

Problem Set 1

1. Show that if $f : R \rightarrow R$ is a strictly increasing function and $u : X \rightarrow R$ is a utility function representing preference relation \succeq , then the function $v : X \rightarrow R$ defined as $v(x) = f(u(x))$ is also a utility function representing preference relation \succeq .
2. Prove that if \succeq is rational and $x \succ y \succeq z$, then $x \succ z$.
3. Let $X = \{x, y, z\}$ and consider the choice structure $(\beta, C(\cdot))$ with

$$\beta = (\{x, y\}, \{y, z\}, \{x, z\}, \{x, y, z\})$$
 And $C(\{x, y\}) = \{x\}$, $C(\{y, z\}) = \{y\}$, $C(\{x, z\}) = \{z\}$, show that $(\beta, C(\cdot))$ violates the WARP.
4. If the WARP holds for the Walrasian demand function $x(p, w)$ then demand should be homogeneous of degree 0 in prices.
5. Assume a consumer having positive wealth w in an economy with two goods only, with prices $p_1 > 0$ and $p_2 > 0$.
 - a) Draw the budget set $B_{p, w}$ and choose a point for consumption \underline{x} that satisfies Walras' Law and the requirements $x_1 > 0$ and $x_2 > 0$. What is the slope of the budget line?
 - b) Suppose that the prices change from \underline{p} to \underline{p}' , $p_1 = p_1'$ and $p_2' > p_2$, and that the new budget line passes through \underline{x} . Draw $B_{p', w'}$ and the new budget set on the same figure you made for (a). What is the new level of wealth w' ? Show Δw on your figure.
 - c) Indicate (on the figure you have drawn) all the points for consumption under prices \underline{p}' and wealth w' that agree with the WARP (they do not necessarily have to satisfy Walras' law).
6. In an economy with two goods, suppose that in period 1 there is equal demand for both goods (i.e. $x_1(p_1, p_2, w) = x_2(p_1, p_2, w)$). Then, in period 2, the price of good 1 increases from p_1 to p_1' , while the price of good 2 decreases from p_2 to p_2' .
 - a) If the Slutsky wealth compensation for this change in prices is zero, find the relation between Δp_1 and Δp_2 . Then show on a diagram the locations of demand for period 2 that do not violate the weak axiom of revealed preference.
 - b) Suppose that in period 3 there is no change in wealth, the price of good 1 further increases to p_1'' , while the price of good 2 remains unchanged (equal to p_2'). Use the diagram you made for (a) to show that any demand in period 3 is consistent with the weak axiom of revealed preference.

7. You are given the following partial information about a consumer's purchases. He consumes only two goods.

	Year 1		Year 2	
	Quantity	Price	Quantity	Price
Good 1	100	100	120	100
Good 2	100	100	?	80

- Over what range of quantities for good 2 consumed in year 2 would you conclude that his behaviour is inconsistent (i.e. it violates the WARP)?
- Over what range of quantities for good 2 consumed in year 2 would you conclude that the consumer's consumption bundle in year 1 is revealed preferred to that in year 2?
- Over what range of quantities for good 2 consumed in year 2 would you conclude that the consumer's consumption bundle in year 2 is revealed preferred to that in year 1?

8. You observe a consumer in two situations: with an income of \$100 he buys 5 units of good 1 at a price of \$10 per unit and 10 units of good 2 at a price of \$5 per unit. With an income of \$175 he buys 3 units of good 1 at a price of \$15 per unit and 13 units of good 2 at a price of \$10 per unit. Do the actions of this consumer conform to the basic axioms of consumer behaviour?

9. Consider a setting where $L=3$ and a consumer whose consumption set is \mathbb{R}^3 . Suppose that his demand function $x(p,w)$ is $x_1(p,w)=p_2/p_3$, $x_2(p,w)=-p_1/p_3$ and $x_3(p,w)=w/p_3$.

- Show that $x(p,w)$ is homogeneous of degree zero in (p,w) and satisfies Walras' law.
- Show that $x(p,w)$ violates the weak axiom.