

Problem Set 2

1. A firm has a fixed cost F_0 and marginal costs

$$c = a + bq$$

where q is output.

- (a) If the firm were a price-taker, what is the lowest price at which it would be prepared to produce a positive amount of output? If the competitive price were above this level, find the amount of output q^* that the firm would produce.
- (b) If the firm is actually a monopolist and the inverse demand function is

$$p = A - \frac{1}{2}Bq$$

(where $A > a$ and $B > 0$) find the expression for the firm's marginal revenue in terms of output. Illustrate the optimum in a diagram and show that the firm will produce

$$q^{**} := \frac{A - a}{b + B}$$

What is the price charged p^{**} and the marginal cost c^{**} at this output level? Compare q^{**} and q^* .

- (c) The government decides to regulate the monopoly. The regulator has the power to control the price by setting a ceiling p_{max} . Plot the average and marginal revenue curves that would then face the monopolist. Use these to show:
- If $p_{max} > p^{**}$ the firm's output and price remain unchanged at q^{**} and p^{**}
 - If $p_{max} < c^{**}$ the firm's output will fall below q^{**}
 - Otherwise output will rise above q^{**} .

2. A monopolist has the cost function

$$C(q) = 100 + 6q + \frac{1}{2}[q]^2$$

- (a) If the demand function is given by

$$q = 24 - \frac{1}{4}p$$

calculate the output-price combination which maximises profits

- (b) Assume that it becomes possible to sell in a separate second market with demand determined by

$$q = 84 - \frac{3}{4}p$$

Calculate the prices which will be set in the two markets and the change in total output and profits from case (a).

- (c) Now suppose that the firm still has access to both markets, but is prevented from discriminating between them. What will be the result?

3. Suppose that a firm owns two plants, each producing the same good. Every plant j 's average cost is given by

$$AC_j(q_j) = a + \beta_j q_j \text{ for } q_j \geq 0, \text{ where } j = \{1, 2\}$$

where coefficient β_j may differ from plant to plant, i.e. if $\beta_1 > \beta_2$ plant 2 is more efficient than plant 1 since its average costs increase less rapidly in output. Assume that you are asked to determine the cost-minimizing distribution of aggregate output $q = q_1 + q_2$, among the two plants (i.e., for a given aggregate output q , how much q_1 to produce in plant 1 and how much q_2 to produce in plant 2.) For simplicity, consider that aggregate output q satisfies $q < \frac{a}{\max_j |\beta_j|}$. (You will be using this condition in part b.)

- (a) If $\beta_j > 0$ for every plant j , how should output be located among the two plants?
- (b) If $\beta_j < 0$ for every plant j , how should output be located among the two plants?
- (c) If $\beta_j > 0$ for some plants and $\beta_i < 0$ for others?

4. A firm has a fixed cost of €400 and a total variable costs = $20q + 0.25 q^2$ where q is output.

- (a) If the firm were a price-taker, what is the lowest price at which it would be prepared to produce a positive amount of output? How much output q^* would it produce at this price? What is the perfectly competitive firm's supply curve?
- (b) If the firm is actually a monopolist and the inverse demand function is $p = 170 - q$. What is the price charged p^{**} and the marginal cost c^{**} at this output. Illustrate the monopoly optimum in a diagram.
- (c) The government decides to regulate the monopoly. The government can set a ceiling of p_{max} . In a separate duplicate graph of b plot the average and marginal revenue curves that would face the monopolist, explaining how output will react to different price ceilings relative to c^{**} and p^{**} .
- (d) Linking to diagram in (b) provide a diagrammatic exposition of monopolistic competition and explain.