Advanced Microeconomic Theory

Chapter 6: Partial and General Equilibrium

Outline

- Partial Equilibrium Analysis
- General Equilibrium Analysis
- Comparative Statics
- Welfare Analysis

Overview...

Competitive equilibrium in a partial equilibrium setting

Competitive equilibrium

- Introduction
- Pareto Optimality & Competitive equilibrium
- Partial equilibrium competitive analysis
- Fundamental theorems of welfare economics
- Welfare analysis in a partial equilibrium model

Pareto Optimality and Competitive Equilibrium

- Consider entire economy in which consumers and firms interact through markets
- Two goals: (1) Introduce *Pareto optimal allocation* and a *competitive (Walrasian) equilibrium,* (2) develop *partial equilibrium model*

Notation

- **Economy**

  I consumers
  J firms
  L goods

$x_i = (x_{1i}, \dots x_{Li})$ •Consumption bundle of L goods consumed by i

$u_i(\cdot)$ •Preferences represented by utility function

$\omega_l \geq 0 \ \forall l = 1, \dots, L$ •Total endowment of good l

Notation

$Y_j \subset \mathbb{R}^L$

- Production set available to firm j

$$y_j = (y_{1j}, \dots, y_{Lj}) \in \mathbb{R}^L$$

- Production netput vector of firm j

$$(y_1, \dots, y_J) \in \mathbb{R}^{LJ}$$

- Production vectors of J firms

$$\omega_l + \sum_j y_{lj}$$

- Total (net) amount of good *l* available

Definition 10.B.1

- An *economic allocation* $(x_1, \ldots, x_I, y_1, \ldots, y_J)$ is a specification of a consumption and production vector for each consumer and producer and is feasible if

Feasible: if total amount of each good consumed does not exceed the total amount available from both initial endowment and production

$$\sum_{i=1}^{I} x_{li} \leq \omega_l + \sum_{j} y_{lj} \ for\ l = 1, \ldots, L.$$

Overview…

Competitive equilibrium in a partial equilibrium setting

Competitive equilibrium

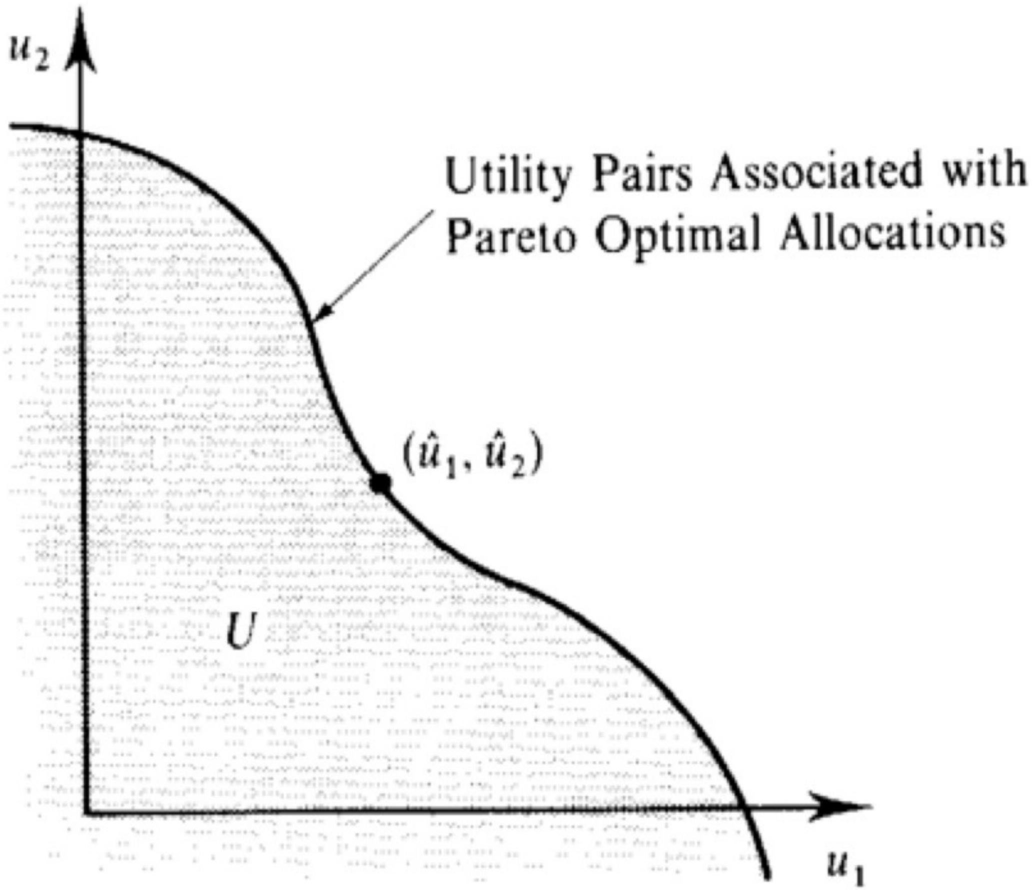
- Introduction
- Pareto Optimality & Competitive equilibrium
- Partial equilibrium competitive analysis
- Fundamental theorems of welfare economics
- Welfare analysis in a partial equilibrium model

Pareto optimality

- Definition 10.B.2: a feasible allocation is $(x_1, \dots, x_I, y_1, \dots, y_J)$ is *Pareto Optimal* if there is no other feasible allocation $(x'_1, \dots, x'_I, y'_1, \dots, y'_J) \forall i$ such that $\mathrm{u_i}(x'_i) \geq u_i(x_i)$ for all i, and $u_i(x'_i) > u_i(x_i)$ for some i.

$$U = \{(u_1, u_2) \in \mathbb{R}^2 \text{ there exists a f.a. } (x_1, \dots, x_I, y_1, \dots, y_J) \text{ s.t. } u_i \leq u_i(x_i) \; for\; i = 1,2\}.$$

Pareto optimal allocations



PO does not ensure equity but only that there is no waste

Overview…

Competitive equilibrium in a partial equilibrium setting

Competitive equilibrium

- Introduction
- Pareto Optimality & Competitive equilibrium
- Partial equilibrium competitive analysis
- Fundamental theorems of welfare economics
- Welfare analysis in a partial equilibrium model

Partial Equilibrium Competitive Analysis

- Marshallian p.e. analysis envisions market for one good: small part of overall economy
- Small size of market facilitates:
  - Wealth effects negligible
  - Prices of other goods approximately unaffected by changes in market
  - Fixity of prices means we can treat expenditure on other goods as single composite commodity (*numeraire*)

Expenditure on good under study reflects small portion of consumer's total expenditure, only a small fraction of additional wealth will be spent on good

Dispersed substitution effects

Partial Equilibrium Analysis

- In a competitive equilibrium (CE), all agents must select an optimal allocation given their resources:
  - Firms choose profit-maximizing production plans given their technology;
  - Consumers choose utility-maximizing bundles given their budget constraint.
- A competitive equilibrium allocation will emerge at a price that makes consumers' purchasing plans to coincide with the firms' production decision (market clearing).

Partial Equilibrium Analysis

- ***Firm***:
  - Given the price vector p^*, firm j's equilibrium output level q_j^* must solve

    $$\max_{q_j \geq 0} \quad p^* q_j - c_j(q_j)$$

    which yields the necessary and sufficient condition

    $$p^* \leq c_j'(q_j^*), \text{ with equality if } q_j^* > 0$$
  - That is, every firm j produces until the point in which its marginal cost, $c_j'(q_j^*)$, coincides with the current market price.

Partial Equilibrium Analysis

- ***Consumers***:
  - Consider a quasilinear utility function

    $$u_i(m_i, x_i) = m_i + \phi_i(x_i)$$

    where m_i denotes the numeraire, and $\phi_i'(x_i) > 0$, $\phi_i''(x_i) < 0$ for all $x_i > 0$.
  - Normalizing, $\phi_i(0) = 0$. Recall that with quasilinear utility functions, the wealth effects for all non-numeraire commodities are zero.

Partial Equilibrium Analysis

- Consumer i's UMP is

$$\max_{m_i \in \mathbb{R}_+,\, x_i \in \mathbb{R}_+} \quad m_i + \phi_i(x_i)$$

$$\text{s.t.} \quad \underbrace{m_i + p^* x_i}_{\text{Total expend.}} \;\leq\; \underbrace{w_{m_i} + \textstyle\sum_{j=1}^{J} \theta_{ij} (\underbrace{p^* q_j^* - c_j(q_j^*)}_{\text{Profits}})}_{\text{Total resources (endowment+profits)}}$$

- The budget constraint must hold with equality (by Walras' law). Hence,

$$m_i = -p^* x_i + \left[w_{m_i} + \textstyle\sum_{j=1}^{J} \theta_{ij} \left(p^* q_j^* - c_j(q_j^*) \right) \right]$$

Partial Equilibrium Analysis

- Substituting the budget constraint into the objective function,

$$\max_{x_i \in \mathbb{R}_+} \quad \phi_i(x_i) - p^* x_i + \left[w_{m_i} + \textstyle\sum_{j=1}^{J} \theta_{ij}\left(p^* q_j^* - c_j(q_j^*)\right)\right]$$

- FOCs wrt x_i yields

$$\phi_i'(x_i^*) \leq p^*, \text{ with equality if } x_i^* > 0$$

- That is, consumer increases the amount he buys of good x until the point in which the marginal utility he obtains exactly coincides with the market price he has to pay for it.

Partial Equilibrium Analysis

- Hence, an allocation $(x_1^*, x_2^*, \dots, x_I^*, q_1^*, q_2^*, \dots, q_J^*)$ and a price vector $p^* \in \mathbb{R}^L$ constitute a CE if:

$$p^* \leq c_j'(q_j^*), \text{ with equality if } q_j^* > 0$$

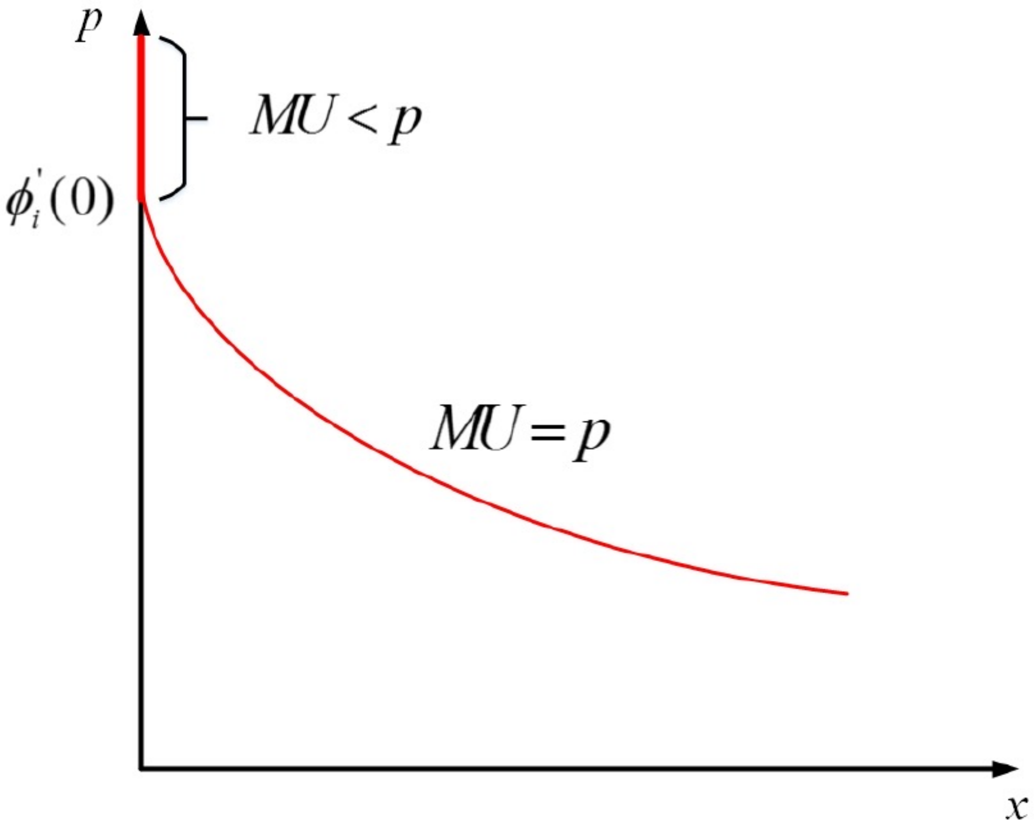
$$\phi_i'(x_i^*) \leq p^*, \text{ with equality if } x_i^* > 0$$

$$\sum_{i=1}^{I} x_i^* = \sum_{j=1}^{J} q_j^*$$

- Note that the these conditions do not depend upon the consumer's initial endowment.

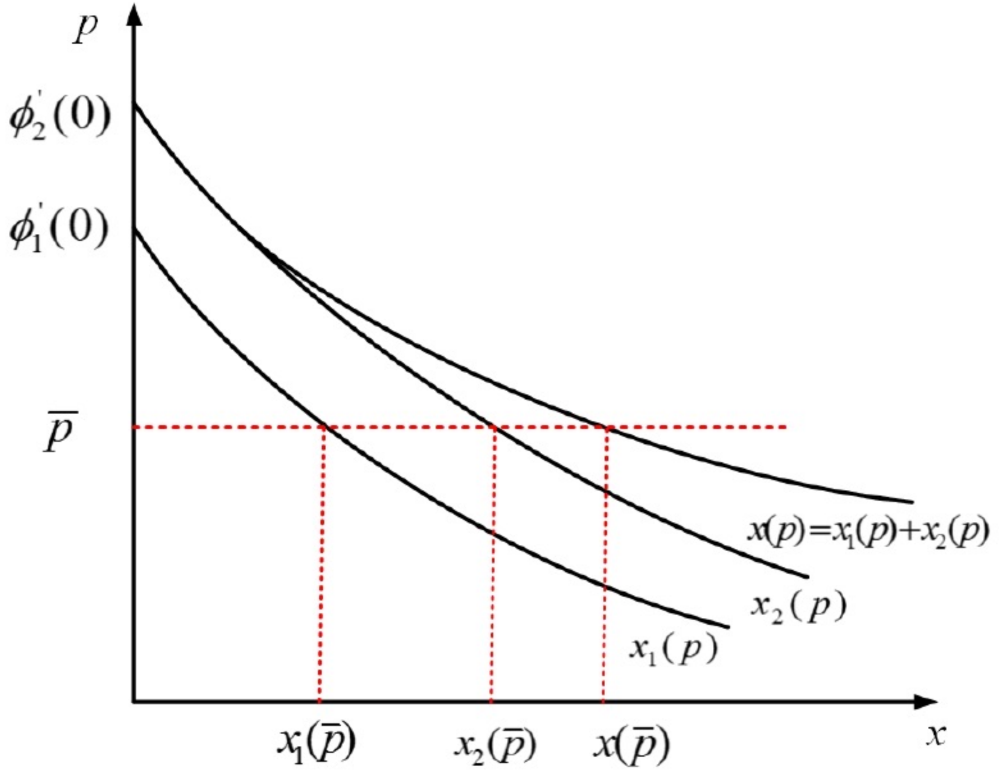
Partial Equilibrium Analysis

- The individual demand curve, where $\phi_i'(x_i^*) \leq p^*$



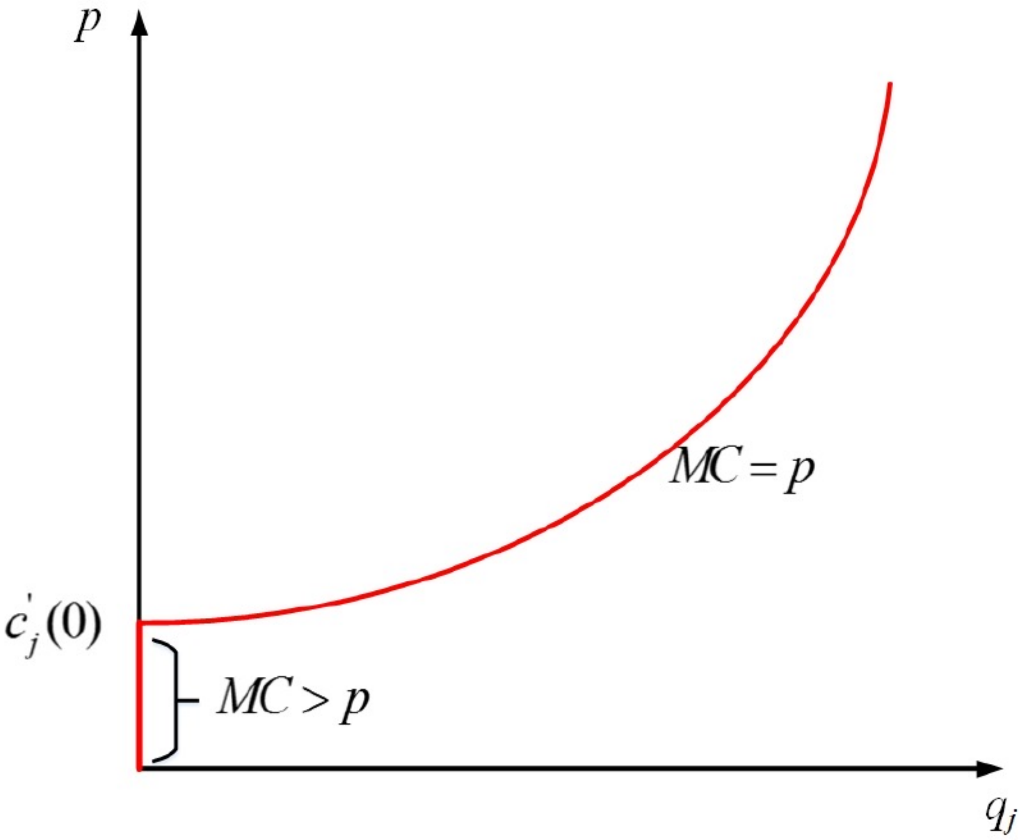
Partial Equilibrium Analysis

- Horizontally summing individual demand curves yields the aggregate demand curve.



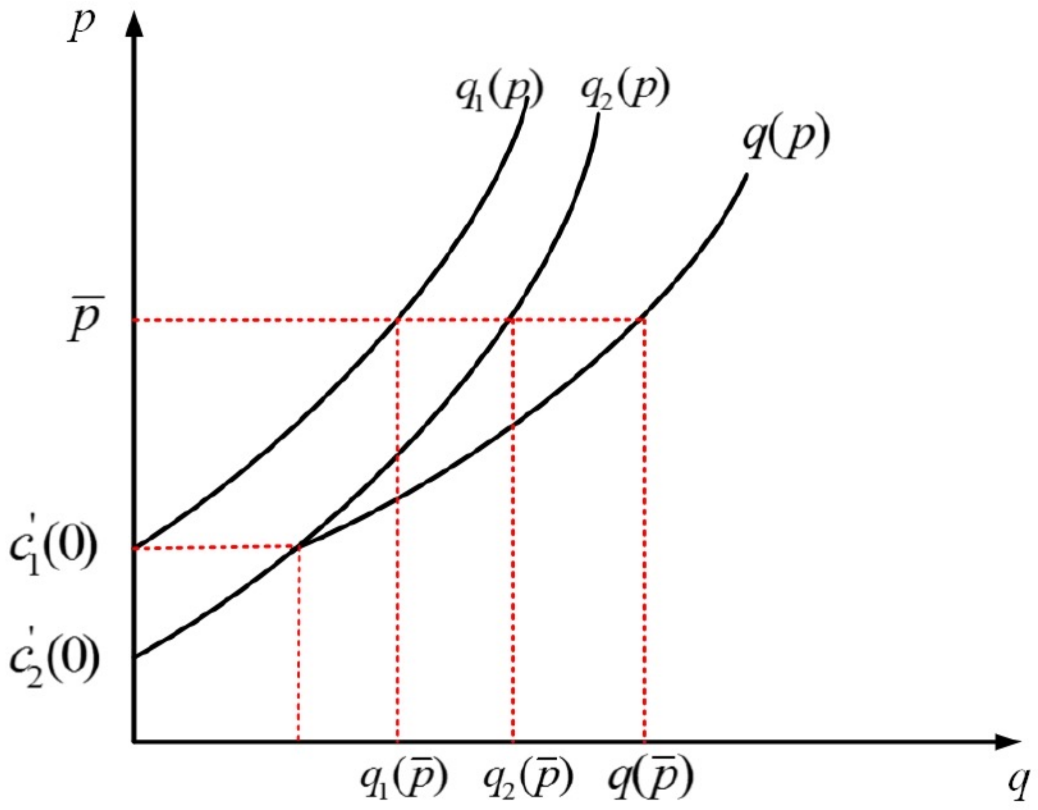
Partial Equilibrium Analysis

- The individual supply curve, where $p^* \leq c_j'(q_j^*)$



Partial Equilibrium Analysis

- Horizontally summing individual supply curves yields the aggregate supply curve.

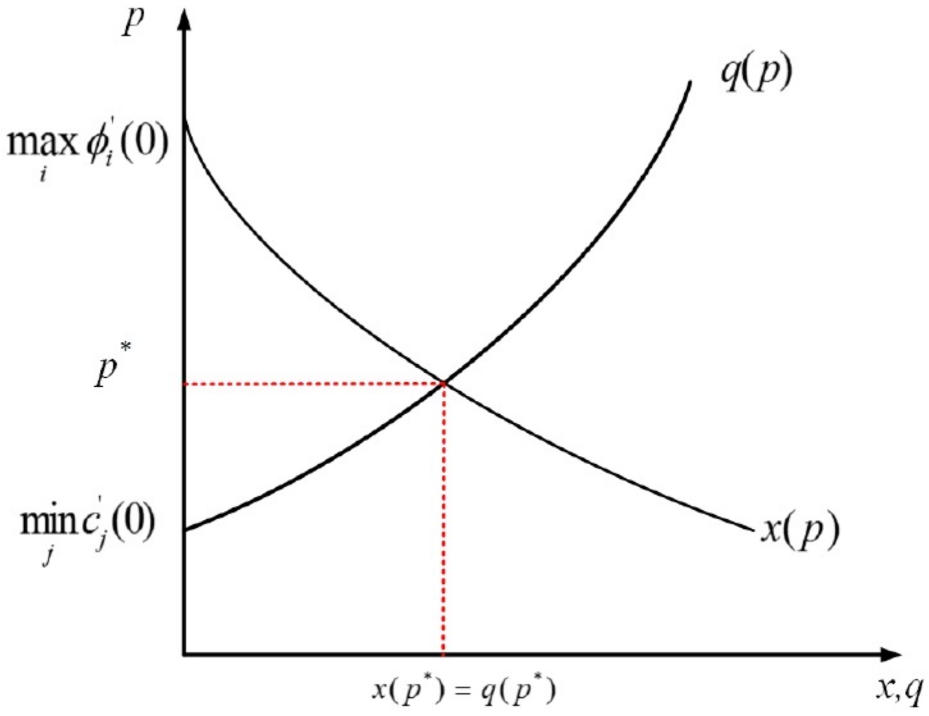


Partial Equilibrium Analysis

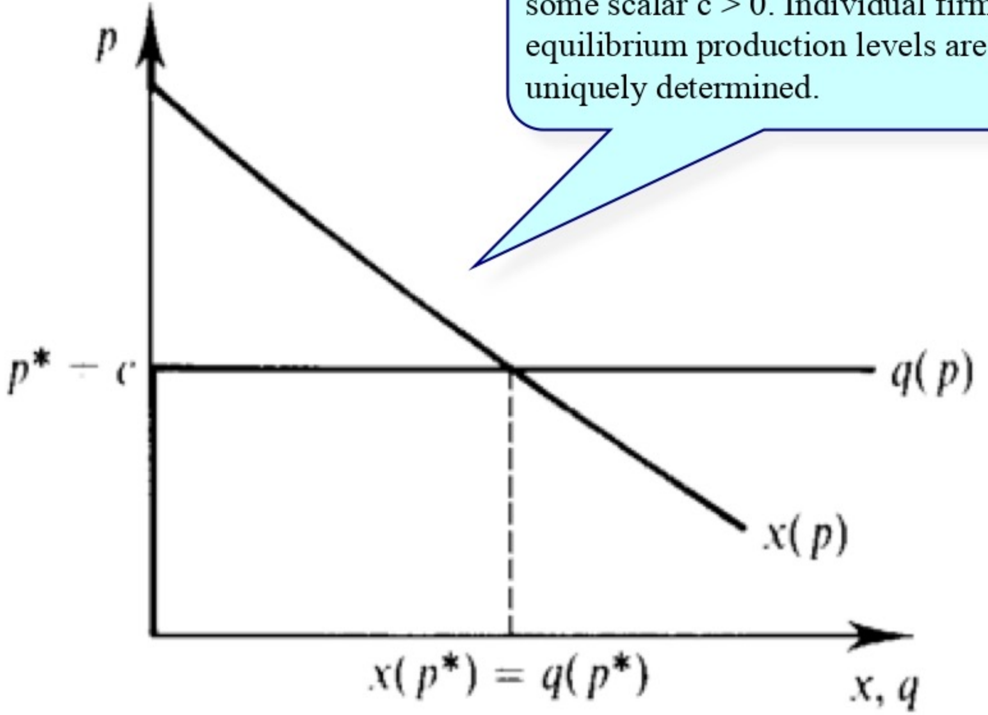
- Superimposing aggregate demand and aggregate supply curves, we obtain the CE allocation of good x.

- To guarantee that a CE exists, the equilibrium price p^* must satisfy

$$\max_i \phi_i'(0) \geq p^* \geq \min_j c_j'(0)$$



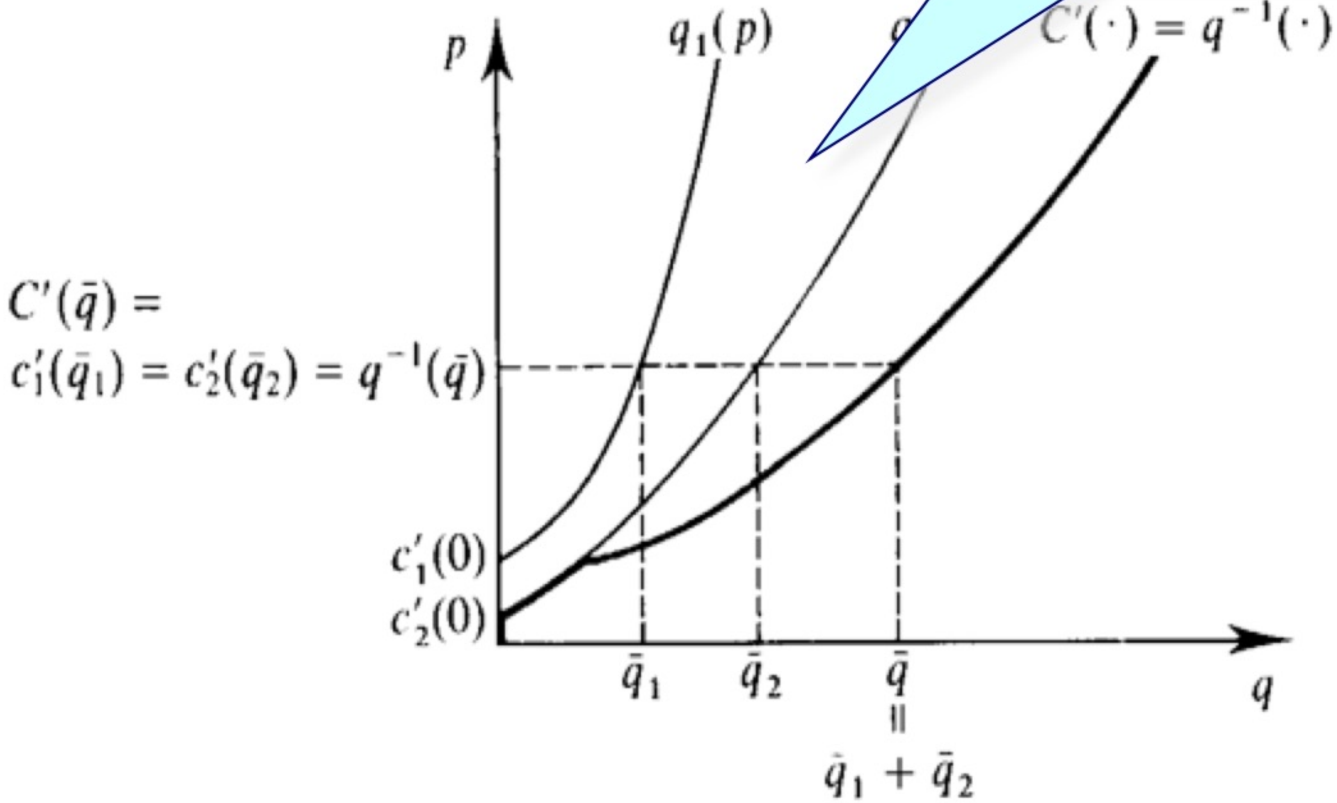
Constant returns case (merely convex)



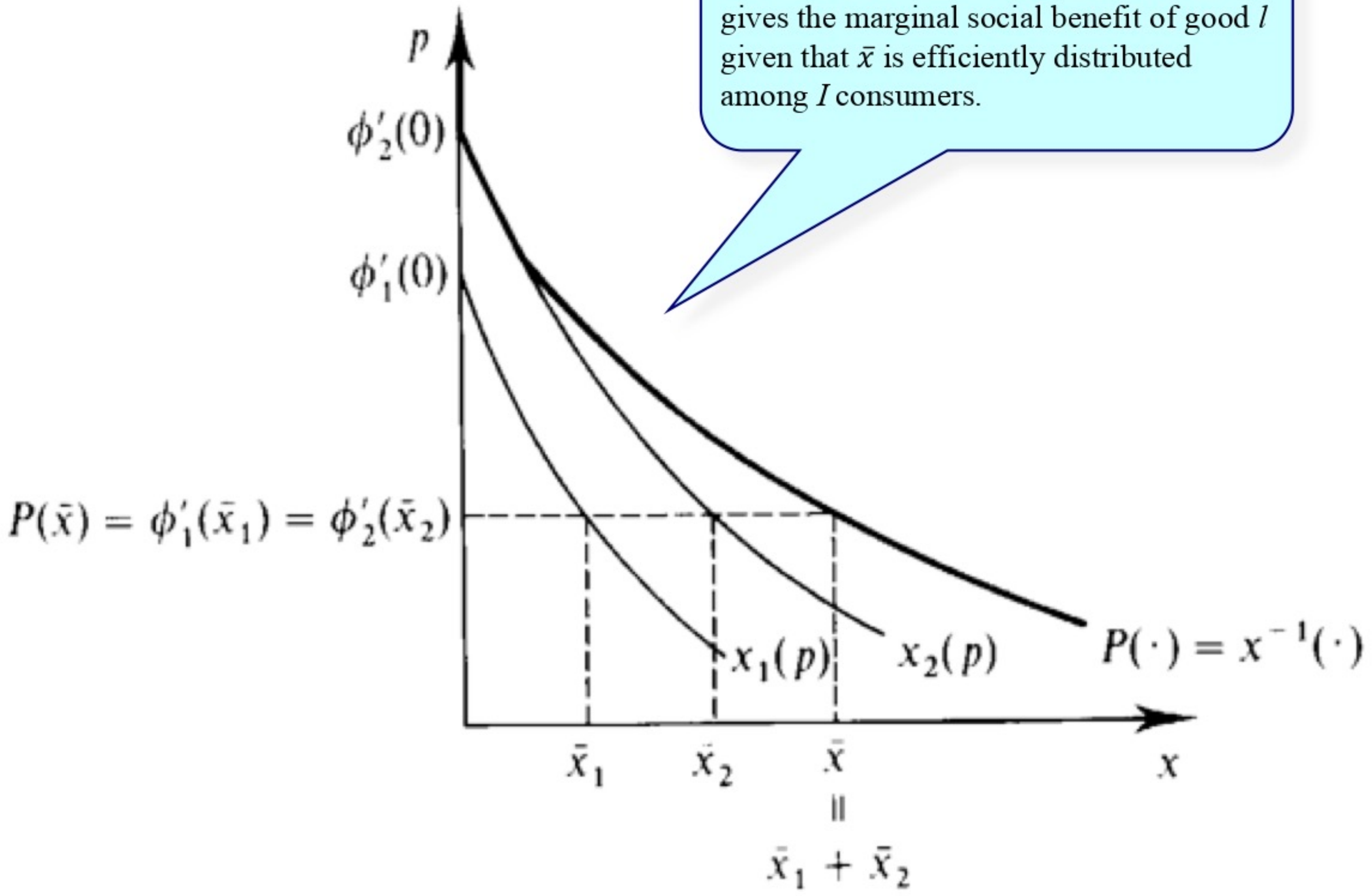
Case where, for all j, $c_j(q_j) = cq_j$ for some scalar $c > 0$. Individual firms' equilibrium production levels are not uniquely determined.

Industry supply as the industry marginal cost function

The inverse of aggregate demand and supply has an interpretation. At any given level of $\bar{q}$, $q^{-1}(\bar{q})$ gives the price that brings forth $\bar{q}$. When each firm faces $q^{-1}(\bar{q}) = p$ aggregate supply is exactly $\bar{q}$ (all firms equate their MC to $q^{-1}(\bar{q})$).



Industry supply as the industry marginal cost function



Inverse demand function $P(\bar{x}) = x^{-1}(\bar{x})$ gives the price that brings forth $\bar{x}$. $P(\bar{x})$ gives the marginal social benefit of good l given that $\bar{x}$ is efficiently distributed among I consumers.

Partial Equilibrium Analysis

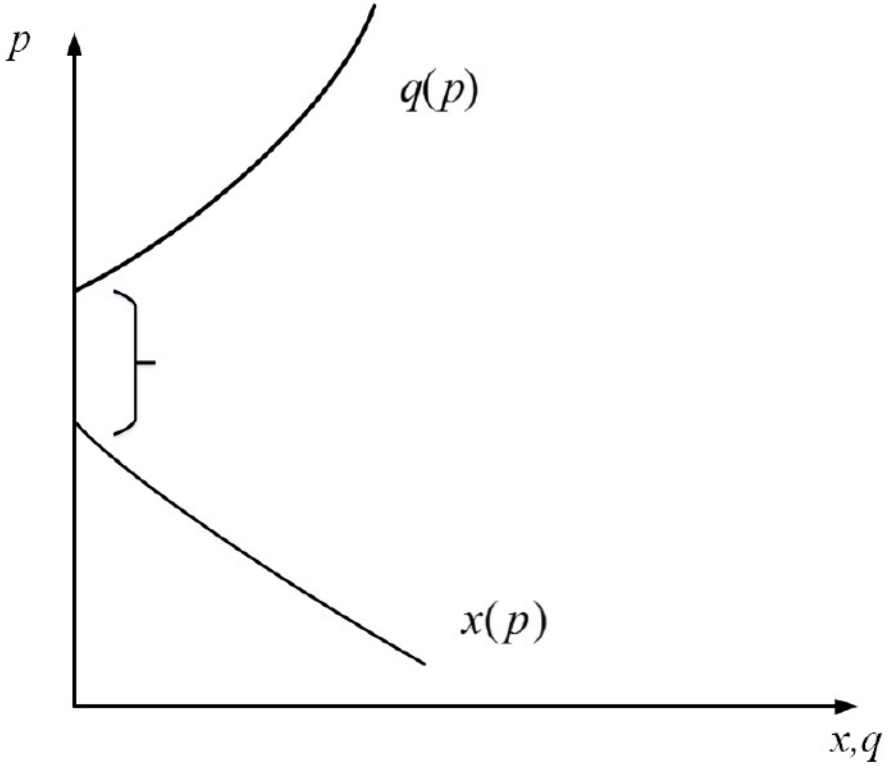
- Also, since $\phi_i'(x_i)$ is downward sloping in x_i, and $c_j'(q_i)$ is upward sloping in q_i, then aggregate demand and supply cross at a unique point.
  - Hence, the CE allocation is unique.

Partial Equilibrium Analysis

- If we have

  $$\max_i \phi_i'(0) < \min_j c_j'(0),$$

  then there is *no* positive production or consumption of good x.



Competitive equilibrium

- Competitive equilibrium involves aggregate output level at which the marginal social benefit of good l is exactly equal to its marginal cost.
- This suggests a social optimality property of the competitive allocation.

Partial Equilibrium Analysis

- ***Example 6.1***:
  - Assume a perfectly competitive industry consisting of two types of firms: 100 firms of type *A* and 30 firms of type *B.*
  - Short-run supply curve of type *A* firm is
$$s_A(p) = 2p$$
  - Short-run supply curve of type *B* firm is
$$s_B(p) = 10p$$
  - The Walrasian market demand curve is
$$x(p) = 5000 - 500p$$

Partial Equilibrium Analysis

- ***Example 6.1*** (continued):
  - Summing the individual supply curves of the 100 type-*A* firms and the 30 type-*B* firms,
$$S(p) = 100 \cdot 2p + 30 \cdot 10p = 500p$$
  - The short-run equilibrium occurs at the price at which quantity demanded equals quantity supplied,
$$5000 - 500p = 500p, \text{ or } p = 5$$
  - Each type-*A* firm supplies: $s_A(p) = 2 \cdot 5 = 10$
  - Each type-*B* firm supplies: $s_B(p) = 10 \cdot 5 = 50$

Comparative Statics

Comparative Statics

- Let us assume that the consumer's preferences are affected by a vector of parameters $\alpha \in \mathbb{R}^M$, where $M \leq L$.
  - Then, consumer i's utility from good x is $\phi_i(x_i, \alpha)$.
- Similarly, firms' technology is affected by a vector of parameters $\beta \in \mathbb{R}^S$, where $S \leq L$.
  - Then, firm j's cost function is $c_j(q_j, \beta)$.

- Notation:
  - $\hat{p}_i(p,t)$ is the effective price paid by the consumer
  - $\hat{p}_j(p,t)$ is the effective price received by the firm
  - Per unit tax: $\hat{p}_i(p,t) = p + t$.
    - Example: $t = \$2$, regardless of the price p
  - Ad valorem tax (sales tax): $\hat{p}_i(p,t) = p + pt = p(1+t)$
    - Example: $t = 0.1$ (10%).

Comparative Statics

- If consumption and production are strictly positive in the CE, then

$$\phi_i'(x_i^*, \alpha) = \hat{p}_i(p^*, t) \text{ for every consumer } i$$

$$c_j'(q_j^*, \beta) = \hat{p}_j(p^*, t) \text{ for every firm } j$$

$$\textstyle\sum_{i=1}^{I} x_i^* = \sum_{j=1}^{J} q_j^*$$

- Then we have $I + J + 1$ equations, which depend on parameter values α, β and t.

- In order to understand how x_i^* or q_j^* depends on parameters α and β, we can use the ***Implicit Function Theorem***.
  - The above functions have to be differentiable.

Comparative Statics

- ***Sales tax*** (Example 6.2):
  - The expression of the aggregate demand is now $x(p+t)$, because the effective price that the consumer pays is actually $p+t$.
  - In equilibrium, the market price after imposing the tax, $p^*(t)$, must hence satisfy

$$x(p^*(t)+t)=q(p^*(t))$$

  - if the sales tax is marginally increased, and functions are differentiable at $p=p^*(t)$,

$$x'(p^*(t)+t)\cdot(p^{*\prime}(t)+1)=q'(p^*(t))\cdot p^{*\prime}(t)$$

Comparative Statics

- Rearranging, we obtain

$$p^{*\prime}(t) \cdot \left[x'(p^*(t)+t) - q'\big(p^*(t)\big)\right]$$
$$= -x'(p^*(t)+t)$$

- Hence,

$$p^{*\prime}(t) = -\frac{x'(p^*(t)+t)}{x'(p^*(t)+t) - q'\big(p^*(t)\big)}$$

- Since $x(p)$ is decreasing in prices, $x'(p^*(t)+t) < 0$, and $q(p)$ is increasing in prices, $q'\big(p^*(t)\big) > 0$,

$$p^{*\prime}(t) = -\frac{x'(p^*(t)+t)}{\underbrace{x'(p^*(t)+t)}_{-} - \underbrace{q'\big(p^*(t)\big)}_{+}} = -\frac{-}{-} = -$$

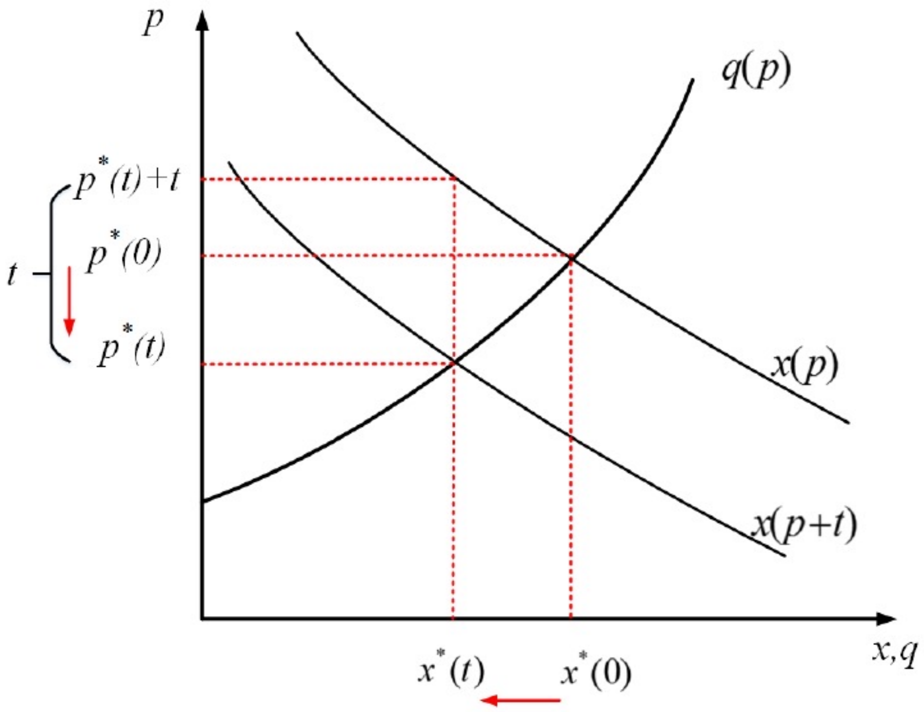
Comparative Statics

- Hence, $p^{*\prime}(t) < 0$.
- Moreover, $p^{*\prime}(t) \in (-1,0]$.
- Therefore, $p^{*}(t)$ decreases in t.
  - That is, the price received by producers falls in the tax, but less than proportionally.
- Additionally, since $p^{*}(t) + t$ is the price paid by consumers, then $p^{*\prime}(t) + 1$ is the marginal increase in the price paid by consumers when the tax marginally increases.
  - Since $p^{*\prime}(t) \overset{<}{\cancel{\geq}} 1$, then $p^{*\prime}(t) + 1 \geq 0$, and consumers' cost of the product also raises less than proportionally.

$p^{*\prime}(t) \geq -1$

Comparative Statics

- *No tax:*
  - CE occurs at $p^*(0)$ and $x^*(0)$
- *Tax:*
  - x^* decreases from $x^*(0)$ to $x^*(t)$
  - Consumers now pay $p^*(t) + t$
  - Producers now receive $p^*(t)$ for the $x^*(t)$ units they sell.



Comparative Statics

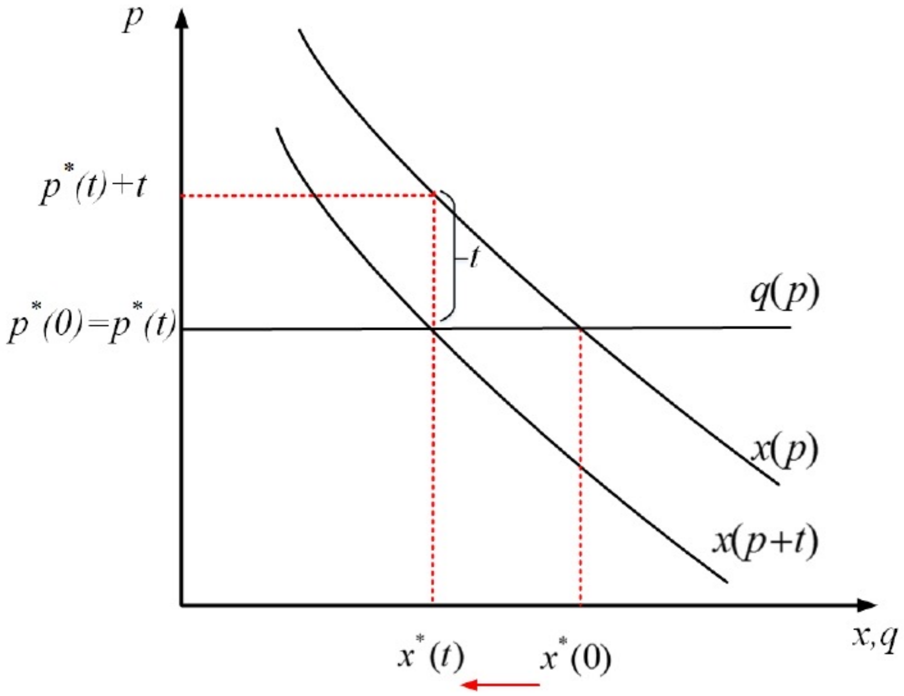
- ***Sales Tax*** (Extreme Cases):
  - *a) The supply is very responsive to price changes, i.e., $q'\left(p^*(t)\right)$ is large.*

$$p^{*\prime}(t) = -\frac{x'(p^*(t)+t)}{x'(p^*(t)+t) - q'\left(p^*(t)\right)} \to 0$$

  - – Therefore, $p^{*\prime}(t) \to 0$, and the price received by producers does not fall.
  - – However, consumers still have to pay $p^*(t) + t$.
    - ▪ A marginal increase in taxes therefore provides an increase in the consumer's price of
$$p^{*\prime}(t) + 1 = 0 + 1 = 1$$
    - ▪ The tax is solely borne by consumers.

Comparative Statics

- A very elastic supply curve
  - The price received by producers almost does not fall.
  - But, the price paid by consumers increases by exactly the amount of the tax.



Comparative Statics

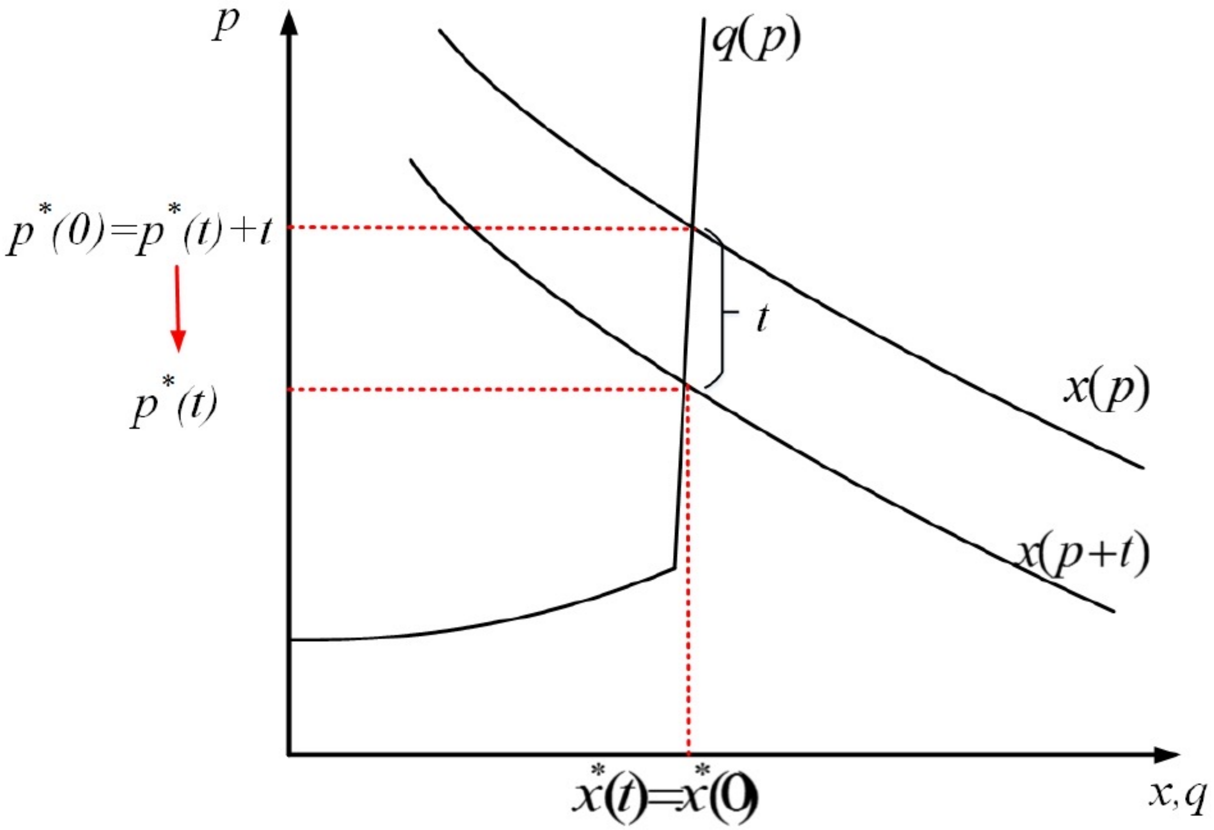
b) *The supply is not responsive to price changes, i.e., $q'(p^*(t))$ is close to zero.*

$$p^{*\prime}(t) = -\frac{x'(p^*(t)+t)}{x'(p^*(t)+t)-q'(p^*(t))} = -1$$

- Therefore, $p^{*\prime}(t) = -1$, and the price received by producers falls by \$1 for every extra dollar in taxes.
  - Producers bear most of the tax burden
- In contrast, consumers pay $p^*(t) + t$
  - A marginal increase in taxes produces an increase in the consumer's price of
    $$p^{*\prime}(t) + 1 = -1 + 1 = 0$$
  - Consumers do not bear tax burden at all.

Comparative Statics

- Inelastic supply curve



Comparative Statics

- ***Example 6.3***:
  - Consider a competitive market in which the government will be imposing an ad valorem tax of t.
  - Aggregate demand curve is $x(p) = Ap^{\varepsilon}$, where $A > 0$ and $\varepsilon < 0$, and aggregate supply curve is $q(p) = ap^{\gamma}$, where $a > 0$ and $\gamma > 0$.
  - Let us evaluate how the equilibrium price is affected by a marginal increase in the tax.

Comparative Statics

- ***Example 6.3*** (continued):
  - The change in the price received by producers at $t = 0$ is

$$p^{*\prime}(0) = -\frac{x'(p^*)}{x'(p^*) - q'(p^*)}$$

$$= -\frac{A\varepsilon p^{*\varepsilon-1}}{A\varepsilon p^{*\varepsilon-1} - a\gamma p^{*\gamma-1}} = -\frac{A\varepsilon p^{*\varepsilon}}{A\varepsilon p^{*\varepsilon} - a\gamma p^{*\gamma}}$$

$$= -\frac{\varepsilon x(p^*)}{\varepsilon x(p^*) - \gamma q(p^*)} = -\frac{\varepsilon}{\varepsilon - \gamma}$$

  - The change in the price paid by consumers at $t = 0$ is

$$p^{*\prime}(0) + 1 = -\frac{\varepsilon}{\varepsilon - \gamma} + 1 = -\frac{\gamma}{\varepsilon - \gamma}$$

Comparative Statics

- ***Example 6.3*** (continued):
  - When $\gamma = 0$ (i.e., supply is perfectly inelastic), the price paid by consumers in unchanged, and the price received by producers decreases be the amount of the tax.
    - That is, producers bear the full effect of the tax.
  - When $\varepsilon = 0$ (i.e., demand is perfectly inelastic), the price received by producers is unchanged and the price paid by consumers increases by the amount of the tax.
    - That is, consumers bear the full burden of the tax.

Comparative Statics

- ***Example 6.3*** (continued):
  - When $\varepsilon \to -\infty$ (i.e., demand is perfectly elastic), the price paid by consumers is unchanged, and the price received by producers decreases by the amount of the tax.
  - When $\gamma \to +\infty$(i.e., supply is perfectly elastic), the price received by producers is unchanged and the price paid by consumers increases by the amount of the tax.

Welfare Analysis

Overview…

Competitive equilibrium in a partial equilibrium setting

- Competitive equilibrium
  - Introduction
  - Pareto Optimality & Competitive equilibrium
  - Partial equilibrium competitive analysis
  - Fundamental theorems of welfare economics
  - Welfare analysis in a partial equilibrium model

The Fundamental Welfare Theorems in a P.E. context

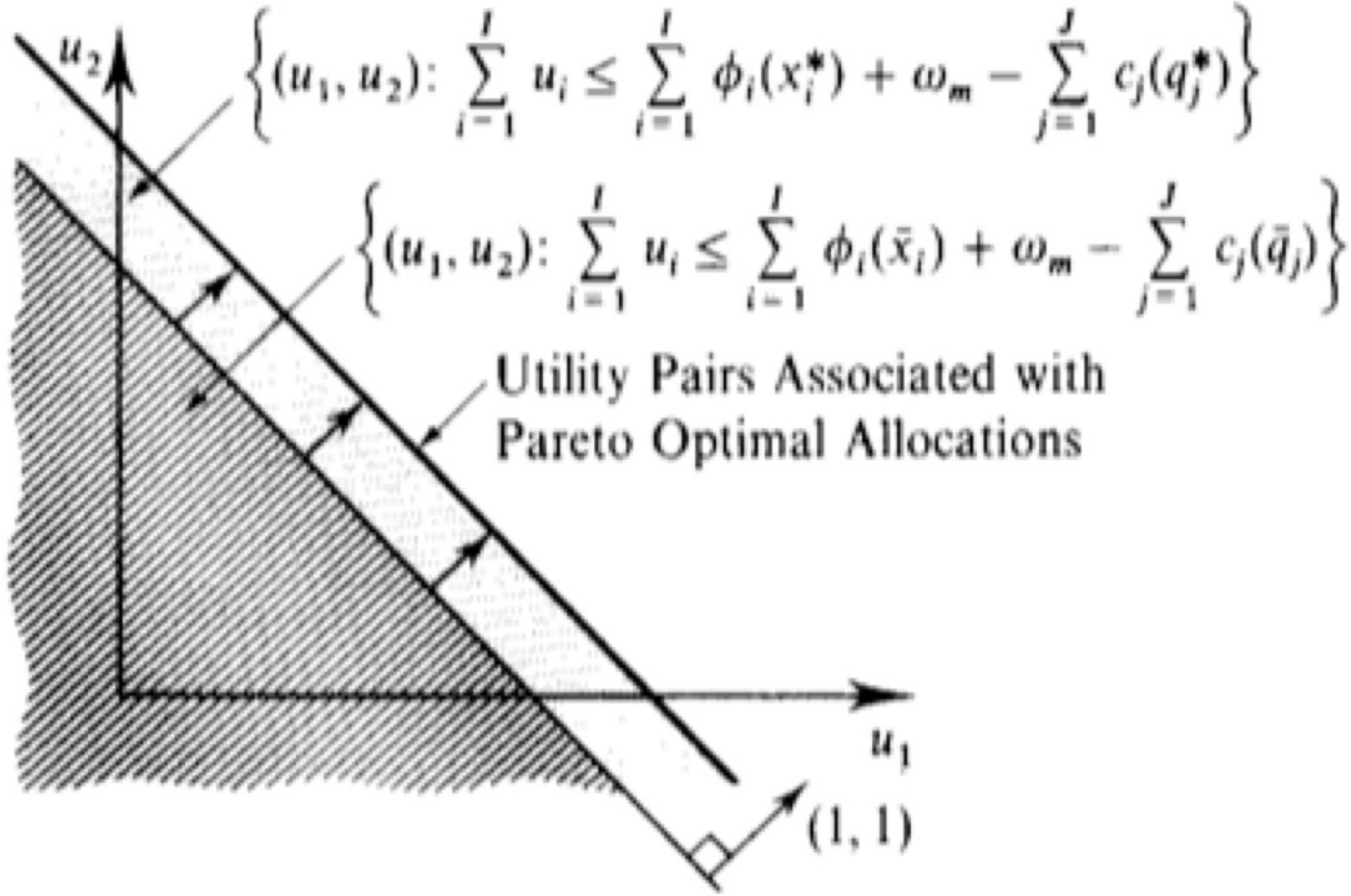
- Study the properties of Pareto optimal allocations in the two-good quasilinear economy
- Establish the link between the set of PO allocations and the set of competitive equilibria
- When consumer preferences are quasilinear, the boundary of the economy’s utility possibility set is linear
- All points on the boudary are associated with consumption allocations that differ only in the distribution of the numeraire among consumers

The Fundamental Welfare Theorems in a P.E. context

- Suppose we fix the consumption and production levels of good *l* at $(\bar{x}_1, \dots, \bar{x}_I, \bar{q}_1, \dots, \bar{q}_J)$
- With these production levels, the total amount of the numeraire available for distribution among consumers is $\omega_m - \sum_j c_j(\bar{q}_j)$.
- Since the quasilinear form allows unlimited utility unit-for-unit transfer of utility across consumers through numeraire transfers the set of attainable utilities is

$$\left\{(u_1, \dots, u_I): \sum_{i=1}^{I} u_i \leq \sum_{i=1}^{I} \phi_i(\bar{x}_i) + \omega_m - \sum_{i=1}^{J} c_j(\bar{q}_j)\right\}$$

Utility possibility set in a quasilinear economy



The Fundamental Welfare Theorems in a P.E. context

- By altering consumption and production levels of good *l* we shift the boundary in a parallel manner.
- Thus ever Pareto Optimal allocation must involve the quantities $(x_1^*, \dots, x_I^*, q_1^*, \dots, q_J^*)$ that extend the boundary as far out as possible
- We call these quantities the optimal consumption and production levels of good *l.*
- As long as these are uniquely determined, Pareto Optimal allocations can differ only in the distribution of the numeraire good.

The Fundamental Welfare Theorems in a P.E. context

- It follows from (10.D.1) that optimal consumption and production levels of good *l* can be obtained from

$$\underset{\substack{(x_1,\ldots,x_I)>0 \\ (q_1,\ldots,q_J)>0}}{Max} \sum_{i=1}^{I} \phi_i(x_i) - \sum_{i=1}^{J} c_j(q_j) + \omega_m \quad (10.D.2)$$

$$s.t. \sum_{i=1}^{I} x_i - \sum_{j=1}^{J} q_j = 0.$$

Aggregate surplus

- The value of the term $\sum_{i=1}^{I} \phi_i(x_i) - \sum_{i=1}^{J} c_j\left(q_j\right)$ is known as the *Marshallian aggregate surplus*
- Essentially the total utility generated from consumption of good *l* less its cost of production
- The optimal consumption and production levels for good *l* maximize this aggregate surplus measure
- Given our convexity assumptions, the FOC of problem (10.D.2) yield necessary and sufficient conditions that characterize the optimal quantities.

Pareto optimality

- If we let μ be the multiplier on the constraints in problem (10.D.2), the *I+J* optimal values $(x_1^*, \dots, x_I^*, q_1^*, \dots, q_J^*)$ and μ satisfy the following *I+J+1* conditions

$$\mu \leq c_j'(q_j^*), with\ equality\ if\ q_j^* > 0\ j = 1, \dots, J. \qquad (10.D.3)$$

$$\phi_i'(x_i^*) \leq \mu, with\ equality\ if\ x_i^* > 0\ j = 1, \dots, J. \qquad (10.D.4)$$

$$\sum_{i=1}^{I} x_i^* = \sum_{j=1}^{J} q_j^* \qquad (10.D.5)$$

First fundamental welfare theorem

- These conditions look familiar. They exactly parallel conditions (10.C.1) to (10.C.3) with μ replacing p^*
- We can immediately infer that any competitive equilibrium outcome is Pareto optimal as we have the same production and consumption levels of Pareto Optimality when $\mu = p^*$.
- This establishes the *first fundamental theorem of welfare economics*

First fundamental welfare theorem

- Proposition 10.D.1. (*The First Fundamental Theorem of Welfare Economics)* If the price p^* and allocation constitute a competitive equilibrium, then this allocation is Pareto optimal.
- Formal expression of Adam Smith's "invisible hand" and the result holds with considerable generality
- Equally important are the conditions under which it fails to holds: markets are complete

Second fundamental welfare theorem

- Proposition 10.D.1. (*The Second Fundamental Theorem of Welfare Economics)* For any Pareto optimal levels of utility $(u_1^*, \ldots, u_I^*)$, there are transfers of the numeraire commodity $(T_1, \ldots, T_I)$ satisfying $\sum_i T_i = 0$, such that competitive equilibrium reached from the endowments $(\omega_{m1} + T_1, \ldots, \omega_{mI} + T_I)$ yields precisely the utilities $(u_1^*, \ldots, u_I^*)$.
- We saw that good *l*'s equilibrium price, it's equilibrium consumption and production levels, and firm's profits are unaffected by changes in consumers' wealth levels.
- A central authority can achieve any PO outcome by endowment transfers and allow 'markets to work"

Price reflects social value

- A critical requirement (in addition to those needed for the 1st welfare theorem) is convexity of preferences and production sets
- Correspondence between p and μ in the equilibrium and PO conditions is noteworthy
- The competitive price is exactly equal to the shadow price on the resource constraint for good l in the PO problem
- We can say that a good's price in c.e. reflects precisely the social value: firms equate their marginal production cost to the marginal social value of its output while consumers their marginal utility (benefit) to price (marginal social cost)

Overview...

Competitive equilibrium

Introduction

Competitive equilibrium in a partial equilibrium setting

Pareto Optimality & Competitive equilibrium

Partial equilibrium competitive analysis

Fundamental theorems of welfare economics

Welfare analysis in a partial equilibrium model

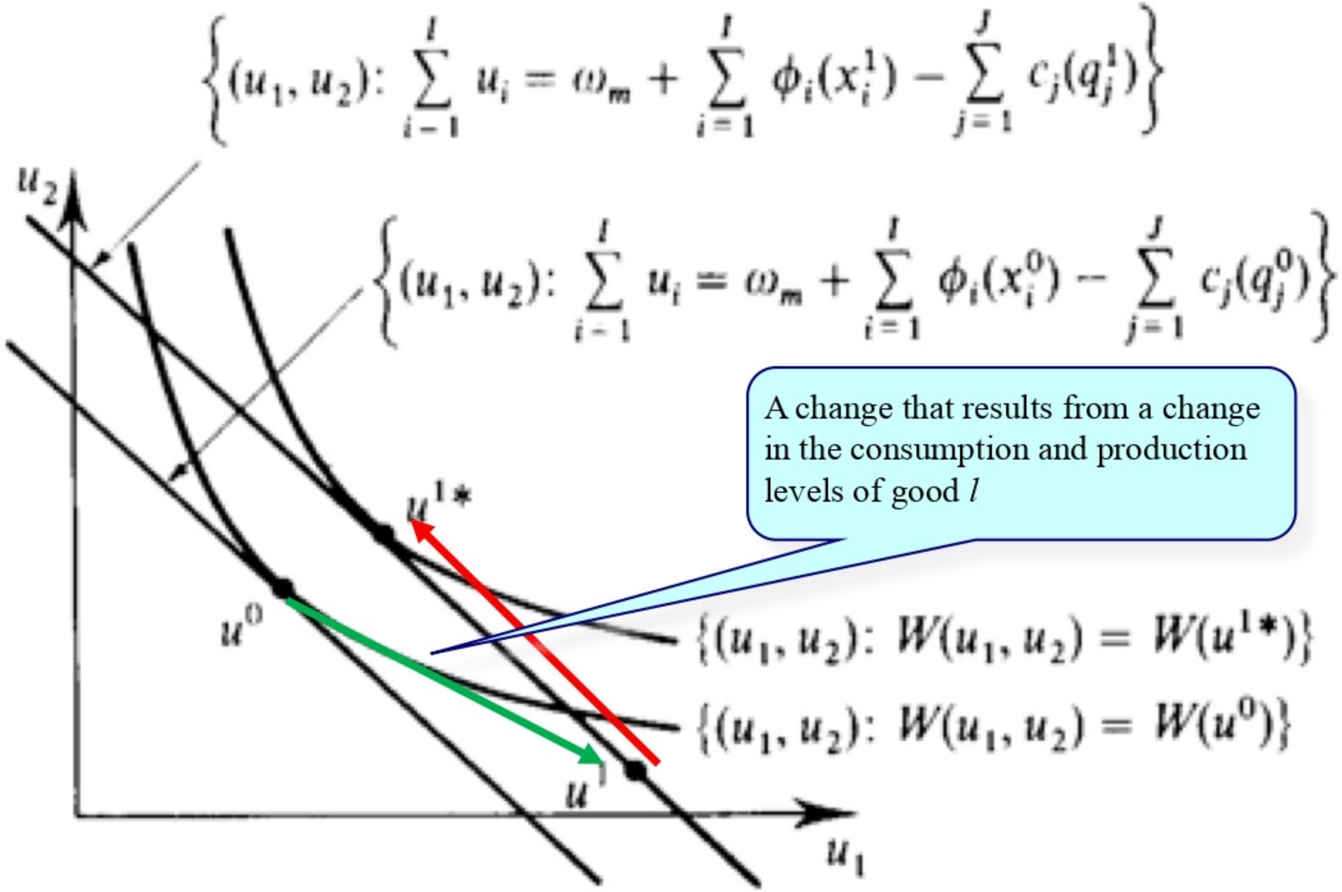
Welfare analysis in a partial equilibrium model

- Of interest to measure changes in levels of social welfare resulting from changes in market conditions (technology, taxes, regulation)
- It is particularly simple to carry out this analysis in the partial equilibrium model: this explains its popularity
- Assume that welfare judgments are embodied in a swf $W(u_1, \dots, u_I)$ assigning social welfare value to every vector $(u_1, \dots, u_I)$

Welfare analysis in a partial equilibrium model

- Assume that some central authority redistributes wealth by means of transfers of the numeraire commodity in order to maximize social welfare
- Critical simplification of quasilinear model is that when central authority redistributes wealth in this mannner: *changes in social welfare can be measured in the Marshallian aggregate surplus for any social welfare function that society may have*

With lump-sum redistribution



A change that results from a change in the consumption and production levels of good *l*

Welfare analysis in a partial equilibrium model

- In short with welfare maximizing lump sum transfers any increase in the Marshallian aggregate surplus results in an increase in welfare
- We can focus on changes in Marshallian aggregate surplus to determine changes in welfare

$$S(x_1, \ldots, x_I, q_1, \ldots, q_j) = \sum_{i=1}^{I} \phi_i(x_i) - \sum_{i=1}^{J} c_j(q_j)$$

Welfare Analysis

- Let us now measure the changes in the aggregate social welfare due to a change in the competitive equilibrium allocation.
- Consider the aggregate surplus

$$S = \sum_{i=1}^{I} \phi_i(x_i) - \sum_{j=1}^{J} c_j(q_j)$$

- Take a differential change in the quantity of good k that individuals consume and that firms produce such that $\sum_{i=1}^{I} dx_i = \sum_{j=1}^{J} dq_j$.
- The change in the aggregate surplus is

$$dS = \sum_{i=1}^{I} \phi_i'(x_i)dx_i - \sum_{j=1}^{J} c_j'(q_j)dq_j$$

Welfare Analysis

- Since
  - $\phi_i'(x_i) = P(x)$ for all consumers; and
    - That is, every individual consumes until MB=p.
  - $c_j'(q_j) = C'(q)$ for all firms
    - That is, every firm's MC coincides with aggregate MC)

then the change in surplus can be rewritten as

$$dS = \sum_{i=1}^{I} P(x)dx_i - \sum_{j=1}^{J} C'(q)dq_j$$

$$= P(x)\sum_{i=1}^{I} dx_i - C'(q)\sum_{j=1}^{J} dq_j$$

Welfare Analysis

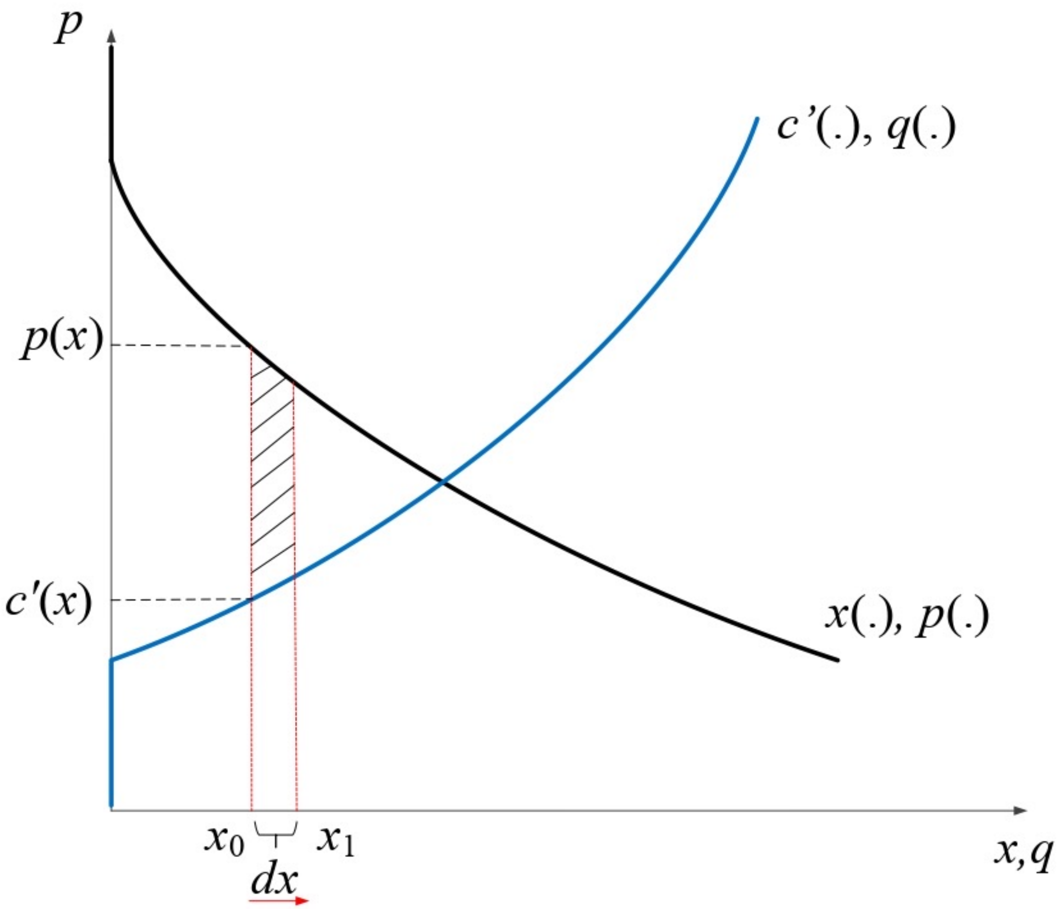
- But since $\sum_{i=1}^{I} dx_i = \sum_{j=1}^{J} dq_j = dx$, and $x = q$ by market feasibility, then

$$dS = [P(x) - C'(q)]dx$$

- *Intuition*:
  - The change in surplus of a marginal increase in consumption (and production) reflects the difference between the consumers' additional utility and firms' additional cost of production.

Welfare Analysis

- Differential change in surplus



Welfare Analysis

- We can also integrate the above expression, eliminating the differentials, in order to obtain the total surplus for an aggregate consumption level of x:

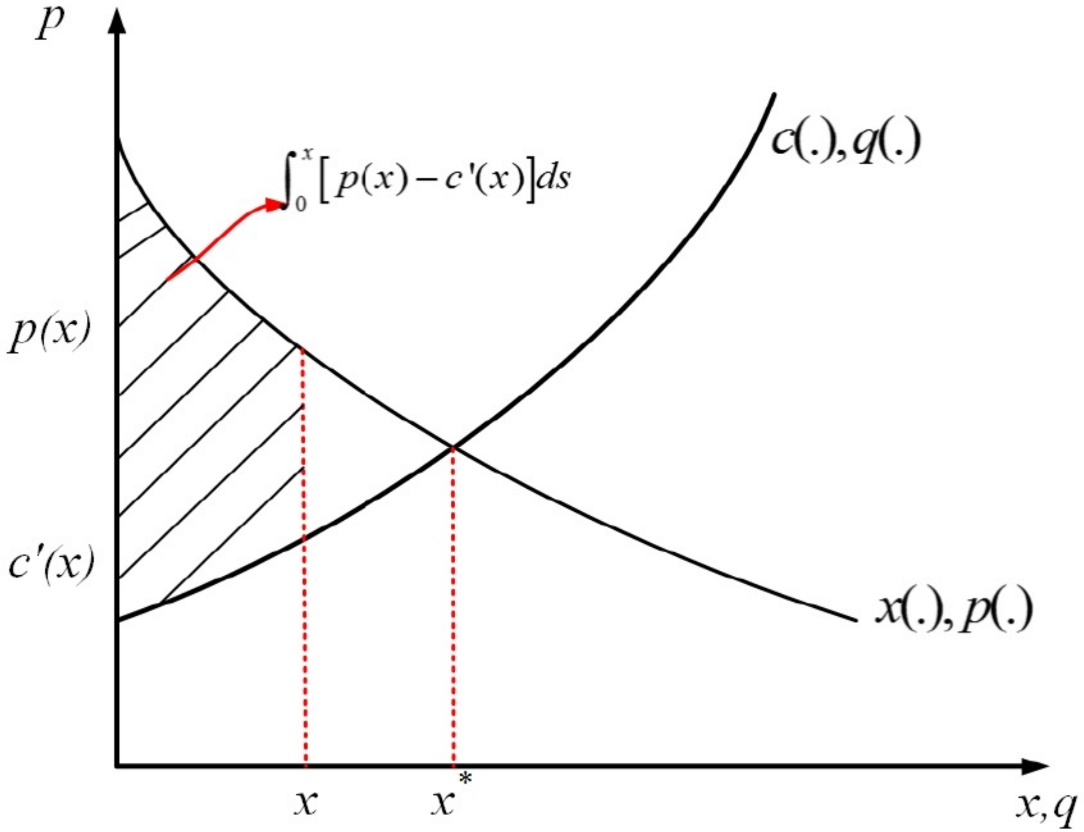
$$S(x) = S_0 + \int_0^x [P(s) - C'(s)]ds$$

where $S_0 = S(0)$ is the constant of integration, and represents the aggregate surplus when aggregate consumption is zero, $x = 0$.

  - $S_0 = 0$ if the intercept of the marginal cost function satisfies $c_j'(0) = 0$ for all J firms.

Welfare Analysis

- Surplus at aggregate consumption x



Welfare Analysis

- For which consumption level is aggregate surplus $S(x)$ maximized?
  - Differentiating $S(x)$ with respect to x,

$$S'(x) = P(x^*) - C'(x^*) \leq 0$$

$$\text{or,} \quad P(x^*) \leq C'(x^*)$$

  - The second order (sufficient) condition is

$$S''(x) = \underbrace{P'(x^*)}_{-} - \underbrace{C''(x^*)}_{+} < 0$$

    - Hence, $S(x^*)$ is concave.
    - Then, when $x^* > 0$, aggregate surplus $S(x)$ is maximized at $P(x^*) = C'(x^*)$.

Welfare Analysis

- Therefore, the CE allocation maximizes aggregate surplus.

- This is the ***First Welfare Theorem***:
  - Every CE is Pareto optimal (PO).