Prerequisites

Almost essential
Firm: Optimisation

Frank Cowell: *Microeconomics*

The Firm: Demand and Supply

MICROECONOMICS
Principles and Analysis
Frank Cowell

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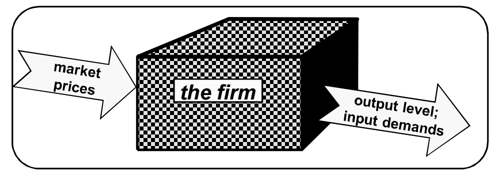
Moving on from the optimum...

- We derive the firm's reactions to changes in its environment.
- These are the *response functions*.
  - We will examine three types of them
  - Responses to different types of market events.
- In effect we treat the firm as a Black Box.



The firm as a "black box"

- Behaviour can be predicted by necessary and sufficient conditions for optimum.
- The FOC can be solved to yield behavioural response functions.
- Their properties derive from the solution function.
- We need the solution function's properties…
- …again and again.

Overview...

Firm: Comparative Statics

- Conditional Input Demand
- Output Supply
- Ordinary Input Demand
- Short-run problem

Response function for stage 1 optimisation

The first response function

- Review the cost-minimisation problem and its solution
- Choose **z** to minimise $\sum_{i=1}^{m} w_i z_i$ subject to $q \leq \phi(\mathbf{z}),\ \mathbf{z} \geq \mathbf{0}$
- The firm's cost function: $C(\mathbf{w}, q) := \min_{\{\phi(\mathbf{z}) \geq q\}} \sum w_i z_i$
- Cost-minimising value for each input:

$$\mathbf{z}_i^* = H^i(\mathbf{w}, q),\ i=1,2,\ldots,m$$

may be a well-defined function or may be a correspondence

vector of input prices

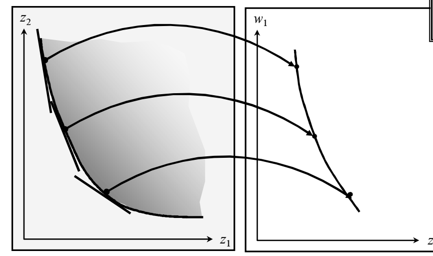
Specified output level

- *The "stage 1" problem*
- *The solution function*
- *H^i is the conditional input demand function.*
- *Demand for input i, conditional on given output level q*

A graphical approach

Mapping into (z_1,w_1)-space

- *Conventional case of Z.*
- *Start with any value of w_1 (the slope of the tangent to Z).*
- *Repeat for a lower value of w_1.*
- *...and again to get...*
- *...the conditional demand curve*



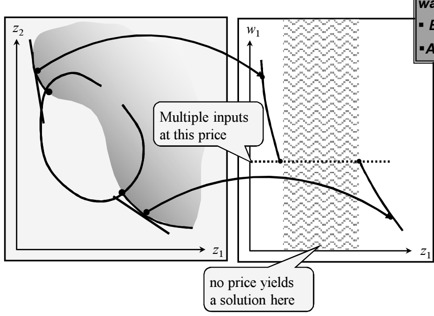
- *Constraint set is convex, with smooth boundary*
- *Response function is a continuous map:*

$H^1(\mathbf{w},q)$

Now try a different case

Another map into (z_1,w_1)-space

- *Now take case of nonconvex Z.*
- *Start with a high value of w_1.*
- *Repeat for a very low value of w_1.*
- *Points "nearby" work the same way.*
- *But what happens in between?*
- *A demand correspondence*



- *Constraint set is nonconvex.*
- *Response is a discontinuous map: jumps in z**
- *Map is multivalued at the discontinuity*

Conditional input demand function

- Assume that single-valued input-demand functions exist.
- How are they related to the cost function?
- What are their properties?
- How are they related to properties of the cost function?

Do you remember these...?

Link to cost function

Using the cost function

The slide: $\frac{\partial C(\mathbf{w}, q)}{\partial w_i}$

Optimal demand for input i

conditional input demand function

- Recall this relation
  $C_i(\mathbf{w}, q) = z_i^*$
- So we have:
  $C_i(\mathbf{w}, q) = H^i(\mathbf{w}, q)$

Second derivative

- Differentiate with respect to w_j
  $C_{ij}(\mathbf{w}, q) = H_j^i(\mathbf{w}, q)$

▪ *...yes, it's Shephard's lemma*

▪ *Link between conditional input demand and cost functions*

▪ *Slope of input demand function*

Two simple results:

Simple result 1

- Use a standard property
  $$\frac{\partial^2(\bullet)}{\partial w_i \partial w_j} = \frac{\partial^2(\bullet)}{\partial w_j \partial w_i}$$
- So in this case
  $C_{ij}(\mathbf{w}, q) = C_{ji}(\mathbf{w}, q)$
- Therefore we have:
  $H_j^i(\mathbf{w}, q) = H_i^j(\mathbf{w}, q)$

▪ *second derivatives of a function "commute"*

▪ *The order of differentiation is irrelevant*

▪ *The effect of the price of input i on conditional demand for input j equals the effect of the price of input j on conditional demand for input i.*

Simple result 2

- Use the standard relationship:
  $C_{ij}(\mathbf{w}, q) = H_j^i(\mathbf{w}, q)$
- We can get the special case:
  $C_{ii}(\mathbf{w}, q) = H_i^i(\mathbf{w}, q)$
- Because cost function is concave:
  $C_{ii}(\mathbf{w}, q) \leq 0$
- Therefore:
  $H_i^i(\mathbf{w}, q) \leq 0$

▪ *Slope of conditional input demand function derived from second derivative of cost function*

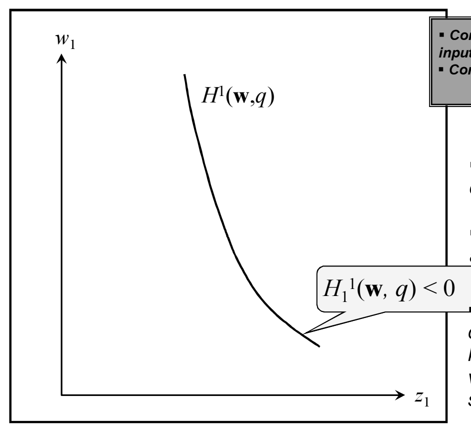
▪ *We've just put j=i*

▪ *A general property*

▪ *The relationship of conditional demand for an input with its own price cannot be positive.*

and so...

Conditional input demand curve



▪ **Consider the demand for input 1**
▪ **Consequence of result 2?**

▪ *"Downward-sloping" conditional demand*

▪ *In some cases it is also possible that* $H_i^i=0$

▪ *Corresponds to the case where isoquant is kinked: multiple* ***w*** *values consistent with same* ***z****.

Link to "kink" figure

For the conditional demand function...

- Nonconvex Z yields discontinuous H
- Cross-price effects are symmetric
- Own-price demand slopes downward.

- (exceptional case: own-price demand could be constant)

Overview...

Response function for stage 2 optimisation

- Firm: Comparative Statics
  - Conditional Input Demand
  - **Output Supply**
  - Ordinary Input Demand
  - Short-run problem

The second response function

- Review the profit-maximisation problem and its solution
- Choose q to maximise: — ▪*The "stage 2" problem*

  $pq - C(\mathbf{w}, q)$
- From the FOC:

  $p \leq C_q(\mathbf{w}, q^*)$ — ▪*"Price equals marginal cost"*

  $pq^* \geq C(\mathbf{w}, q^*)$ — ▪*"Price covers average cost"*
- profit-maximising value for output:

  $q^* = S(\mathbf{w}, p)$ — ▪ *S is the supply function*

  (input prices; output price)

▪*(again it may actually be a correspondence)*

Supply of output and output price

- Use the FOC: — ▪*"marginal cost equals price"*

  $C_q(\mathbf{w}, q) = p$
- Use the supply function for q: — ▪*Gives an equation in w and p*

  $C_q(\mathbf{w}, S(\mathbf{w}, p)) = p$
- Differentiate with respect to p — ▪*Use the "function of a function" rule*

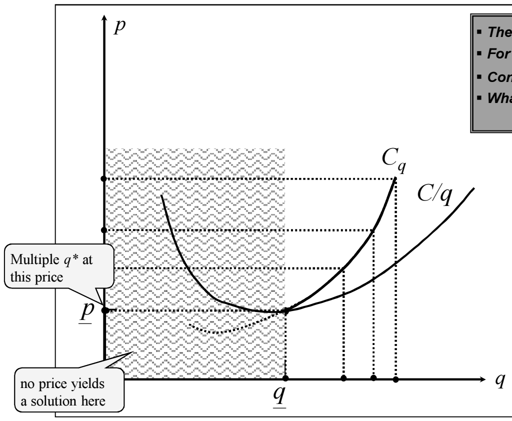
  $C_{qq}(\mathbf{w}, S(\mathbf{w}, p))\, S_p(\mathbf{w}, p) = 1$

  (Differential of S with respect to p)
- Rearrange: — ▪*...e of the supply function.*

  $$S_p(\mathbf{w}, p) = \frac{1}{C_{qq}(\mathbf{w}, q)}.$$

  (Positive if MC is increasing.)

The firm's supply curve



- The firm's AC and MC curves.
- For given p read off optimal q*
- Continue down to $\underline{p}$
- What happens below $\underline{p}$

- *Supply response is given by $q=S(\mathbf{w},p)$*
- *Case illustrated is for ϕ with first IRTS, then DRTS. Response is a <u>discontinuous</u> map: jumps in q**
- *Map is multivalued at the discontinuity*

Supply of output and price of input j

- Use the FOC:

$$C_q(\mathbf{w}, S(\mathbf{w}, p)) = p$$

- *Same as before: "price equals marginal cost"*

- Differentiate with respect to w_j

$$C_{qj}(\mathbf{w}, q^*) + C_{qq}(\mathbf{w}, q^*)\, S_j(\mathbf{w}, p) = 0$$

- *Use the "function of a function" rule again*

- Rearrange:

$$S_j(\mathbf{w}, p) = -\frac{C_{qj}(\mathbf{w}, q^*)}{C_{qq}(\mathbf{w}, q^*)}$$

Remember, this is positive

- *Supply of output must fall with w_j if marginal cost increases with w_j.*

For the supply function...

- Supply curve slopes upward.
- Supply decreases with the price of an input, if MC increases with the price of that input.
- Nonconcave ϕ yields discontinuous S.
- IRTS means ϕ is nonconcave and so S is discontinuous.

Overview...

Response function for combined optimisation problem

- Firm: Comparative Statics
  - Conditional Input Demand
  - Output Supply
  - **Ordinary Input Demand**
  - Short-run problem

The third response function

- Recall the first two response functions:

$$z_i^* = H^i(\mathbf{w},q)$$

▪*Demand for input i, conditional on output q*

$$q^* = S(\mathbf{w}, p)$$

▪*Supply of output*

- Now substitute for q^* :

$$z_i^* = H^i(\mathbf{w}, S(\mathbf{w}, p))$$

▪*Stages 1 & 2 combined…*

- Use this to define a new function:

$$D^i(\mathbf{w},p) := H^i(\mathbf{w}, S(\mathbf{w}, p))$$

input prices

output price

▪*Demand for input i (unconditional)*

▪*Use this relationship to analyse further the firm's response to price changes*

Demand for *i* and the price of output

- Take the relationship

$$D^i(\mathbf{w}, p) = H^i(\mathbf{w},$$

"function of a function" rule again

- Differentiate with respect to p:

$$D_p^i(\mathbf{w}, p) = H_q^i(\mathbf{w}, q^*)\, S_p(\mathbf{w}, p)$$

▪*D^i increases with p iff H^i increases with q. Reason? Supply increases with price (S_p>0).*

- But we also have, for any q:

$$H^i(\mathbf{w}, q) = C_i(\mathbf{w}, q)$$
$$H_q^i(\mathbf{w}, q) = C_{iq}(\mathbf{w}, q)$$

▪*Shephard's Lemma again*

- Substitute in the above:

$$D_p^i(\mathbf{w}, p) = C_{qi}(\mathbf{w}, q^*) S_p(\mathbf{w}, p)$$

▪*Demand for input i (D^i) increases with p iff marginal cost (C_q) increases with w_i.*

Demand for *i* and the price of *j*

- Again take the relationship

$$D^i(\mathbf{w}, p) = H^i(\mathbf{w}, S(\mathbf{w}, p)).$$

- Differentiate with respect to w_j:

"function of a function" rule yet again

$$D_j^i(\mathbf{w}, p) = H_j^i(\mathbf{w}, q^*) + H_q^i(\mathbf{w}, q^*) S_j(\mathbf{w}, p)$$

- Use Shephard's Lemma again:

$$H_q^i(\mathbf{w}, q) = C_{iq}(\mathbf{w}, q) = C_{qi}(\mathbf{w}, q)$$

- Use this and the previous r[illegible] p) to give a decomposition into a "substitu[illegible] effect" and an "output effect":

"substitution effect"

"output effect"

$D_j^i(\mathbf{w}, p) =$ [redacted]

Results from decomposition formula

- Take the general relationship:

$D_j^i(\mathbf{w}, p) =$ [redacted]

We already know this is symmetric in i and j.

Obviously symmetric in i and j.

▪*The effect w_i on demand for input j equals the effect of w_j on demand for input i.*

- Now take the special case where $j = i$:

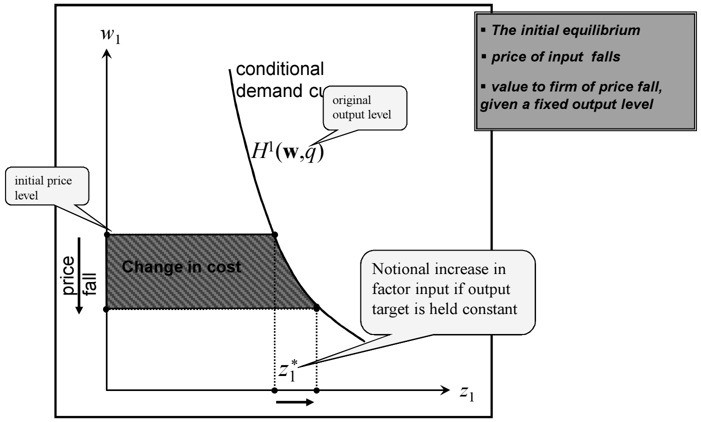
$D_i^i(\mathbf{w}, p) =$ [redacted]

We already know this is negative or zero.

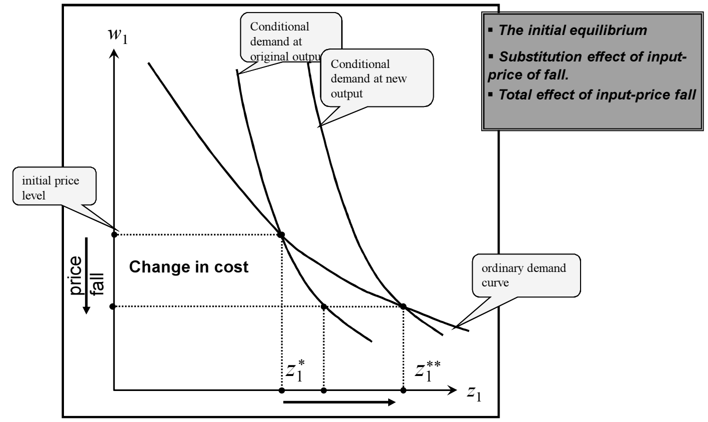
cannot be positive.

▪*If w_i increases, the demand for input i cannot rise.*

Input-price fall: substitution effect



Input-price fall: total effect



The ordinary demand function...

- Nonconvex *Z* may yield a discontinuous *D*
- Cross-price effects are symmetric
- Own-price demand slopes downward
- Same basic properties as for *H* function

Overview...

Optimisation subject to side-constraint

- Firm: Comparative Statics
  - Conditional Input Demand
  - Output Supply
  - Ordinary Input Demand
  - **Short-run problem**

The short run...

- This is not a moment in time but…
- … is defined by additional constraints within the model
- Counterparts in other economic applications where we sometimes need to introduce side constraints

The short-run problem

- We build on the firm's standard optimisation problem
- Choose q and $\mathbf{z}$ to maximise

$$\Pi := pq - \sum_{i=1}^{m} w_i z_i$$

- subject to the standard constraints:

$$q \le \phi(\mathbf{z})$$

$$q \ge 0, \mathbf{z} \ge \mathbf{0}$$

- But we add a *side condition* to this problem:

$$z_m = \bar{z}_m$$

- Let $\bar{q}$ be the value of q for which $z_m = \bar{z}_m$ would have been freely chosen in the unrestricted cost-min problem…

The short-run cost function

$$\tilde{C}(\mathbf{w}, q, \bar{z}_m) := \min_{\{z_m = \bar{z}_m\}} \Sigma\, w_i z_i$$

- *The solution function with the side constraint.*

- Short-run demand for input i:

$$\tilde{H}^i(\mathbf{w}, q, \bar{z}_m) = \tilde{C}_i(\mathbf{w}, q, \bar{z}_m)$$

- *Follows from Shephard's Lemma*

- Compare with the ordinary cost function

$$C(\mathbf{w}, q) \le \tilde{C}(\mathbf{w}, q, \bar{z}_m)$$

- *By definition of the cost function. We have "=" if $q = \bar{q}$.*

- So, dividing by q:

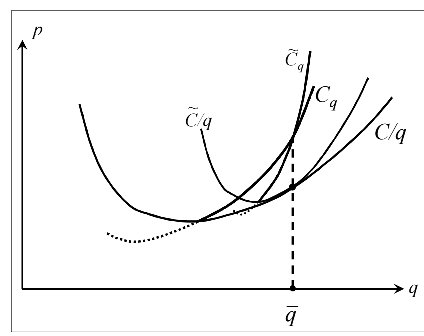
$$\frac{C(\mathbf{w}, q)}{q} \le \frac{\tilde{C}(\mathbf{w}, q, \bar{z}_m)}{q}$$

- *Short-run AC ≥ long-run AC. SRAC = LRAC at $q = \bar{q}$*

Supply curves

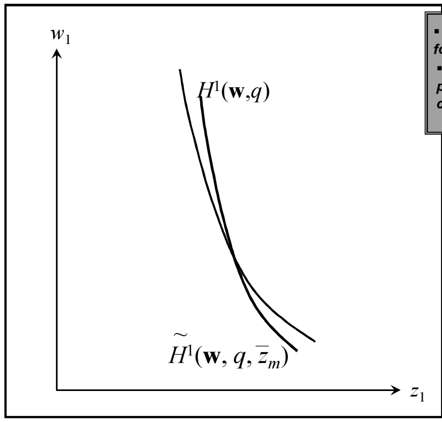
MC, AC and supply in the short and long run

- *AC if all inputs are variable*
- *MC if all inputs are variable*
- *Fix an output level.*
- *AC if input m is now kept fixed*
- *MC if input m is now kept fixed*
- *Supply curve in long run*
- *Supply curve in short run*



- *SRAC touches LRAC at the given output*
- *SRMC cuts LRMC at the given output*
- *The supply curve is steeper in the short run*

Conditional input demand



- *The original demand curve for input 1*
- *The demand curve from the problem with the side constraint.*

- *"Downward-sloping" conditional demand*
- *Conditional demand curve is steeper in the short run.*

Key concepts

- Basic functional relations
- price signals → <u>firm</u> → input/output responses

Review	• $H^i(\mathbf{w},q)$	demand for input *i*, conditional on output
Review	• $S(\mathbf{w},p)$	supply of output
Review	• $D^i(\mathbf{w},p)$	demand for input *i* (unconditional)

And they all hook together like this:

• $H^i(\mathbf{w}, S(\mathbf{w},p)) = D^i(\mathbf{w},p)$

What next?

- Analyse the firm under a variety of <u>market conditions</u>.
- Apply the analysis to the <u>consumer's optimisation problem</u>.