

Problem Set 2 for Social Choice

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1. As in the time sharing example by Moulin, we have k radio stations, n agents and n_i supporters of each radio station. This means that $\sum_{i=1}^k n_i = n$. We want to derive the time share $\{x_1, x_2, \dots, x_k\}$ that the agents will use under the Nash collective welfare function.
2. Consider the following preference profiles for $n = 200$ agents for the social outcomes A, B and C:

	$n_1 = 70$	$n_2 = 66$	$n_3 = 64$
Top	A	B	C
Middle	C	A	B
Bottom	B	C	A

- i) Which outcome would be chosen under:
 - a) Plurality
 - b) Borda Count
 - c) Condorcet Rule
 - ii) What would happen under Borda Count and Plurality voting if outcome C were no longer available? Which basic axiom of social welfare functions do these two voting schemes violate?
3. From Moulin (p. 132 ex. 4.4): We have the following preferences for the outcomes a, b, c, d with $n = 7$ agents:

$n_1 = 3$	$n_2 = 2$	$n_3 = 2$
c	b	a
b	a	d
a	d	c
d	c	b

- i) Derive the majority relation according to Condorcet and break the cycle to its weakest link.
 - ii) Compute the Borda Count and find an outcome such that when it is no longer available to the agents, the social ordering of the Borda Count is completely reversed.
4. Suppose we have 4 social outcomes, A, B, C and D and a social welfare function for which the social ordering R^S is given by nature, that is $A \succ_S B \succ_S C \succ_S D$. This means that no matter what the preferences of the agents of the society are, the SWF always ranks the alternatives as above.

- i) Does this social welfare function satisfy IIA?
 ii) Does it satisfy Dictatorship?
 iii) Does it satisfy the Pareto Principle? Explain.
5. Suppose that a family consists of a mother and two children, namely Ann and Bob. The possible alternatives for (any) Saturday's dinner are pizza, burgers or souvlaki. Their mother wants to find a social choice rule that would enable her to consistently rank her children's choices about Saturday's dinner and eventually make a choice.
- i) This Saturday, Ann's preferences are $p \succ_A b \succ_A s$ and Bob's are $b \succ_B p \succ_B s$. Their mother says to Bob: "let's eat pizza and let your sister get her way for today. I promise that next Saturday, if there is the same conflict again, I will resolve it in favor of you and we will have burgers." However, Bob knows that there is no possible way for his mother to be consistent with Arrow's axioms in her decision and yet choose burgers next Saturday. Why is that? What must the mother choose next Saturday (if the same conflict comes up) if she wants to be consistent with IIA, UD, Transitivity and the Pareto Principle?
- ii) Suppose that the mother announces the following social welfare function for any Saturday's dinner: "If Bob prefers $b \succ_B s$ then the social ordering will be his own ordering; otherwise, the social ordering will be Ann's individual ranking." Does this rule satisfy IIA, Dictatorship and the Pareto Principle? Explain.
6. For the following preference profiles for agents A and B derive the social ordering consistent with Arrow's criteria (Transitivity, Pareto Principle, IIA and UD) and explain your results. We know that the social ordering for the first from the left profile is $z \succ_S y \succ_S x$, as well as that agent A's preferences are on the horizontal axis and B's on the vertical axis.

3		y	
2	z		
1			x
	1	2	3

3			x
2		z	
1	y		
	1	2	3

3	z		
2			y
1		x	
	1	2	3

3	x		
2			z
1		y	
	1	2	3