Four principles of Distributive Justice

- "Equals should be treated equally, and unequals unequally, in proportion to relevant similarities and differences" Aristotle, *Nicomachean Ethics*
- In modern rendition…first step toward the formal definition of distributive fairness
- Consider *benevolent dictator* (firm, parent, judge) seeking a *reasoned* compromise of conflicting distributional interests

Four elementary principles

- *Equal treatment of equals* clear-cut principle: if two persons have identical characteristics in all dimensions *relevant* to the allocation problem at hand they should receive the same treatment
- *Unequal treatment of unequals* is a vague principle
- Four elementary ideas at heart of most discussions of distributive justice: *exogenous rights, compensation, reward* and *fitness*

The canonical story

- A flute that must be given to one of four children:
  - 1st child has fewer toys than other three so by *compensation* principles should receive the flute
  - 2nd worked hard to clean it so should receive it as *reward*
  - *3rd* child's father owns the flute so he has the *right* to claim it.
  - 4th child is a flutist so the flute must go to him because all enjoy the music (*fitness* argument)

Compensation and Ex Post Equality

- When differences in individual characteristics deemed relevant to fairness, the two ideas of *compensation* and *reward* come into play
- Certain differences in individual characteristics are involuntary, morally unjustified, and affect the distribution of a *higher-order* characteristic that we deem to equalize
- This justifies *unequal* share of resources in order to *compensate* for the involuntary differences

Compensation

- *Nutritional needs* differ for infants, pregnant women, and adult males => different share of food
- The ill *need* medical care to become *as* healthy…
- The handicapped *need* more resources to enjoy certain “primary” goods
- Economic needs are the central justification of redistributive policies (tax breaks, welfare support, medical aid programs)

Compensation

$$v_i = u_i(y_i)$$

$$u_i(y_i) = u_j(y_j) \Rightarrow y_i > y_j$$

v_i Higher order characteristic enjoyed by i, e.g., satisfaction of nutritional needs

u_i Transforms share into index

y_i Resource, e.g., food

e.g., i pregnant woman, j elderly male, pregnant woman requires more food to receive *equal* nourishment

Reward

- Differences in individual characteristics are morally relevant when they are viewed as voluntary and agents are held *responsible* for them.
- Past sacrifices justify a larger share of resources today (veterans)
- Past wrongdoings a lesser share (no free healthcare for substance abuse, no organ transplant for criminal, countries that polluted bear higher costs)

Reward

- A central question of political philosophy is the fair reward of individual productive contributions
- Lockean argument entitles me the fruit of my own labor => but no precise rule when difficult to separate contributions (externality/ jointness)
  - sharing joint costs or surplus generated by the cooperation

Exogenous rights

- Certain principles guiding the allocation of resources are entirely *exogenous* to the consumption of these resources and to the responsibility of the consumers in their production.
- Flute story: ownership is independent from the consumption of the flute (and the related questions who needs it?, who deserves it?, who will make the best use of it?)

Exogenous Rights

- Equal treatment of equals is archetypal example of an *exogenous right*
  - E.g., "one person, one vote" (doesn't favor any elector, anonymous equal weight)
    - Could argue that some difference should have bearing on weight: *conscientious* versus *whimsical* citizen
    - Medieval religious assemblies gave more weight to senior members, voting rights commonly linked to wealth throughout 19^{th} Century
- Basic rights such as political rights, the freedom of speech and of religion, access to education

Exogenous rights

- Equal exogenous rights correspond to *equality ex ante*, in the sense that we have an equal claim to the resources regardless of the way they affect our welfare and that of others.
- Eg., ability to vote and weight of one's vote, duty to be drafted, access to public beach
- Examples of unequal rights are also numerous and important, e.g., private ownership, status from social standing and seniority, shareholders in a publicly traded firm, creditors in American bankruptcy law are prioritized

Fitness

- Resources must go to whomever makes the best use of them, flutes to the best flutist, the cake to the glutton…
- *Fitness* justifies unequal allocation of the resources independently of needs, merit or rights.
- Fitness can be expressed in two conceptually different ways, *sum-fitness* and *efficiency-fitness*

Fitness: Sum Fitness

- The concept of sum-fitness relies on the notion of utility (measurement of higher order characteristic)
- The central object is the function transforming resources into utility, e.g., health level if resource is medical care, pleasure if resource is food
- Sum-fitness allocates resources so as to maximize total utility
- Sum-fitness is a *fairness* principle

Sum fitness: flute example

$$n \cdot a_i + b$$

a_i objective quality of hearing flute
b is pleasure from playing same for all
n number of children

In this case sum fitness will give the flute to the most talented. Compensation may allow all children to take a turn playing (depends on values of b and a)

Fitness: Efficiency fitness

- The more general concept of efficiency-fitness (or simply efficiency, or Pareto-optimality) is the central normative requirement of collective rationality
- Efficiency fitness typically imposes much looser constraints than sum-fitness on the allocation of resources,
- e.g., compatible with sum-fitness in form of classical utilitarianism, compensation in form of egalitarian collective utility and compromises between these extremes

Lifeboat example

- Allocation of single indivisible good
- Access to a lifeboat when sinking (medical triage, allocation of organs, immigration policies)
- Seats in boat must be rationed:
  - Exogenous rights: draw lots (equality), keep good citizens (ranking)
  - Compensation: let the strong men swim (equalizing chance of survival)
  - Reward /punish the one who causes boat to sink
  - Fitness: Keep woman as they can bear children, or children as they have more years to live

Examples

- Consider some further examples where we assume equal exogenous rights (namely difference in claims is the only reason to give different shares to agents)
- Fitness plays no role as either every agent wants more of the good or every agent wants less of the bad
  - efficiency-fitness automatically satisfied
  - Identify agent's share with welfare => sum-fitness automatically satisfied
- So discussion bears on principles of compensation and reward

Exogenous rights examples

xi is i's claim
t is commodity (good or bad)

$t \neq x_N = \sum_i x_i$

t < x_N deficit
t > x_N excess

e.g., prescription drugs, bankruptcy (xi bankrupt firm's debt to creditor, t liquidation value), good xi is agent I's opp. Cost of joining the venture (standalone salary) t total revenue

Example: Joint Venture Surplus. Tresa pianist and David violinist. Work as full time duo. Before duo Teresa earning

Teresa: $50K/yr
David $100K/yr

Together net revenue $210K/yr

What is the fair split?

Joint Venture: Excess

(1) *Proportional* solution
(2) *Equal surplus*
(3) *Uniform gains*

$$y_i = \frac{x_i}{\sum_N x_j} t \qquad (1)$$

$$y_i = x_i + \frac{1}{n}(t - \sum_N x_j) \qquad (2)$$

$$y_i = \max\{\lambda, x_i\} \text{ where } \lambda \text{ computed}$$

$$\sum_N \max\{\lambda, x_i\} = t \qquad (3)$$

Proportional solution: $y_i = \frac{x_i}{\sum_N x_j} t$

e.g., standalone as a proxy of their respective contributions

Teresa 70K = $y_T = \frac{50}{150} 210$

David 140K $y_D = \frac{100}{150} 210$

Equal surplus:

Eg. Standalone as the "status quo ante". They divide the surplus equally 210-150=60 (30,30)

Teresa 50 +30=80K $y_i = x_i + \frac{1}{n}(t - \sum_N x_j) = 130 = 50 + \frac{1}{2}(210 - 150)$

David 100+30=130K

Uniform Gains:

Egalitarian criterion one step further. Standalone sets floor on an agent's share (no one should be penalized for joining a cooperative venture). Except for this constraint the revenue is split equally.

$$y_i = \max\{\lambda, x_i\} \text{ where } \lambda \text{ computed}$$

$$\sum_N \max\{\lambda, x_i\} = t \quad \max\{105, 50\} + \max\{105, 100\} = 210$$

Teresa 105K
David 105K

Joint Venture: Deficit

(1) *Proportional* solution
(4) Uniform gains
(5) Uniform losses (eq sur)

$$y_i = \frac{x_i}{\sum_N x_j} t \qquad (1)$$

$$y_i = \min\{\lambda, x_i\}$$

$$\sum_N \min\{\lambda, x_i\} = t \qquad (4)$$

$$y_i = \max\{x_i - \mu, 0\}$$

$$\sum_N \max\{x_i - \mu, 0\} = t \qquad (5)$$

Example: Joint Venture Deficit

Proportional solution given by the same formula.

Say revenue 90K.
Teresa 30K
David 60K

$$y_T = \frac{50}{150} * 90$$

$$y_D = \frac{100}{150} * 90$$

$$y_i = \frac{x_i}{\sum_N x_j} t$$

Example: Joint Venture Deficit

Uniform gains (revenue 90K)

min{45, 50}+min{45,100}=90

$$y_i = \min\{\lambda, x_i\}$$

$$\sum_N \min\{\lambda, x_i\} = t \qquad (4)$$

Teresa 45K
David 45K

Equals surplus becomes "uniform losses"

$$y_i = \max\{x_i - \mu, 0\}$$

with 90K deficit is 60K

$$\sum_N \max\{x_i - \mu, 0\} = t \qquad (5)$$

max{50-30,0}+max{100-30,0}=90K

Teresa 20K
David 70K

Uniform Losses with high deficit, e.g., total revenue 40K so deficit 110K

Max{50-60,0}+max{100-60,0}=40K
Teresa 0K
David 40K

$$y_i = \max\{x_i - \mu, 0\}$$

$$\sum_N \max\{x_i - \mu, 0\} = t \qquad (5)$$

Example: Joint Venture Deficit

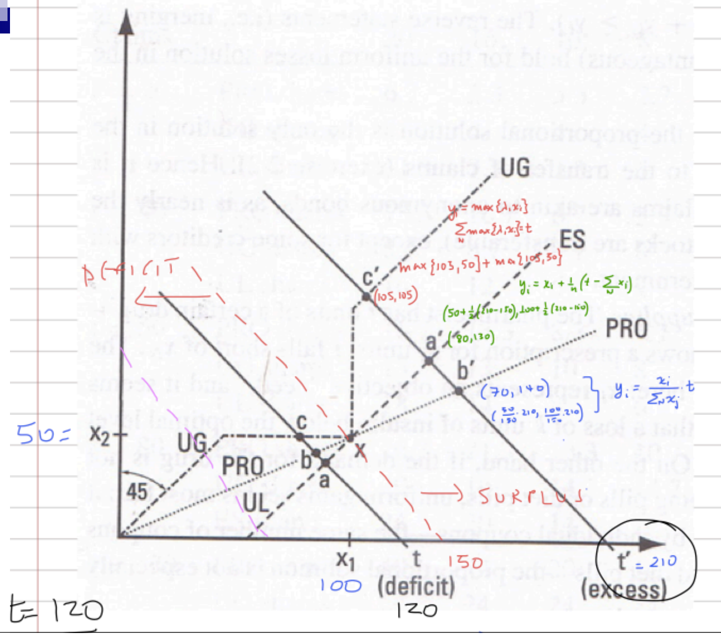
Uniform gains (revenue 120K similar behavior for deficit between 100 and 150)

min{70,50}+min{70,100}=50+70=120

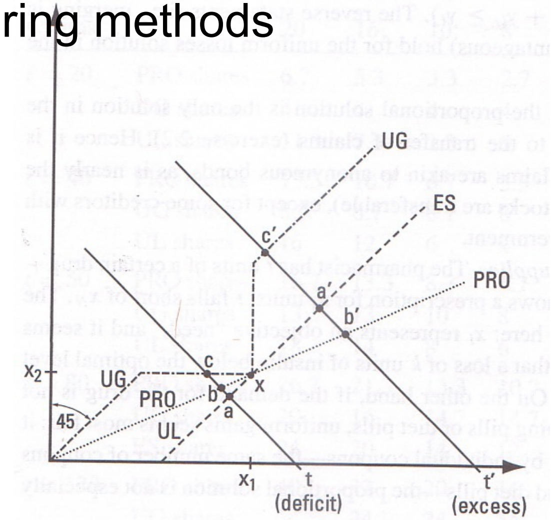
Teresa 50K
David 70K

$$y_i = \min\{\lambda, x_i\}$$

$$\sum_N \min\{\lambda, x_i\} = t \qquad (4)$$



Three basic rationing/surplus-sharing methods



Equal Sacrifice in Taxation

- A Deficit problem (N,t,x) can always be interpreted as a taxation problem where x_i is agent i's taxable income, y_i his income net of tax, t is total after tax income, and x_N-t is the total tax levied
- Property *fair ranking* places some minimal equity constraints on tax shares:

$$x_i \le x_j \Rightarrow y_i \le y_j \quad and \quad x_i - y_i \le x_j - y_j$$

Progressivity and Regressivity

$$prog: \quad x_i \le x_j \Rightarrow \frac{x_i - y_i}{x_i} \le \frac{x_j - y_j}{x_j}$$

$$reg: \quad x_i \le x_j \Rightarrow \frac{x_i - y_i}{x_i} \ge \frac{x_j - y_j}{x_j}$$

Equal sacrifice

- J.S.Mill first introduced concept
- An equal sacrifice method is defined by fixing a concave reference utility function u, which is increasing and continuous and for all i:

$$y_i > 0 \Rightarrow u(x_i) - u(y_i) = \max_j \{u(x_j) - u(y_j)\}$$

Equal sacrifice

- An equal sacrifice method always meets half of the fair ranking property (right part)
- The other half is satisfied iff u is a *concave* function
- U-equal sacrifice yields the proportional solution with the log function
- u-equal sacrifice method is progressive iff u is more concave than the log function and regressive iff u is less concave than the log function

$$u(z) = \log z \Rightarrow \frac{x_i}{y_i} = \frac{x_j}{y_j}$$

Two families of reference utilities

A solution (N,t,x)->r(N,t,x) is scale invariant if r(N,λt,λx)=λr(N,t,x), where a is concave and increasing. The scale invariant equal sacrifice methods correspond to the following two families of reference utility functions.

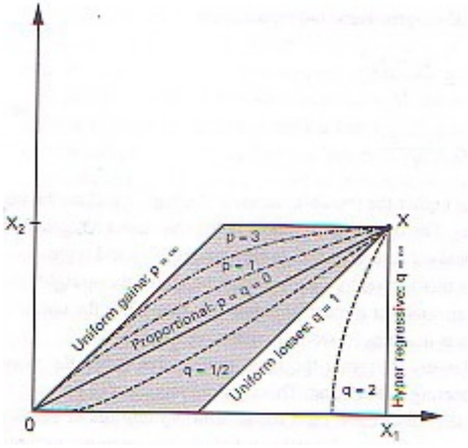
U_p method converges to ug as p arbitrarily large

U^q method converges to ul as p goes to 0

$$u_p(z) = -1/z^p \qquad 0 < p < +\infty$$

$$u^q(z) = z^q \qquad 0 < q < 1$$

Fair ranking taxation schedules



Sum-Fitness and Equality

Classical Utilitarian $\quad Find \quad y_i \geq 0 \max ing \sum_i u_i(y_i) \; under \; y_N$

Egalitarian* $\quad Find \quad y_i \geq 0 \; st \; y_N = t, \; y_i > 0 \Rightarrow u_i(y_i) = \min_j u_j(y_j)$

- Principles of compensation and sum-fitness come into play in the simple utilitarian model of resource allocation
- This model is a prelude to the more general welfarist approach
- The benevolent dictator must share t units of resources between n agents, and each agent has her own u fn to “produce” utility from resources