Game Theory: Basics

MICROECONOMICS

Principles and Analysis

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Introduction

- Focus on conflict and cooperation.
- Provides fundamental tools for microeconomic analysis.
- Offers new insights on concepts and techniques we have discussed earlier:
  - Optimisation.
  - Equilibrium.
- Begin with basic ingredients of a game…

Ingredients of a game

- The players:
  - Firms?
  - Consumers?
  - Household members?
  - …
- Objectives of players:
  - Profits?
  - Utility?
  - …
- Rules of play:
  - Available actions
  - Information
  - Timing
  - …
- The outcomes:
  - List of monetary payoffs?
  - List of utility levels?
  - …

Details follow

Actions

- Actions could be almost anything of economic interest.
- Important to specify the *action space* as part of the rules of the game
- Some examples:

Type of action	*Action space*
Quantity of a good	Nonnegative numbers $[0,\infty)$
Product price	Nonnegative numbers $[0,\infty)$
Location	Set of places {London, Paris, New York…}
Participation	A simple pair {[+], [–]} or {Left, Right} or {Bid, No Bid} …

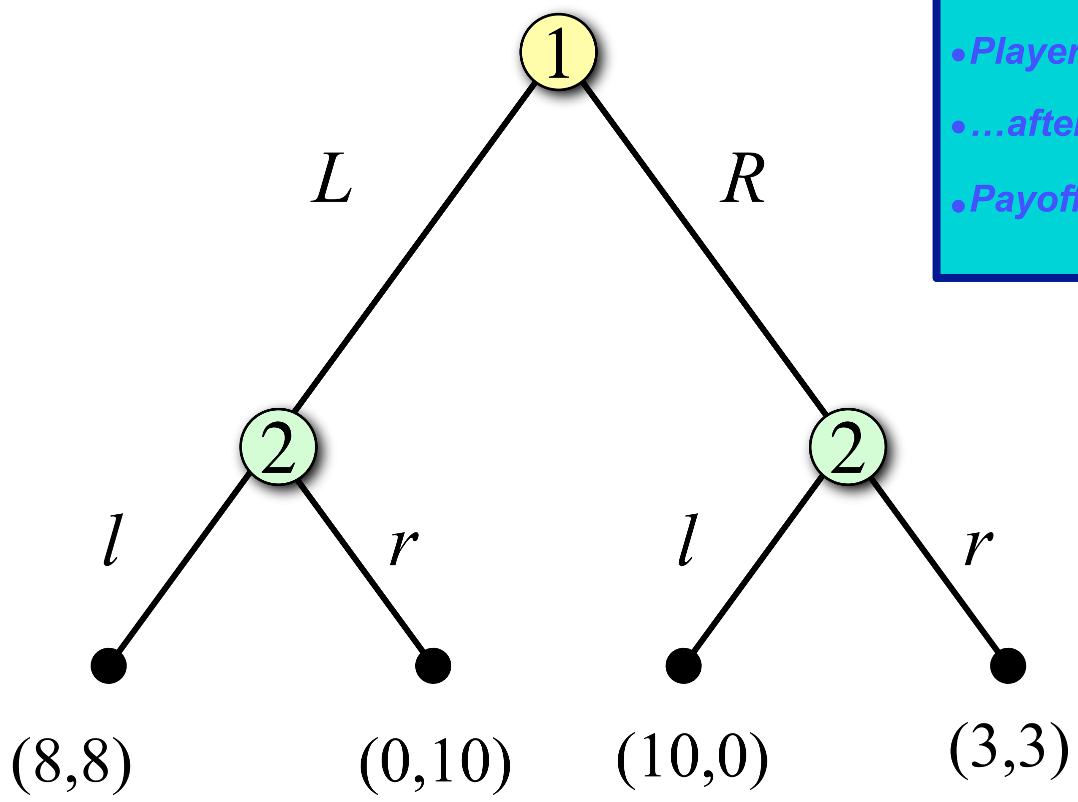
How to represent a game

- Two main approaches:
  - *Extensive form*
    - Unfold the game like a story
    - See the economic interaction as a kind of sequence.
  - *Normal form*
    - Represent the interaction as a snapshot
    - Encapsulate details of the game in a single table.
- Each form has its uses in analysing the economic issues modelled by a game.
- Consider a simple example…

Extensive form

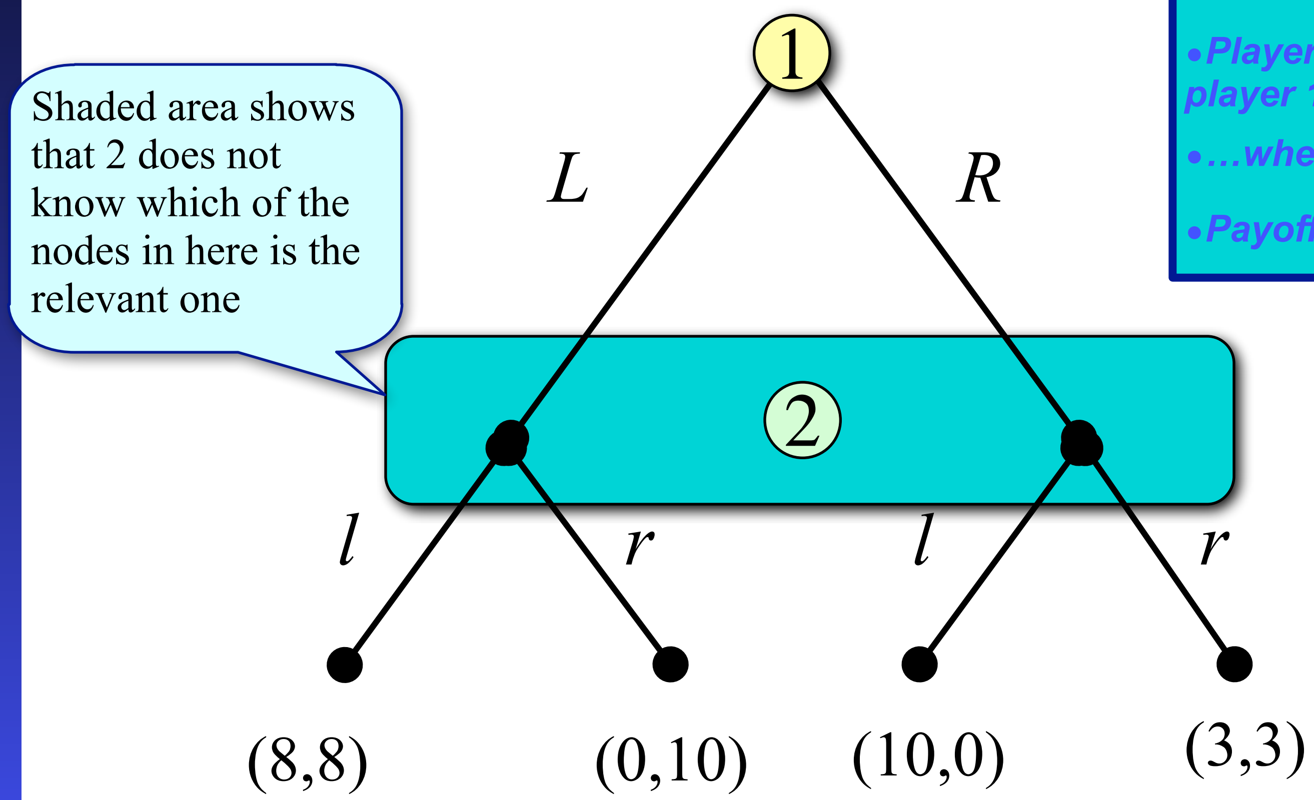
- Take two players:
  - Numbered for convenience.
  - Player 1 does not necessarily have precedence over 2.
- Each can take one of two actions:
  - Player 1 can choose *L*eft or *R*ight.
  - Player 2 can choose *l*eft or *r*ight.
- Outcomes just represented by numbers (utility levels).
- Consider the rules of play:
  - Do the players take turns?
  - Or do they have to move at the same time?

Extensive Form: Sequential



- *Player 1 chooses L or R*
- *Player 2 chooses l or r…*
- *…after player 1's choice*
- *Payoffs, given as (u_1, u_2)*

Extensive Form: Simultaneous



- *Player 1 chooses L or R*
- *Player 2 does not know player 1's choice…*
- *…when choosing l or r*
- *Payoffs as before*

Normal form

- Also known as *strategic* form.
- Represents the game as a consequences table
- Need to specify
  - The players
  - The strategies available to each
  - The payoff to each consequent upon each possible combination of strategies
- A difficulty: as yet we have not defined "strategy"
- But we can get some insight a simplified version of the normal form
  - For now we restrict attention to cases where the "action" can be used instead of "strategy"

Normal form example

- We use the game form we have represented earlier.
- Two players
- Each can take one of two actions
- Same payoffs as before
- Focus on the case of simultaneous play
  - A key assumption for the simplification we are using

Normal form

- *A table for two players*
- *Player 1 chooses L or R*
- *Player 2 chooses l or r*
- *Payoffs, given as u_1, u_2*
- *Outcome of choices L,r*

Player 1			
	L	8,8	0,10
	R	10,0	3,3
		l	*r*

Player 2

How to solve the game?

- A simple approach
- It is sometimes easy to see what actions could *not* be optimal
- In which case, let's eliminate them
- Could what is left be a solution?

The method in action

- Let's illustrate the method with four examples.
- Examples 2-4 are well-known cases:
  - We will identify them by their usual nam
  - …although the names will not mean muc
  - But if you want a brief explanation click on the "**story**" button, top left of the screen.
- This is only a first glimpse of some important paradigms.
- The examples will be taken up in further presentations.

Click the button again to close the window

The examples

Each game:

- … is symmetric
  - You can interchange the two players
- … has a simple action space
  - Binary: i.e. every one consists of {Left, Right}, {[+],[–]}, etc.
- …has a simple payoff structure
  - Purely ordinal (numbers in *italics* to remind us of this)
  - Four utility levels for each person {*0*, *1*, *2*, *3*}

    worst → *0*; best → *3*
  - This will have to be changed in later developments
- … raises some interesting questions:
  - Can you spot what the solution must be?
  - Are there apparent solutions that the method misses?
  - Does it matter that the game is played simultaneously rather than sequentially?

Story

A trivial game?

Although this game is trivial in terms of game theory it can be seen as a caricature of an important economic phenomenon. We can consider it as a form of positive externality. If each player (firm or household) chooses [+] this reinforces the payoff to the other player of choosing [+] rather than [-]. In this case the model of the externality is so simple that a decision taken on an individual myopic basis automatically leads to the best possible outcome for each player. It is clear what the outcome would be.

game

Player 1

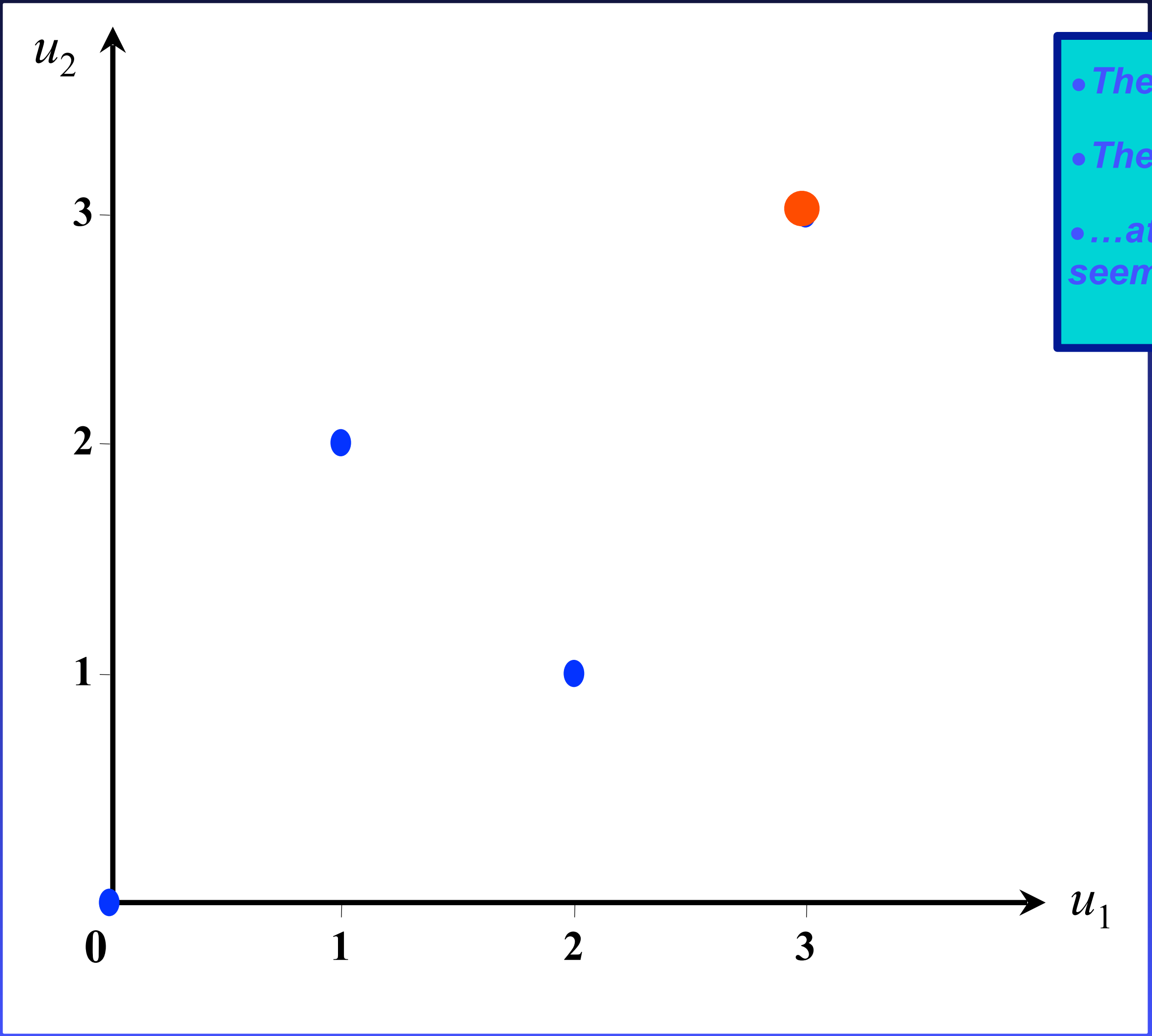
	[+]	[–]
	3,3	1,2
[–]	2,1	0,0

Player 2

- *A "contribution game"*
- *Each player can choose [+] or [–].*
- *Payoffs, given as (u_1, u_2)*
- *If 2 plays [+] then 1 gets more by playing [+]*
- *If 2 plays [–] then 1 also does better by playing [+]*
- *So player 1 eliminates [–]*
- *For similar reasons 2 eliminates [–]*
- *The solution*

Same result would emerge whether play is simultaneous or sequential

Trivial game: outcomes



- *The payoffs*
- *The solution…*
- *…at this salient point seems reasonable*

Story

Prisoner's Dilemma

Two prisoners are each suspected of a serious crime. They are held in separate cells, unable to communicate with each other or the outside world. Each is invited to confess and implicate the other prisoner. Each is aware of the following consequences:

•If one prisoner alone confesses then he is given complete immunity (utility level 3) and the other prisoner is gets life imprisonment (utility level 0).

• If both confess both get a substantial sentence, but less than life (utility level 1).

•If neither confesses there is enough evidence to convict them both of some minor violation for which they are fined (utility level 2).

If there were some way of each guaranteeing to the other that he would not confess then they could secure for themselves the outcome (2,2). But, under the circumstances, no such enforceable guarantee is possible. Each has an incentive to confess immediately, for fear of being implicated by the other. The solution is (1,1).

Economic relevance: it forms the basis of a class of problems where social and private interests are in direct opposition to private interests, the most relevant of which is the provision of public goods. Suppose citizens have to decide on one of two actions:

•action [+] – contribute private resources to provision of a public good.

•action [–] – not to contribute any resources.

Each citizen might well like to see that a public good is provided, even at the expense of his own contribution to pay for it. If each person's socially responsible action [+] could be guaranteed then this would produce the outcome (2,2). But if the good is genuinely public (once provided no-one can be excluded from enjoying its benefits) then each person would prefer that someone else incur the financial burden and would selfishly take the action [–] – this produces the outcome (1,1).

s dilemma”

2,2	0,3
3,0	1,1
+]	[–]

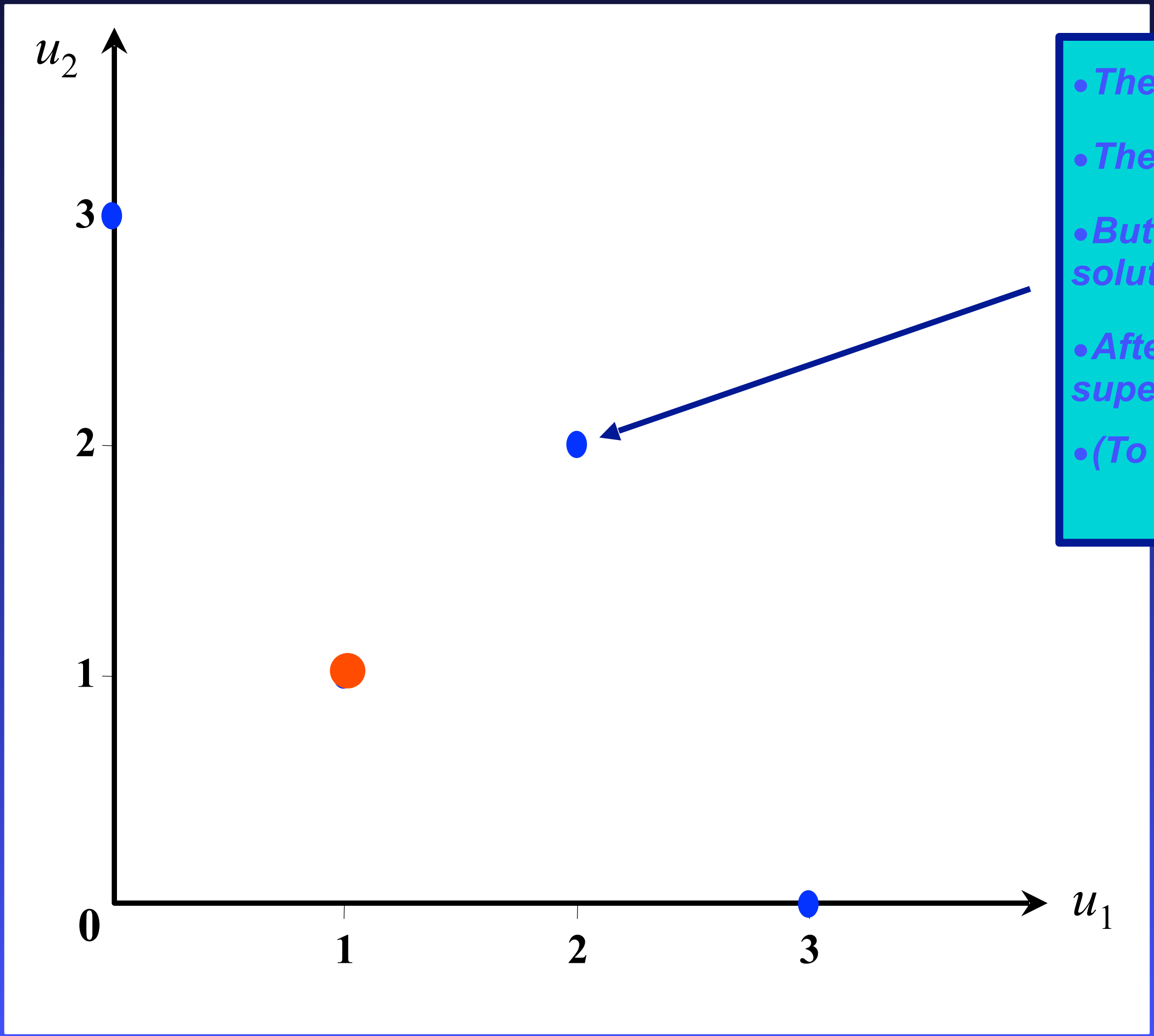
Player 2

- A “contribution game”
- Each player can choose [+] or [–].
- Payoffs, given as (u_1, u_2)
- If 2 plays [+] then 1 gets more by playing [–]
- If 2 plays [–] then 1 also does better by playing [–]
- So player 1 eliminates [+]
- For similar reasons 2 eliminates [+]
- The solution?

outcome from simultaneous or sequential play.

- But there is something odd about this…

Prisoner's Dilemma: outcomes



- *The payoffs*
- *The "solution"*
- *But why isn't <u>this</u> a solution…?*
- *After all, it's Pareto-superior to the "solution"*
- *(To be continued…)*

Story

Battle of the Sexes

A couple want to decide on an evening's entertainment. He prefers to go to the West End (there's a new play); she wants to go to the East End (dog races). If they go as a couple each person gets utility level 2 if it is his/her preferred activity and 1 otherwise. However, for each person the evening would be ruined if the partner were not there to share it (utility level 0).

The model is appropriate to situations where networking or synergy is crucial but location, for example, is unimportant..

le of the sexes”

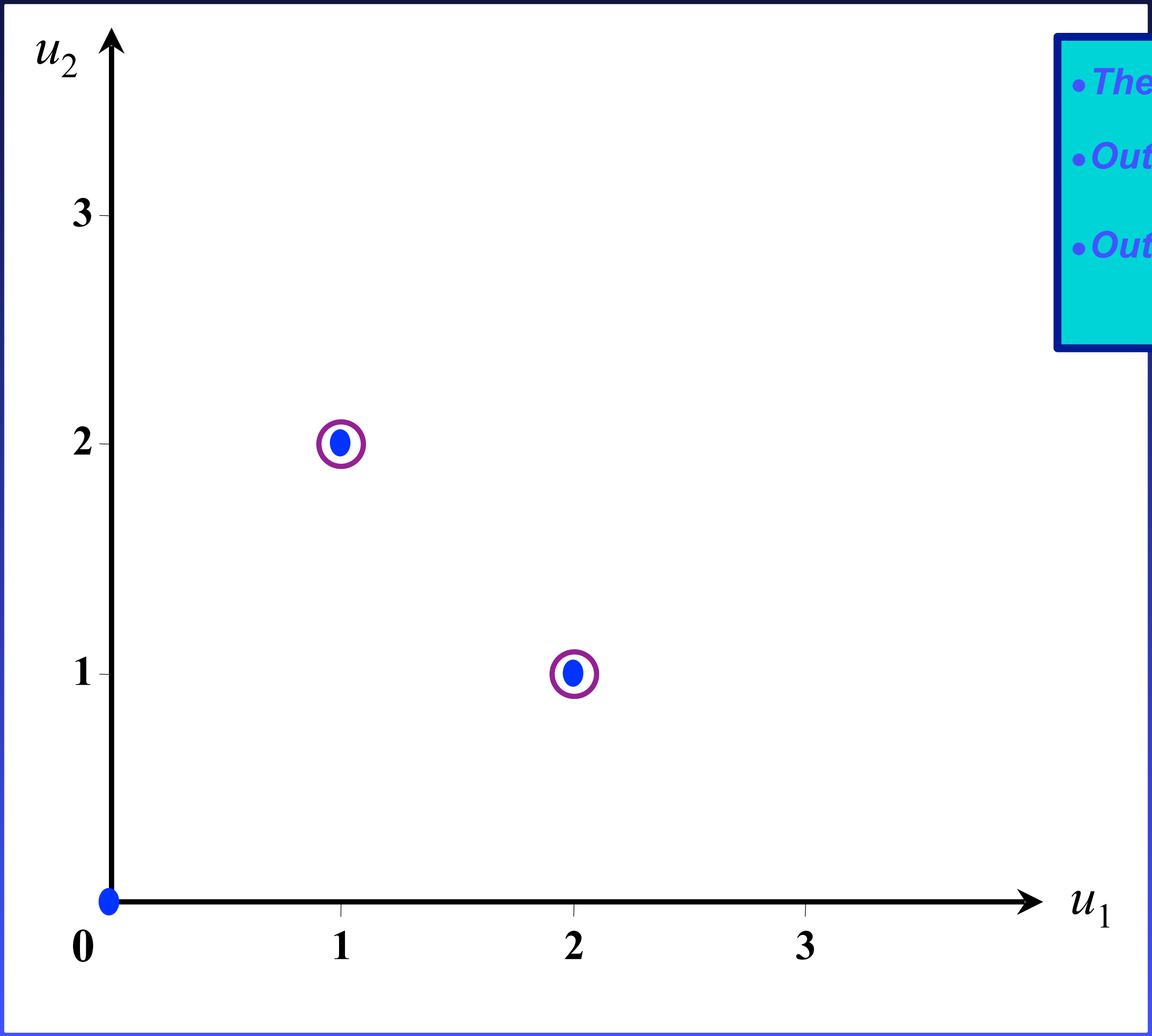
	West	East
est	2,1	0,0
ast	0,0	1,2

Player 2

- *A “coordination game”*
- *Each player can choose West or East.*
- *Payoffs, given as (u_1, u_2)*
- *If 2 plays W then 1 gets more by playing W*
- *But if 2 plays E then 1 does better by playing E*
- *So, no elimination*

- *Outcome is clear if one player has “first move”: (2,1) or (1,2)*
- *But what happens if they move simultaneously?*

Battle of sexes: outcomes



- The payoffs
- Outcome if 1 moves first
- Outcome if 2 moves first

Story

Chicken

Two tearaways drive their cars at each other:

•If neither driver swerves, both end up dead (utility level 0)

•If one driver swerves out of the way then he is regarded as "chicken" (utility level 1) and the other acquires extraordinary social esteem (utility level 3).

•If both drivers swerve, both look embarrassed (utility level 2).

Clearly, if either driver *knows* what the other will do there is an incentive to do the opposite.

Economic relevance: it is the basis of a class of problems where social and private interests are partially in opposition. An application is the provision of public goods. Suppose citizens have to decide on one of two actions:

•action [+] – contribute private resources to provision of an essential public good.

•action [–] – not to contribute any resources.

Each citizen wants to see that a public good is provided, even at his own (partial) expense. Each knows that if no resources are provided there is no public good – outcome (0,0). He would prefer it if someone else pays for the public good and he can "free ride" – outcome (3,1). But if he knows that no-one else will pay for the good he is willing to do it himself.

n”

	[+]	[–]
	2,2	1,3
	3,1	0,0

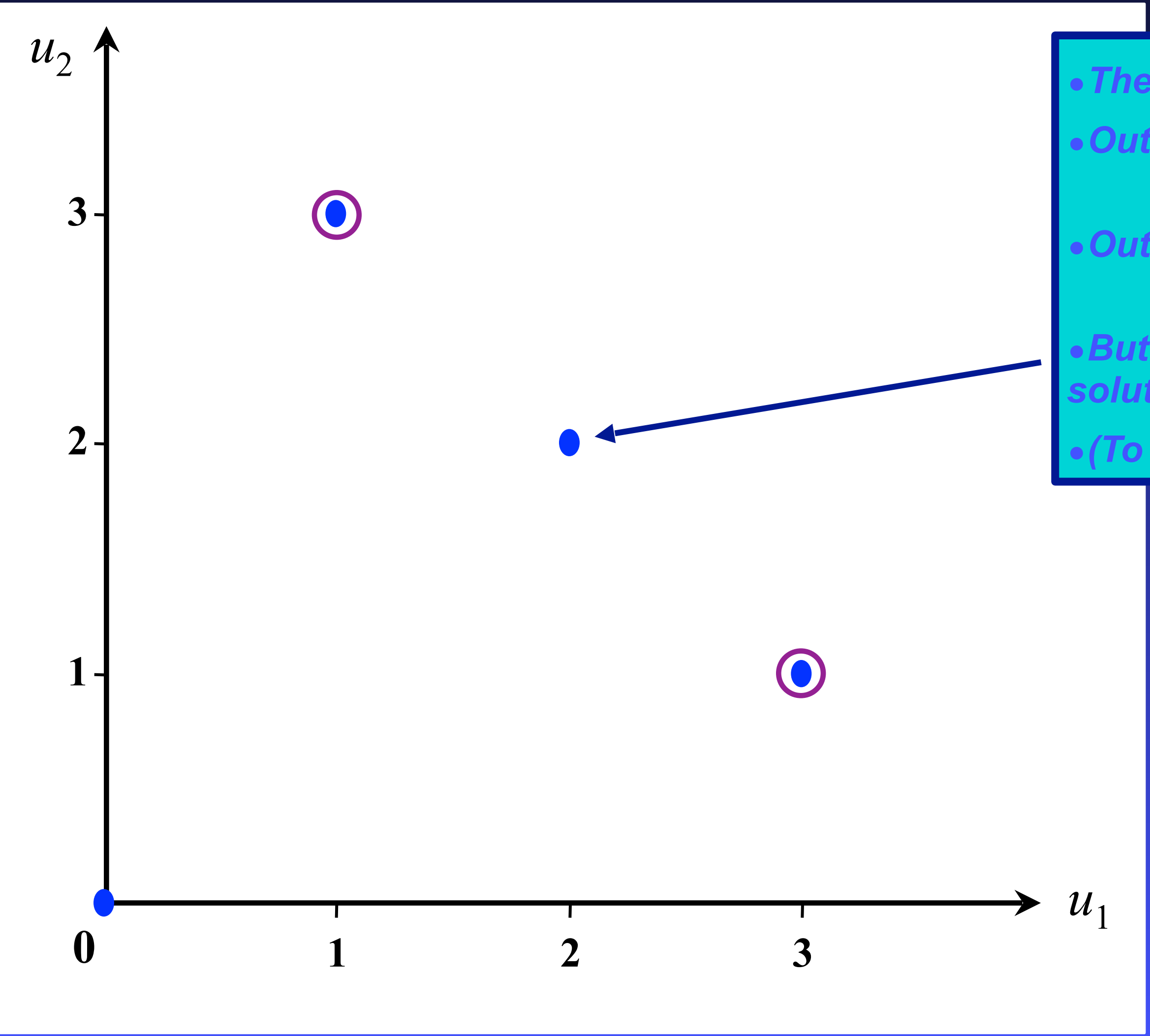
Player 2

- Another “contribution game”
- Each chooses [+] or [–].
- Payoffs, given as (u_1, u_2)
- If 2 plays [+] then 1 gets more by playing [–]
- But if 2 plays [–] then 1 does better by playing [+]
- Again no elimination

r has “first move” outcome is (3,1) or (1,3)

ppens in the case of a simultaneous move?

Chicken: outcomes



- *The payoffs*
- *Outcome if 1 moves first*
- *Outcome if 2 moves first*
- *But why isn't this a solution…?*
- *(To be continued…)*

Review: basic concepts

Review
- Actions
  - Must specify carefully the action space

Review
- Order of play
  - Simultaneous or sequential?

Review
- Representation of the game
  - Extensive form
  - Normal form

Review
- Elimination
  - First steps toward a solution

What next?

- Introduce strategy
- The role of information – a first look
- Examine the meaning of rationality
- Formalise equilibrium
- This is handled in *Game Theory: Strategy and Equilibrium*