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# LECTURE 4

## MICROECONOMIC THEORY

## CONSUMER THEORY

## Consumer Welfare

**Lecturer: Andreas Papandreou**

# USING CONSUMER THEORY

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- ❑ Consumer analysis is not just a matter of consumers' reactions to prices.
- ❑ We pick up the effect of prices on incomes on attainable utility - consumer's welfare.
- ❑ This is useful in the design of economic policy, for example.
  - The tax structure?
- ❑ We can use a number of tools that have become standard in applied microeconomics
  - price indices?

# OVERVIEW...

*Interpreting the outcome of the optimisation in problem in welfare terms*

Consumer welfare

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graph TD; A[Consumer welfare] --- B[Utility and income]; A --- C[CV and EV]; A --- D[Consumer's surplus];
```

Utility and income

CV and EV

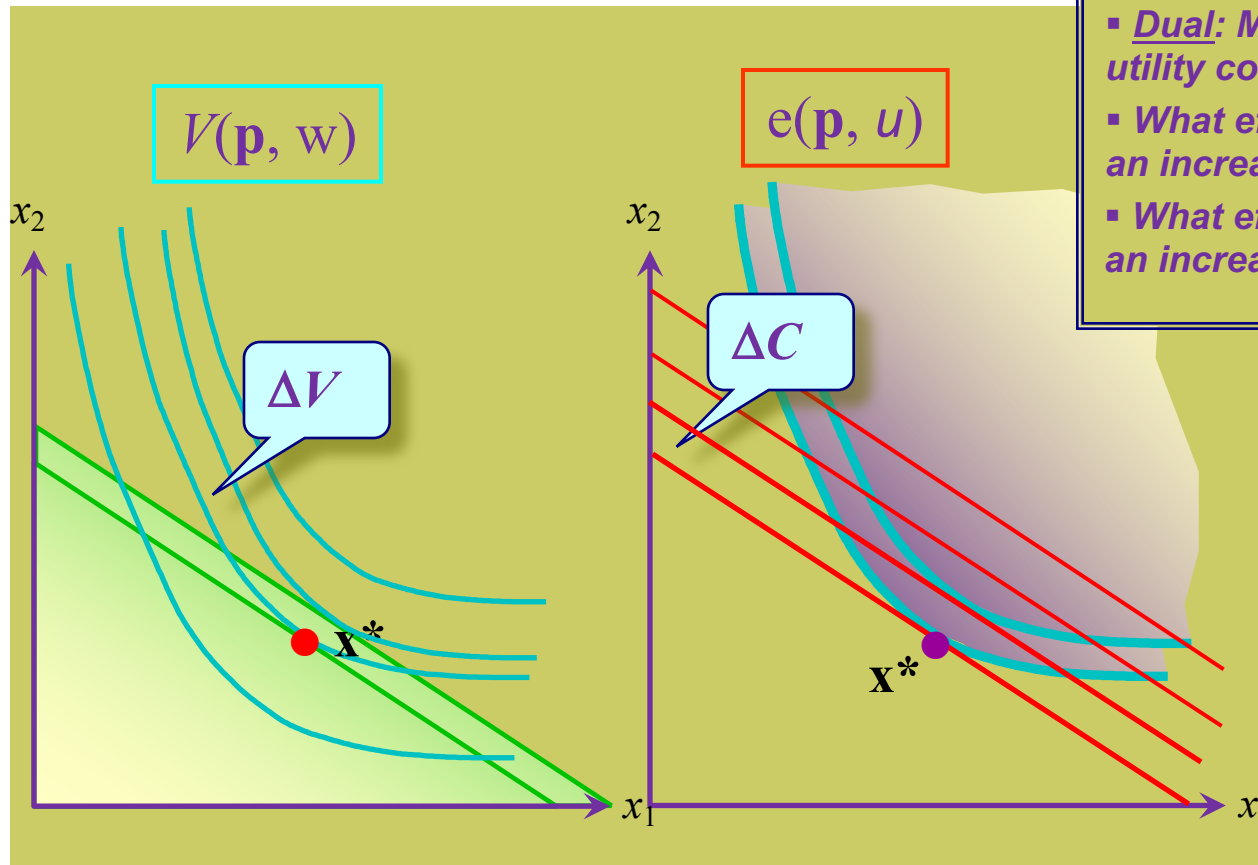
Consumer's surplus

# HOW TO MEASURE A PERSON'S “WELFARE”?

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- We could use some concepts that we already have.
- Assume that people know what's best for them...
- ...So that the preference map can be used as a guide.
- We need to look more closely at the concept of “maximised utility”...
- ...the indirect utility function again.

# THE TWO ASPECTS OF THE PROBLEM



- **Primal:** Max utility subject to the budget constraint
- **Dual:** Min cost subject to a utility constraint
- What effect on max-utility of an increase in budget?
- What effect on min-cost of an increase in target utility?

Interpretation  
of Lagrange  
multipliers

# UTILITY AND INCOME: SUMMARY

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- ❑ This gives us a framework for the evaluation of marginal changes of income...
- ❑ ...and an interpretation of the Lagrange multipliers
- ❑ The Lagrange multiplier on the income constraint (primal problem) is the marginal utility of income.
- ❑ The Lagrange multiplier on the utility constraint (dual problem) is the marginal cost of utility.
- ❑ But does this give us all we need?

# UTILITY AND INCOME: LIMITATIONS

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- This gives us some useful insights but is limited:
  1. We have focused only on marginal effects
    - infinitesimal income changes.
  2. We have dealt only with income
    - not the effect of changes in prices
- We need a general method of characterising the impact of budget changes:
  - valid for arbitrary price changes
  - easily interpretable
- For the essence of the problem re-examine the basic diagram.

# OVERVIEW...

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Consumer welfare

Utility and  
income

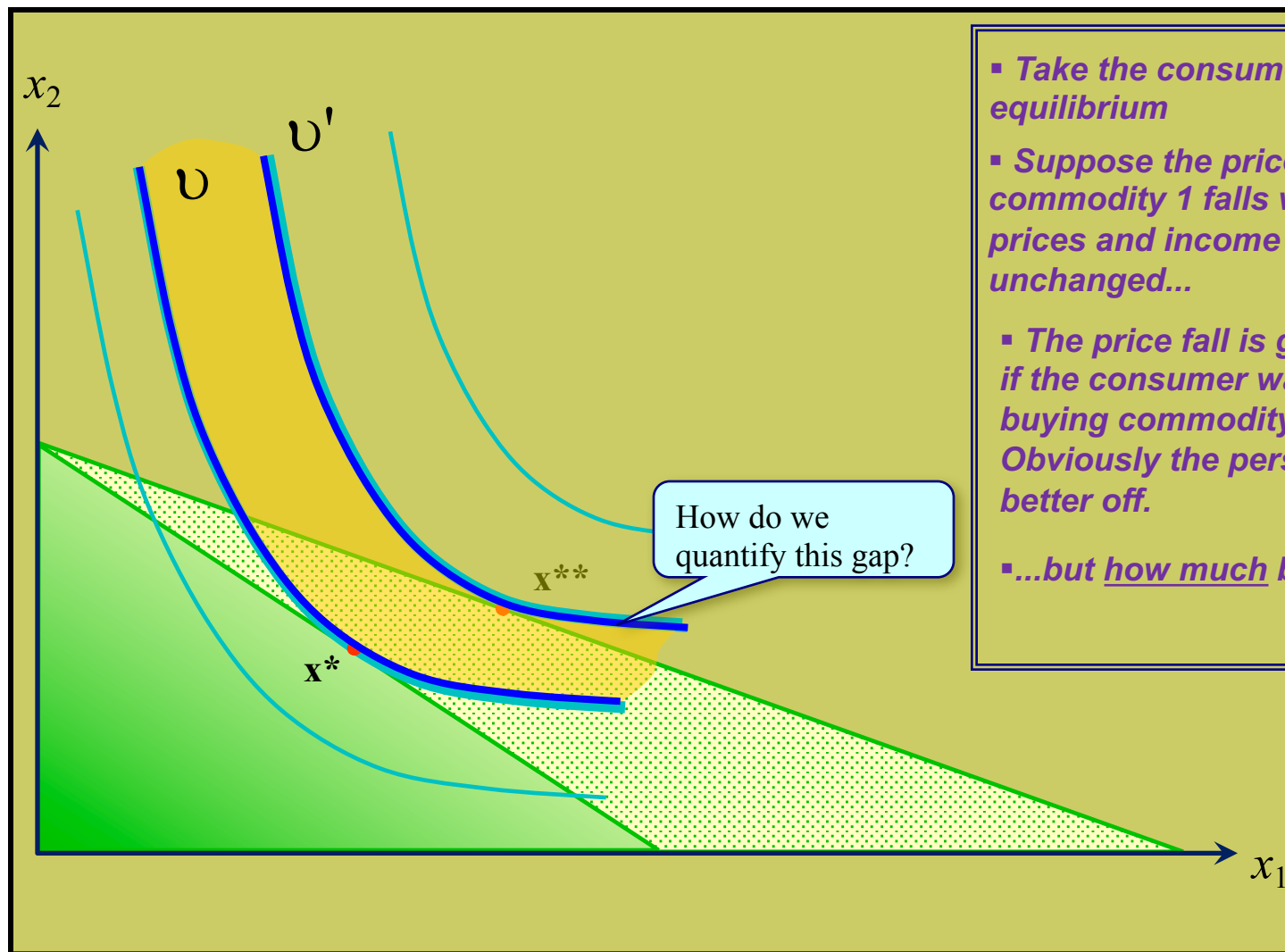
CV and EV

Consumer's  
surplus

*Exact money  
measures of  
welfare*



# THE PROBLEM...



- Take the consumer's equilibrium
- Suppose the price of commodity 1 falls while other prices and income remain unchanged...
- The price fall is good news if the consumer was actually buying commodity 1. Obviously the person is better off.
- ...but how much better off?

# APPROACHES TO VALUING UTILITY CHANGE

- Three things that are not much use:

1.  $v'$

Utility differences

depends on the **units** of the  $U$  function

Utility ratios

depends on the **origin** of the  $U$  function

•  $u$

some distance function

depends on the **cardinalisation** of the  $U$  function

1.  $d(v', v)$

- A more productive idea:

- *Use income not utility as a measuring rod*
  1. *To do the transformation we use the  $V$  function*
  2. *We can do this in (at least) two ways...*

# STORY NUMBER 1

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- Suppose  $\mathbf{p}$  is the original price vector and  $\mathbf{p}'$  is vector after good 1 becomes cheaper.
- This causes utility to rise from  $v$  to  $v'$ .
  - $v = v(\mathbf{p}, w)$
  - $v' = v(\mathbf{p}', w)$
- Express this rise in money terms?
  - What hypothetical change in income would bring the person back to the starting point?
  - (and is this the right question to ask...?)
- Gives us a standard definition....

# IN THIS VERSION OF THE STORY WE GET THE *COMPENSATING VARIATION*

$$u = V(\mathbf{p}, w)$$

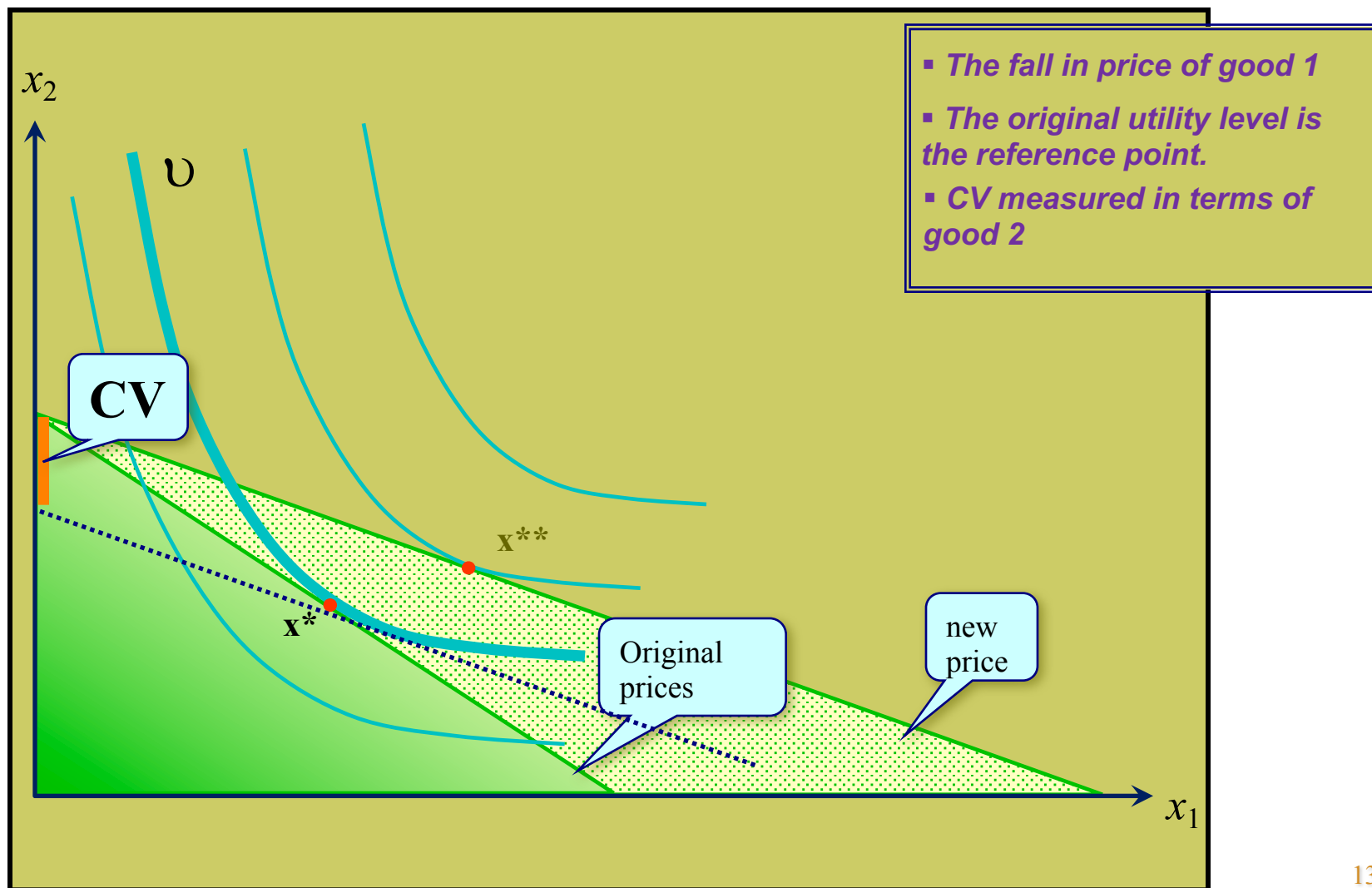
the original utility level at prices  $\mathbf{p}$  and income  $w$

$$u = V(\mathbf{p}', w - \text{CV})$$

the original utility level restored at new prices  $\mathbf{p}'$

- *The amount CV is just sufficient to “undo” the effect of going from  $\mathbf{p}$  to  $\mathbf{p}'$ .*

# THE COMPENSATING VARIATION



# CV – ASSESSMENT

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- ❑ The CV gives us a clear and interpretable measure of welfare change.
- ❑ It values the change in terms of money (or goods).
- ❑ But the approach is based on one specific reference point.
- ❑ The assumption that the “right” thing to do is to use the original utility level.
- ❑ There are alternative assumptions we might reasonably make. For instance...

# HERE'S STORY NUMBER 2

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- Again suppose:
  - $\mathbf{p}$  is the original price vector
  - $\mathbf{p}'$  is the price vector after good 1 becomes cheaper.
- This again causes utility to rise from  $u$  to  $u'$ .
- But now, ask ourselves a different question:
  - Suppose the price fall had never happened
  - What hypothetical change in income would have been needed ...
  - ...to bring the person to the *new* utility level?

# IN THIS VERSION OF THE STORY WE GET THE *EQUIVALENT* *VARIATION*

$$u' = v(\mathbf{p}', w)$$

the utility level at new prices  $\mathbf{p}'$  and income  $w$

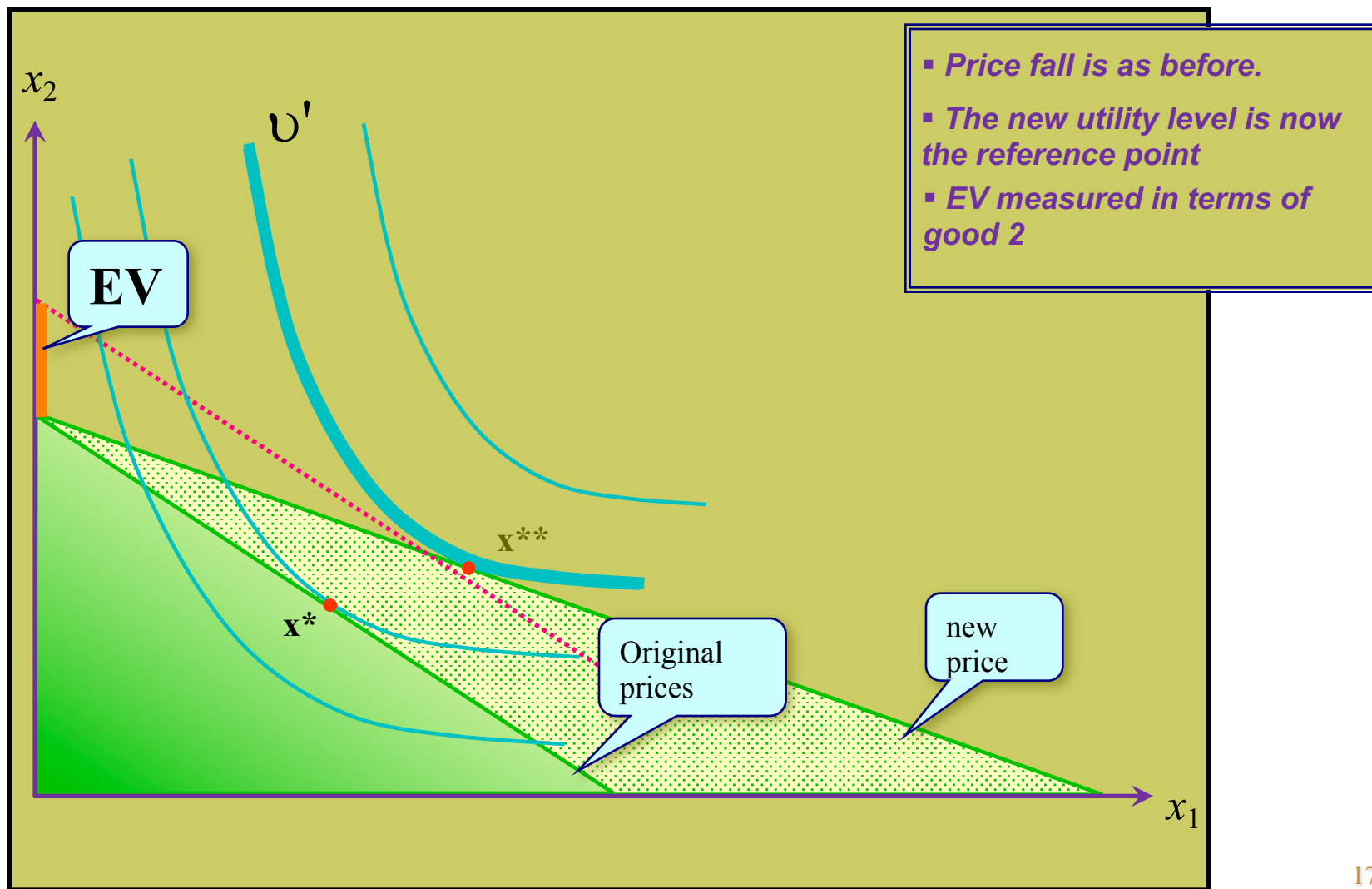
$$u' = v(\mathbf{p}, w + \mathbf{EV})$$

the new utility level reached at original prices  $\mathbf{p}$

- *The amount EV is just sufficient to “mimic” the effect of going from  $\mathbf{p}$  to  $\mathbf{p}'$ .*



# THE EQUIVALENT VARIATION



# OVERVIEW...

Consumer welfare

Utility and  
income

CV and EV

Consumer's  
surplus

*A simple,  
practical  
approach?*

# ANOTHER EQUIVALENT FORM FOR CV

Prices  
before

Reference  
utility level

Pr  
after

- Use the cost-difference definition
- $$CV(\mathbf{p} \rightarrow \mathbf{p}') = C(\mathbf{p}, v) - C(\mathbf{p}', v)$$

(-) change in cost of hitting utility level  $v$ . If positive we have a welfare *increase*.

- Assume that the price of good 1 changes from  $p_1$  to  $p_1'$  while other prices remain unchanged. Then we can rewrite the above as:

(Just using the definition of a definite integral)

Hicksian (compensated)  
demand for good 1

- Further rewrite as:

$$CV(\mathbf{p} \rightarrow \mathbf{p}') = \int_{p_1'}^{p_1} H^1(\mathbf{p}, v) dp_1$$

You're right. It's using Shephard's lemma again

***So CV can be seen as an area under the compensated demand curve***

# ANOTHER (EQUIVALENT) FORM FOR CV

Prices before

Reference utility level

Prices after

- Use the cost-difference definition
- $$CV(\mathbf{p} \rightarrow \mathbf{p}') = C(\mathbf{p}, v) - C(\mathbf{p}', v)$$

(-) change in cost of hitting utility level  $v$ . If positive we have a welfare *increase*.

- Assume that the price of good 1 changes from  $p_1$  to  $p_1'$  while other prices remain unchanged. Then we can rewrite the above as:

(the CV can be found by integrating the cost function over a sequence of small changes in prices from  $\mathbf{p}$  to  $\mathbf{p}'$ )

$$CV(\mathbf{p} \rightarrow \mathbf{p}') = \int_{p_1'}^{p_1} dC$$

Hicksian (compensated) demand for good 1

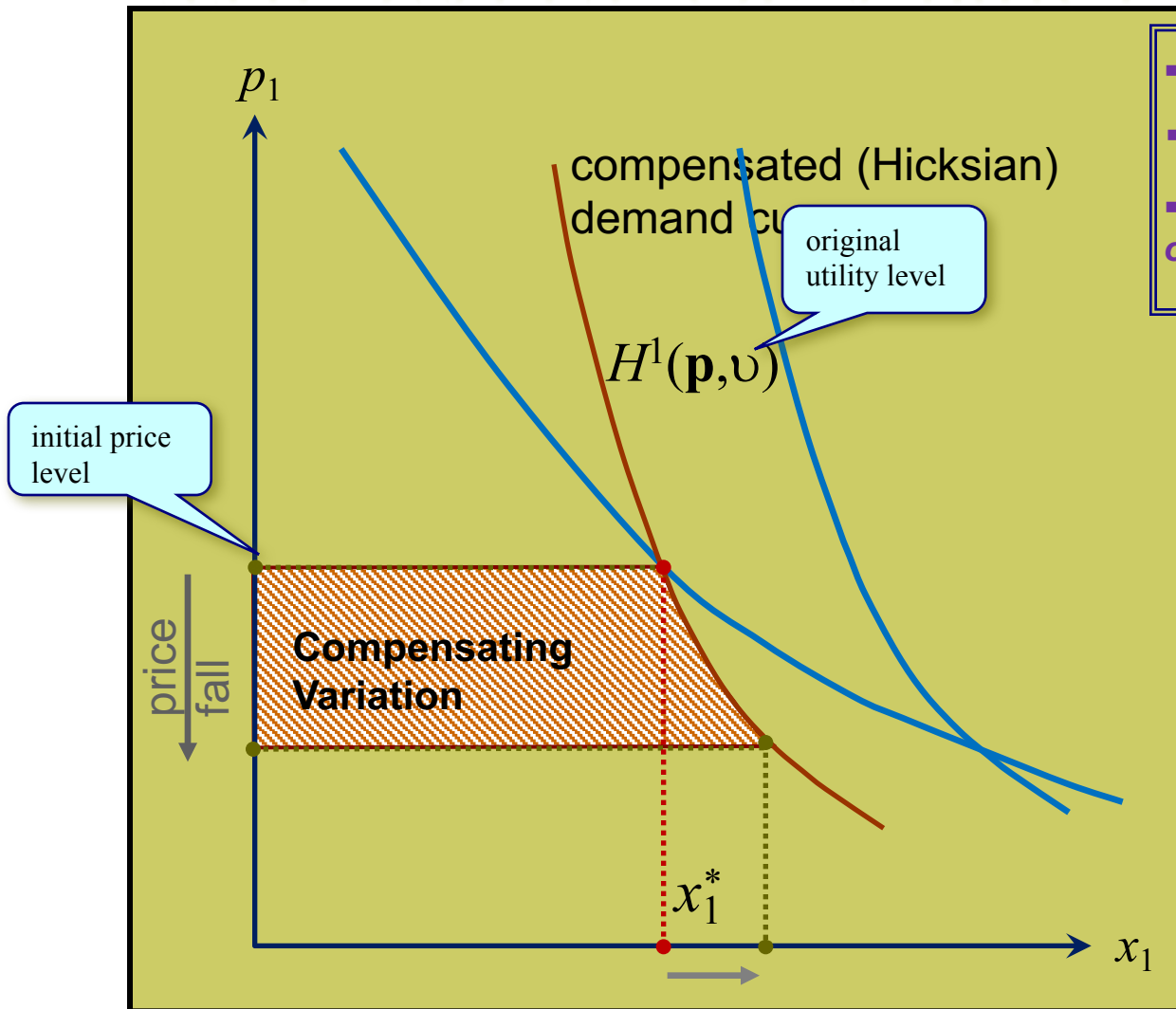
- Further rewrite as:

$$CV(\mathbf{p} \rightarrow \mathbf{p}') = \int_{p_1'}^{p_1} H^1(\mathbf{p}, v) dp_1$$

You're right. It's using Shephard's lemma again

**So CV can be seen as an area under the compensated demand curve**

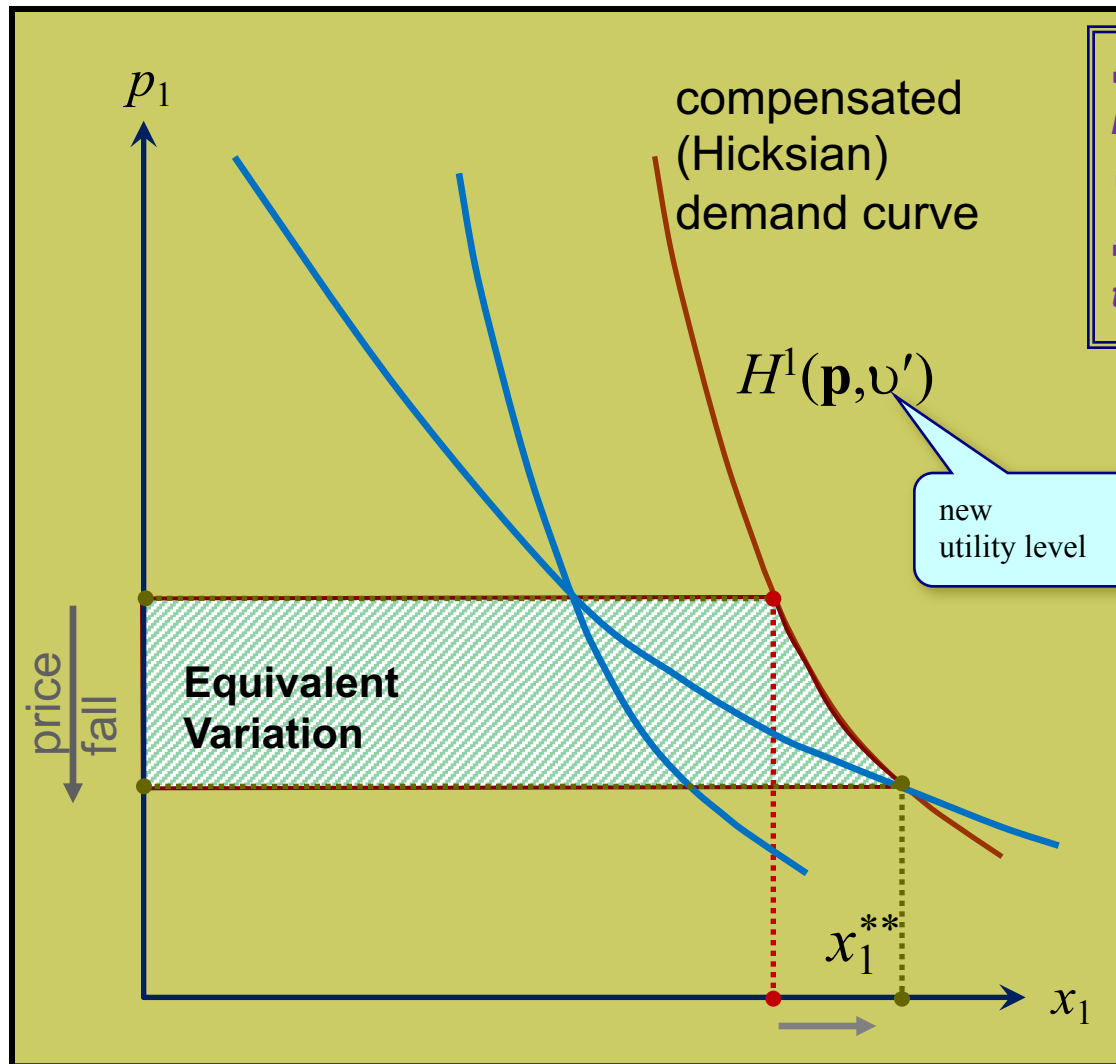
# COMPENSATED DEMAND AND THE VALUE OF A PRICE FALL



- *The initial equilibrium*
- *price fall: (welfare increase)*
- *value of price fall, relative to original utility level*

- *The CV provides an exact welfare measure.*
- *But it's not the only approach*

# COMPENSATED DEMAND AND THE VALUE OF A PRICE FALL (2)

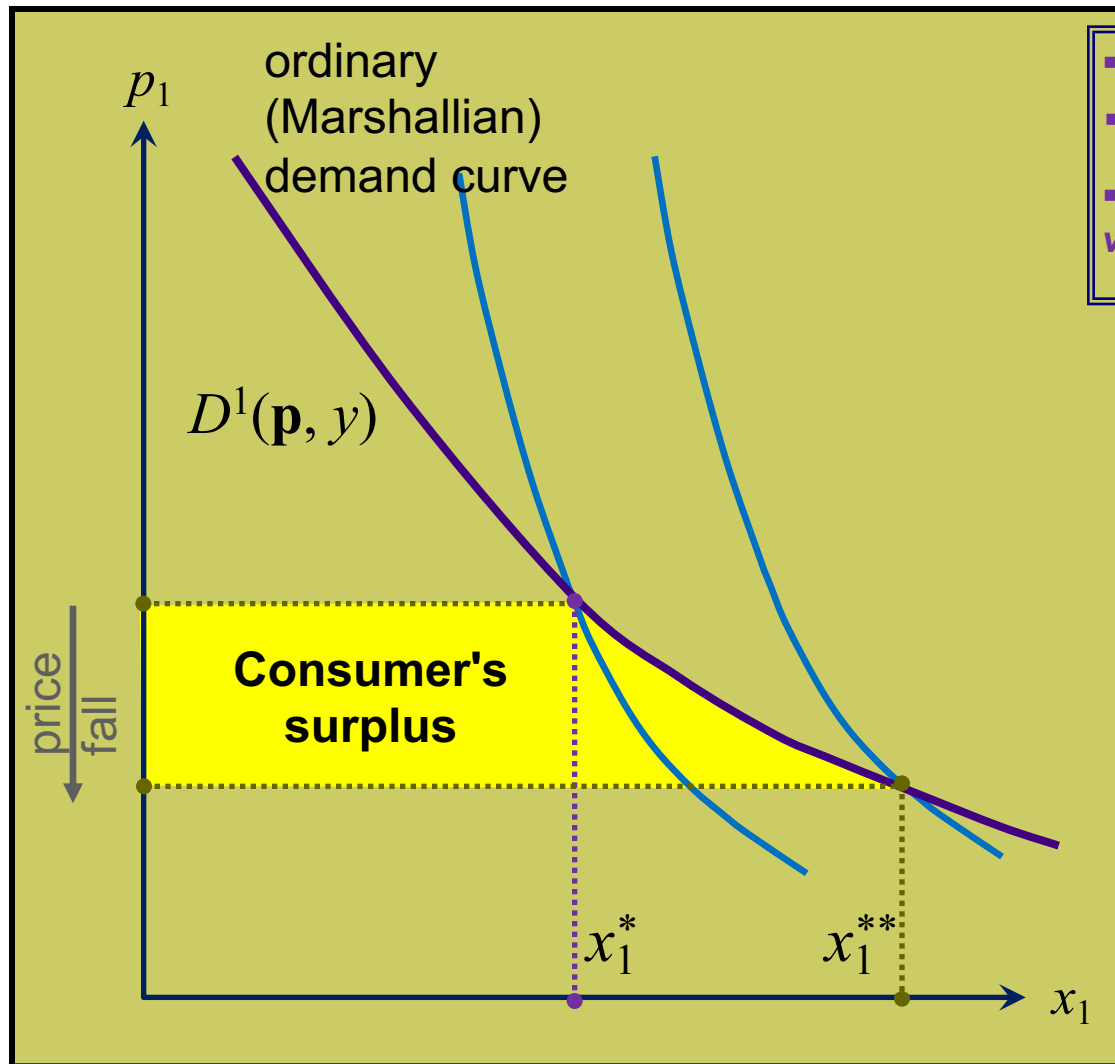


- As before but use new utility level as a reference point
- price fall: (welfare increase)
- value of price fall, relative to new utility level

new utility level

- The EV provides another exact welfare measure.
- But based on a different reference point
- Other possibilities...

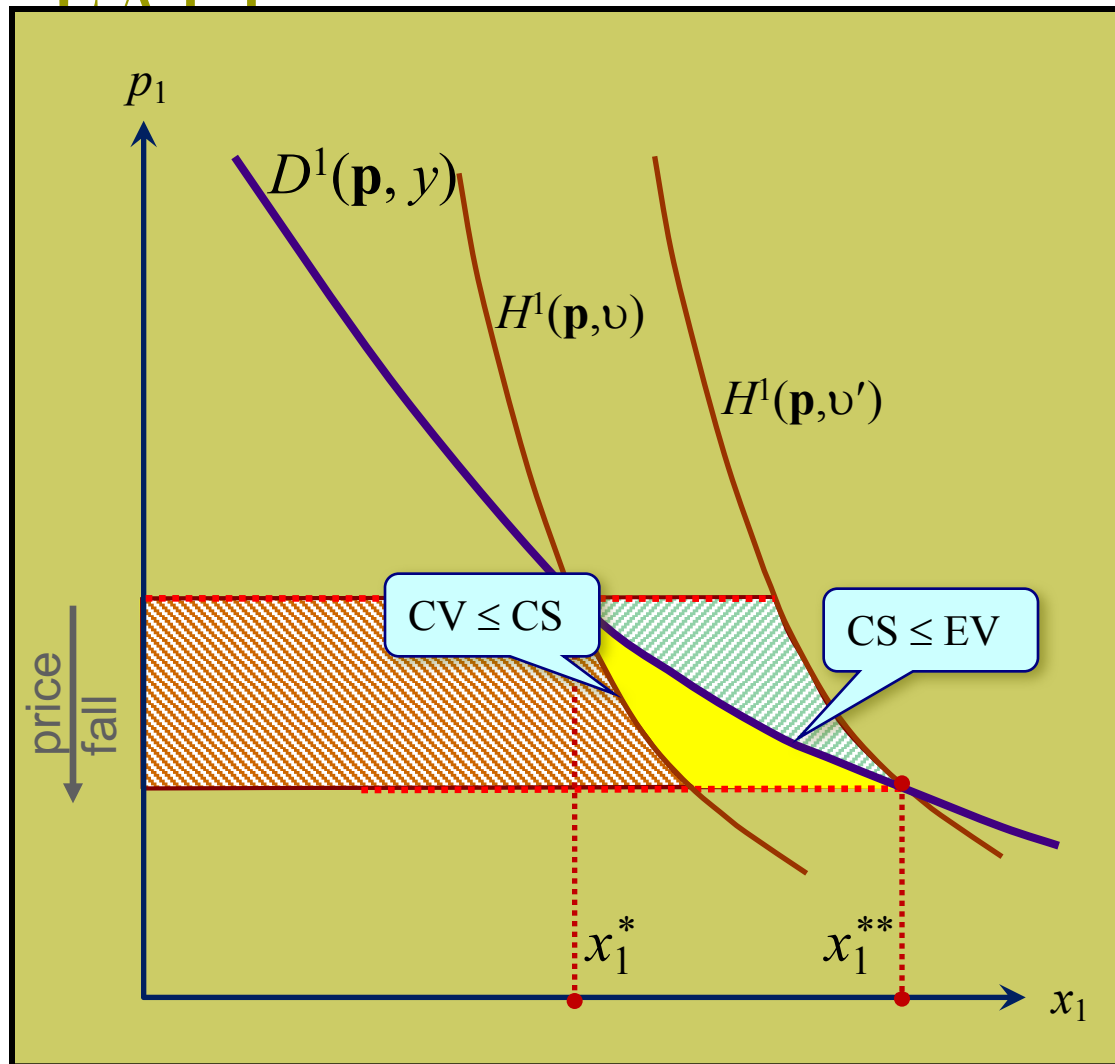
# ORDINARY DEMAND AND THE VALUE OF A PRICE FALL



- *The initial equilibrium*
- *price fall: (welfare increase)*
- *An alternative method of valuing the price fall?*

▪ *CS provides an approximate welfare measure.*

# THREE WAYS OF MEASURING THE BENEFITS OF A PRICE FALL



- Summary of the three approaches.
- Conditions for normal goods
- So, for normal goods:  
 $CV \leq CS \leq EV$
- For inferior goods:  
 $CV > CS > EV$



# SUMMARY: KEY CONCEPTS

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- ❑ Interpretation of Lagrange multiplier
- ❑ Compensating variation
- ❑ Equivalent variation
  - CV and EV are measured in monetary units.
- ❑ Consumer's surplus
  - The CS is a convenient approximation
  - For normal goods:  $CV \leq CS \leq EV$ .
  - For inferior goods:  $CV > CS > EV$ .