Problem Set 3 2016

1. Suppose that we have a farm that produces oranges next to a farm that produces honey. The total cost functions of the two firms are $C_0(q_o, q_h) = \frac{{q_o}^2}{100} - q_h$ and $C_h(q_h) = \frac{{q_h}^2}{100}$ respectively. The two firms operate as price takers in perfectly competitive markets with $p_o = 4$ and $p_h = 2$.
   i) How much honey and how many oranges are being produced if the two firms operate independently of one another?
   ii) Suppose that the two firms merge. Find the optimal amounts of honey and oranges in this new setting and compare it with the Pareto Optimal outcome.s

2. We have two agents, with utility functions over a numeraire good m_i, a private good x_i and an externality h. That is, $u_1(m_1, x_1, h) = m_1 + 10\ln(1 + x_1) + 5h - h^2$and $u_2(m_1, x_1, h) = m_2 + 5\ln(1 + x_2) - h^2$.
   i) Derive the agents' indirect utility functions depending on their wealth level w and on the price of the private good p (hint: you need to maximize each agent's utility function subject to their budget constraint for given prices).
   ii) Given that p and w will not change in our partial equilibrium setting, derive the utility functions of the agents depending only on the level of the externality and find the externality level that agent 1 will choose to generate.
   iii) Find the Pareto Optimal amount of externality.
   iv) Suppose that agent 2 is given the right to an externality-free environment so that agent 1 must pay a price p_h to agent 2 for each unit of externality that she generates. Also, assume that this price is formed in a competitive market. Derive the necessary conditions for equilibrium in the externality market and find the price and level of the externality.

3. (From MWG) A certain lake can be freely accessed by fishermen. When a boat is sent to catch fish, its cost is $r > 0$. The total quantity of fish caught (Q) is a function of the total boats sent (b), that is $Q = f(b)$, with each boat getting $\frac{f(b)}{b}$ fish. We assume concavity of the production function, that is $f'(b) > 0$and $f''(b) < 0$. We also assume a competitive market where the fish are sold at a given price $p > 0$.
   i) Characterize the equilibrium number of boats that are sent fishing, if fishermen are allowed freely to fish in the lake (hint: First, find the Total Cost function and the Profits function).

ii) Characterize the optimal number of boats that are sent fishing.

4. We have two consumers, A and B with utility functions $U_A = \log x_A + logG$ and $U_B = \log x_A + logG$ respectively. $x_i$represents the amount of a private good that each person consumes, while G represents the total amount of a public good that is offered and $g_A + g_B = G$, with g_A and g_B being the contributions of the two agents for the public good. Also, we assume that both the prices of the public and the private good equal to 1.
   i) Find the best response functions of A and B for the public good. That is, we are looking for a function that, for each person, gives the optimal amount of public good that she demands, with respect to the other person's quantity demanded.
   ii) Solve for the Nash Equilibrium. What will be the optimal contributions to the public good? Show your answer on a diagram with the agents' reaction functions.
   iii) Find the socially optimal (Pareto Optimal) level of the public good and compare it with your previous result.

5. (From MWG) In an economy we have J firms and I individuals. Each firm j generates a level of externality h_j and its profits depend on that externality, that is $\pi_j = \pi_j(h_j)$, while each individual's derived utility function depends in general on the externalities generated by all of the firms, that is $U_i = \Phi_i(h_1, h_2, \dots, h_J) + w_i$. In this case we do not have a homogeneous externality.
   i) Using the proper FOCs derive the Pareto Optimal amounts of externalities for the economy as well as the amounts that will be generated in a competitive equilibrium.
   ii) What tax/subsidy can restore efficiency?

6. (From MWG) Suppose that consumer i's preferences can be represented by the utility function $u_i(x_{1i}, \dots, x_{Li}) = \sum_l log(x_{li})$ (Cobb Douglas preferences).
   i) Derive his demand for good l. What is the wealth effect?
   ii) What happens to the wealth effect as we increase the number of goods? (Calculate the limit as L goes to infinity)

7. (From MWG) Consider an economy with two goods, one consumer and one firm. The initial endowment of the numeraire is $\omega_m > 0$ and the initial endowment of good l is 0. The consumer's quasilinear utility function is $u(x, m) = m + \varphi(x)$, where $\varphi(x) = \alpha + \beta\ln(x)$, with $\alpha, \beta > 0$. The firm's cost function is $c(q) = \sigma q$, with $\sigma > 0$. Also assume that the consumer

receives all the profits of the firm. Both the consumer and the firm act as price takers. Finally, $p_m = 1$ and denote $p_l = p$.

i) Derive the consumer's and the firm's first order conditions.

ii) Derive the competitive price and output of good l. How do these vary with α, β, σ?