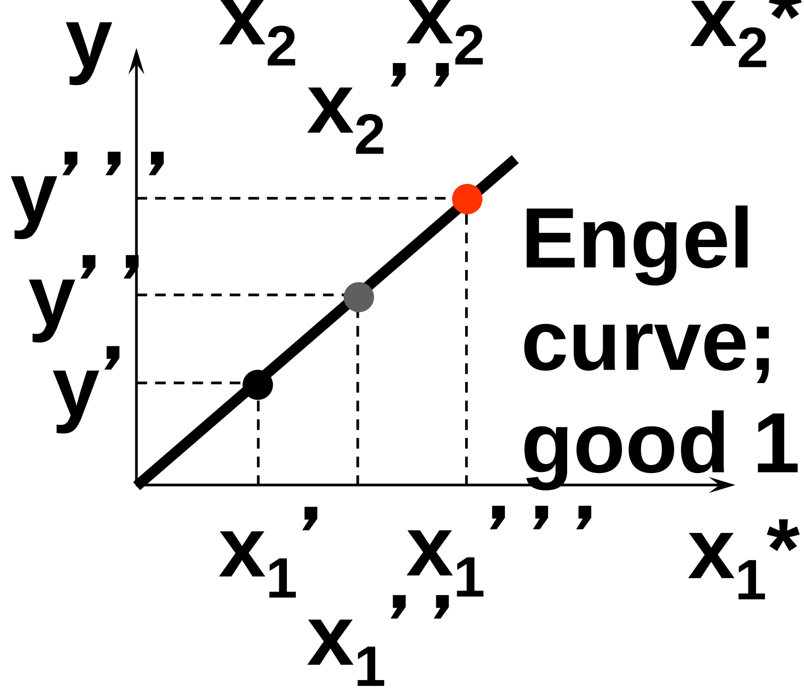
Fixed p_1 and p_2.

$$y = (p_1 + p_2)x_2^*$$



Engel curve; good 2

$$y = (p_1 + p_2)x_1^*$$



Engel curve; good 1

Income Changes and Perfectly-Substitutable Preferences

- **Another example of computing the equations of Engel curves; the perfectly-substitution case.**

$$U(x_1, x_2) = x_1 + x_2.$$

- **The ordinary demand equations are**

Income Changes and Perfectly-Substitutable Preferences

$$x_1^*(p_1, p_2, y) = \begin{cases} 0 & , \text{if } p_1 > p_2 \\ y/p_1 & , \text{if } p_1 < p_2 \end{cases}$$

$$x_2^*(p_1, p_2, y) = \begin{cases} 0 & , \text{if } p_1 < p_2 \\ y/p_2 & , \text{if } p_1 > p_2. \end{cases}$$

Income Changes and Perfectly-Substitutable Preferences

$$x_1^*(p_1,p_2,y) = \begin{cases} 0 & ,\text{if } p_1 > p_2 \\ y/p_1 & ,\text{if } p_1 < p_2 \end{cases}$$

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Suppose $p_1 < p_2$. Then

Income Changes and Perfectly-Substitutable Preferences

$$x_1^*(p_1, p_2, y) = \begin{cases} 0 & , \text{if } p_1 > p_2 \\ y/p_1 & , \text{if } p_1 < p_2 \end{cases}$$

$$x_2^*(p_1, p_2, y) = \begin{cases} 0 & , \text{if } p_1 < p_2 \\ y/p_2 & , \text{if } p_1 > p_2. \end{cases}$$

Suppose $p_1 < p_2$. Then $x_1^* = \dfrac{y}{p_1}$ **and** $x_2^* = 0$

Income Changes and Perfectly-Substitutable Preferences

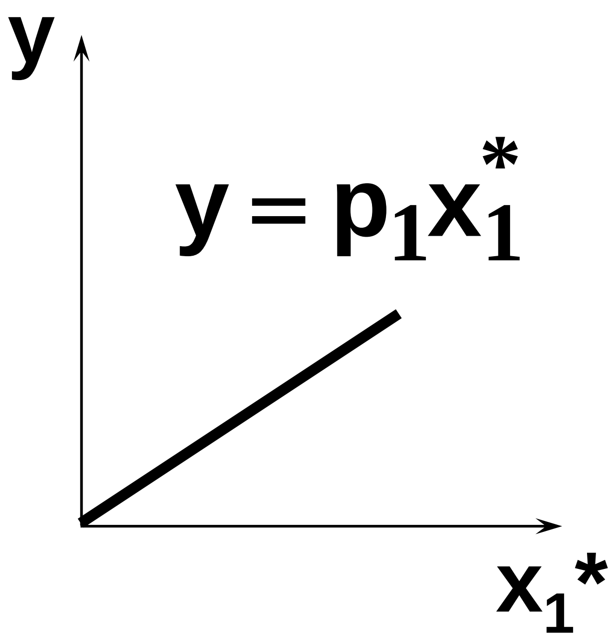
$$x_1^*(p_1, p_2, y) = \begin{cases} 0 & , \text{if } p_1 > p_2 \\ y / p_1 & , \text{if } p_1 < p_2 \end{cases}$$

$$x_2^*(p_1, p_2, y) = \begin{cases} 0 & , \text{if } p_1 < p_2 \\ y / p_2 & , \text{if } p_1 > p_2. \end{cases}$$

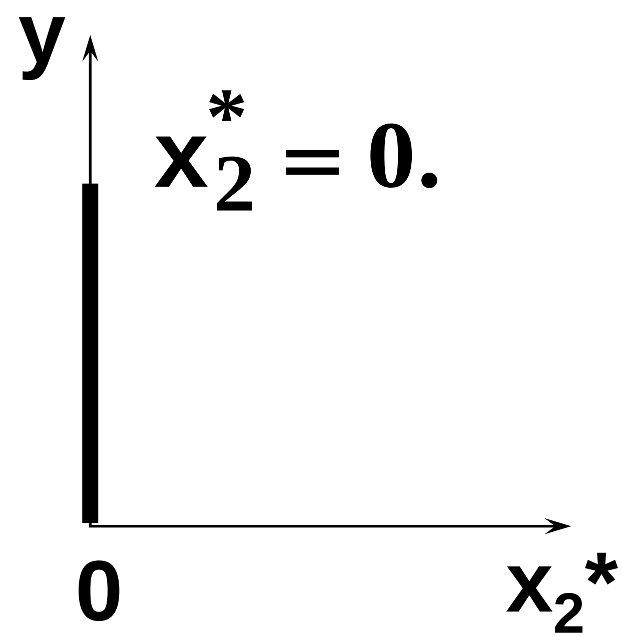
Suppose $p_1 < p_2$. Then $x_1^* = \dfrac{y}{p_1}$ and $x_2^* = 0$

➡ $y = p_1 x_1^*$ and $x_2^* = 0.$

Income Changes and Perfectly-Substitutable Preferences



Engel curve for good 1



Engel curve for good 2

Income Changes

- **In every example so far the Engel curves have all been straight lines? Q: Is this true in general?**
- **A: No. Engel curves are straight lines if the consumer's preferences are homothetic.**

Homotheticity

- **A consumer's preferences are homothetic if and only if**

$$(x_1,x_2) \prec (y_1,y_2) \Leftrightarrow (kx_1,kx_2) \prec (ky_1,ky_2)$$

for every k > 0.

- **That is, the consumer's MRS is the same anywhere on a straight line drawn from the origin.**

Income Effects -- A Nonhomothetic Example

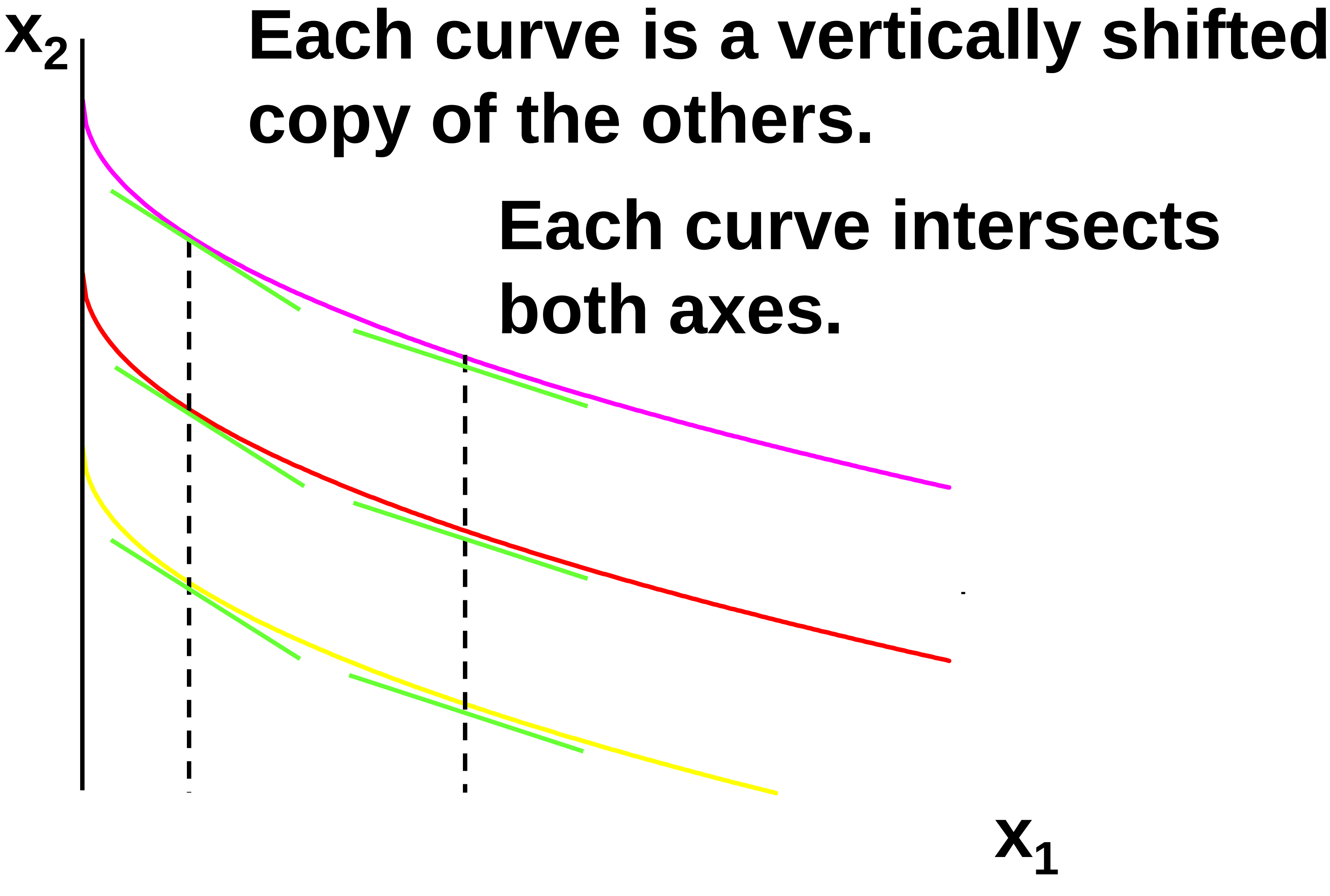
- **Quasilinear preferences are not homothetic.**

$$U(x_1, x_2) = f(x_1) + x_2.$$

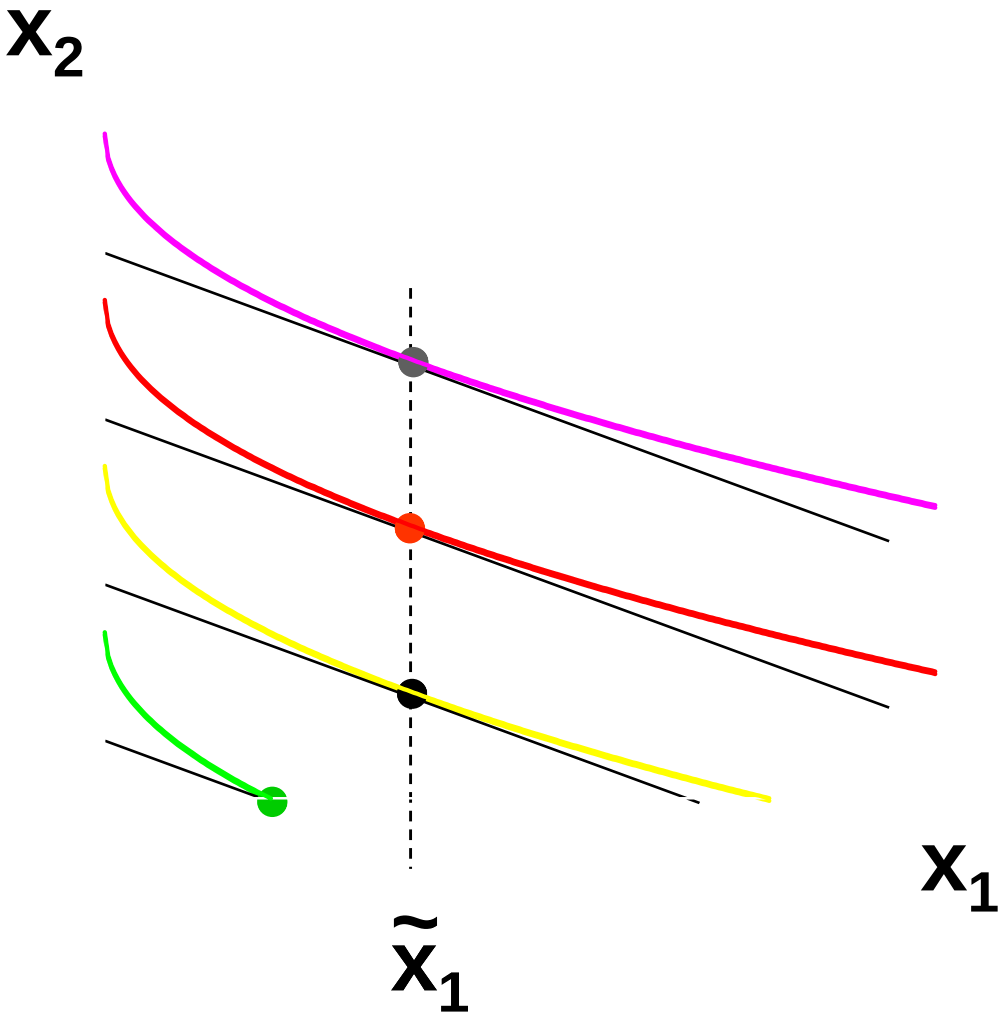
- **For example,**

$$U(x_1, x_2) = \sqrt{x_1} + x_2.$$

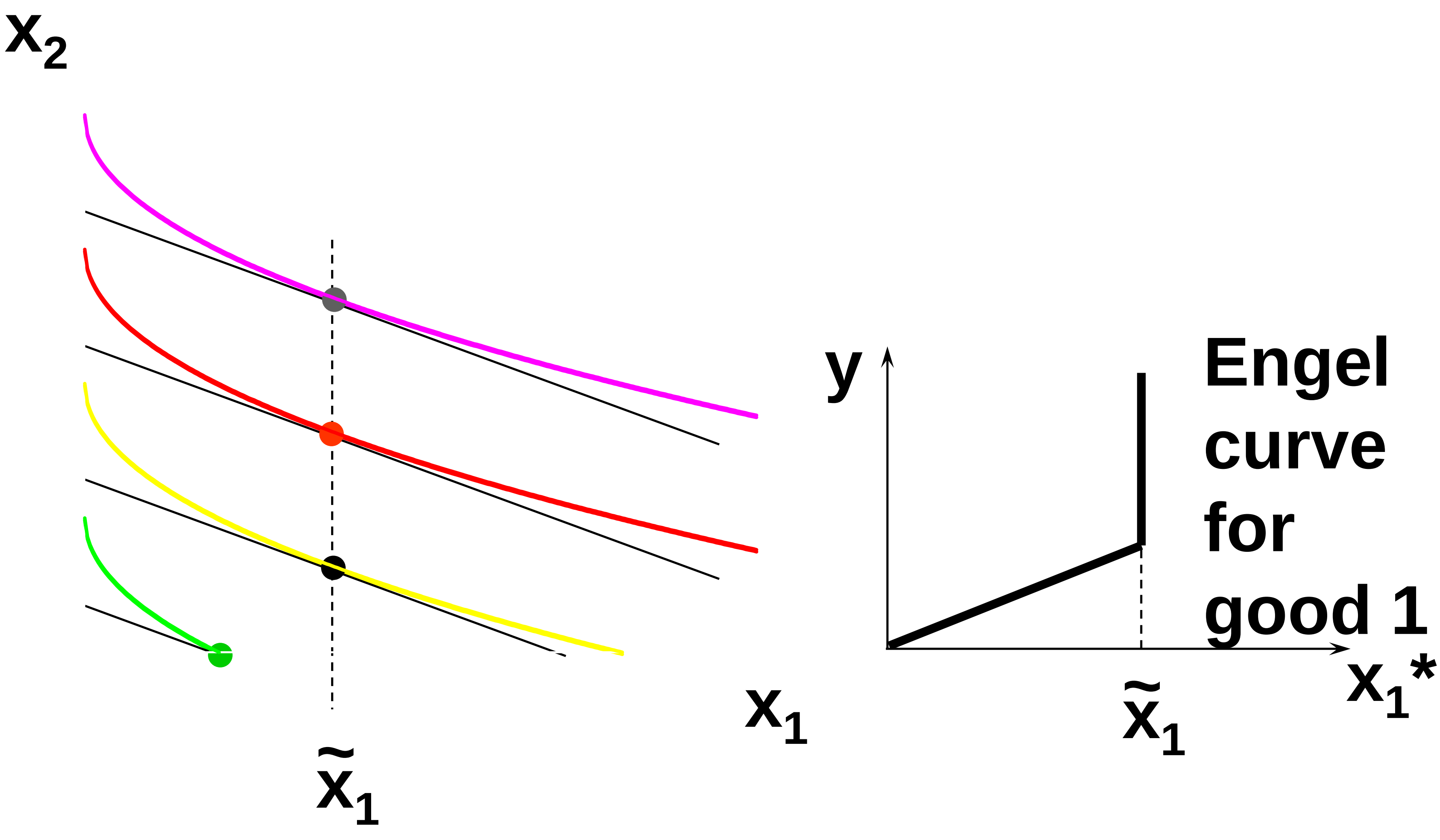
Quasi-linear Indifference Curves



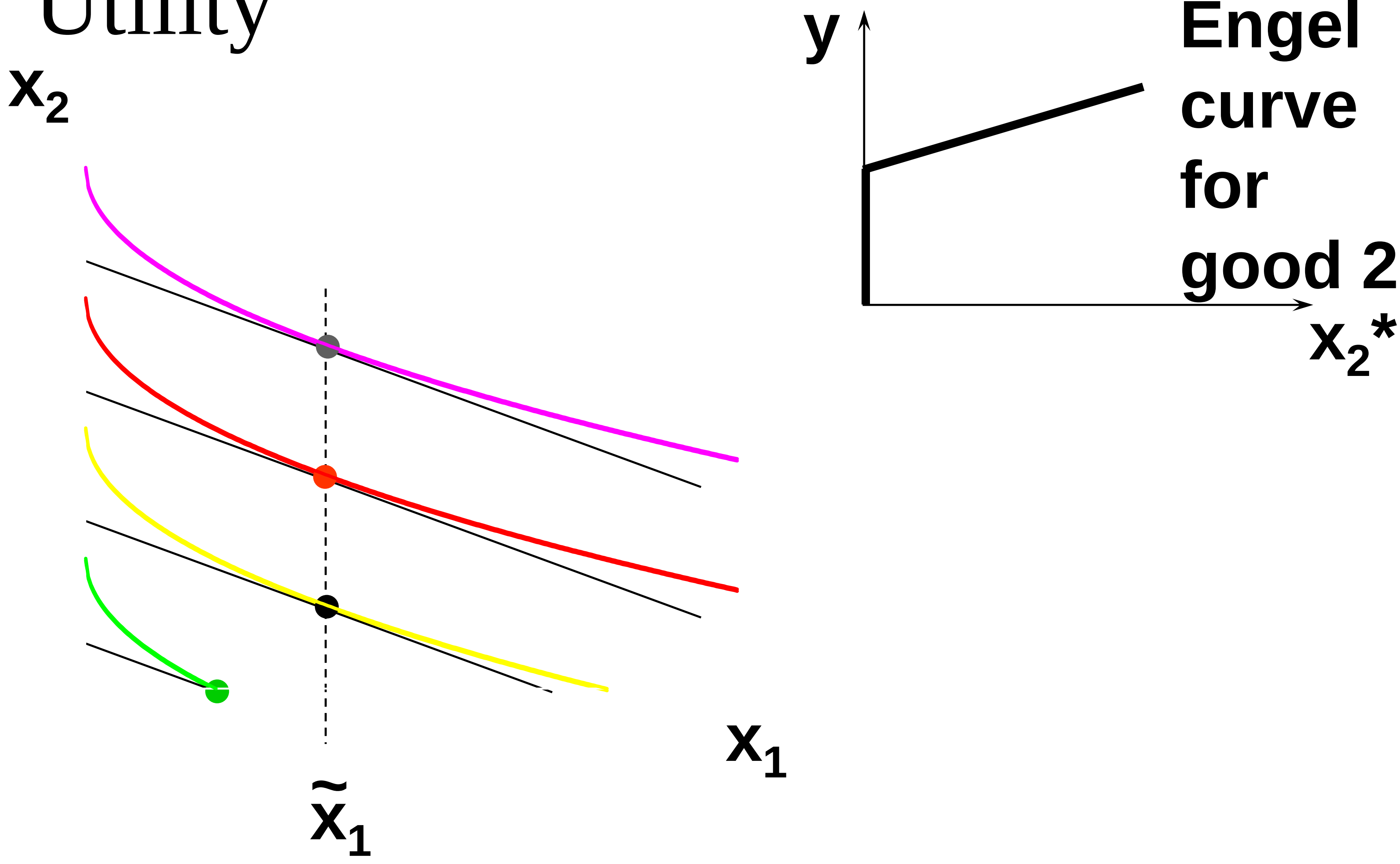
Income Changes; Quasilinear Utility



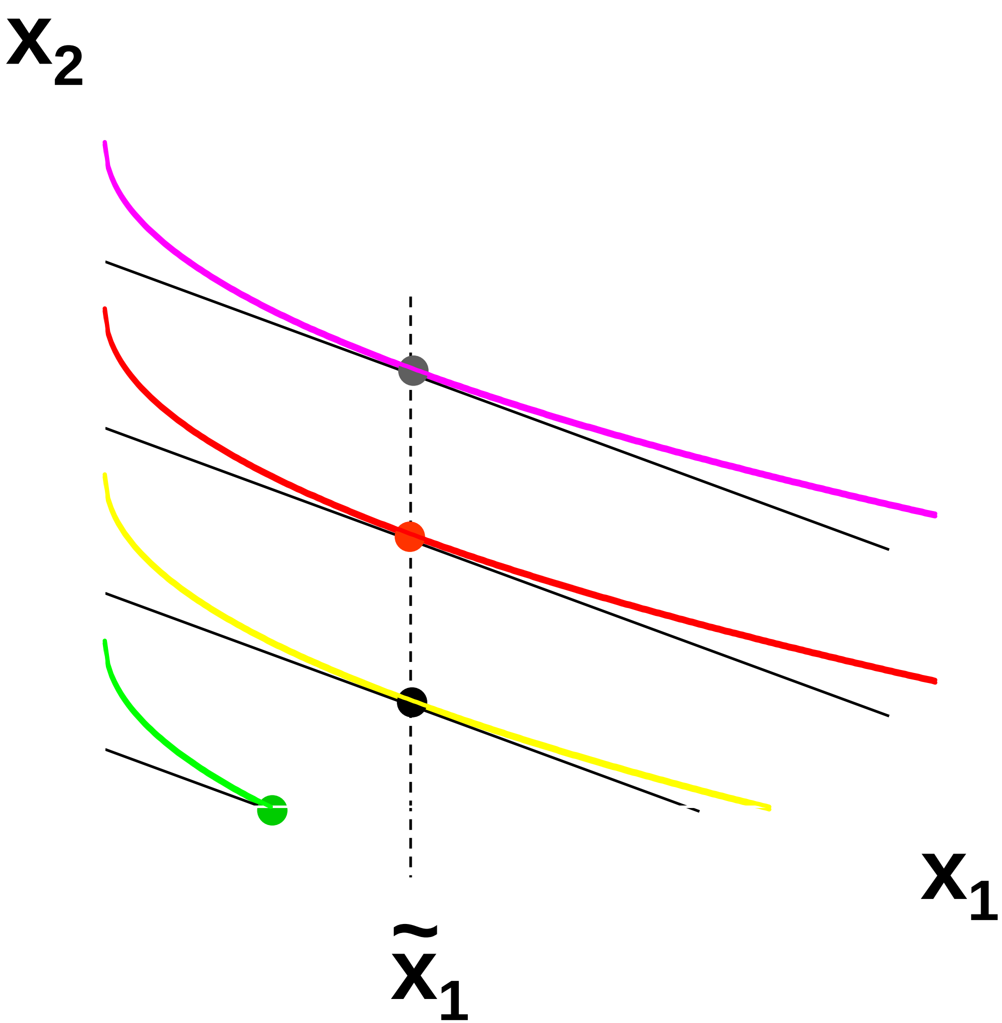
Income Changes; Quasilinear Utility

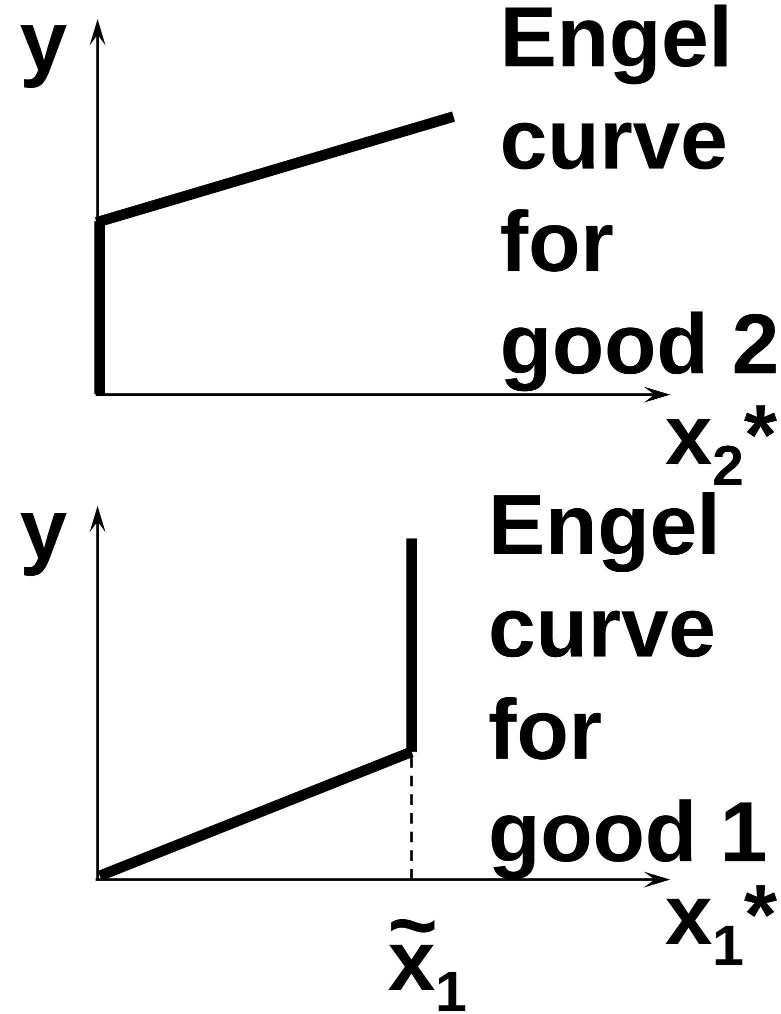


Income Changes; Quasilinear Utility



Income Changes; Quasilinear Utility





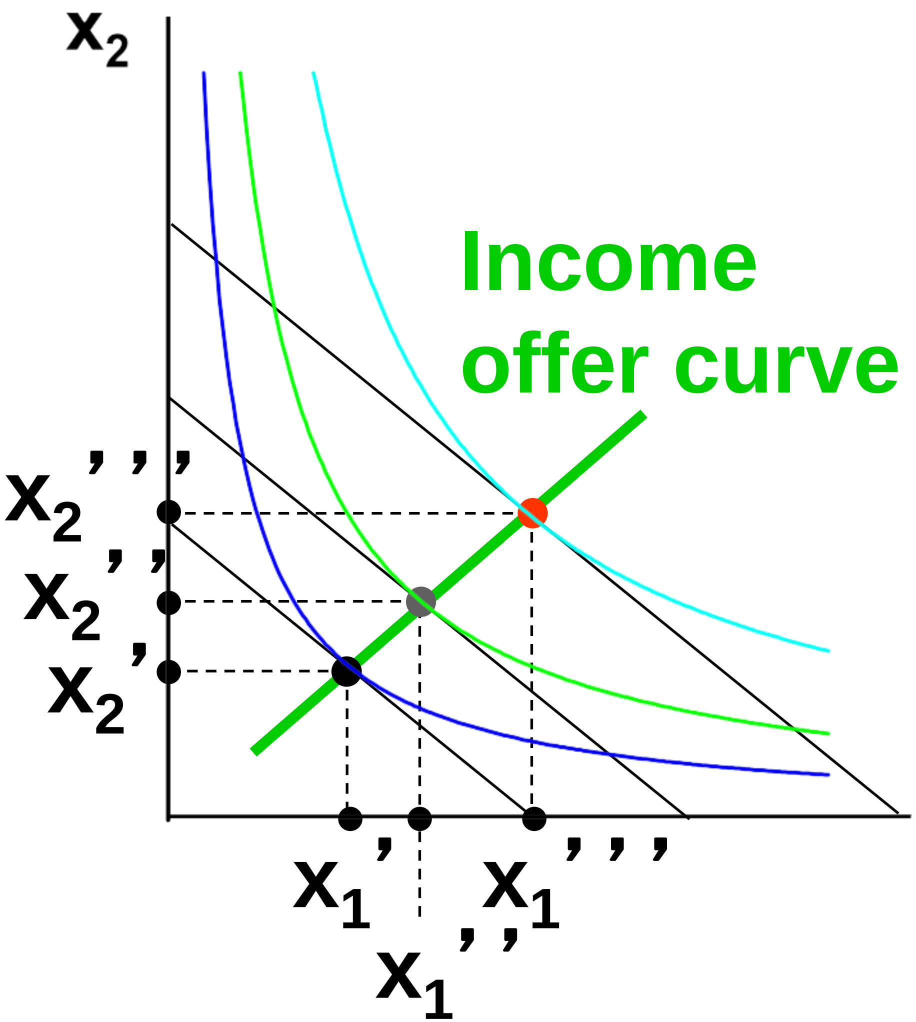
Income Effects

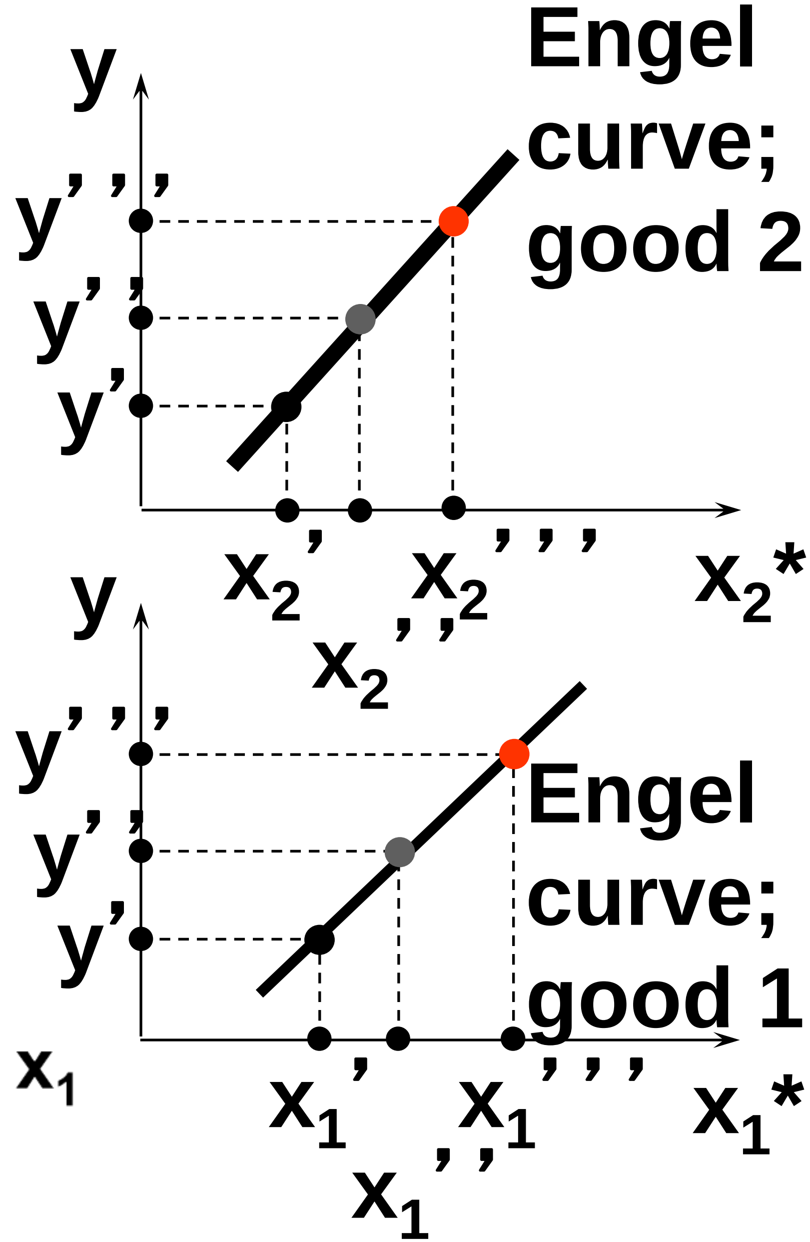
- A good for which quantity demanded rises with income is called normal.
- Therefore a normal good's Engel curve is positively sloped.

Income Effects

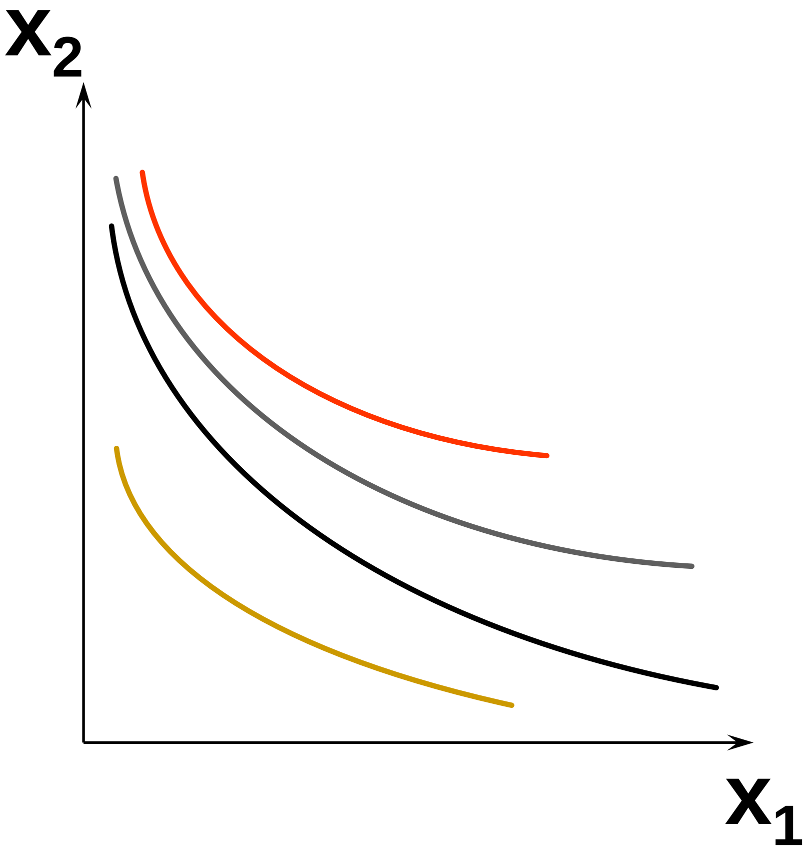
- **A good for which quantity demanded falls as income increases is called income inferior.**
- **Therefore an income inferior good’s Engel curve is negatively sloped.**

Income Changes; Goods 1 & 2 Normal

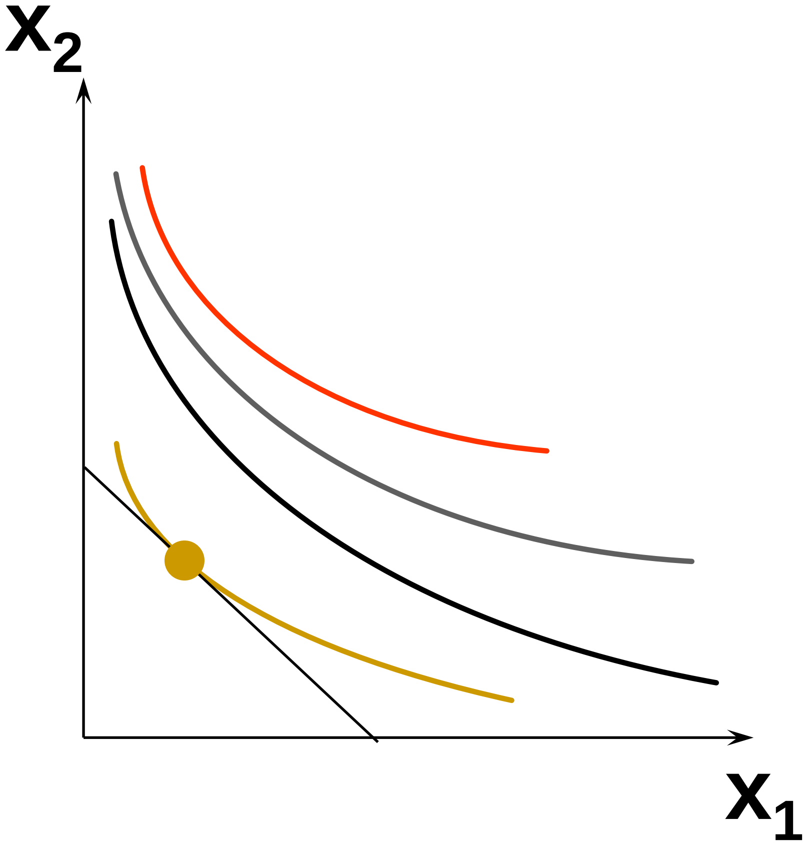




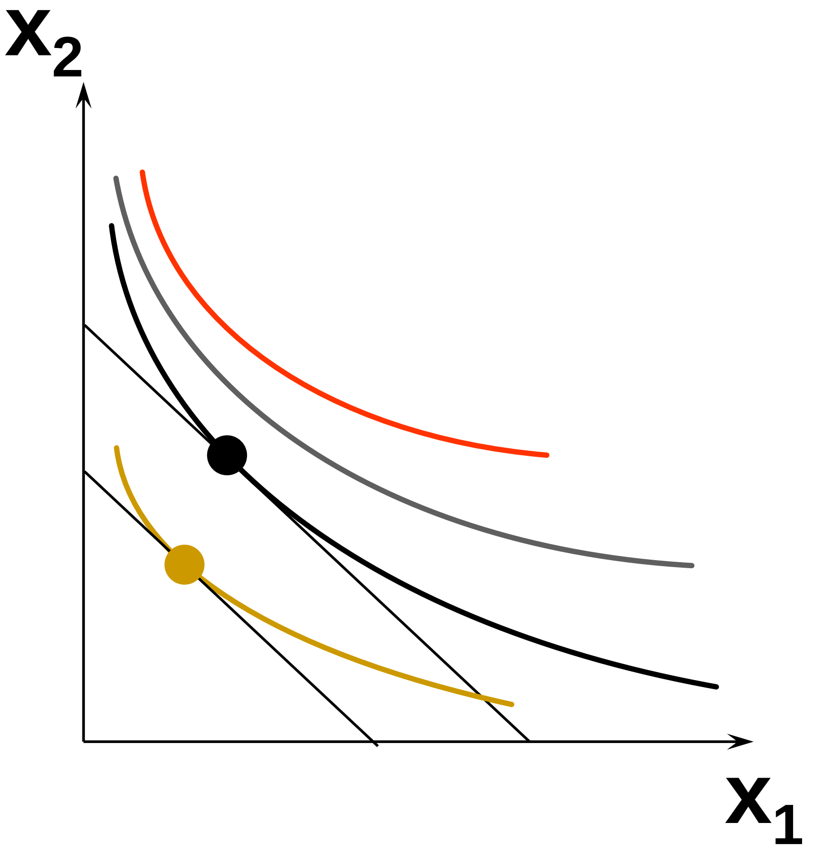
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



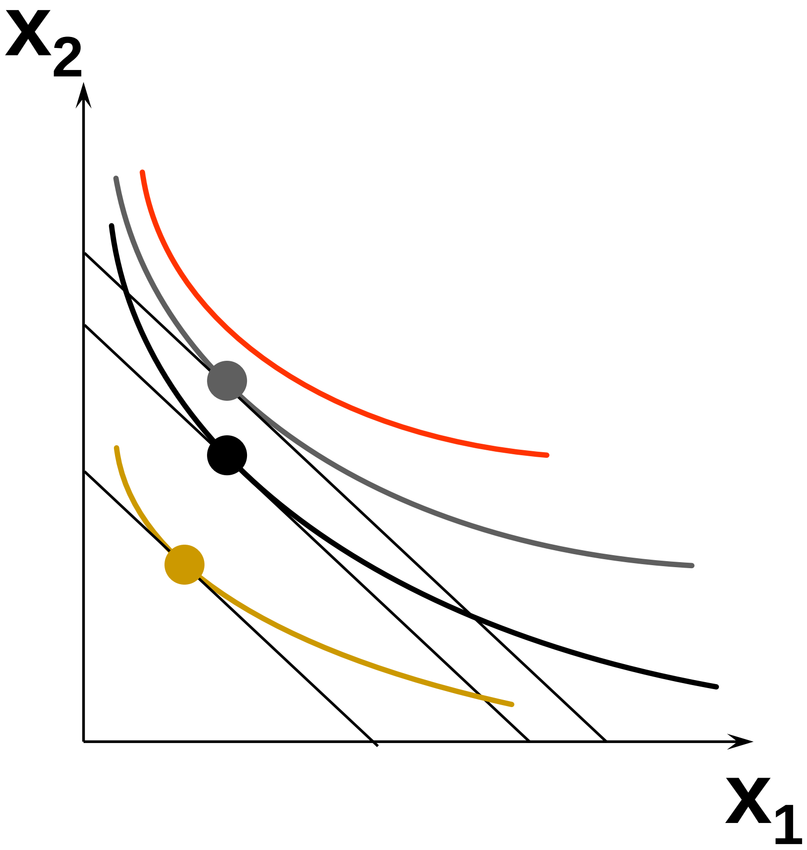
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



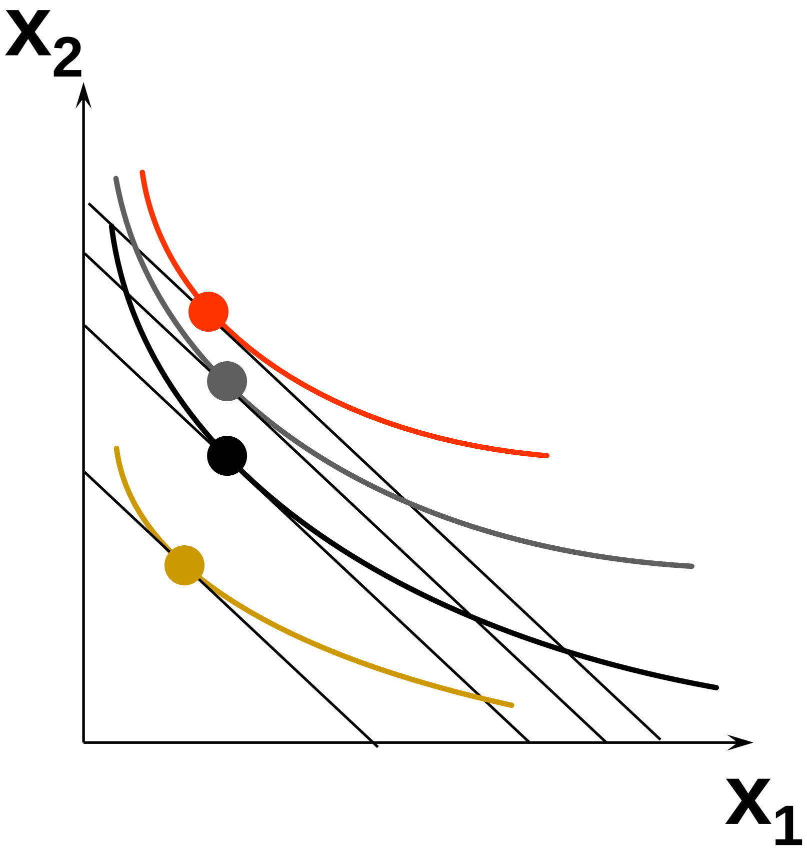
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



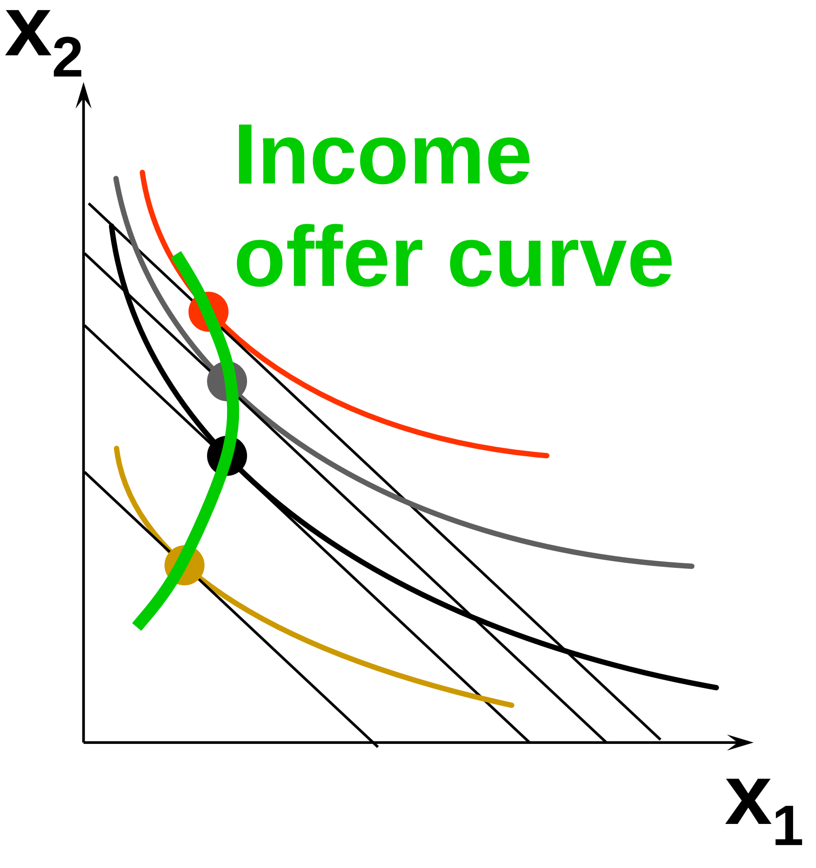
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



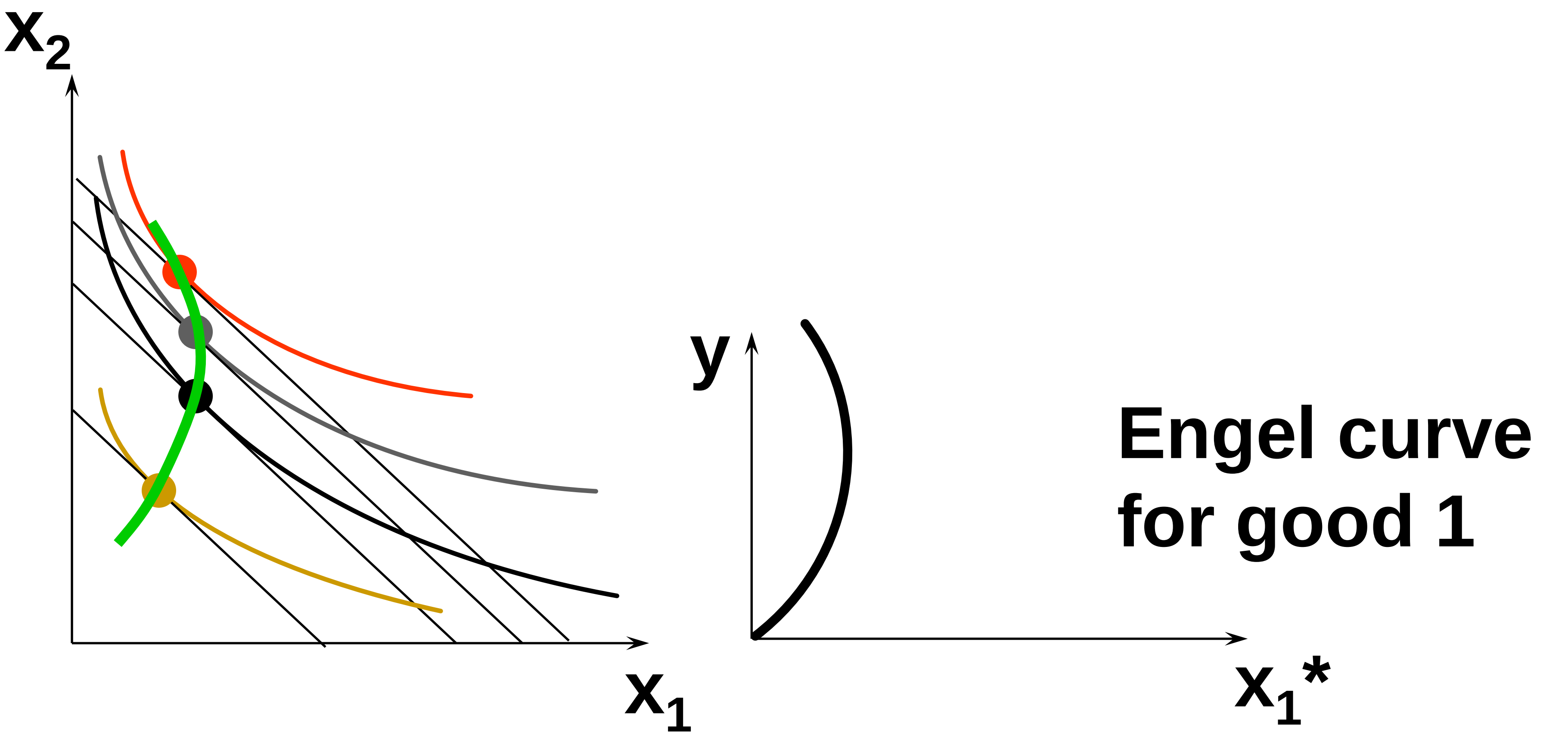
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



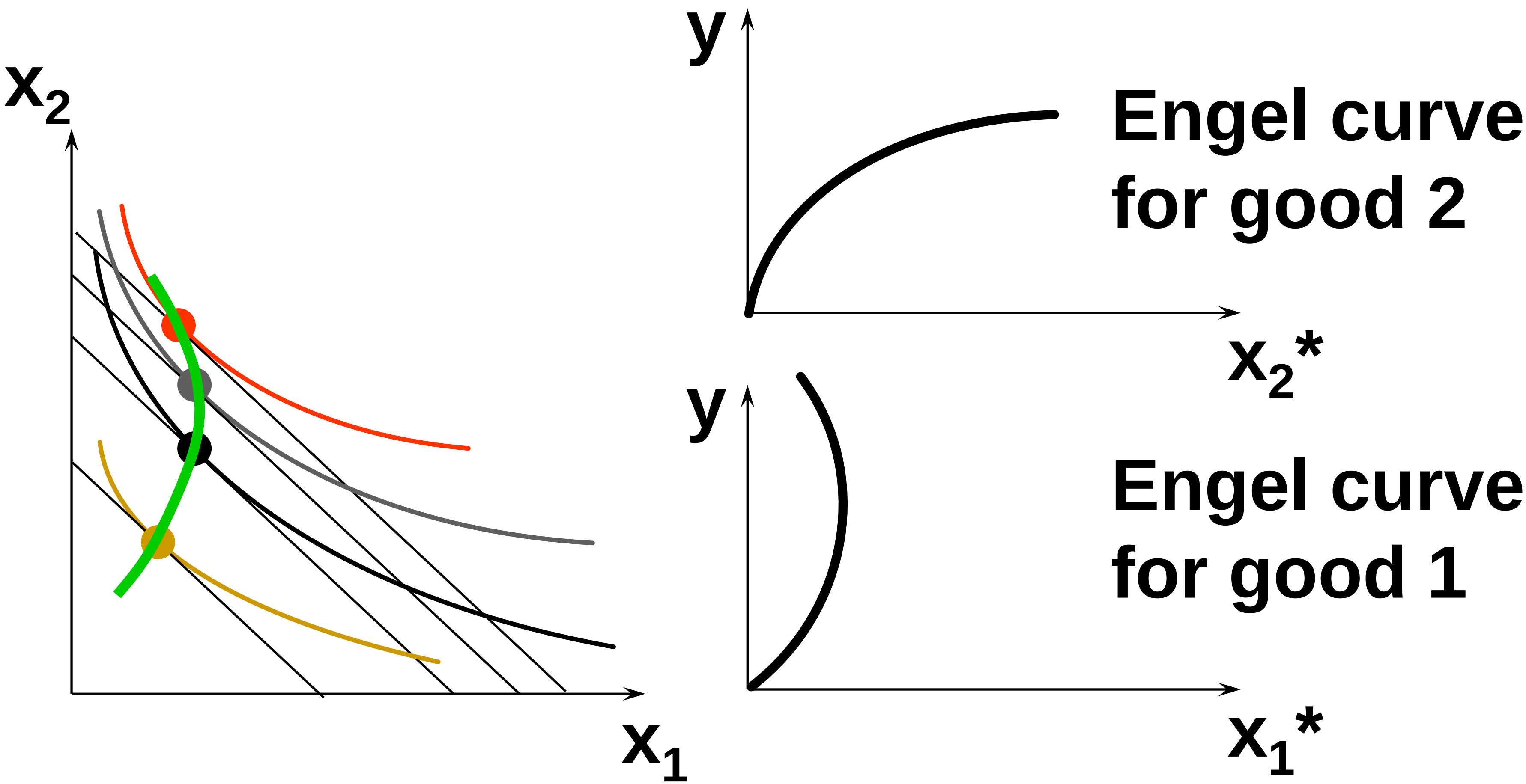
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior

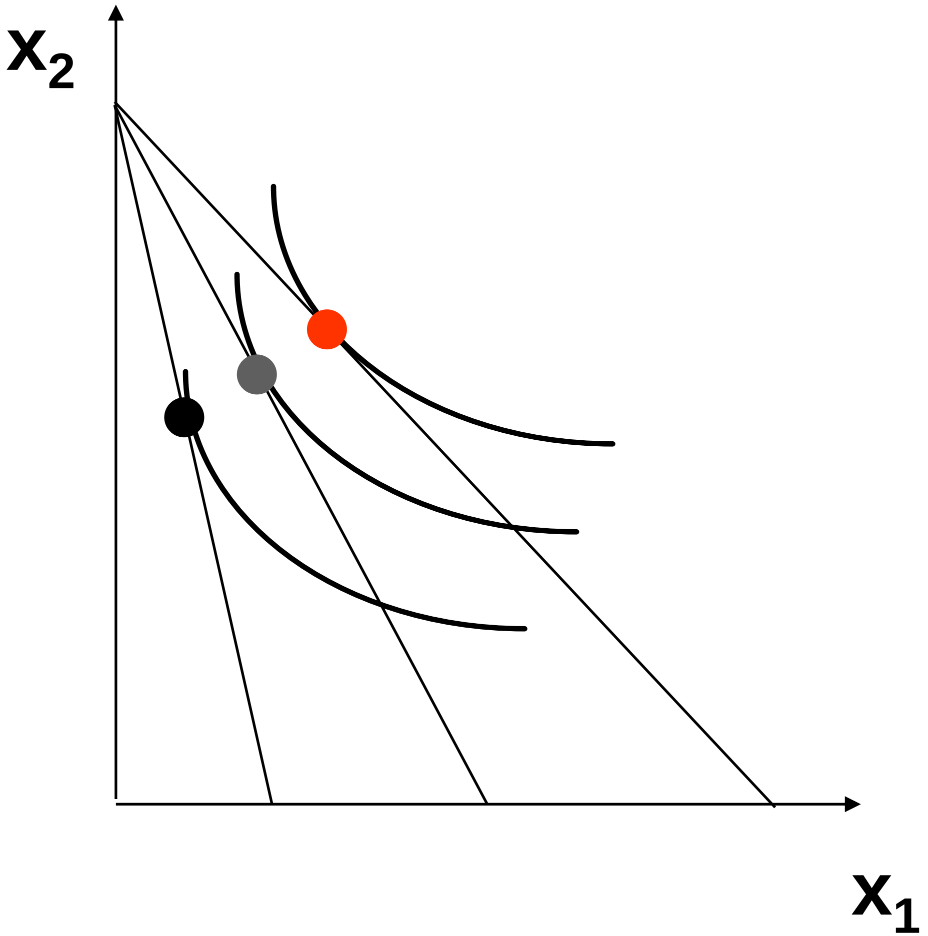


Ordinary Goods

- **A good is called ordinary if the quantity demanded of it always increases as its own price decreases.**

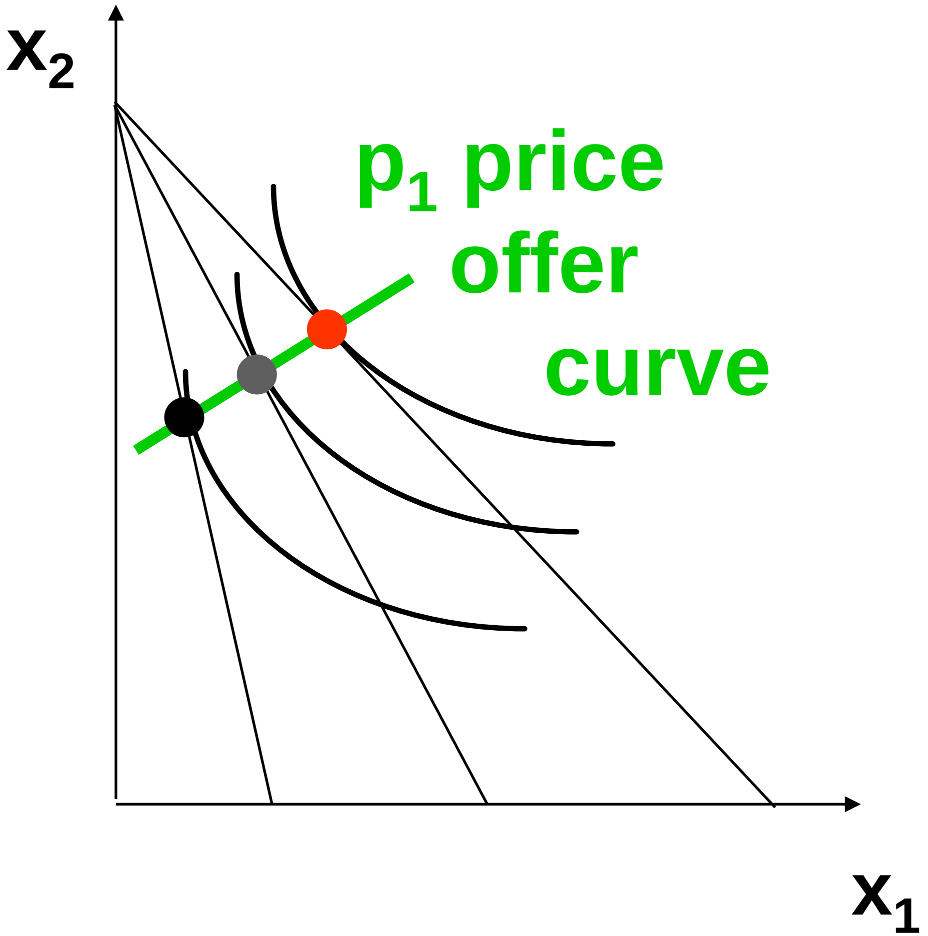
Ordinary Goods

Fixed p_2 and y.

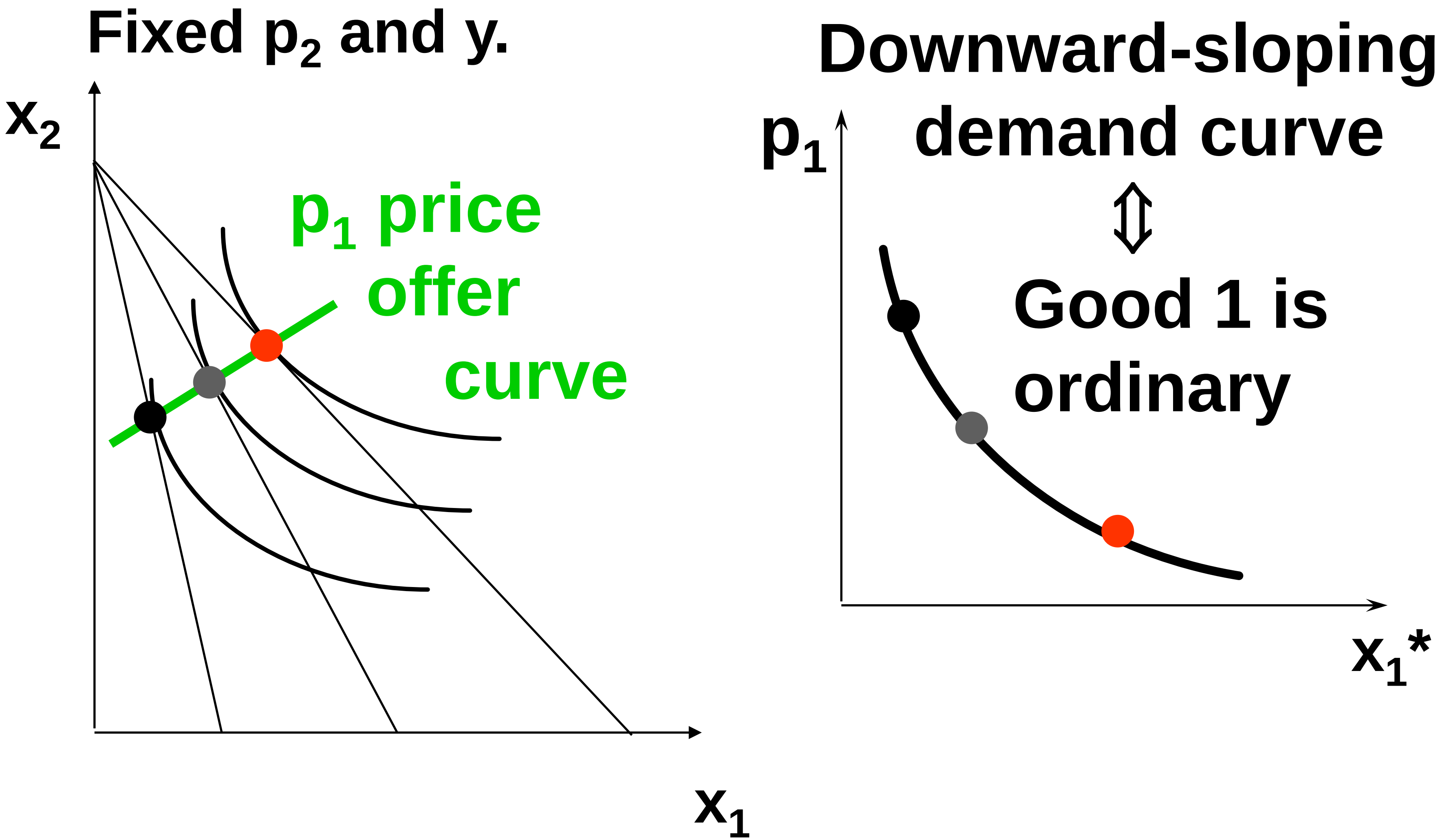


Ordinary Goods

Fixed p_2 and y.



Ordinary Goods

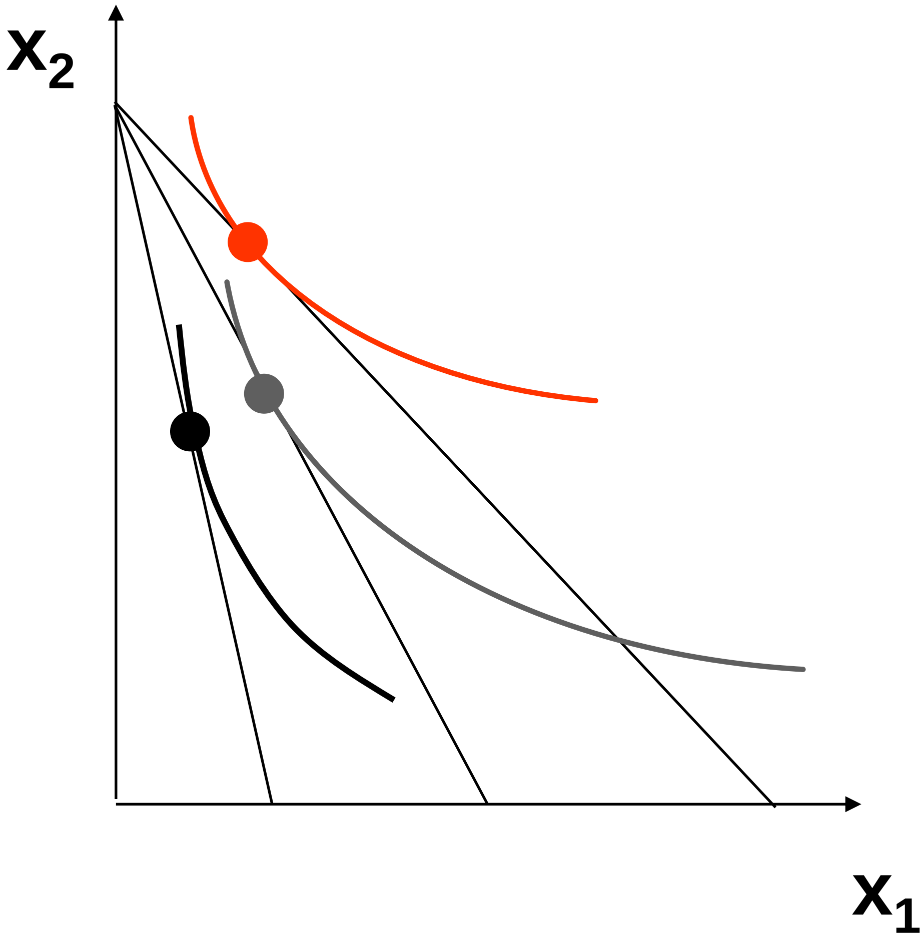


Giffen Goods

- **If, for some values of its own price, the quantity demanded of a good rises as its own-price increases then the good is called Giffen.**

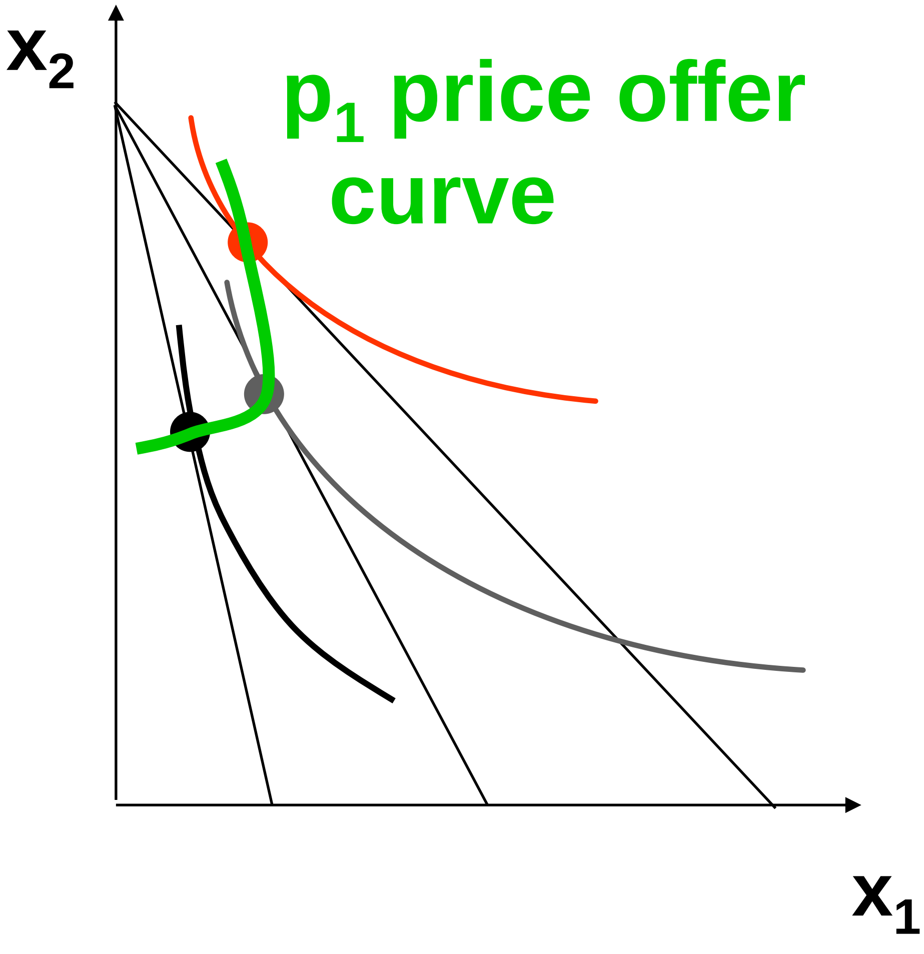
Ordinary Goods

Fixed p_2 and y.



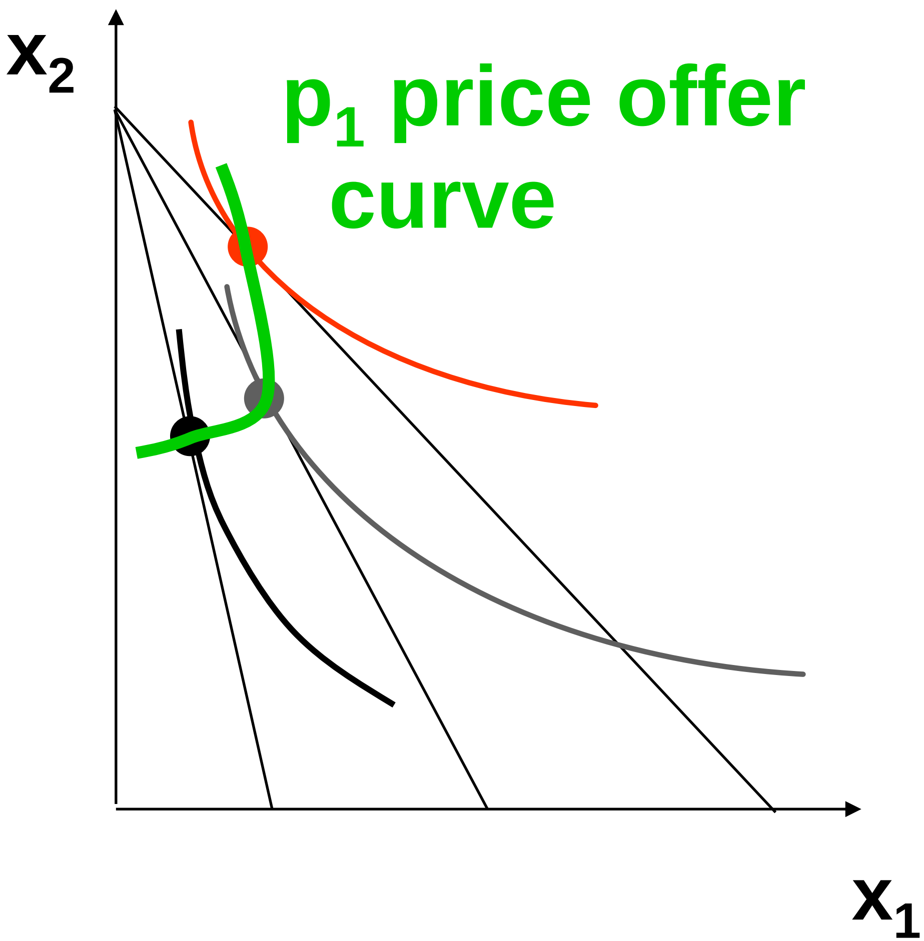
Ordinary Goods

Fixed p_2 and y.



Ordinary Goods

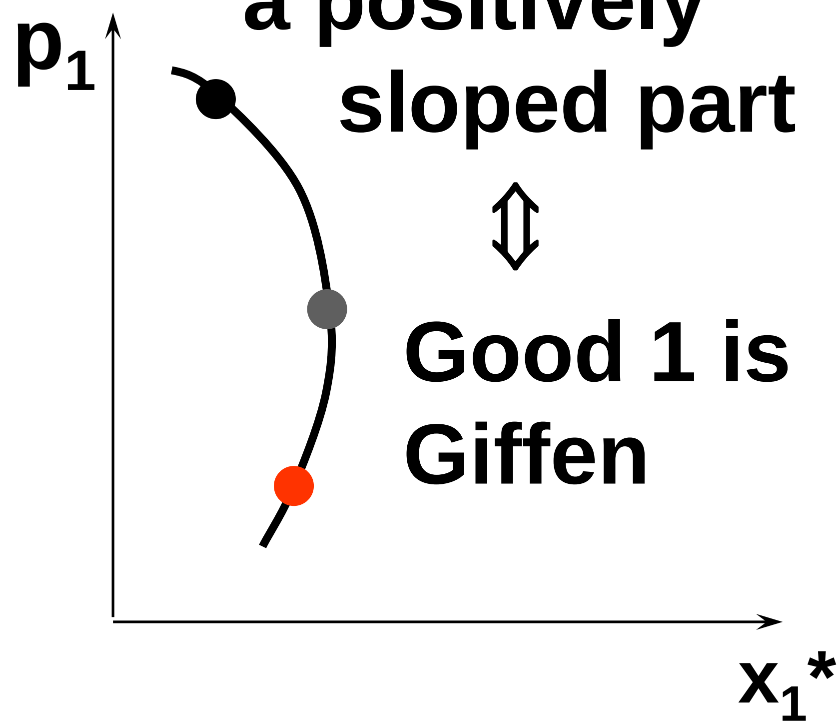
Fixed p_2 and y.



Demand curve has a positively sloped part

⇕

Good 1 is Giffen



Cross-Price Effects

- If an increase in p_2
  - increases demand for commodity 1 then commodity 1 is a gross substitute for commodity 2.
  - reduces demand for commodity 1 then commodity 1 is a gross complement for commodity 2.

Cross-Price Effects

A perfect-complements example:

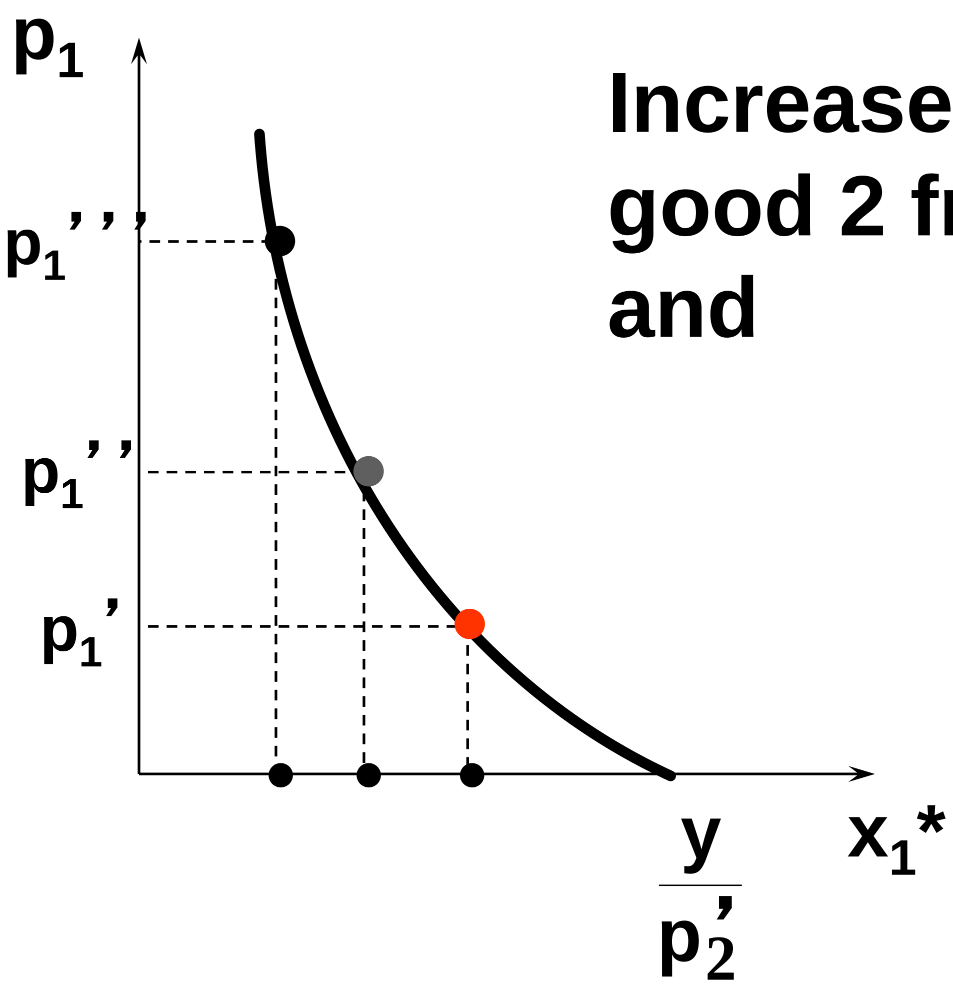
$$x_1^* = \frac{y}{p_1 + p_2}$$

so

$$\frac{\partial x_1^*}{\partial p_2} = -\frac{y}{(p_1 + p_2)^2} < 0.$$

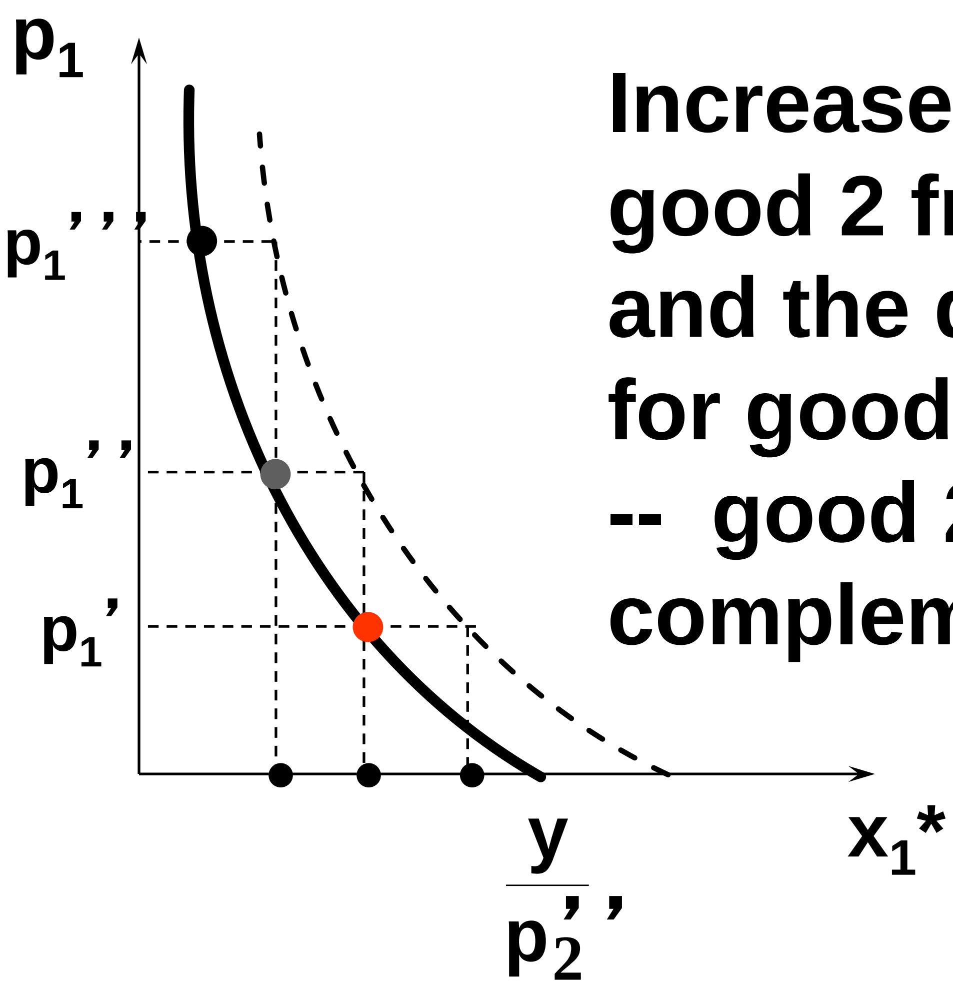
Therefore commodity 2 is a gross complement for commodity 1.

Cross-Price Effects



Increase the price of good 2 from p_2' to p_2'' and

Cross-Price Effects



Increase the price of good 2 from p_2' to p_2'' and the demand curve for good 1 shifts inwards -- good 2 is a complement for good 1.

Cross-Price Effects

A Cobb- Douglas example:

$$x_2^* = \frac{by}{(a+b)p_2}$$

so

Cross-Price Effects

A Cobb- Douglas example:

$$x_2^* = \frac{by}{(a+b)p_2}$$

so

$$\frac{\partial x_2^*}{\partial p_1} = 0.$$

Therefore commodity 1 is neither a gross complement nor a gross substitute for commodity 2.

Slutsky Equation

Effects of a Price Change

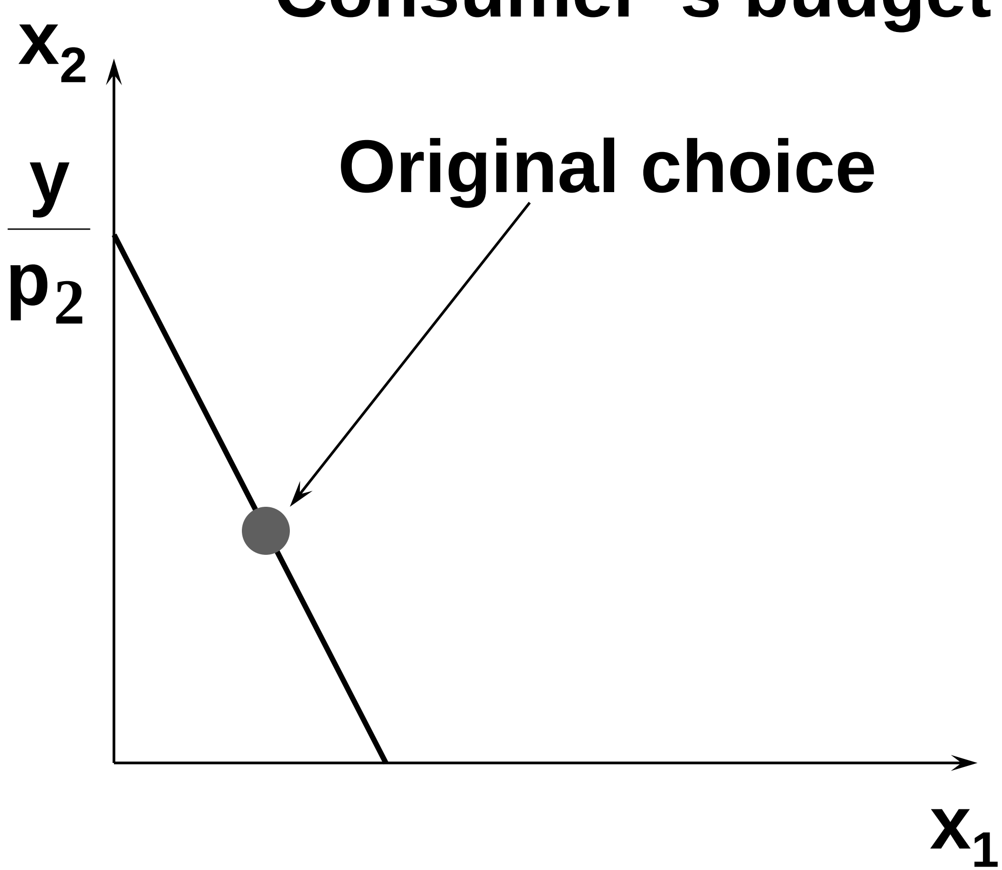
- **What happens when a commodity' s price decreases?**
  - **Substitution effect: the commodity is relatively cheaper, so consumers substitute it for now relatively more expensive other commodities.**

Effects of a Price Change

– **Income effect: the consumer's budget of $y can purchase more than before, as if the consumer's income rose, with consequent income effects on quantities demanded.**

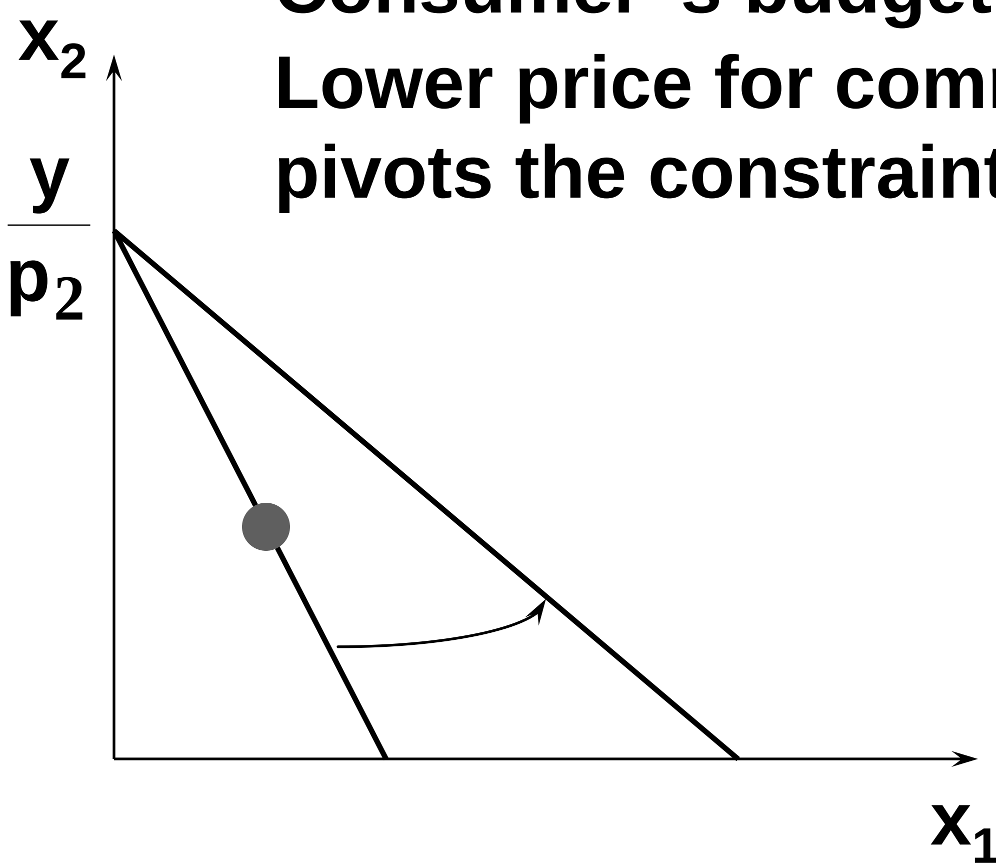
Effects of a Price Change

Consumer's budget is $y.



Effects of a Price Change

Consumer's budget is $y.
Lower price for commodity 1 pivots the constraint outwards.

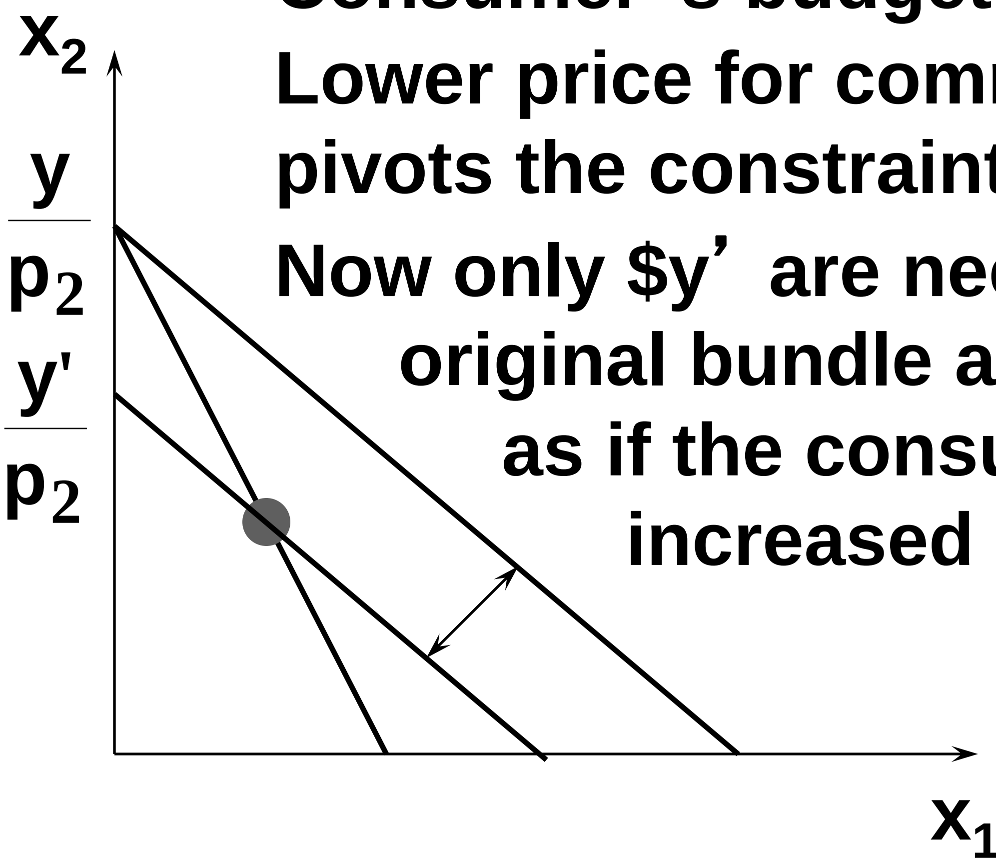


Effects of a Price Change

Consumer's budget is \$y.

Lower price for commodity 1 pivots the constraint outwards.

Now only \$y' are needed to buy the original bundle at the new prices, as if the consumer's income has increased by \$y - \$y'.



Effects of a Price Change

- **Changes to quantities demanded due to this ‘extra’ income are the income effect of the price change.**

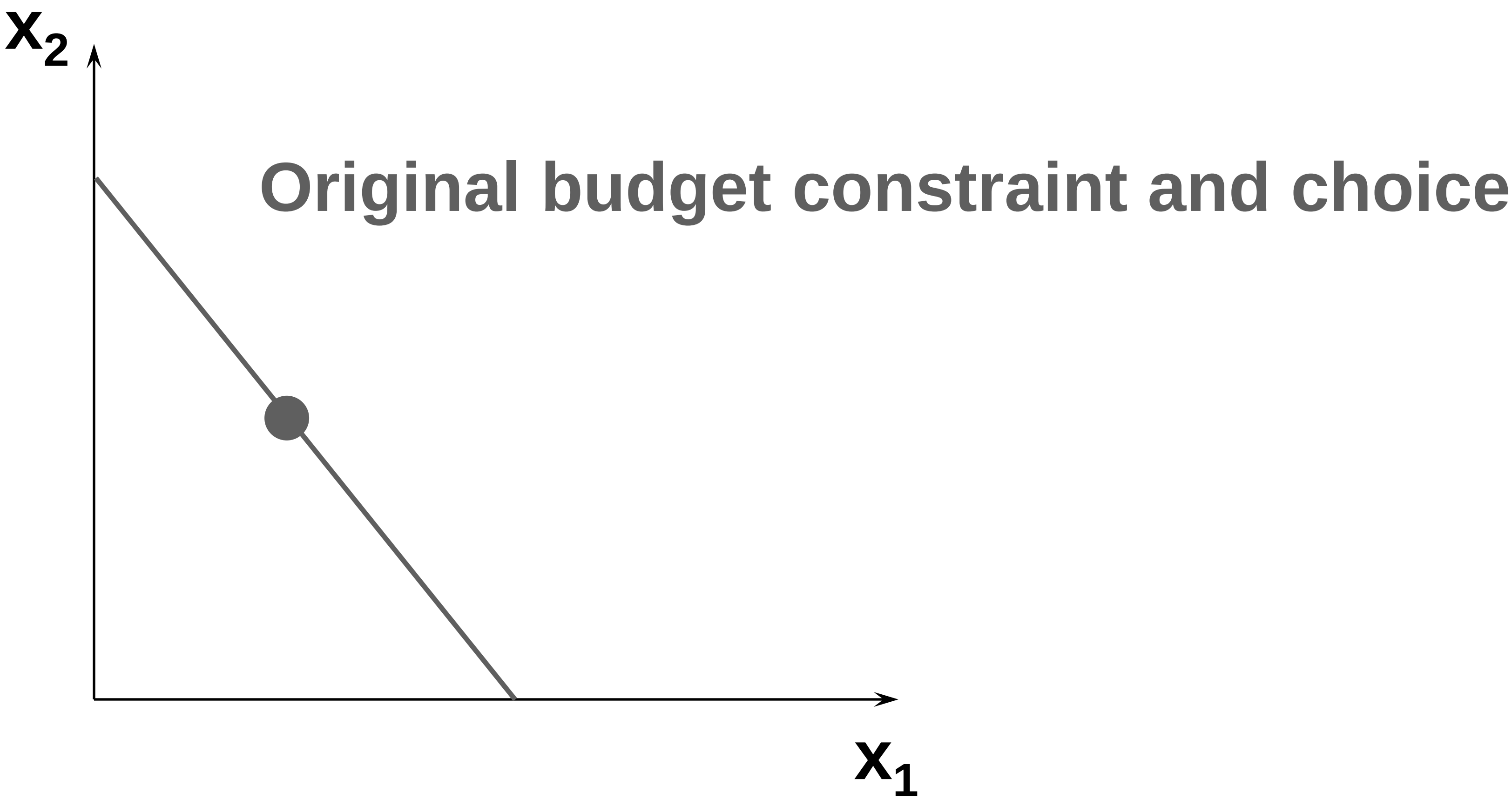
Effects of a Price Change

- **Slutsky discovered that changes to demand from a price change are always the sum of a pure substitution effect and an income effect.**

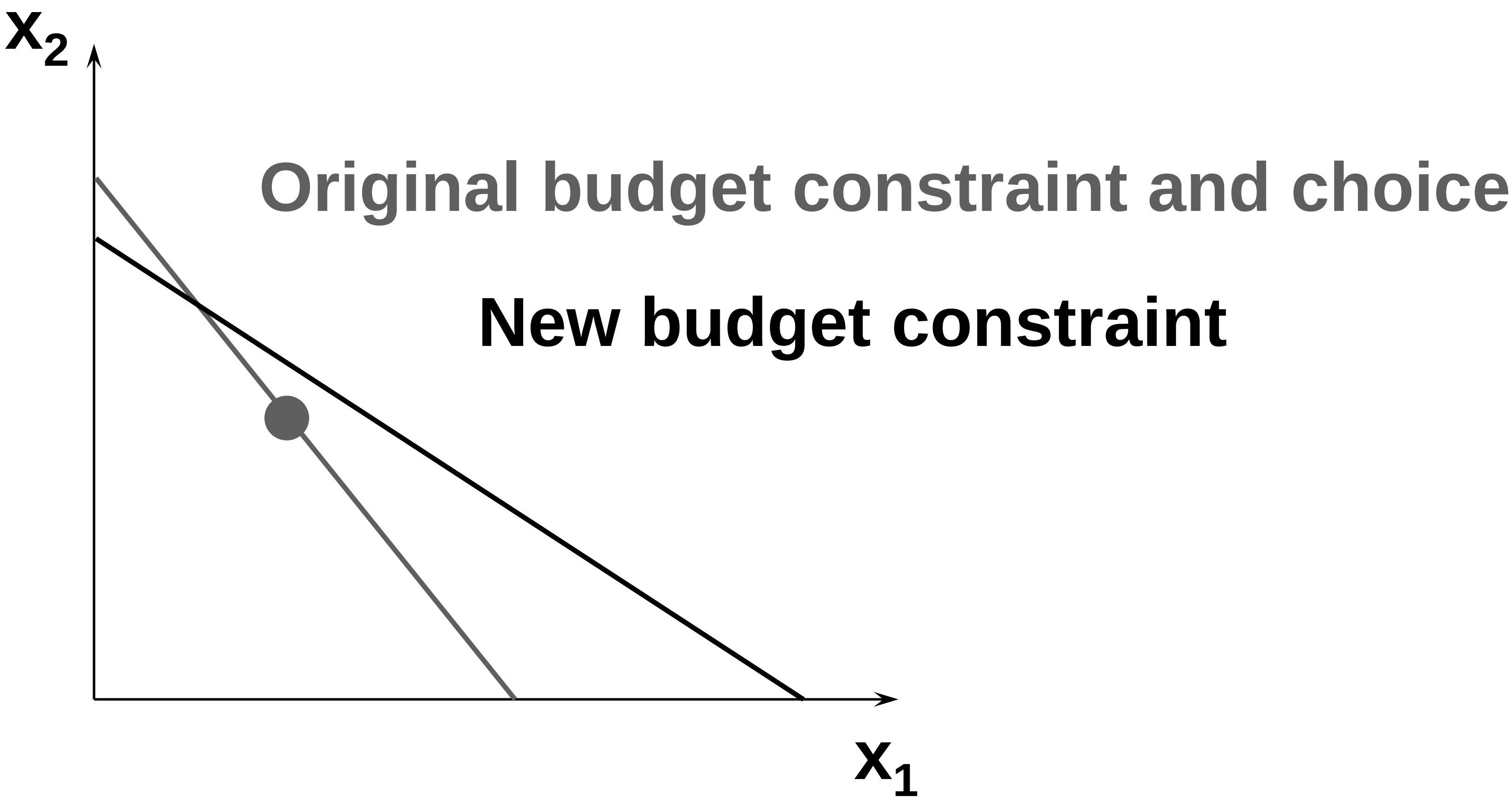
Real Income Changes

- Slutsky asserted that if, at the new prices,
  - less income is needed to buy the original bundle then “real income” is increased
  - more income is needed to buy the original bundle then “real income” is decreased

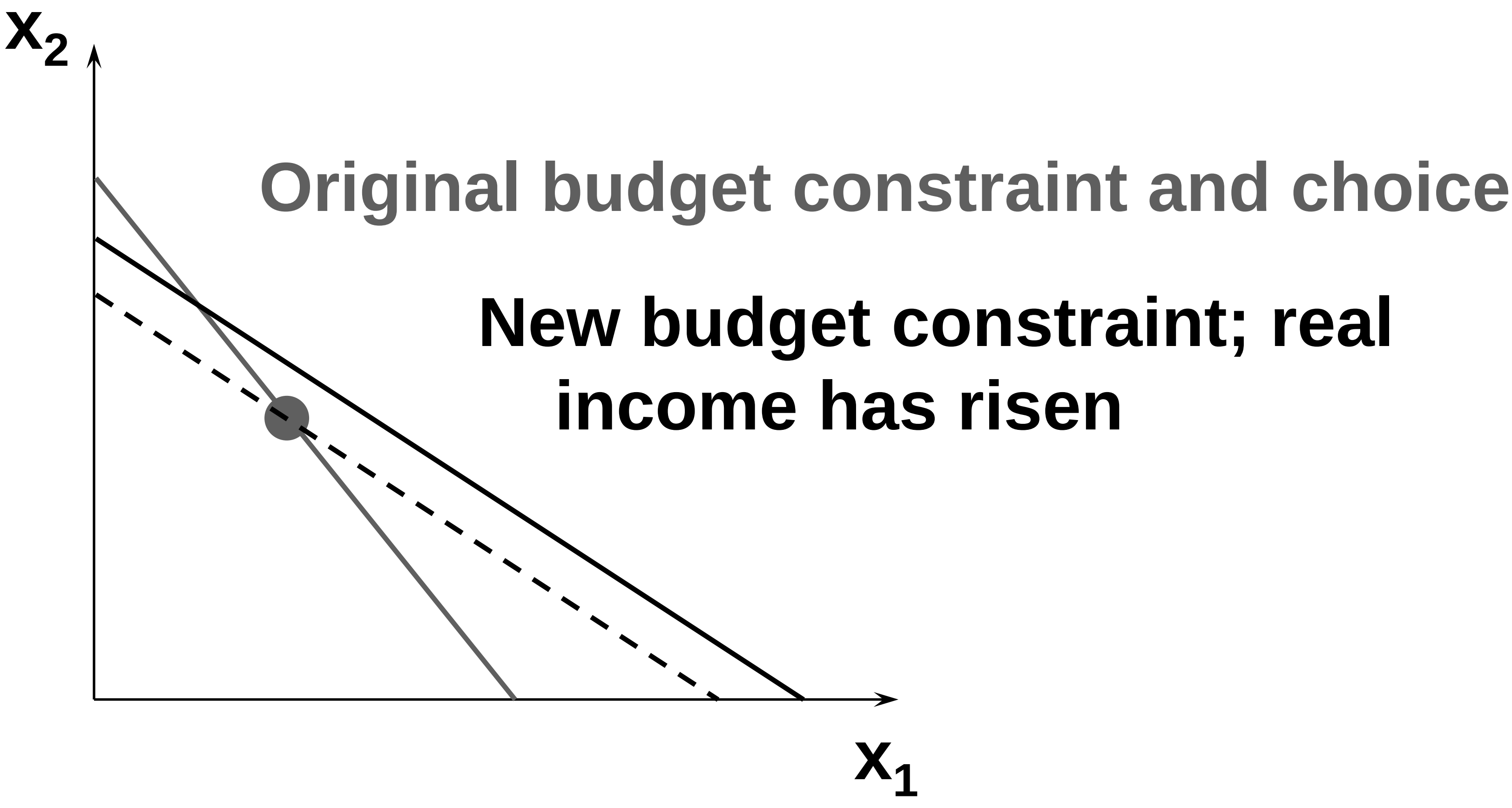
Real Income Changes



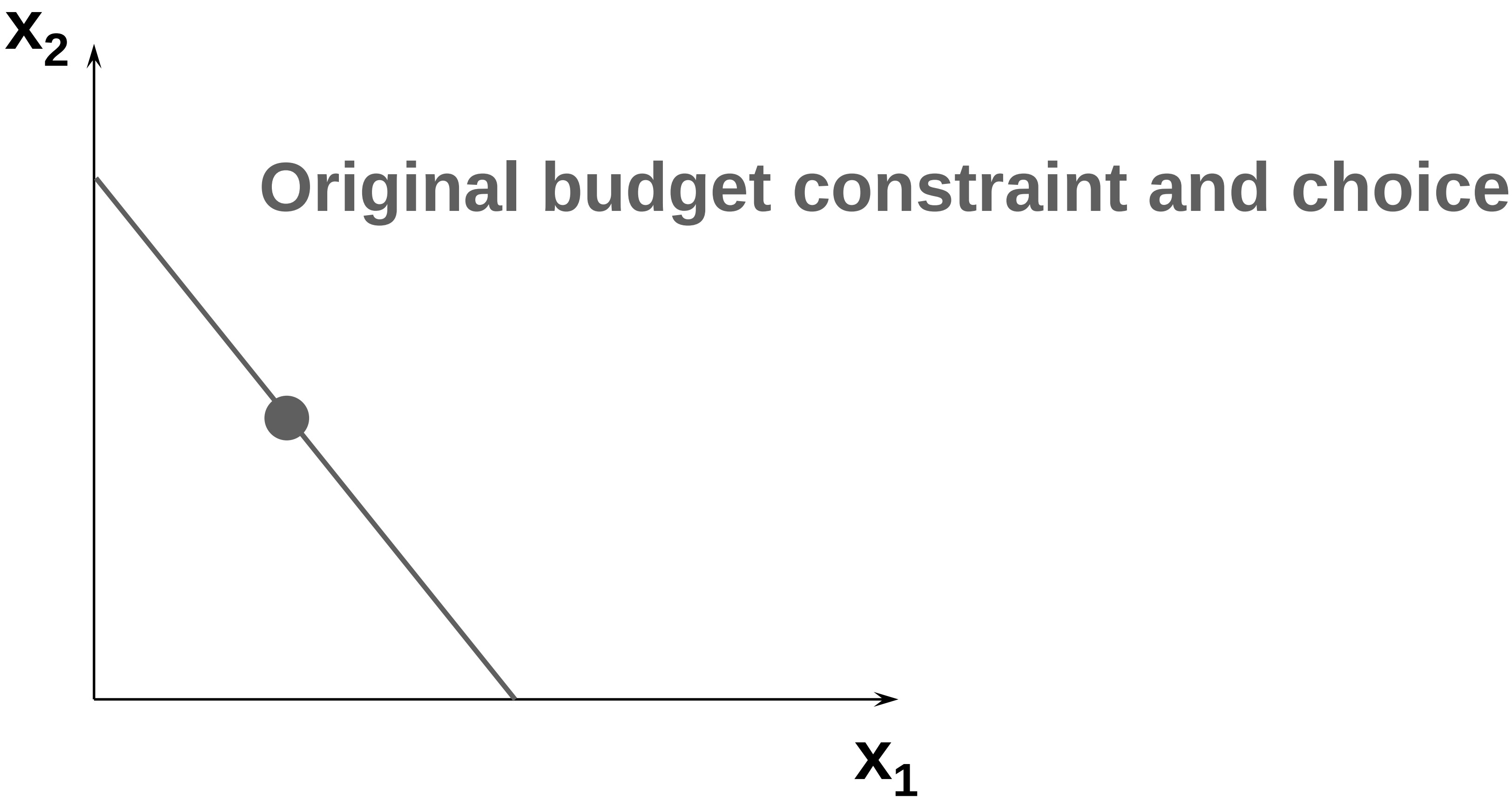
Real Income Changes



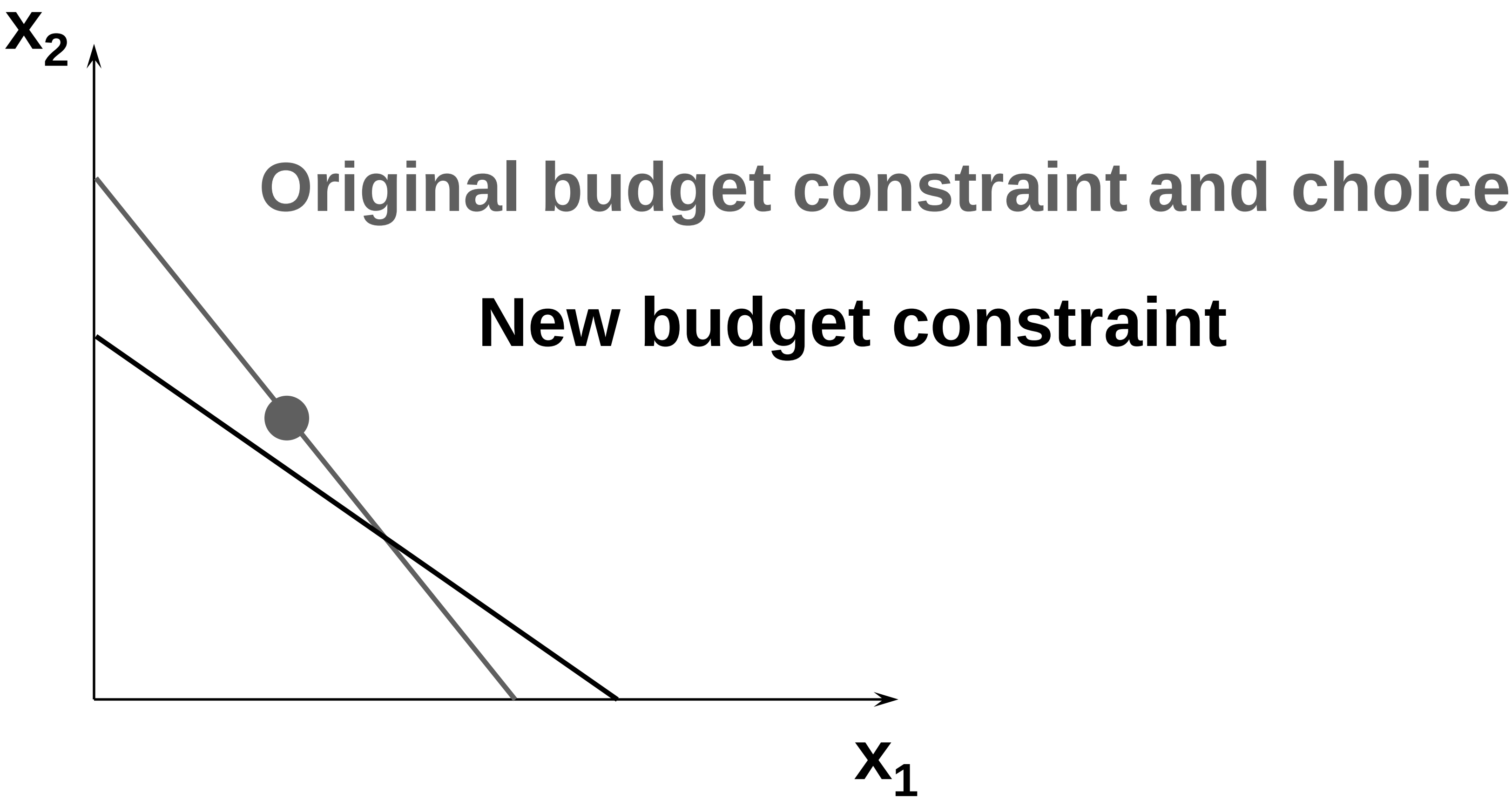
Real Income Changes



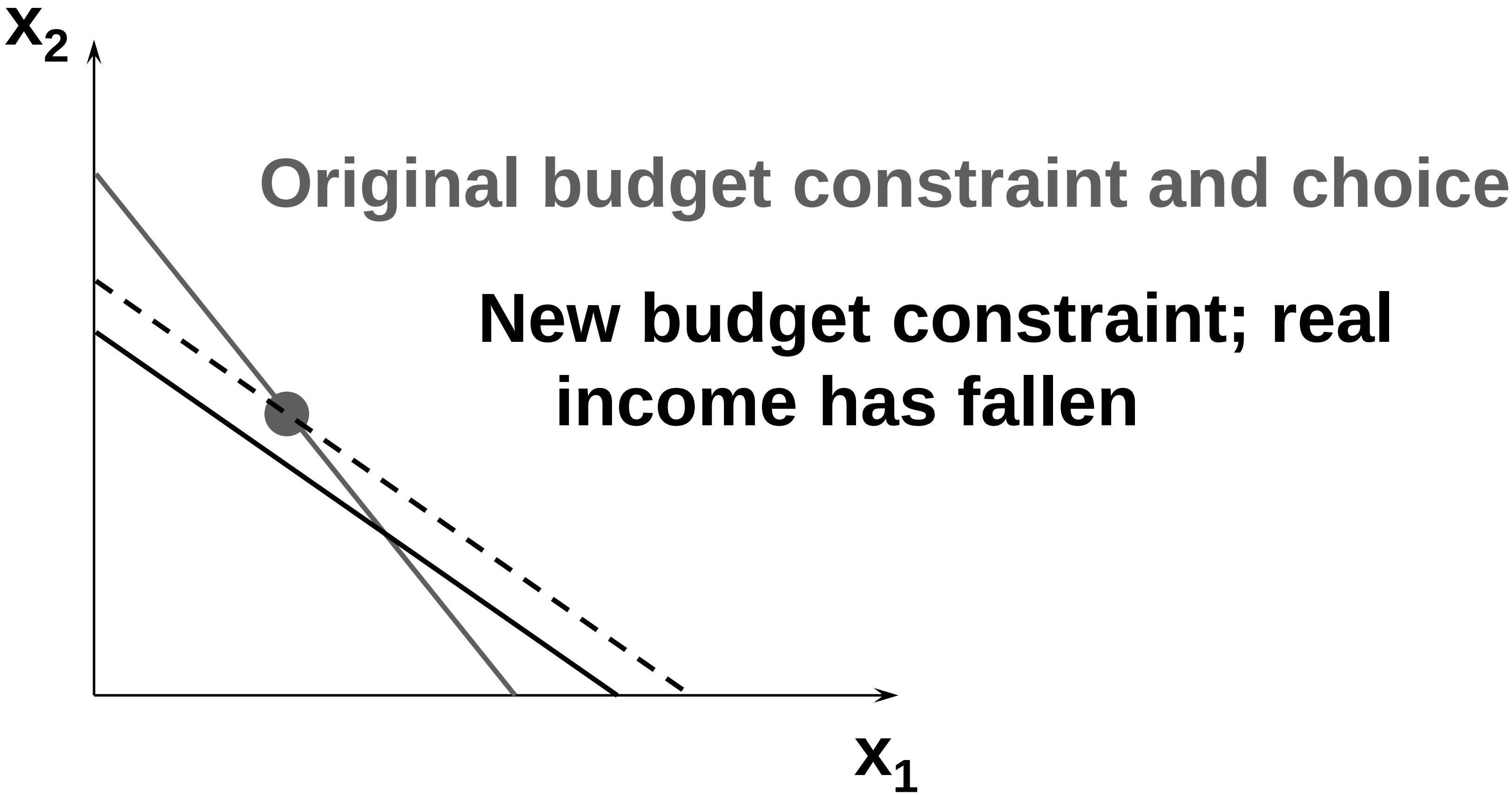
Real Income Changes



Real Income Changes



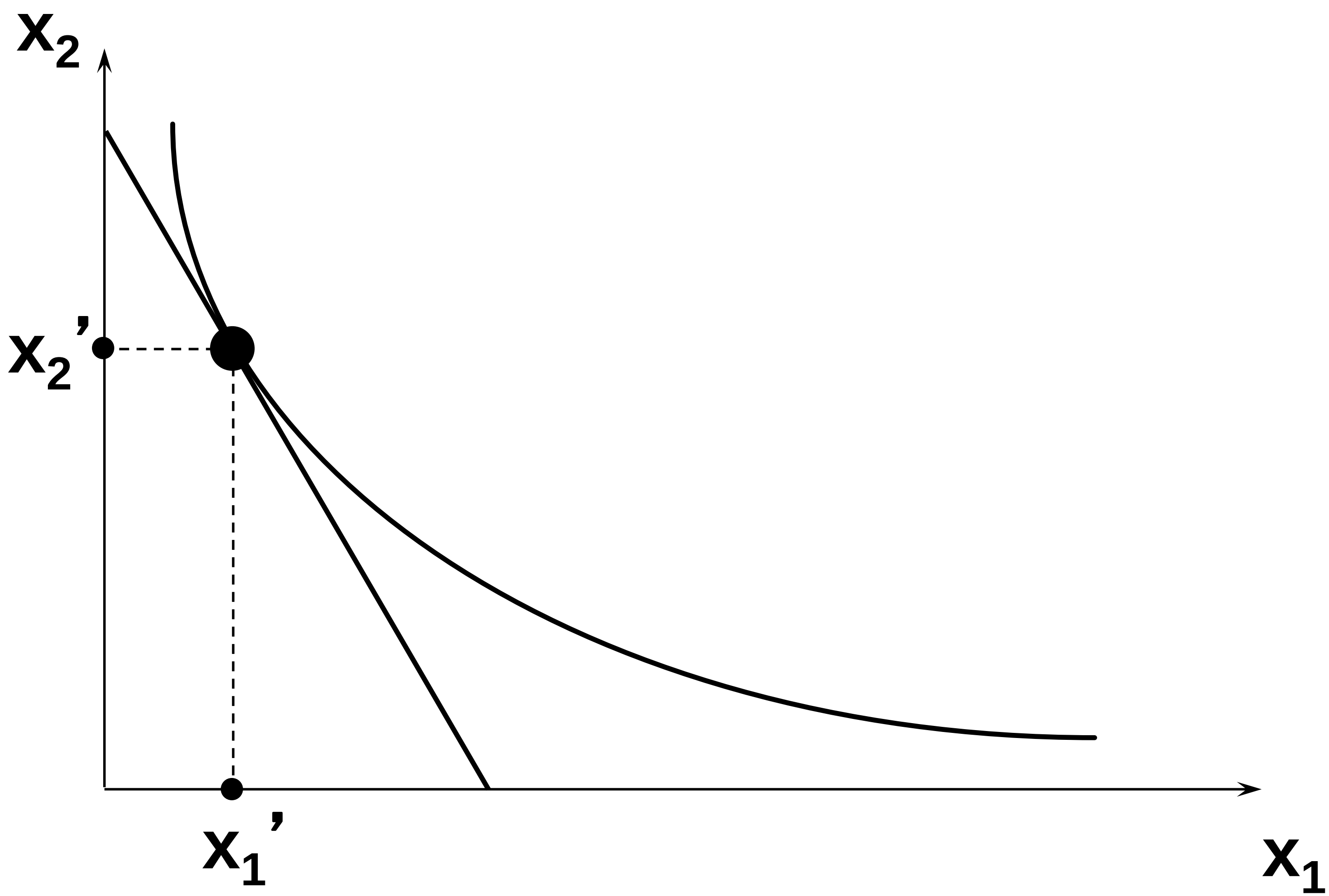
Real Income Changes



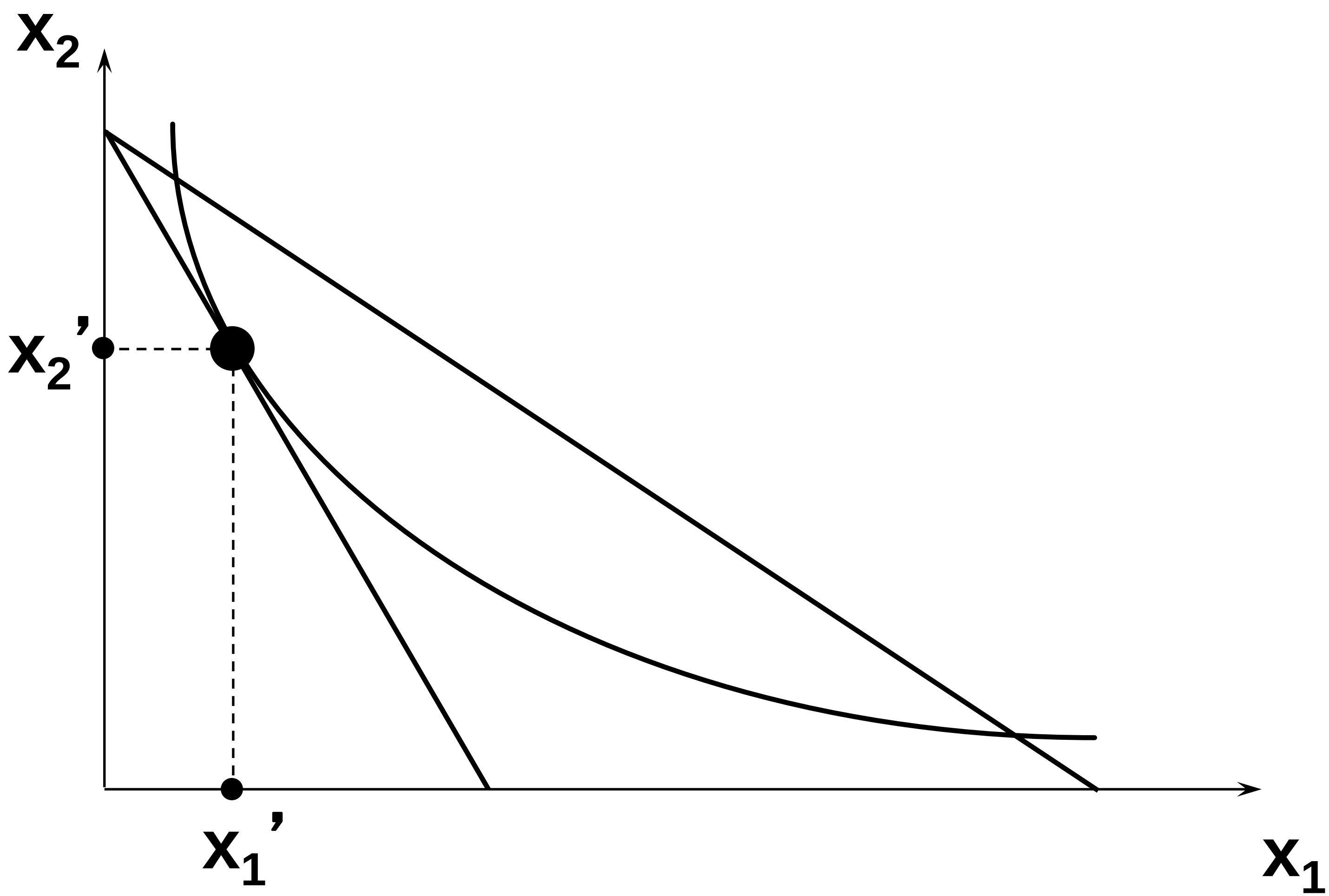
Pure Substitution Effect

- **Slutsky isolated the change in demand due only to the change in relative prices by asking “What is the change in demand when the consumer’s income is adjusted so that, at the new prices, she can only just buy the original bundle?”**

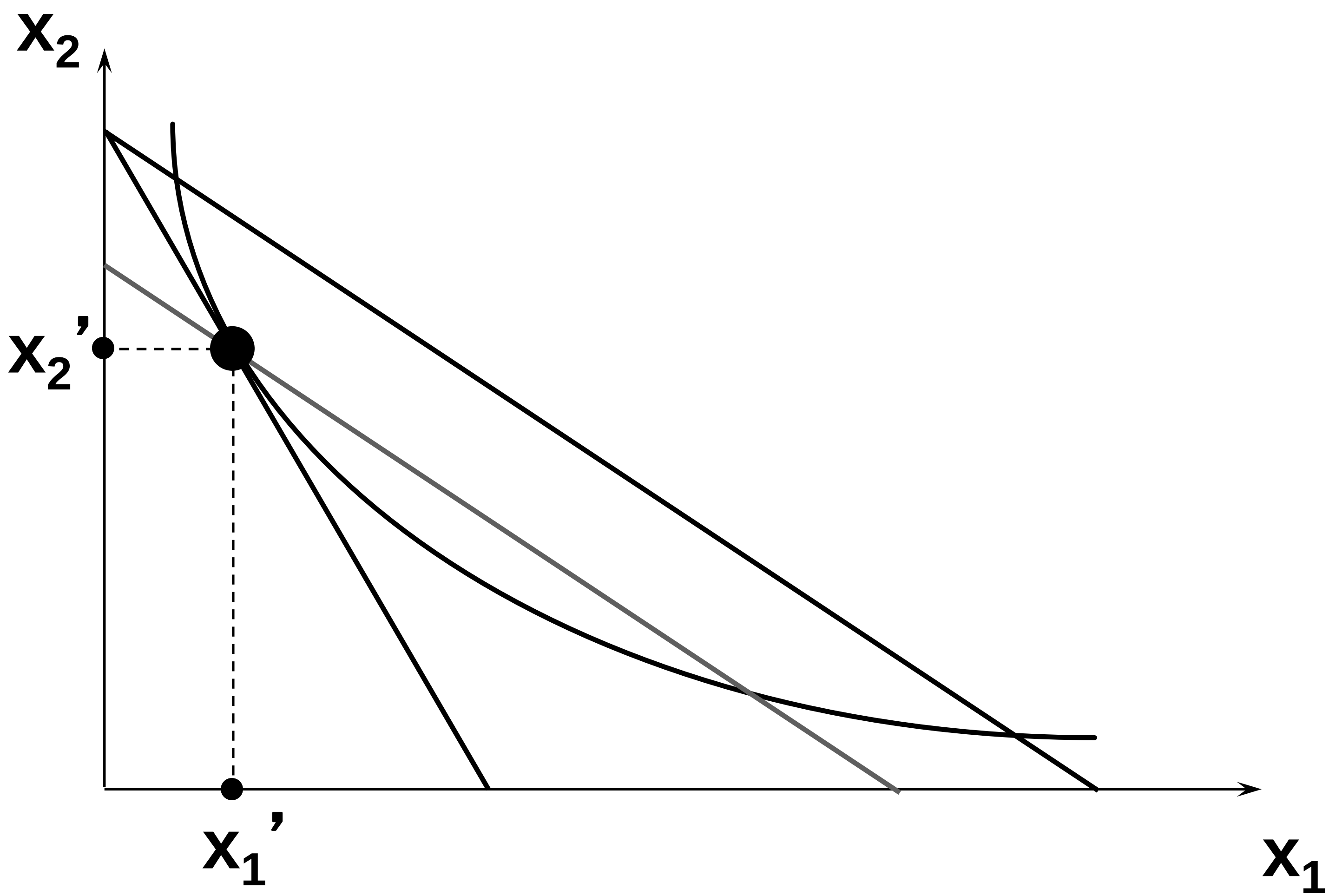
Pure Substitution Effect Only



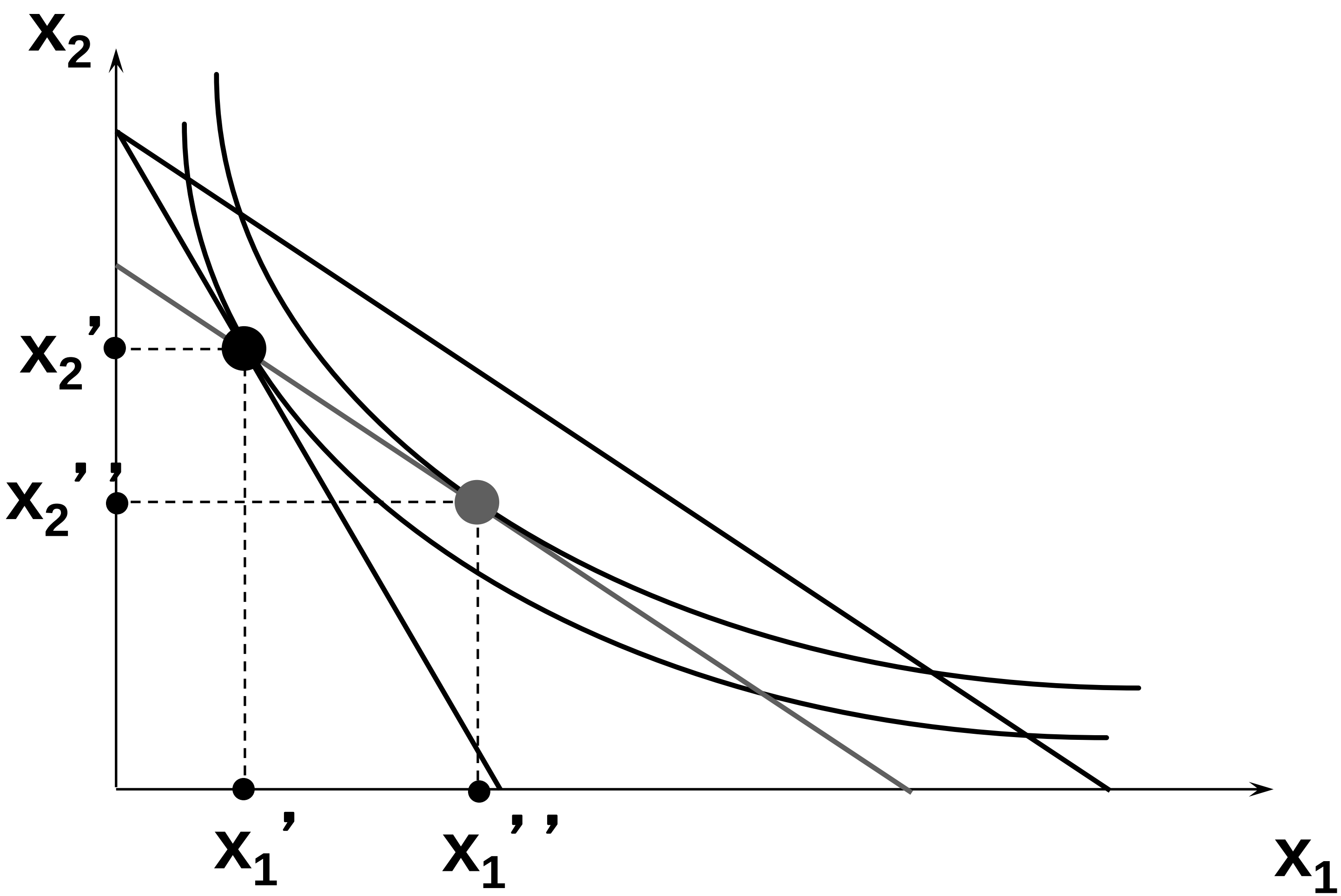
Pure Substitution Effect Only



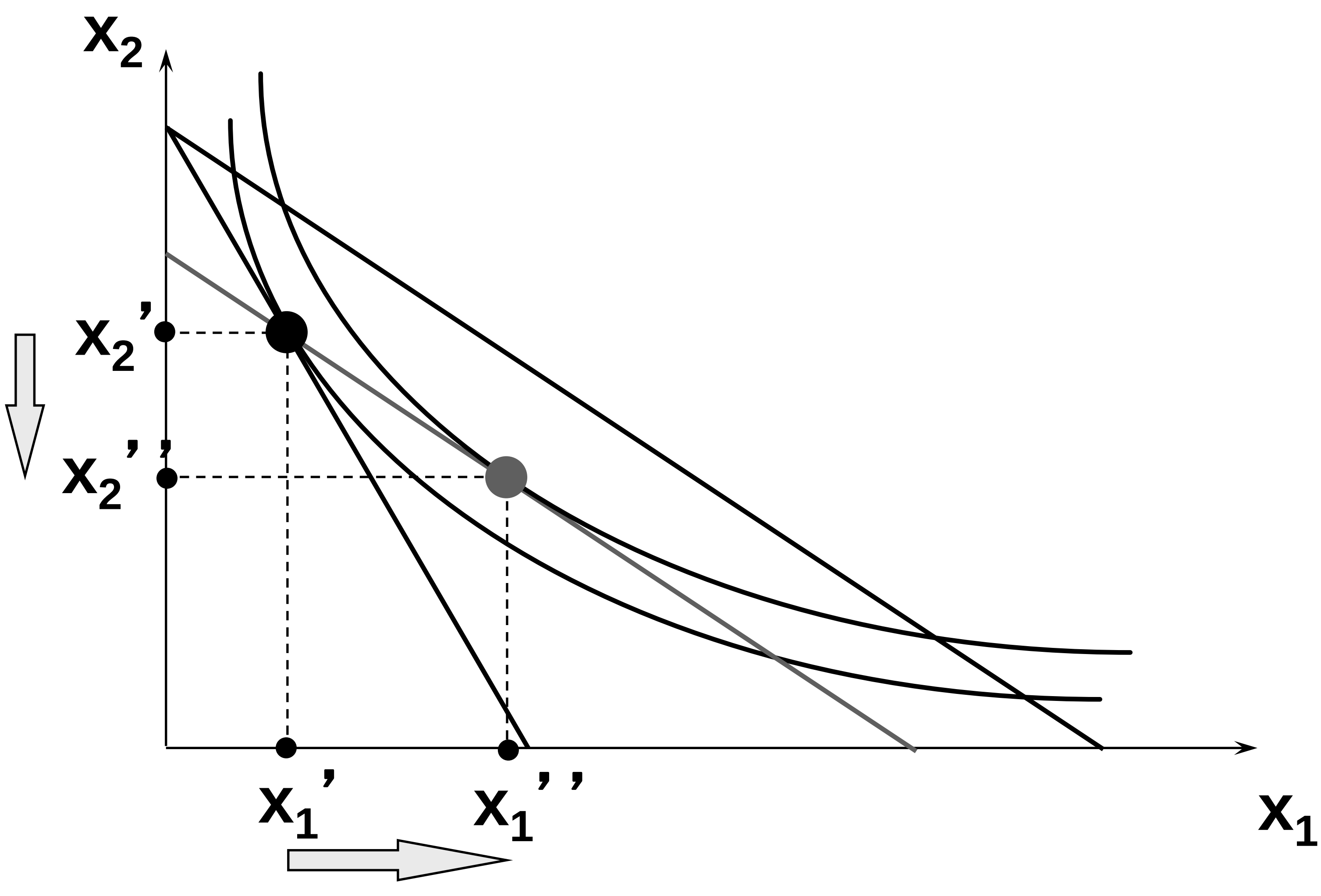
Pure Substitution Effect Only



Pure Substitution Effect Only

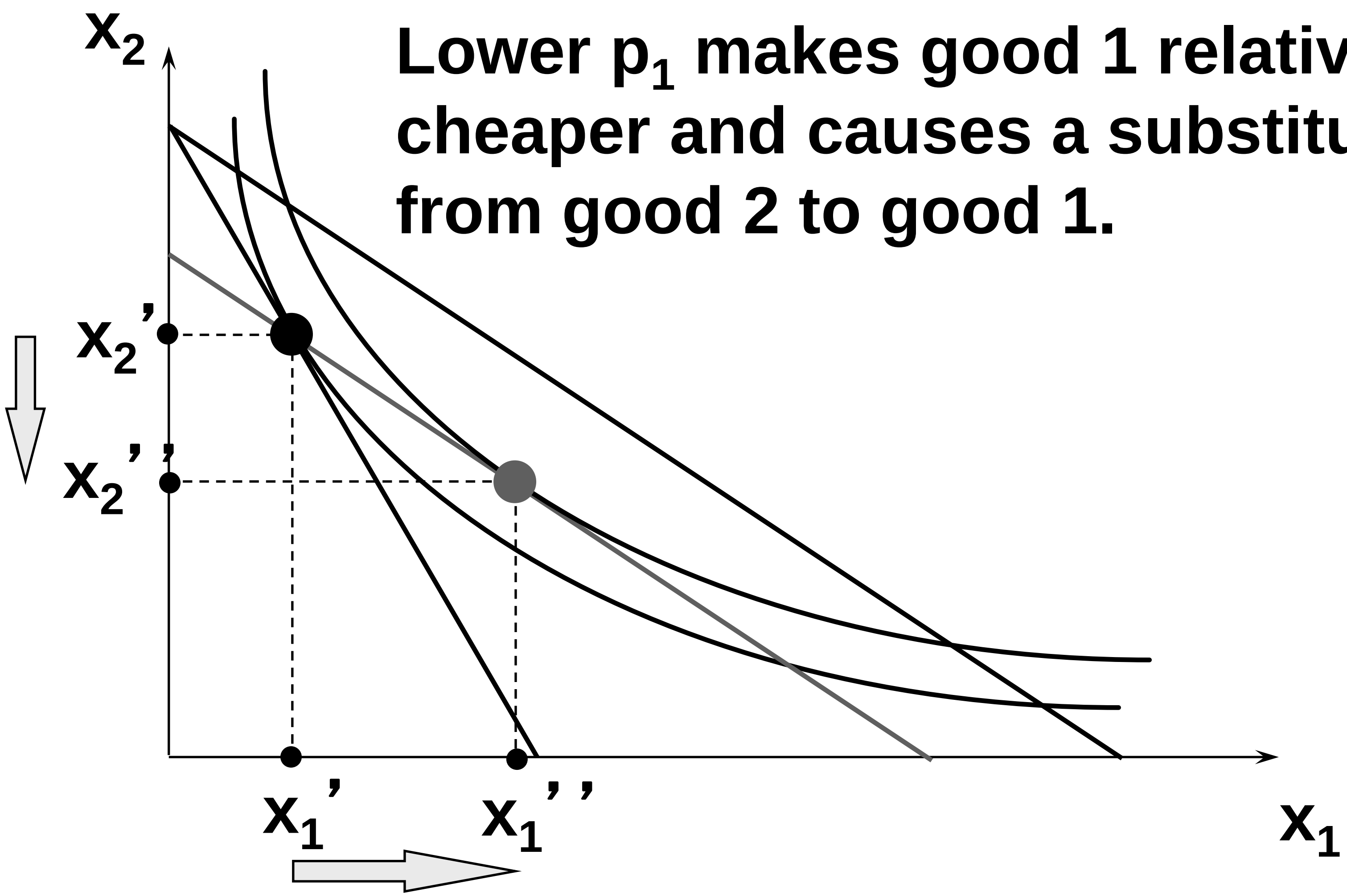


Pure Substitution Effect Only



Pure Substitution Effect Only

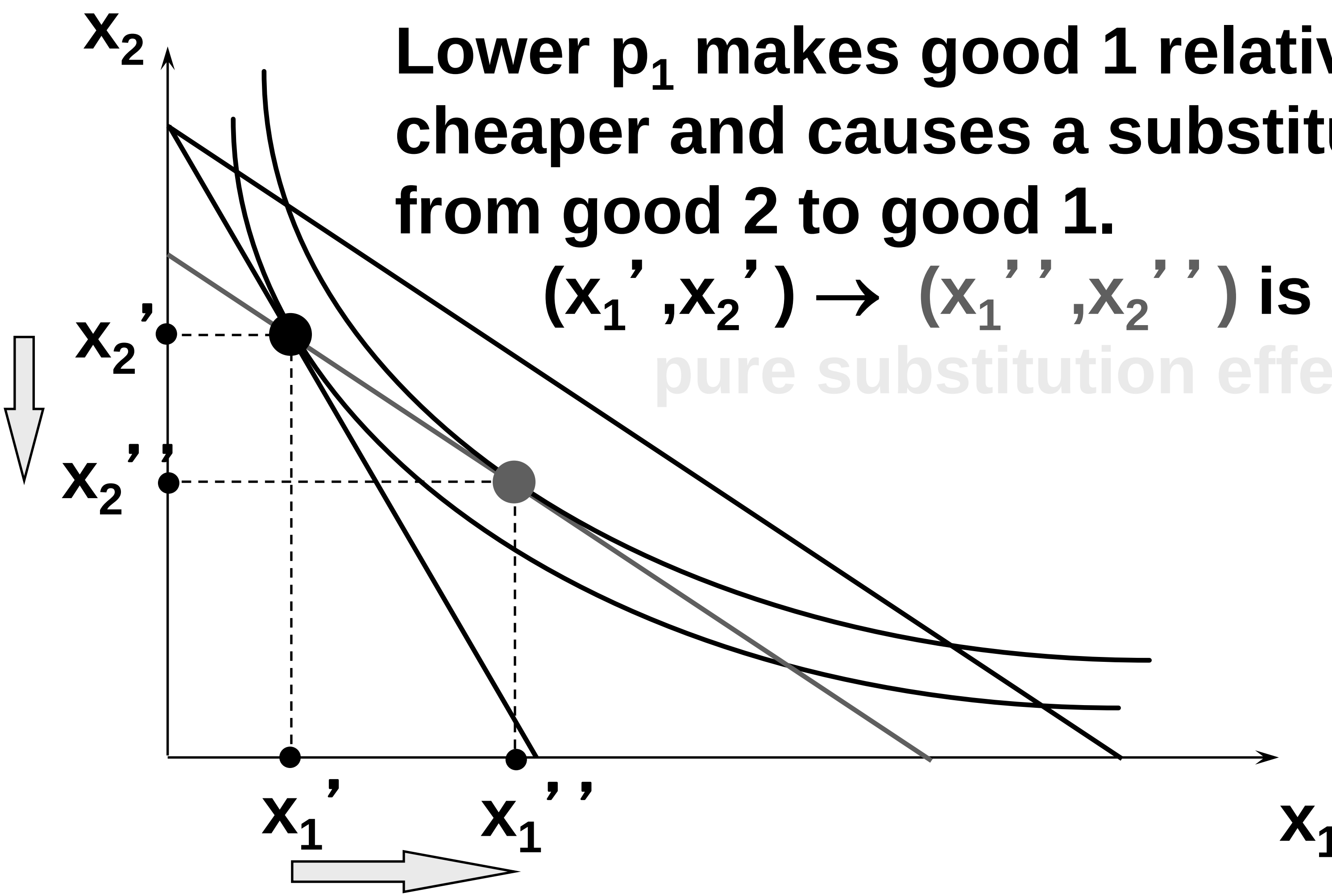
Lower p_1 makes good 1 relatively cheaper and causes a substitution from good 2 to good 1.



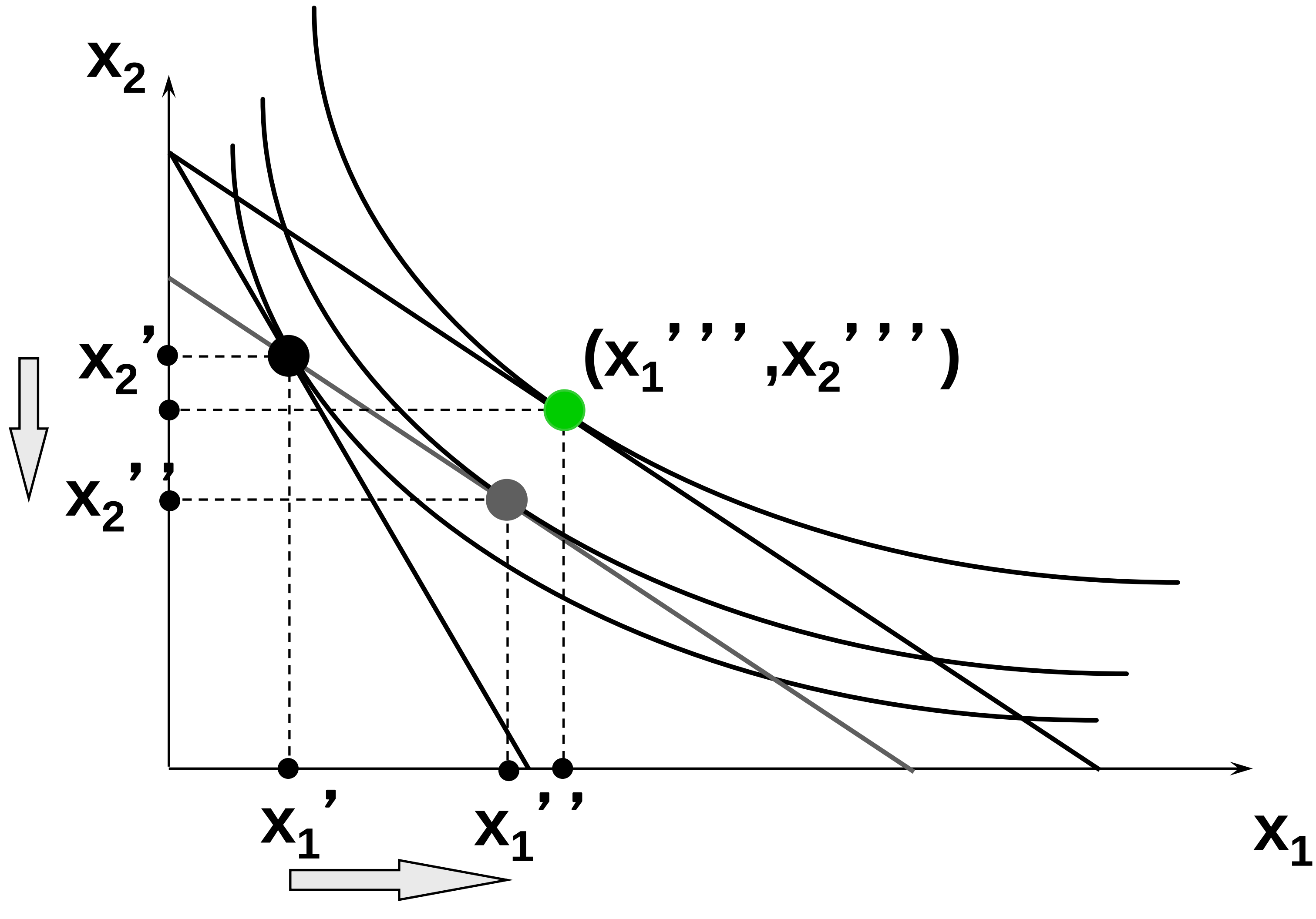
Pure Substitution Effect Only

Lower p_1 makes good 1 relatively cheaper and causes a substitution from good 2 to good 1.

$(x_1', x_2') \rightarrow (x_1'', x_2'')$ is the pure substitution effect.



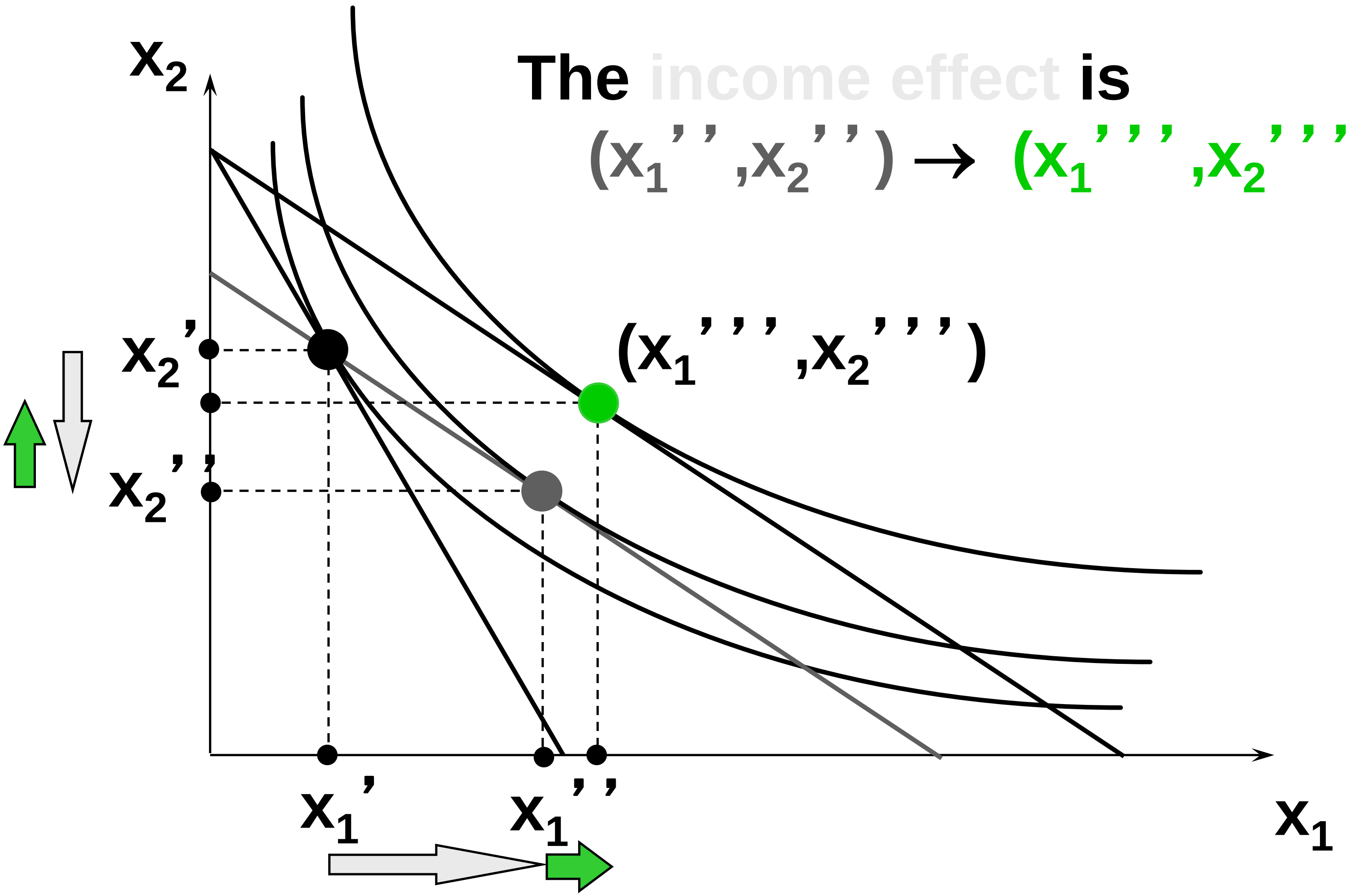
And Now The Income Effect



And Now The Income Effect

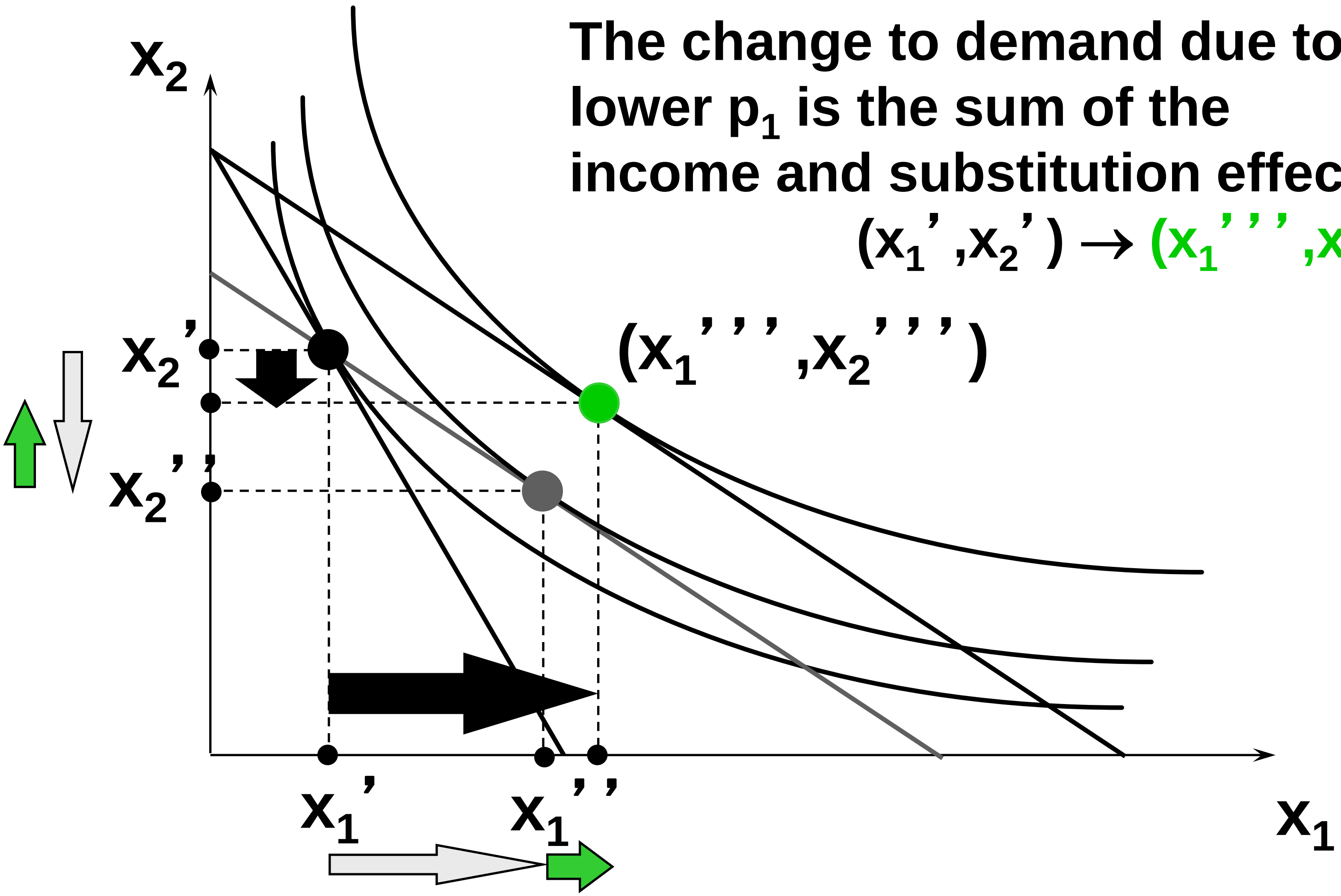
The income effect is

$(x_1'', x_2'') \rightarrow (x_1''', x_2''')$.



The Overall Change in Demand

The change to demand due to lower p_1 is the sum of the income and substitution effects, $(x_1', x_2') \rightarrow (x_1''', x_2''')$.

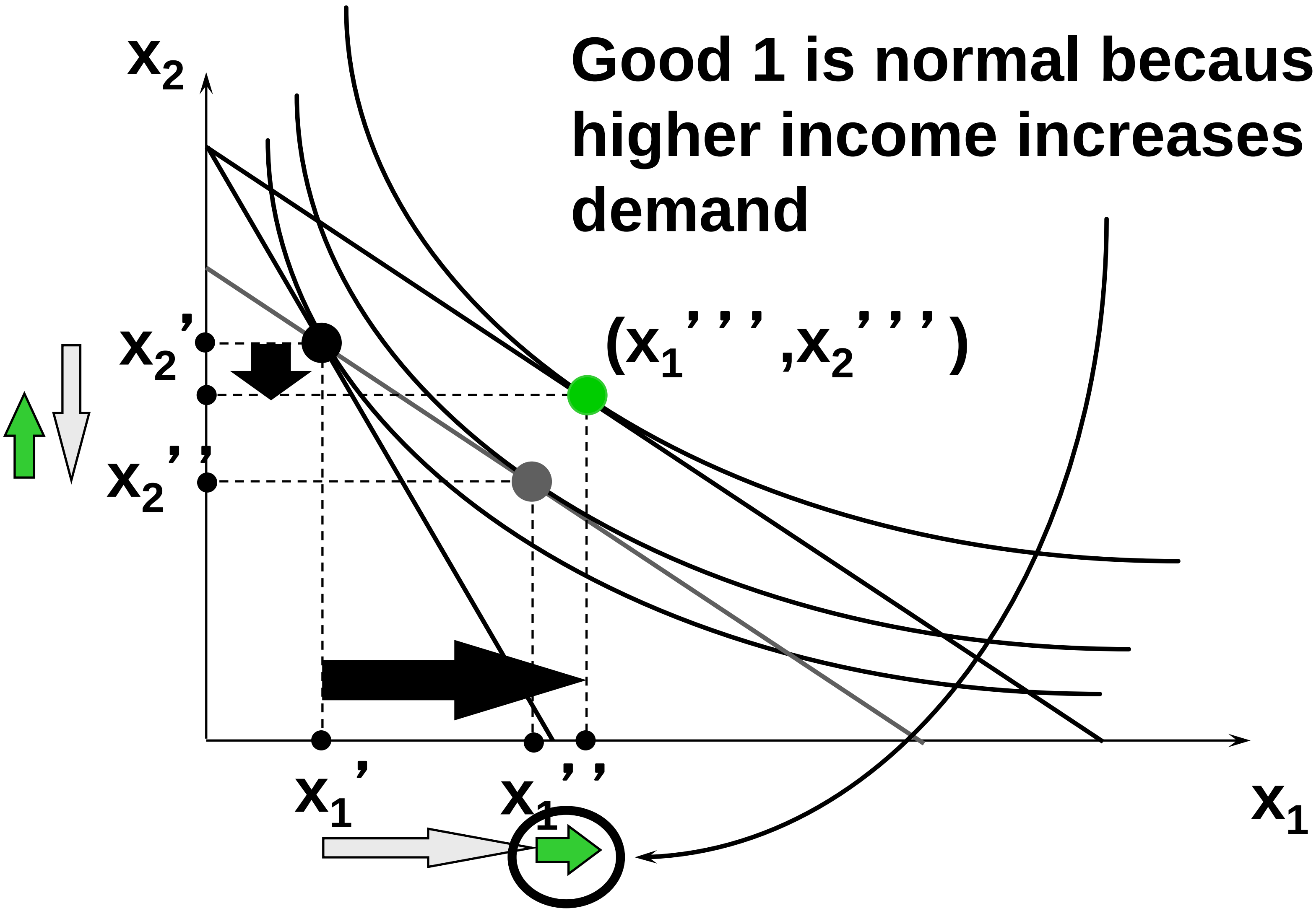


Slutsky's Effects for Normal Goods

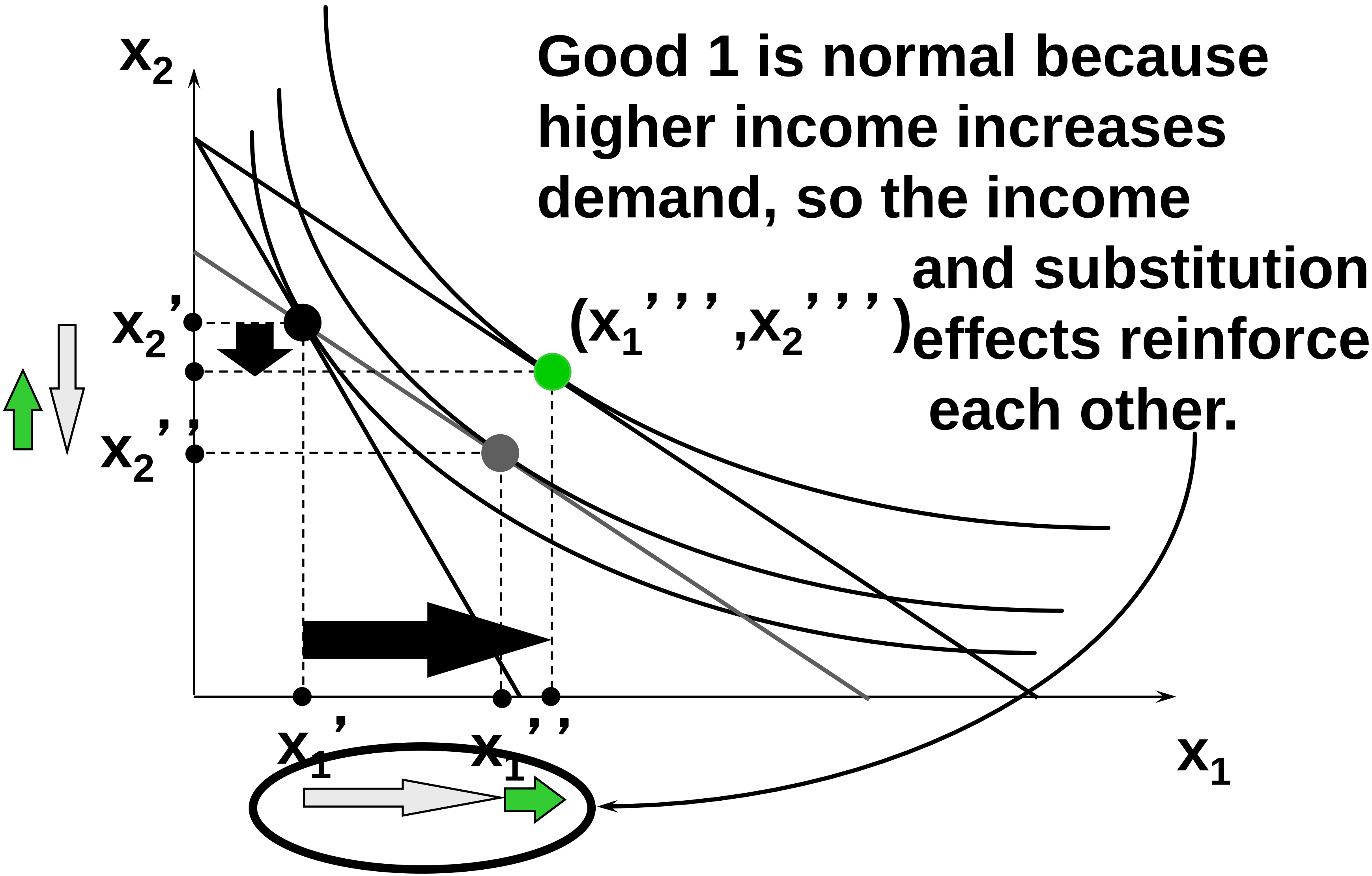
- **Most goods are normal (i.e. demand increases with income).**
- **The substitution and income effects reinforce each other when a normal good's own price changes.**

Slutsky's Effects for Normal Goods

Good 1 is normal because higher income increases demand



Slutsky's Effects for Normal Goods



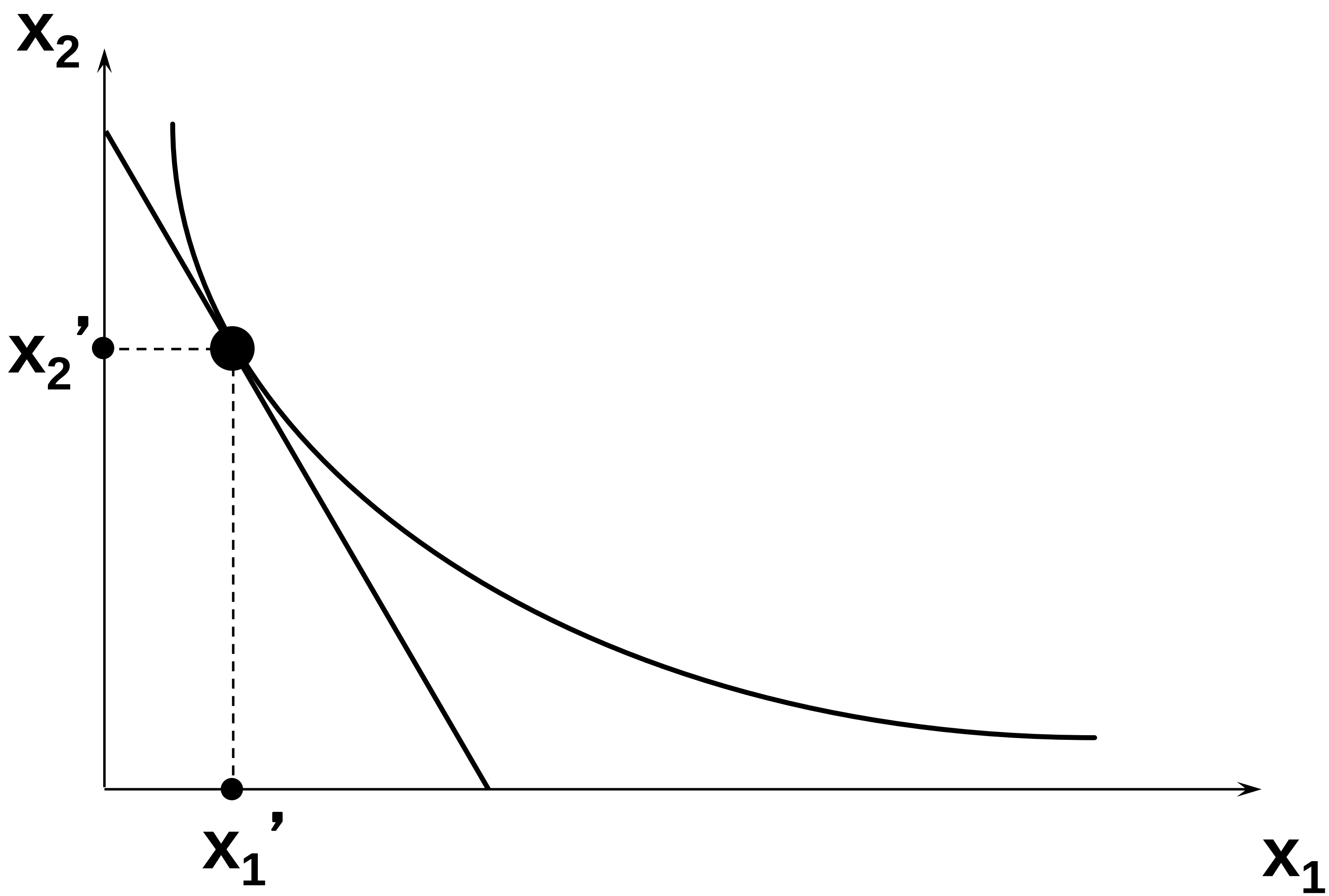
Slutsky’s Effects for Normal Goods

- **Since both the substitution and income effects increase demand when own-price falls, a normal good’s ordinary demand curve slopes down.**
- **The Law of Downward-Sloping Demand therefore always applies to normal goods.**

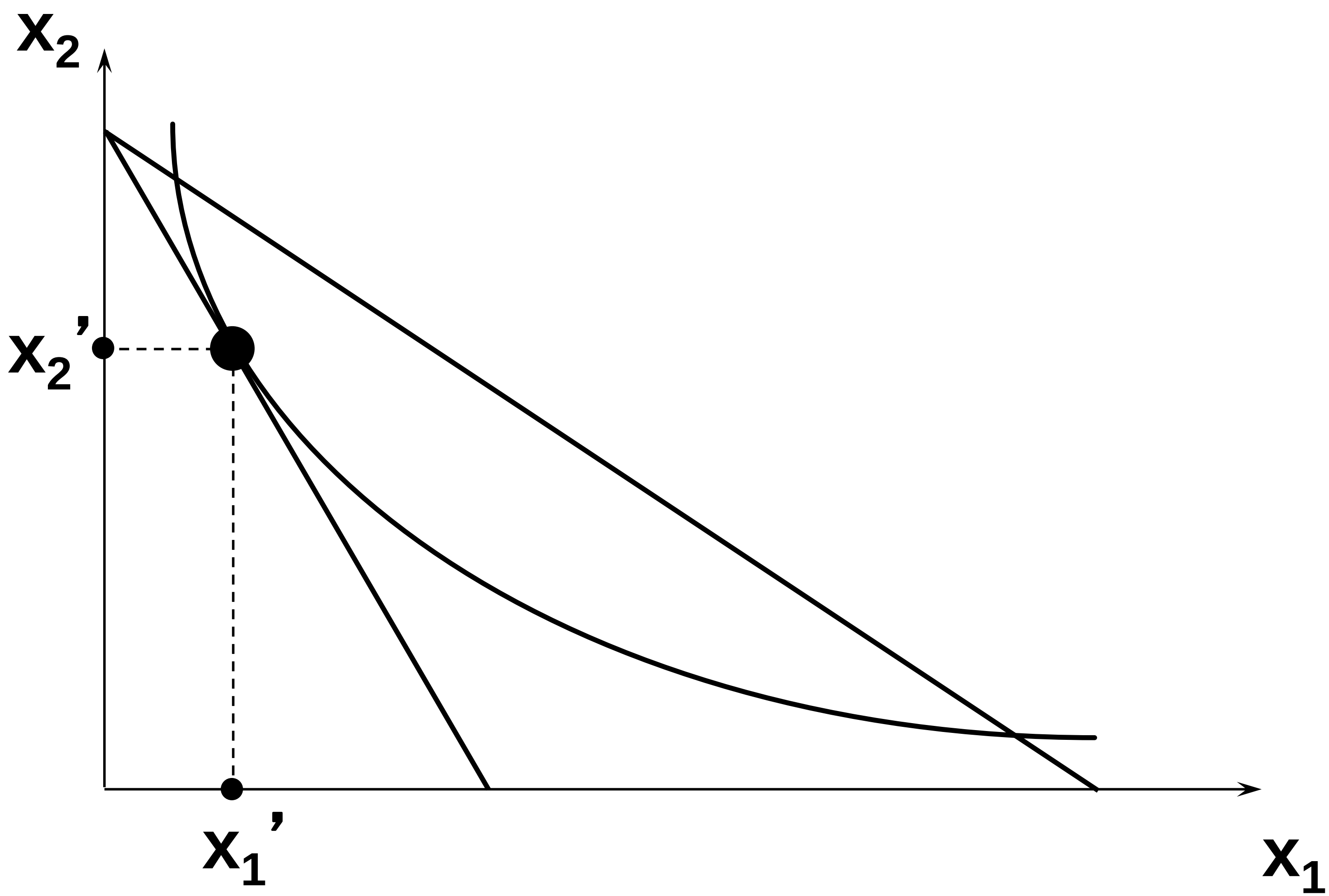
Slutsky's Effects for Income-Inferior Goods

- **Some goods are income-inferior (i.e. demand is reduced by higher income).**
- **The substitution and income effects oppose each other when an income-inferior good's own price changes.**

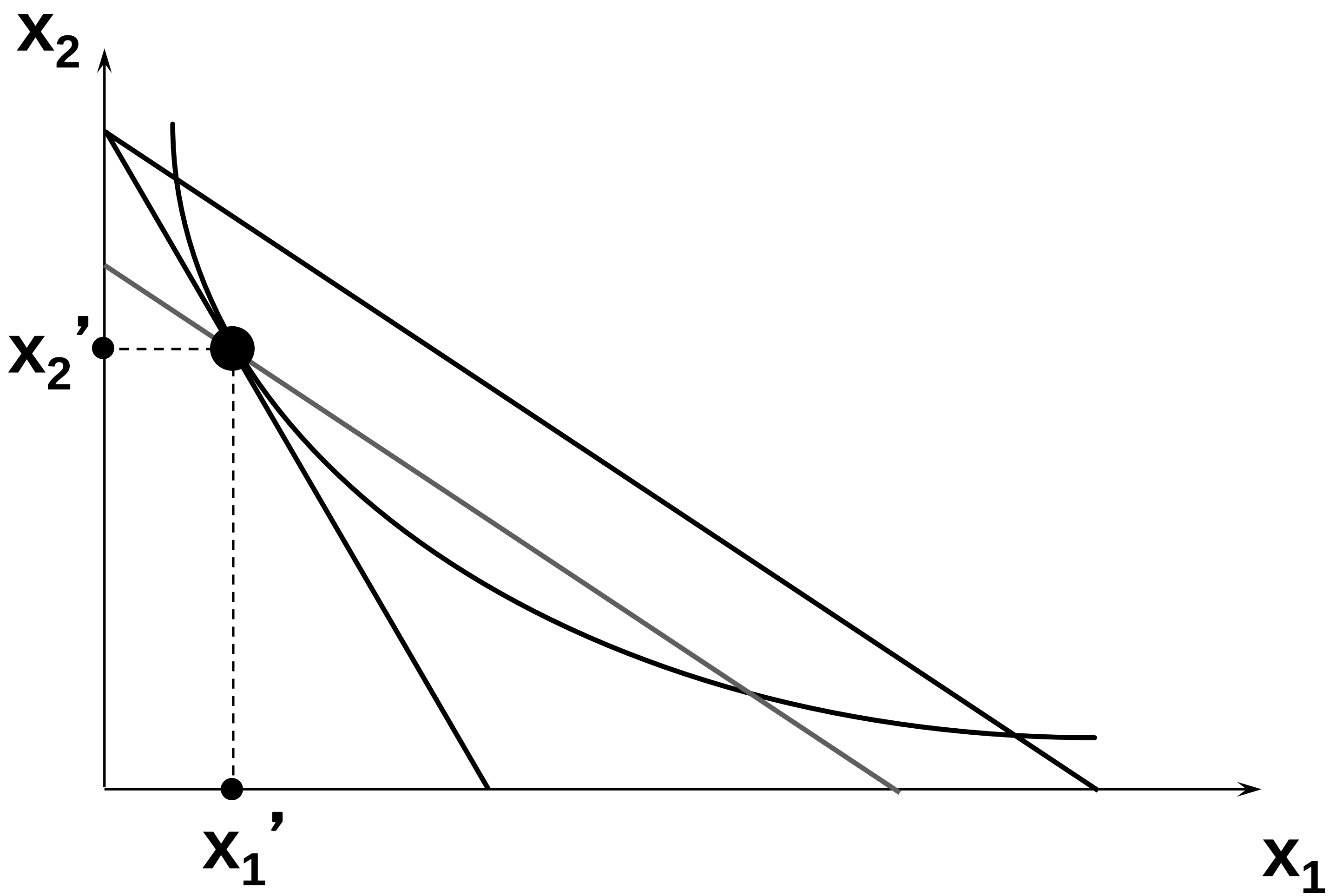
Slutsky's Effects for Income-Inferior Goods



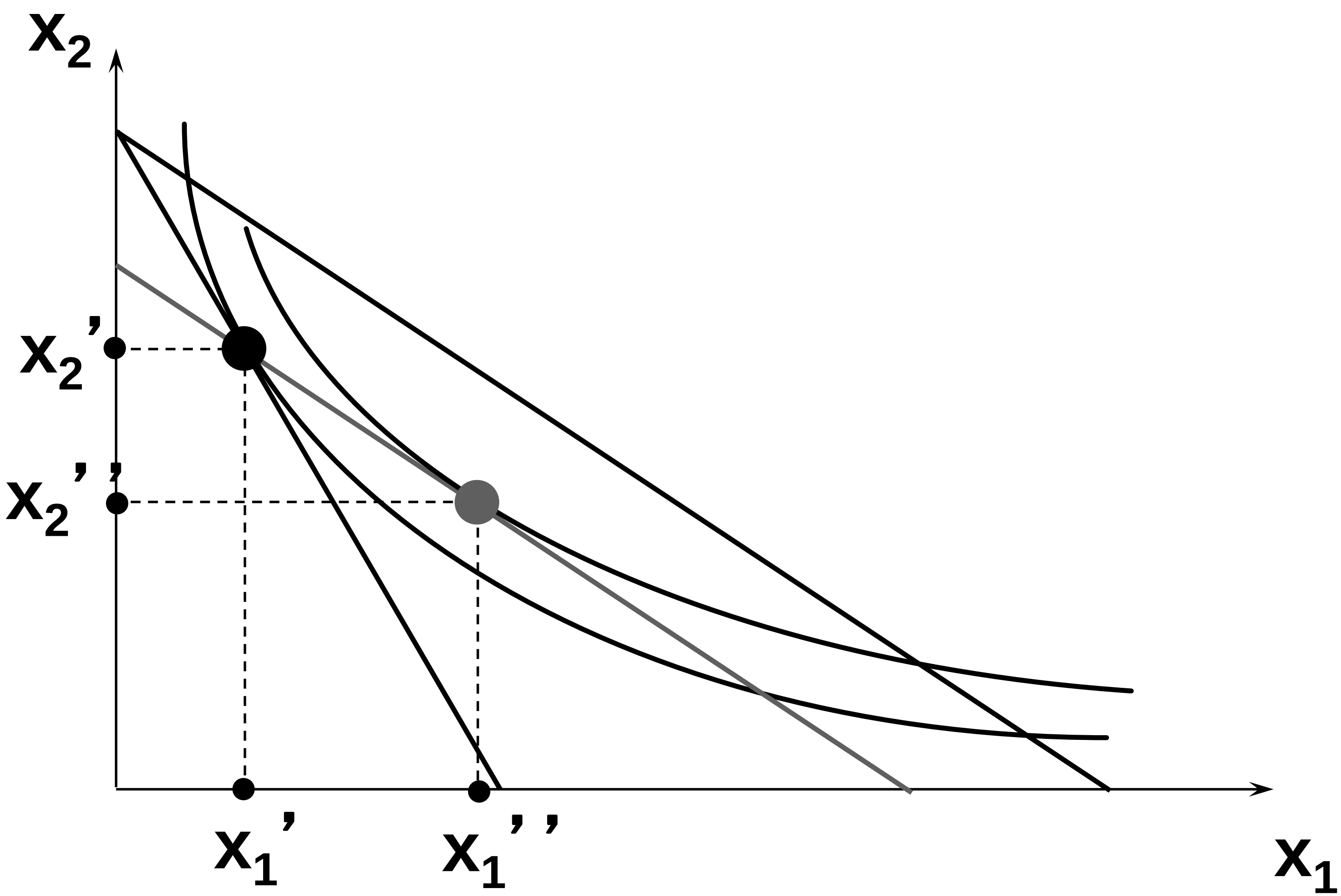
Slutsky's Effects for Income-Inferior Goods



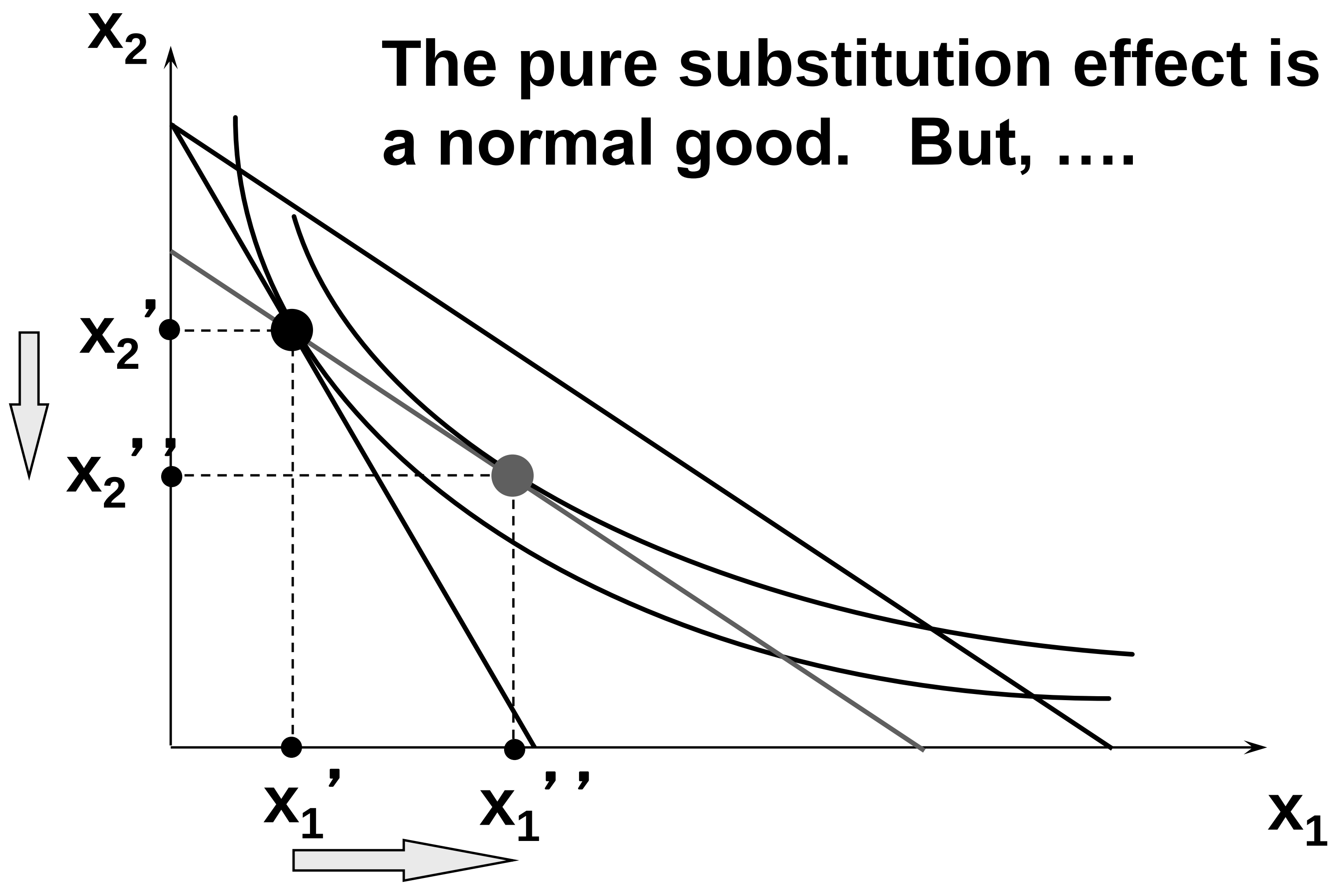
Slutsky's Effects for Income-Inferior Goods



Slutsky's Effects for Income-Inferior Goods

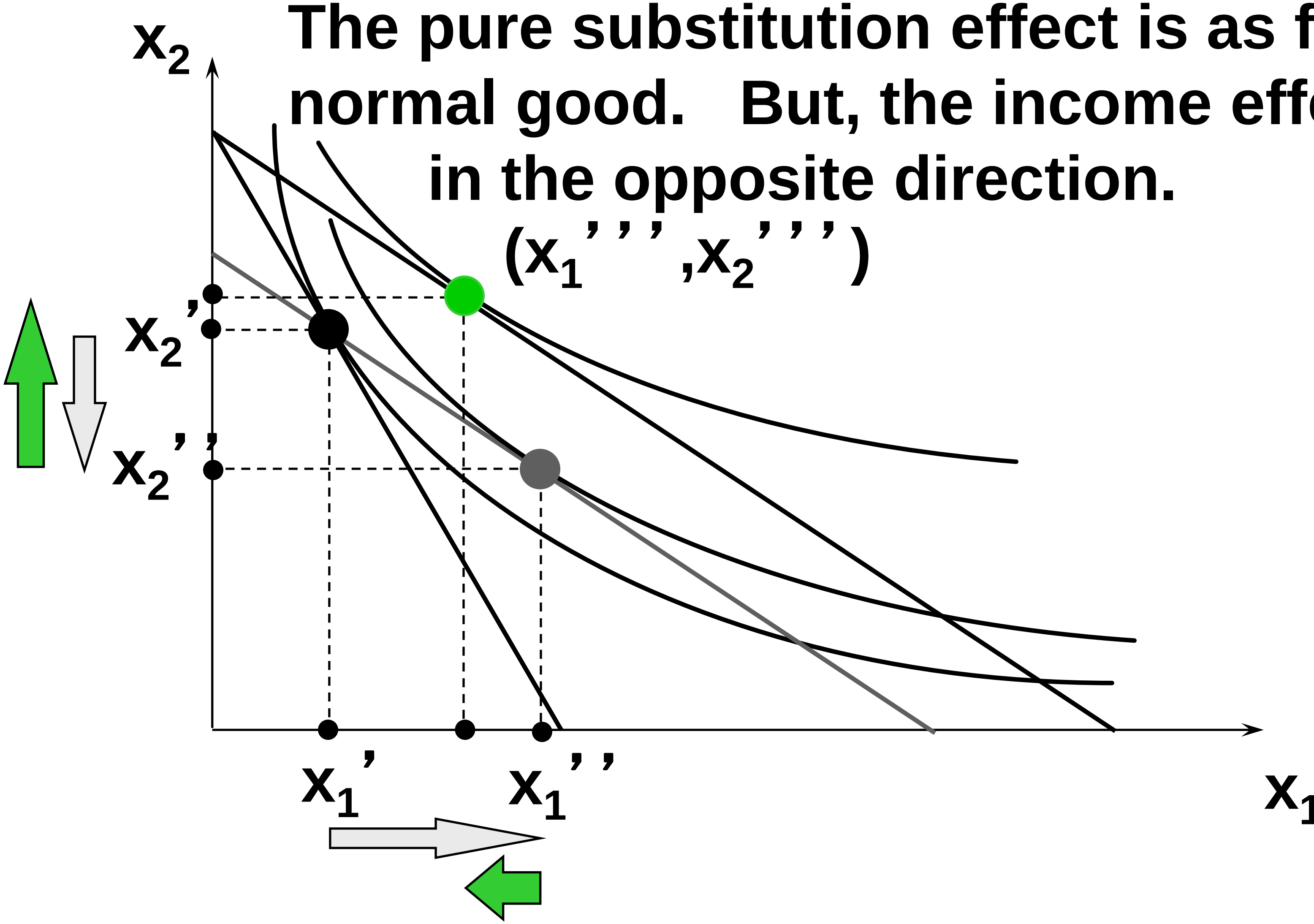


Slutsky's Effects for Income-Inferior Goods

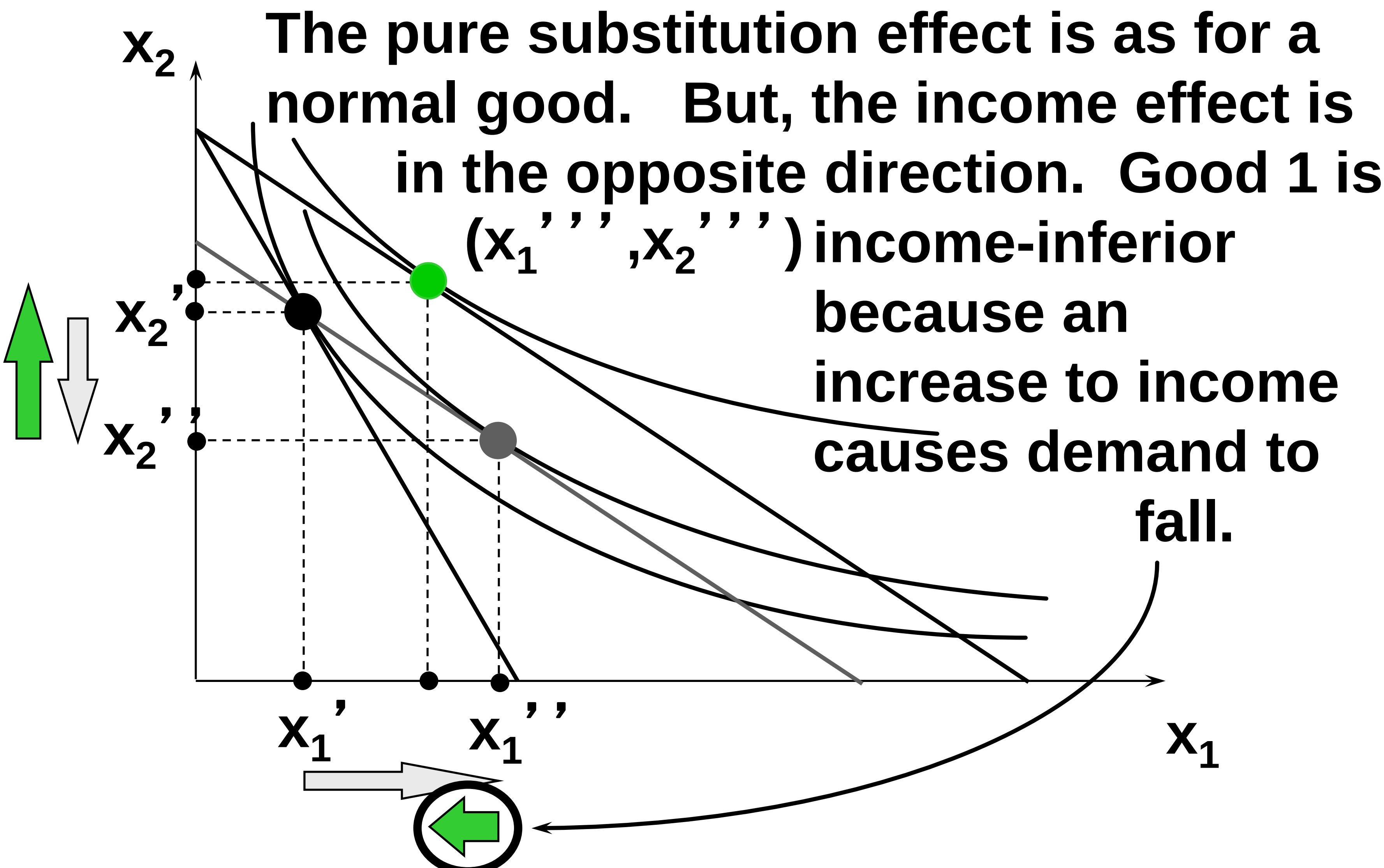


Slutsky's Effects for Income-Inferior Goods

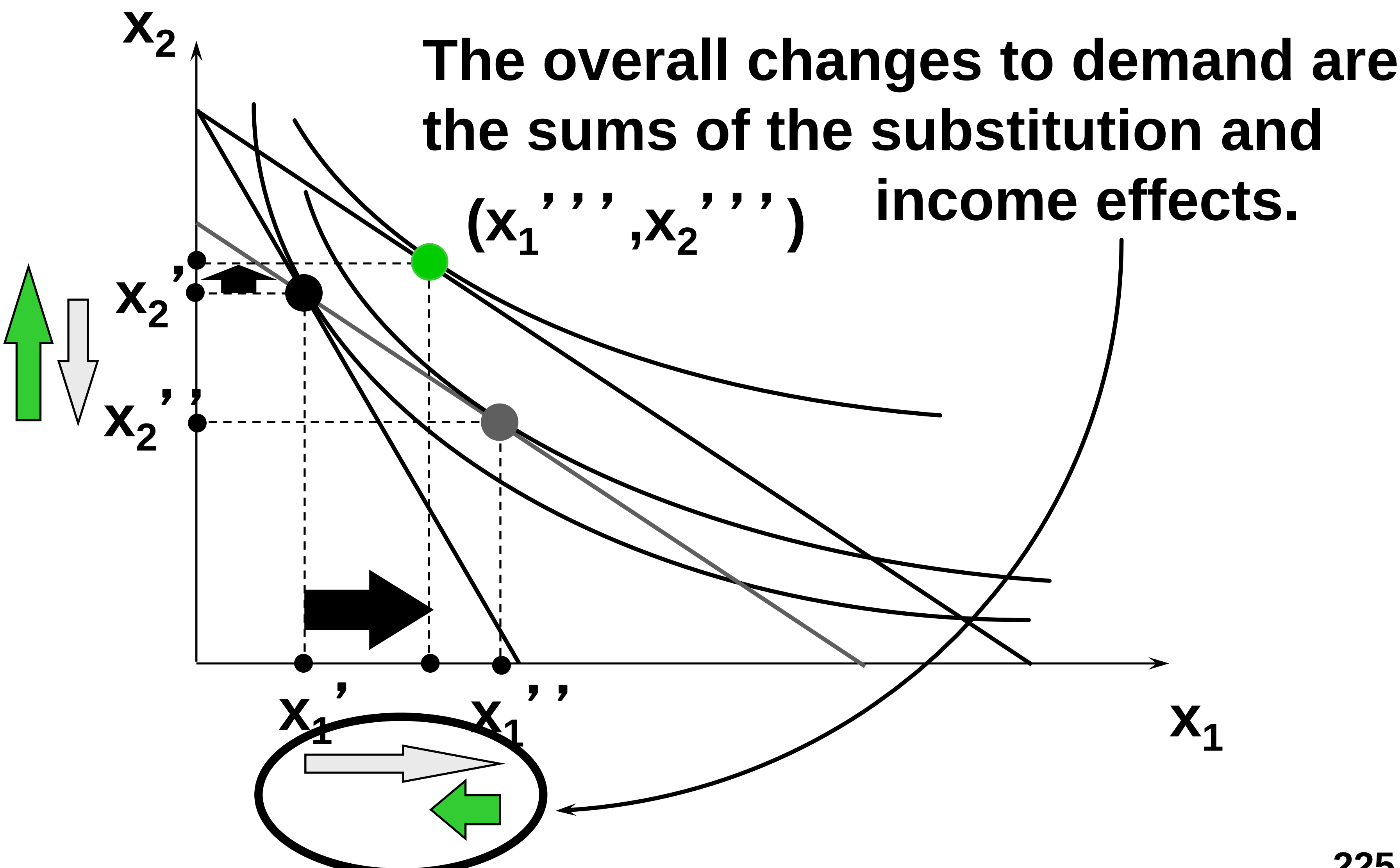
The pure substitution effect is as for a normal good. But, the income effect is in the opposite direction.



Slutsky's Effects for Income-Inferior Goods



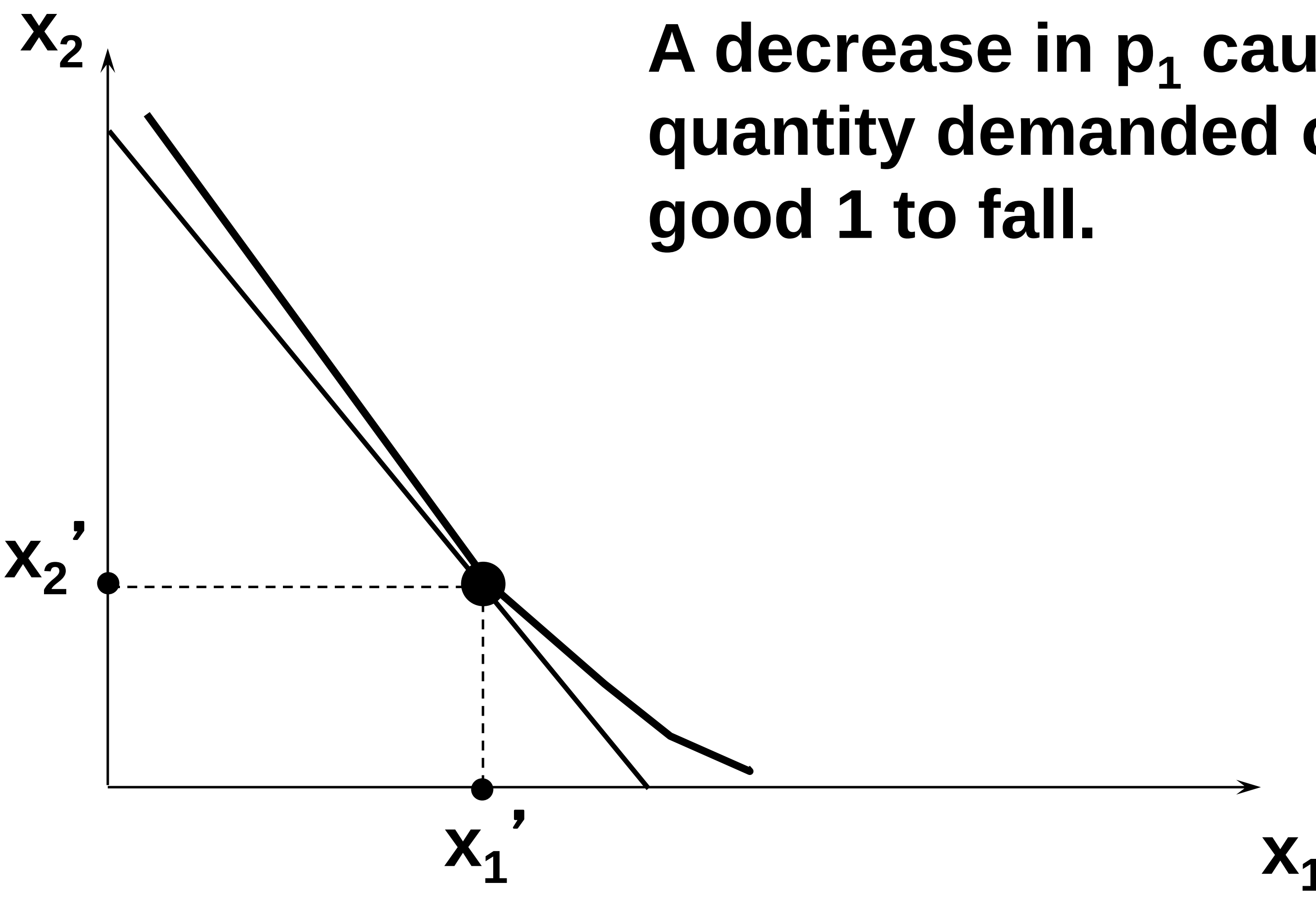
Slutsky's Effects for Income-Inferior Goods



Giffen Goods

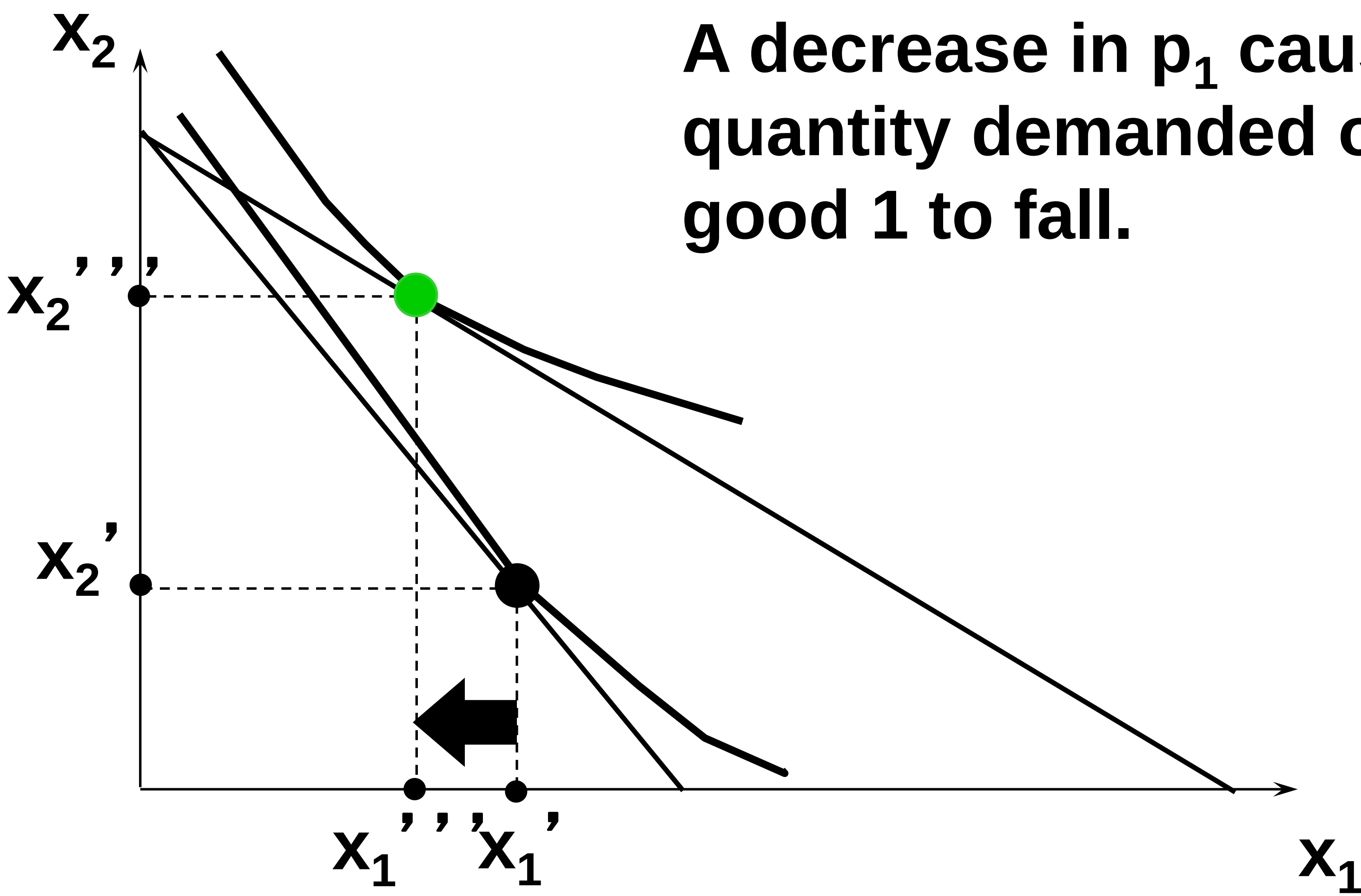
- **In rare cases of extreme income-inferiority, the income effect may be larger in size than the substitution effect, causing quantity demanded to fall as own-price rises.**
- **Such goods are Giffen goods.**

Slutsky's Effects for Giffen Goods



A decrease in p_1 causes quantity demanded of good 1 to fall.

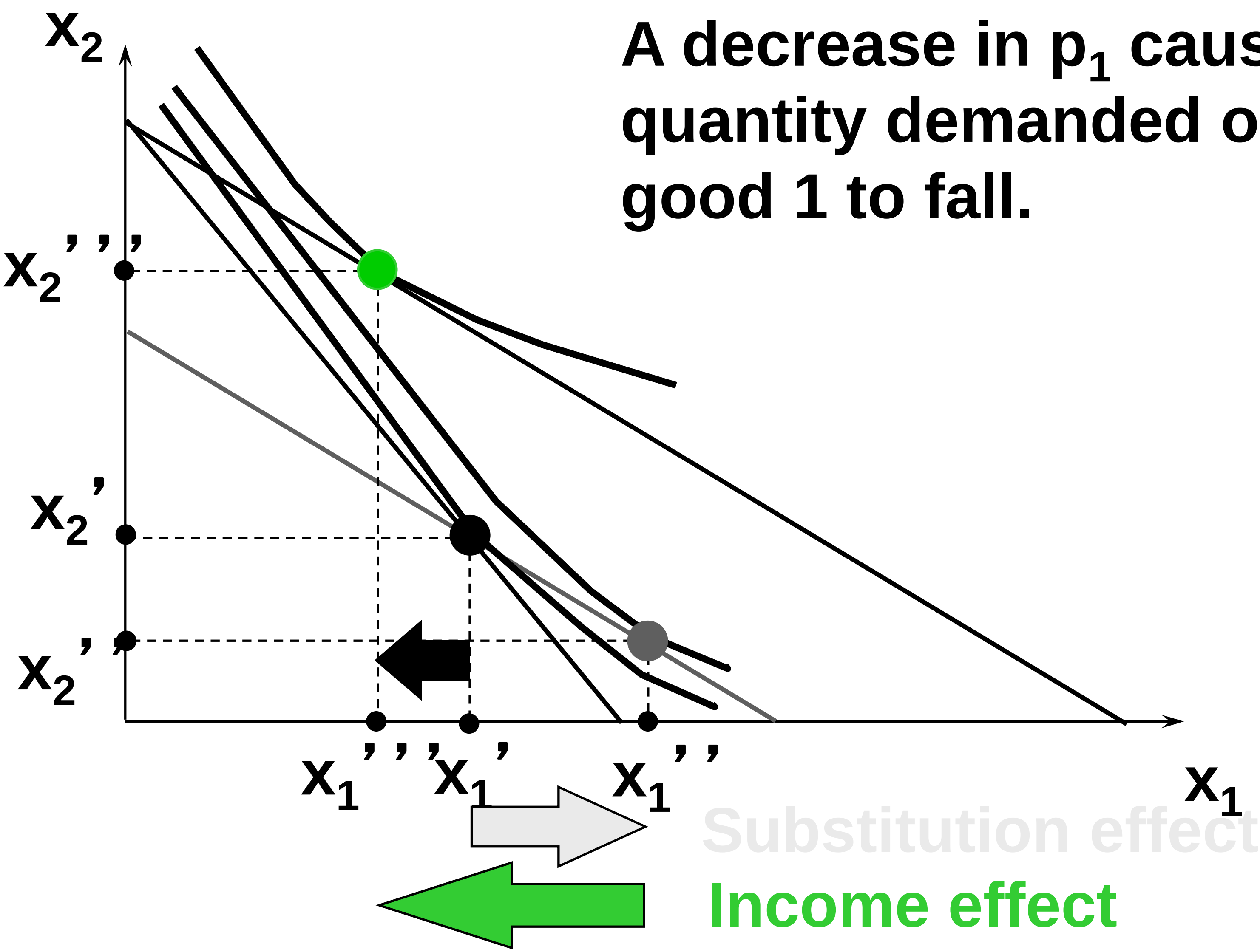
Slutsky's Effects for Giffen Goods



A decrease in p_1 causes quantity demanded of good 1 to fall.

Slutsky’s Effects for Giffen Goods

A decrease in p_1 causes quantity demanded of good 1 to fall.



Slutsky's Effects for Giffen Goods

- **Slutsky's decomposition of the effect of a price change into a pure substitution effect and an income effect thus explains why the Law of Downward-Sloping Demand is violated for extremely income-inferior goods.**