LECTURE 2

MICROECONOMIC THEORY
CONSUMER THEORY
Consumer Choice

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Consumer choice

(in the spirit of the choice-based approach)

Fundamental decision unit: the *consumer*

Definition (Market)
The "place" where demand and supply meet.
A setting in which consumers can buy products at known prices
(or, equivalently, trade goods at known exchange rates).

Question: How do consumers make constrained choices?

Consumer choice: decision theory when individuals face given market prices.

Consumer choice: basic concepts Commodities

- Commodities (goods and services) – what is available for purchase in the market

  - Finite number *L* of divisible goods (commodities)
  - $\mathbb{R}^L_+$ is the *commodity space*
  - $X \subset \mathbb{R}^L_+$ is the *consumption set*
  - $x \in X$ is a *consumption vector* or *consumption bundle*

Consumer choice: basic concepts Commodities

Commodities: goods and services available in an economy.

– In principle many distinctions possible, e.g. commodities consumed
  - at different time points
  - in different states of nature (e.g. umbrella with/without rain) should, in principle, be viewed as different commodities

– The extent to which aggregation – across time, space, … – may be appropriate depends:
  - on the specific economic question under consideration
  - and on the economic data to which the model is being applied

Consumer choice: basic concepts
Consumption set

- The number of commodities is finite and equal to L (indexed by $\ell = 1, \ldots, L$).
- A *commodity vector* (or *commodity bundle*) is a list of amounts of the different commodities,

$$x = \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ x_L \end{bmatrix}$$

Consumer choice: basic concepts
Consumption set

$$x = \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ x_L \end{bmatrix}$$

- With a total of L commodities, x is then a point in the L-dimensional commodity space.
- Consumption bundle may be described with a commodity bundle.
- • Notation: in this lecture, x always represents the above commodity vector, while x_i is a number that denotes the consumption of commodity i

Consumer choice: basic concepts
Consumption set

The consumption set (X): subset of the commodity space. Limitations may result from physical or institutional restrictions.

Elements of X are bundles that an individual may consume given the context's *physical constraints*.

EXAMPLES

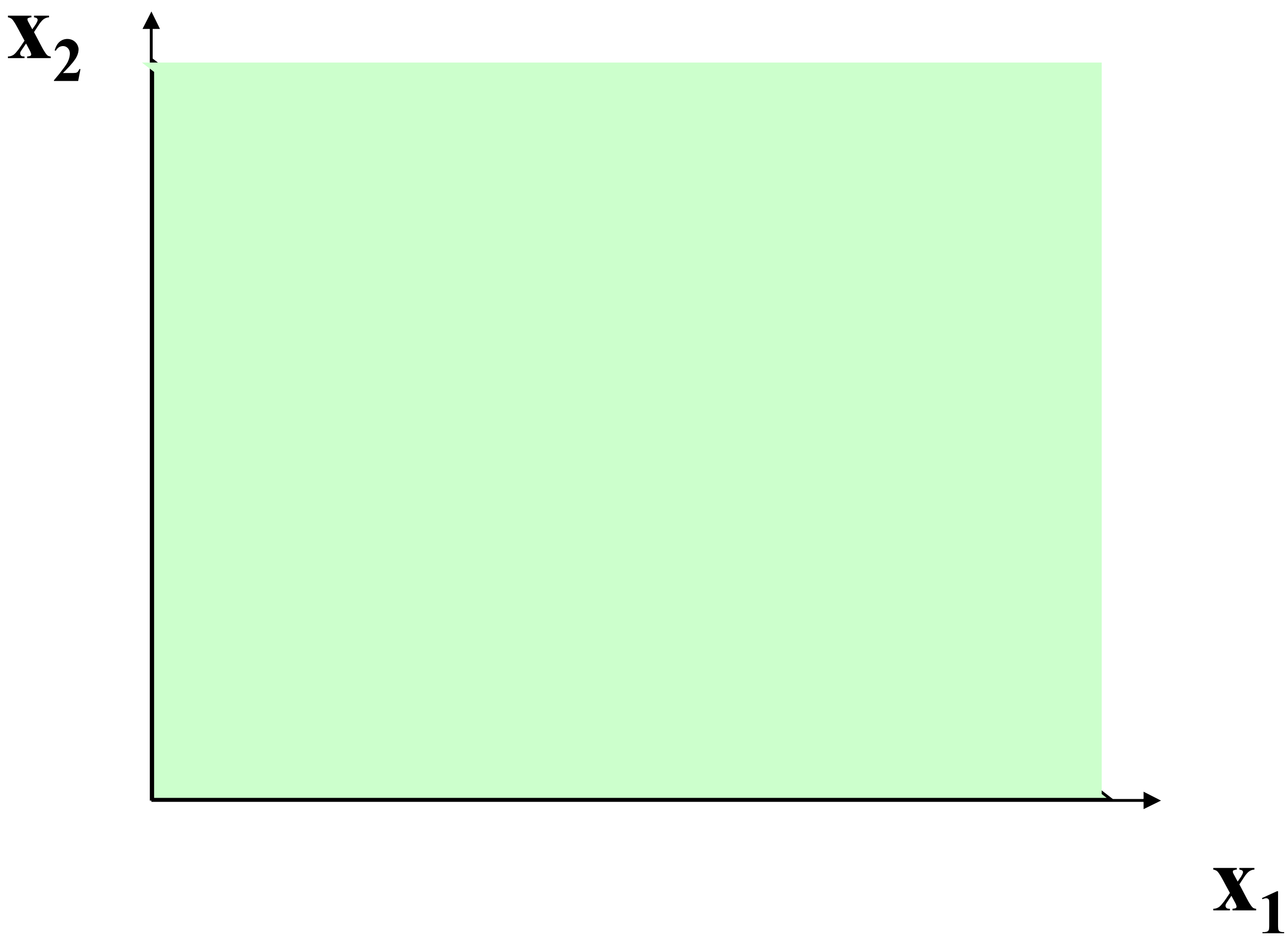
Consumption of bread and leisure: $X = \left\{(b, l) \in \mathbb{R}^2_+ : l \leq 24\right\}$

Minimum consumption of white or brown bread (survival consumption): $X = \left\{(w, b) \in \mathbb{R}^2_+ : w + b \geq 4\right\}$

$X = \mathbb{R}^L_+ = \left\{x \in \mathbb{R}^L : x_l \geq 0,\ l = 1 \ldots L\right\}$

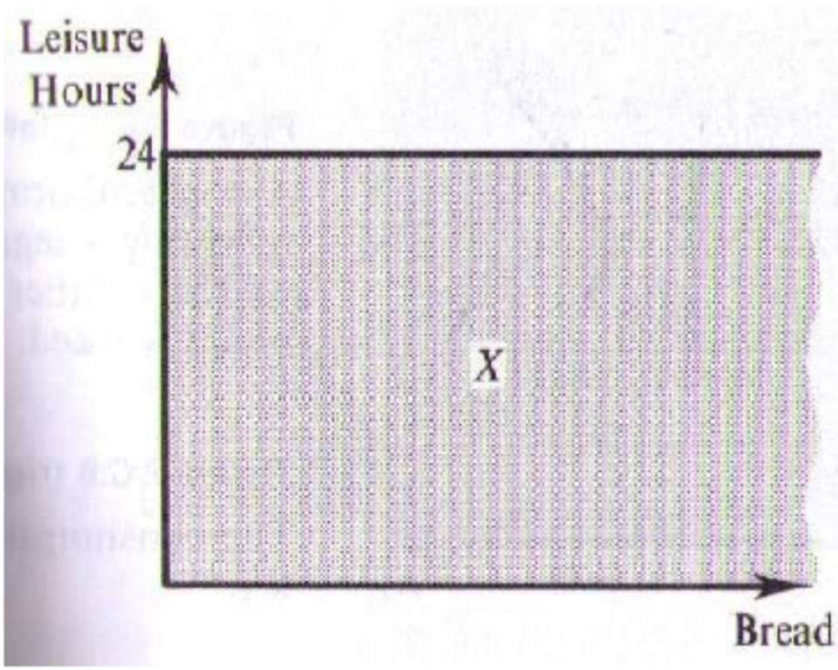
Consumer choice: basic concepts
Consumption set

Example 1: L = 2, consumption of both commodities must be non-negative.
MOST GENERAL CASE



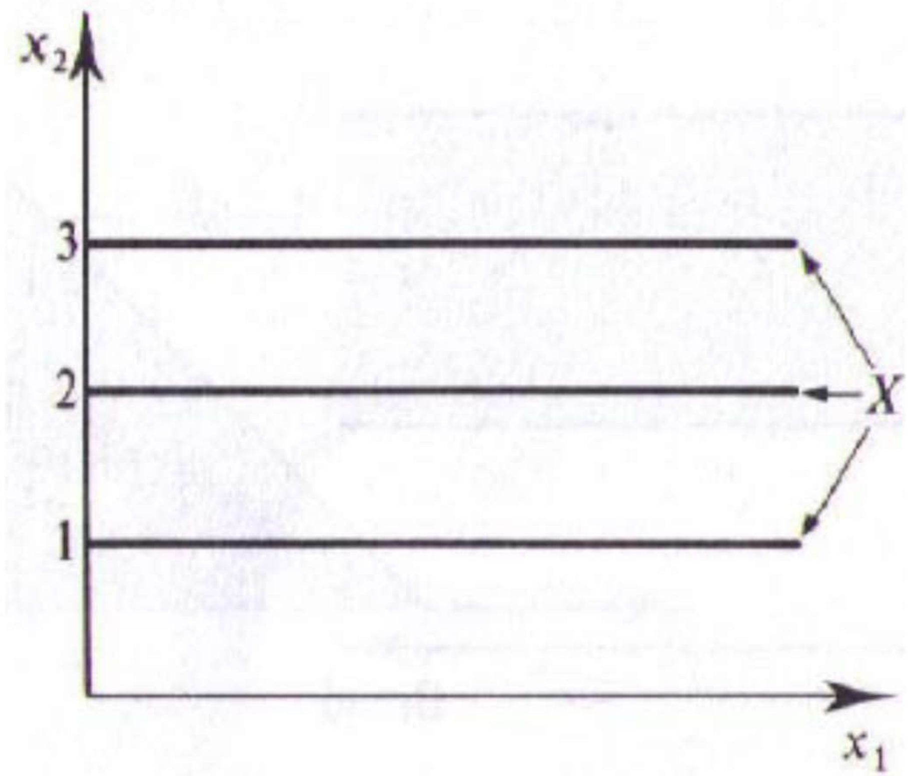
Consumer choice: basic concepts
Consumption set

Example 2: possible consumption levels of bread and leisure in a day. Both levels must be non-negative, consumption ≤ 24 for leisure



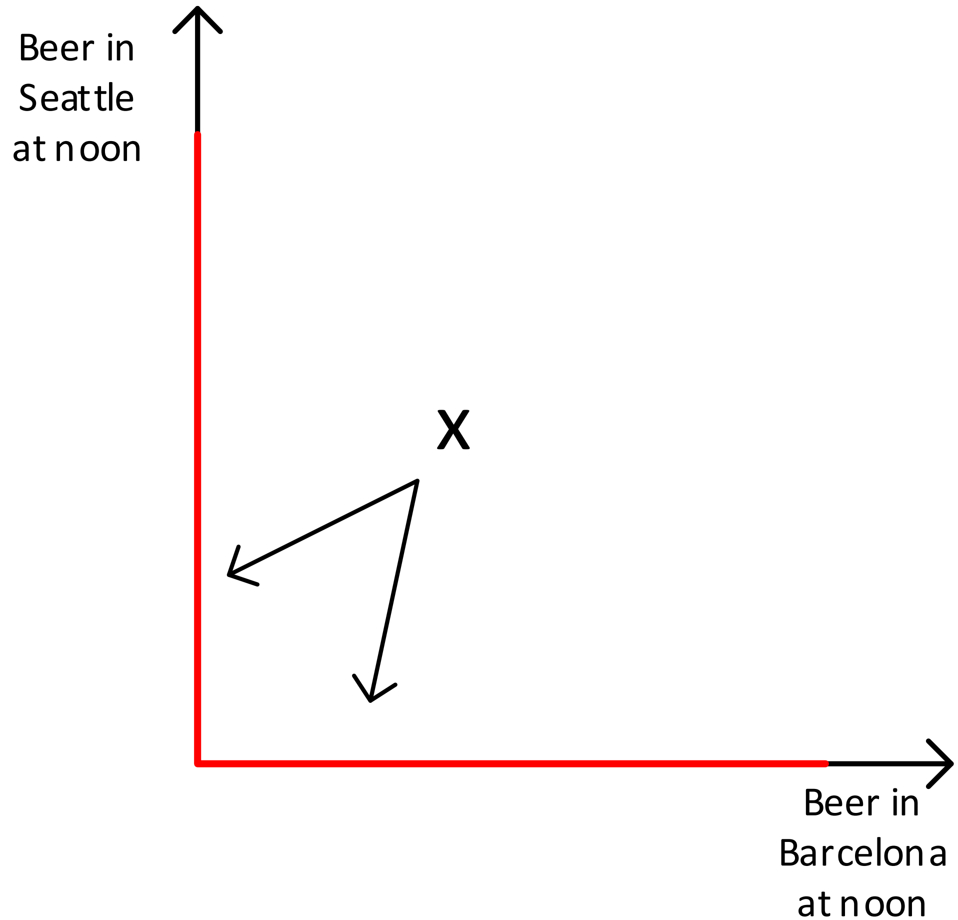
Consumer choice: basic concepts
Consumption set

Example 3: good 1 is perfectly divisible, but consumption of good 2 only in nonnegative integer amounts.



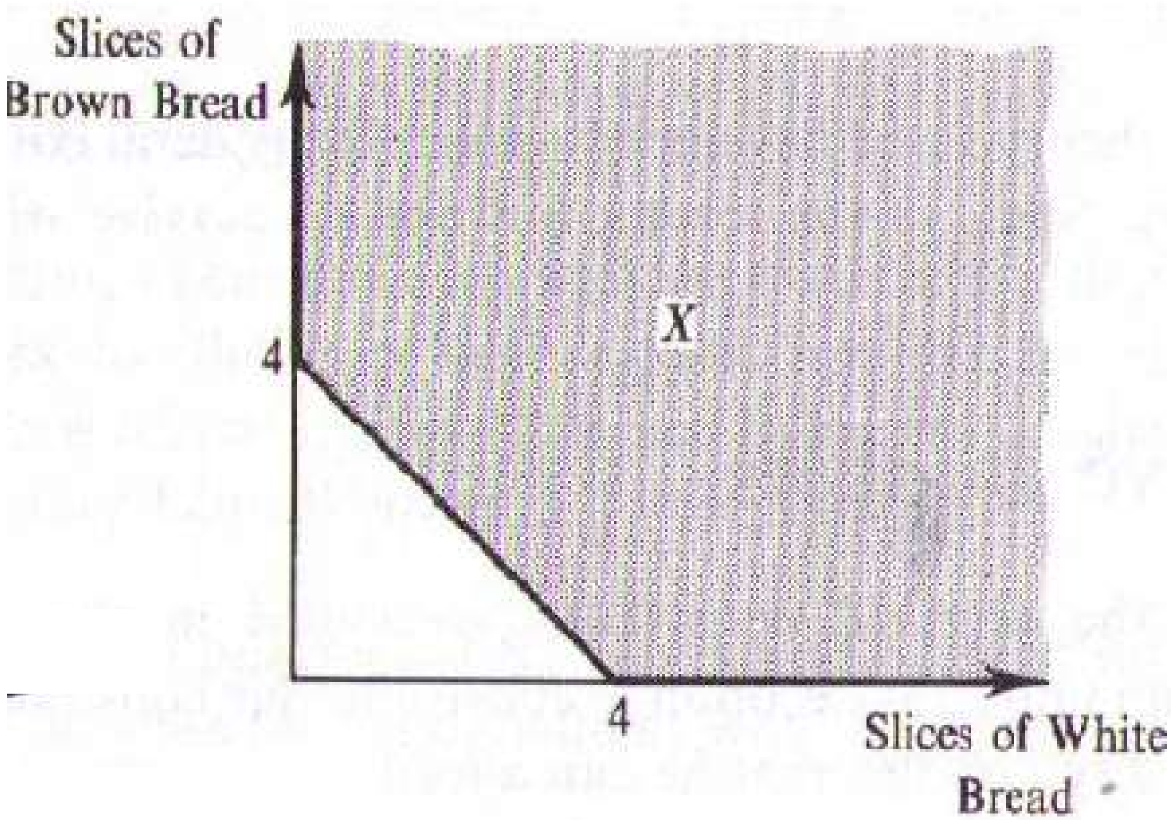
Consumer choice: basic concepts
Consumption set

Example 4: consumption of one good may make consumption of another good impossible (you cannot drink beer at the same time in Seattle and in Barcelona).



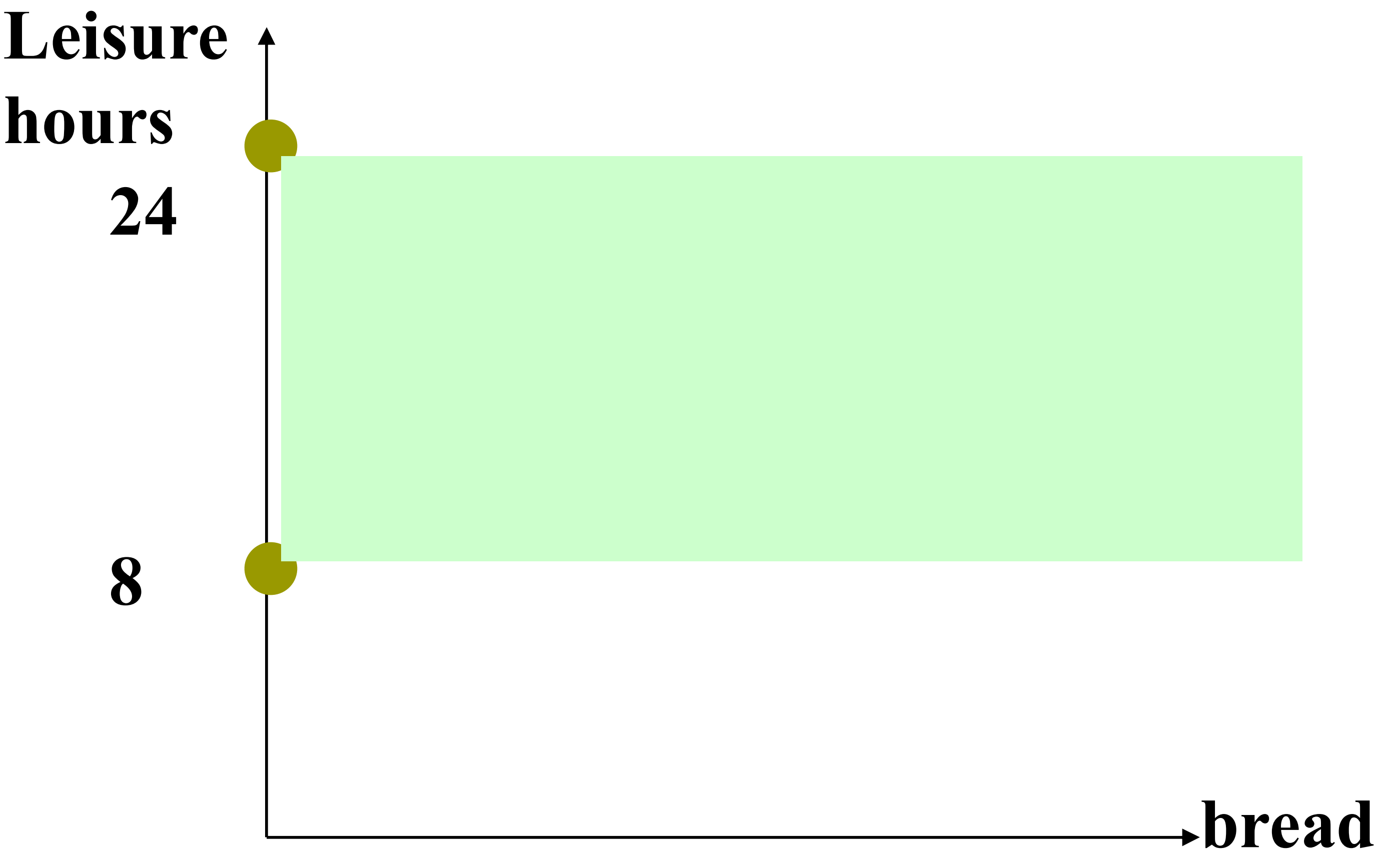
Consumer choice: basic concepts
Consumption set

Example 5: the consumer requires a minimum of 4 slices of bread a day to survive and there are two types of bread, white and brown.

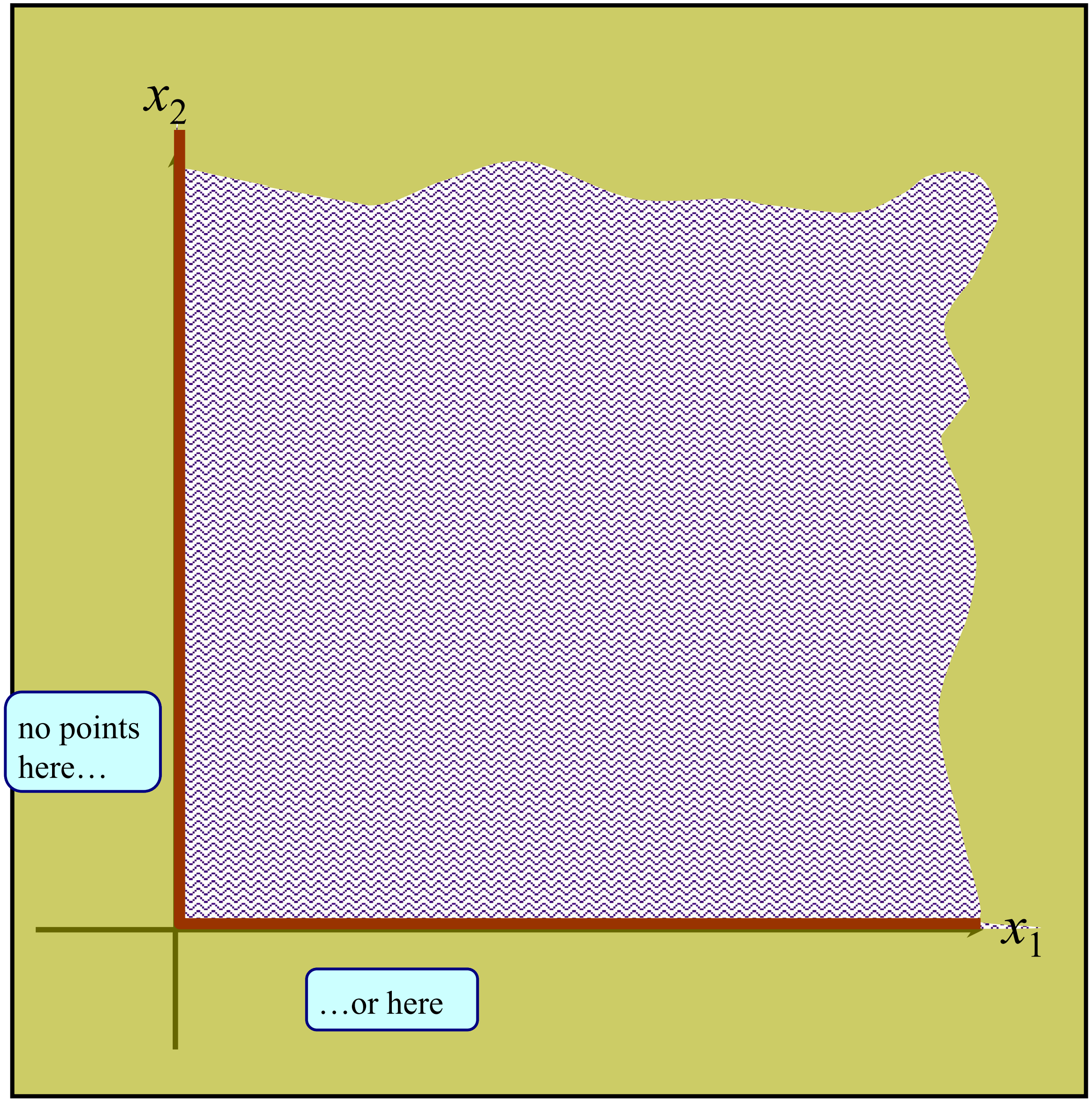


Consumer choice: basic concepts
Consumption set

The above are physical constraints. We could also have institutional constraints (e.g. you cannot work more than 16 hours a day). Example 2 would change to :



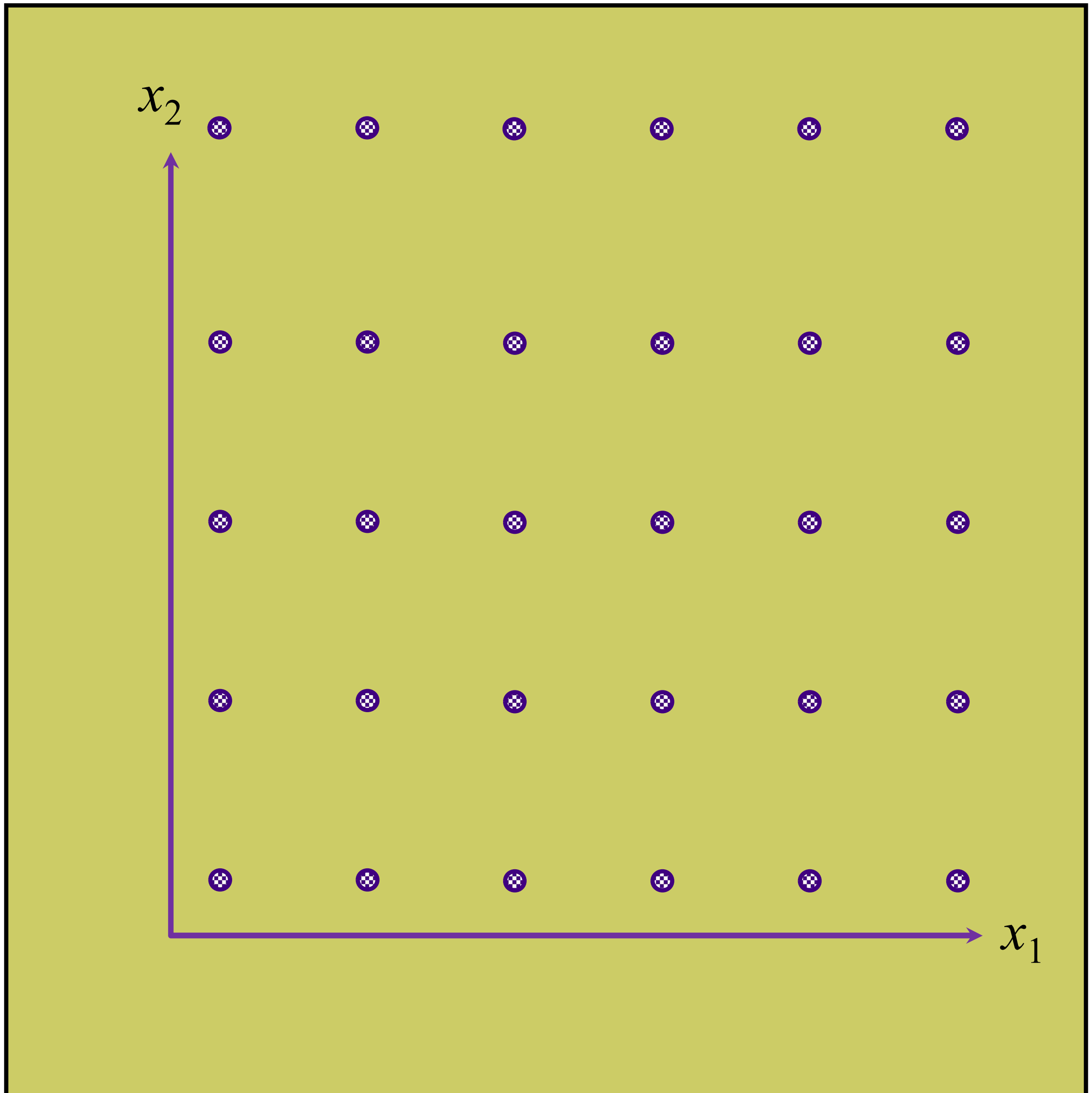
THE SET X: STANDARD ASSUMPTIONS



- **_Axes indicate quantities of the two goods x_1 and x_2._**
- **_Usually assume that X consists of the whole non-negative orthant._**
- **_Zero consumptions make good economic sense_**
- **_But negative consumptions ruled out by definition_**

- *Consumption goods are (theoretically) divisible…*
- *…and indefinitely extendable…*
- *But only in the ++ direction*

RULES OUT THIS CASE...



- *Consumption set X consists of a countable number of points*

- *Conventional assumption does not allow for indivisible objects.*
- *But suitably modified assumptions may be appropriate*

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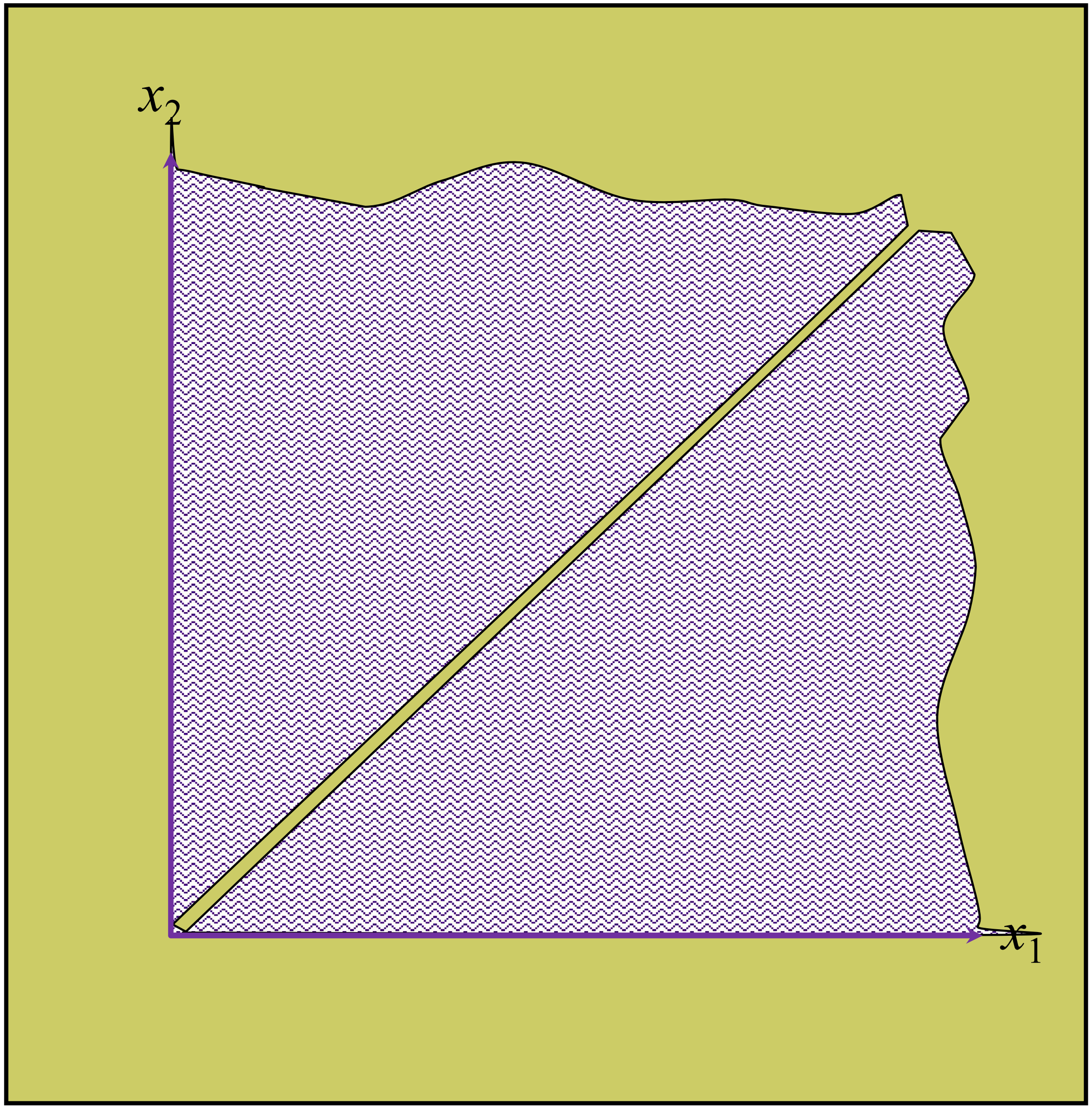
... AND THIS



- *Consumption set X has holes in it*

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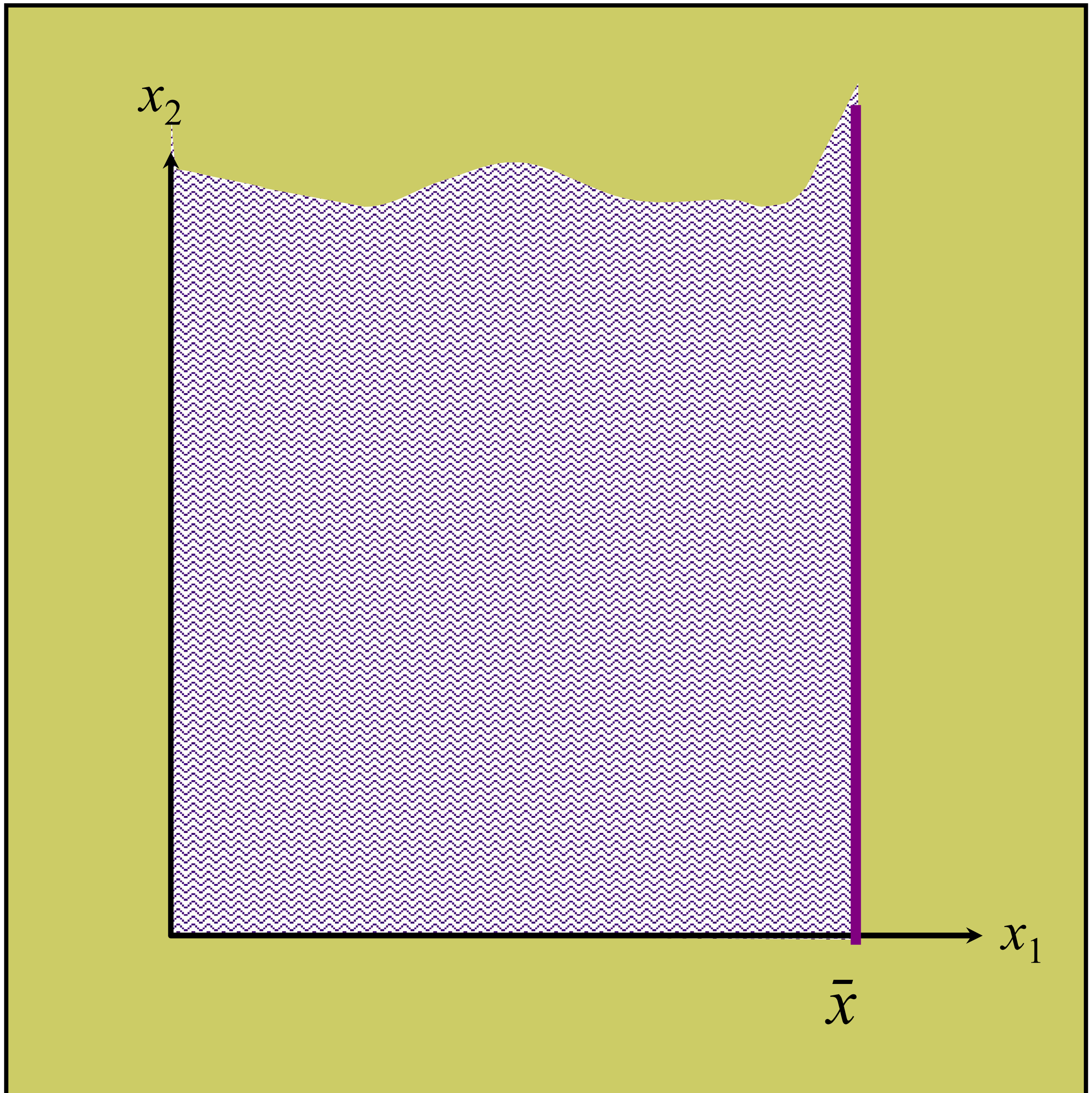
… AND THIS



- Consumption set X has the restriction $x_1 < \bar{x}$

- Conventional assumption does not allow for physical upper bounds
- But there are several economic applications where this is relevant

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Consumer choice: basic concepts
Consumption set

- For the rest of the course we will adopt the simplest and most general form of consumption set, i.e. the set of all non-negative bundles of commodities

$$X = \mathrm{R}^L_+ = \{x \in \mathrm{R}^L : x_\ell \geq 0 \; for \; \ell = 1, \ldots, L\}$$

This is a convex set: if x and x' are an element of the set R^L_+, then the bundle x'' = α x + (1 - α)x' is also an element of this set for any $\alpha \in [0,1]$.

In the following, we will usually take R^L_+ as the consumption set

note: aggregation may help to convexify the consumption set, e.g. bread consumed over a longer period in example 3.

Consumer choice: basic concepts
Consumption set

- Intuitively, a consumption set is convex if, for any two bundles that belong to the set, we can construct a straight line connecting them that lies completely within the set.

Consumer choice: basic concepts Budget set

In addition to physical constraints, the consumer also faces an *economic constraint*: his consumption choice is limited to those commodity bundles that he can *afford*.

assumption 1: commodities are traded at prices

- **Price space,** $p \in \mathbb{R}^L_+$.

$$p = \begin{bmatrix} 2 \\ 1 \\ 3 \\ 4 \end{bmatrix} \equiv \text{price vector}$$

which are publicly quoted. Completeness of markets (???)

• Notation: p always represents the above price vector,

while p_l is a number that denotes the price of commodity l

– usually we assume $p_l > 0$ for all l

– but, in principle we may have $p_l < 0$, e.g. for "bads" (e.g. pollution)

Consumer choice: basic concepts
Budget set

1) All commodities can be traded in a market, at prices that are publicly observable.
   - This is the principle of completeness of markets
   - It discards the possibility that some goods cannot be traded, such as pollution.
2) Prices are strictly positive for all L goods, i.e., $p \gg 0$ for every good k.
   - Some prices could be negative, such as pollution.

Consumer choice: basic concepts
Budget set

assumption 2: consumers are price-takers

Effectively, we assume linear prices: price per unit not a function of how much you buy.

w: a consumer's wealth level, i.e. a number (usually assumed to be strictly positive)

Price taking assumption: a consumer's demand for all L goods represents a small fraction of the total demand for the good.

The Walrasian (or competitive) budget set:

$$B_{p,w} = \{x \in \mathbb{R}_+^L : p \cdot x \leq w\}$$

= all consumption bundles that are affordable.

Consumer choice: basic concepts
Budget set

- Notation: a dot · between two vectors always represents the inner product of these two vectors. For example $p{\cdot}x$, is the number

$$p \cdot x = \begin{bmatrix} p_1 \\ \cdot \\ \cdot \\ p_L \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ x_L \end{bmatrix} = p_1 x_1 + \cdots + p_L x_L$$

- **Example:**

$$p = \begin{bmatrix} 2 \\ 1 \\ 3 \\ 4 \end{bmatrix}, \; x = \begin{bmatrix} 3 \\ 5 \\ 2 \\ 8 \end{bmatrix}$$

then

$$px = (2 \times 3) + (1 \times 5) + (3 \times 2) + (4 \times 8) = 49.$$

Consumer choice: basic concepts
Budget set

- Consumer's problem: Given p and w, "choose a consumption bundle x from $B_{p,w}$".
- When all wealth is exhausted: the set

$$\{x \in \mathbb{R}^L_+ : p \cdot x = w\}$$

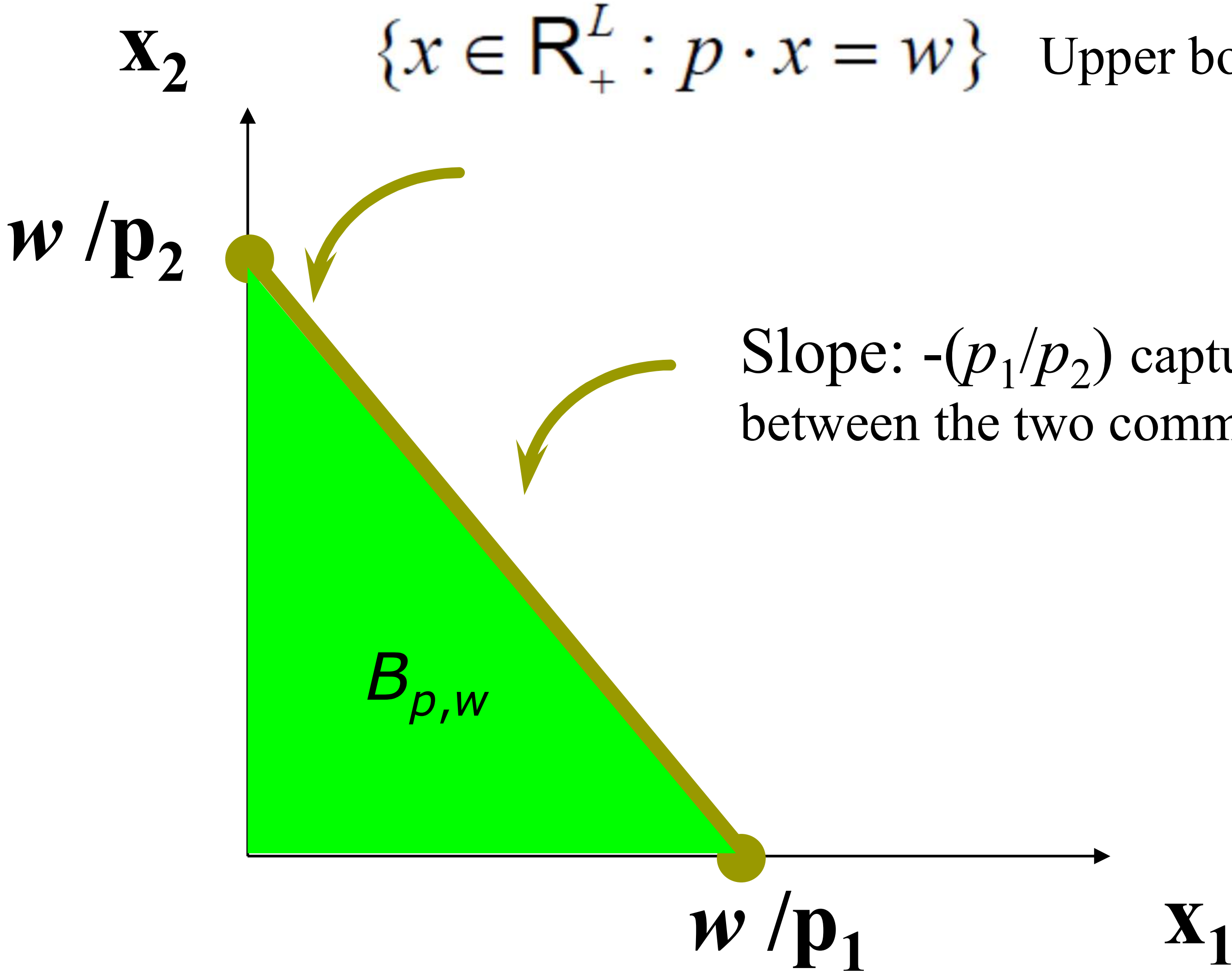
of just affordable bundles is called budget hyperplane

– If L=2 it is called the budget line.

Consumer choice: basic concepts
Budget set

Example for two goods:

$\{x \in \mathrm{R}^L_+ : p \cdot x = w\}$ Upper boundary of the budget set



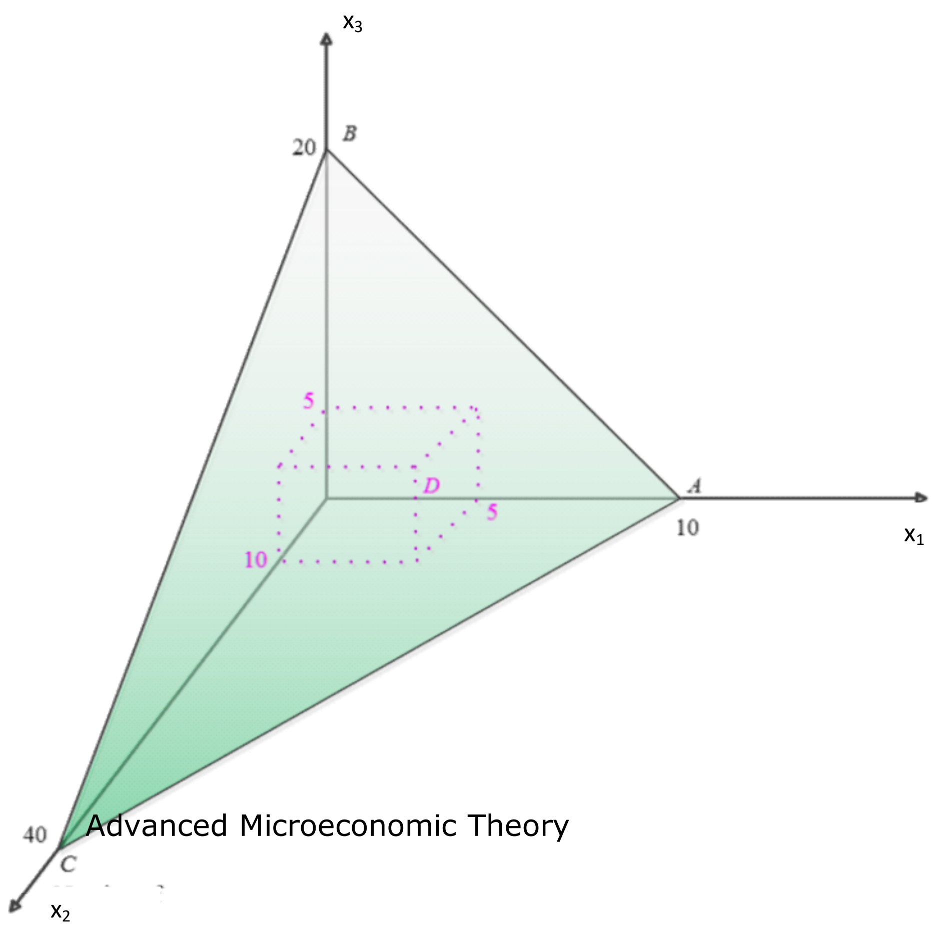
Slope: $-(p_1/p_2)$ captures the rate of exchange between the two commodities

Consumer choice: basic concepts
Budget set

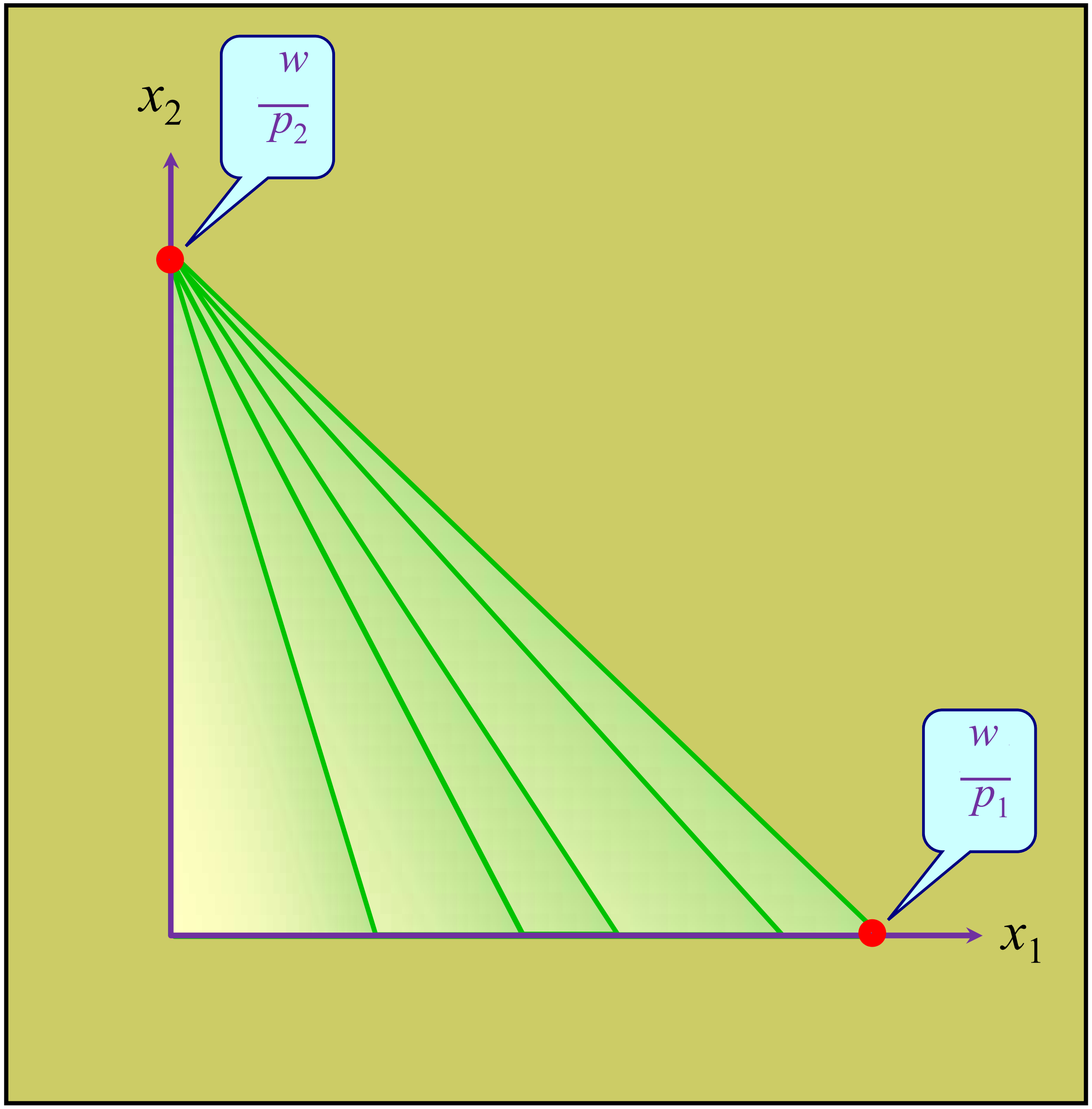
- ***Example for three goods***:

$$B_{p,w} = \{x \in \mathbb{R}^3_+ : p_1x_1 + p_2x_2 + p_3x_3 \leq w\}$$

  - The surface $p_1x_1 + p_2x_2 + p_3x_3 = w$ is referred to as the "Budget hyperplane"



CASE 1: FIXED NOMINAL INCOME



- ***Budget constraint determined by the two end-points***
- ***Examine the effect of changing p_1 by "swinging" the boundary thus…***

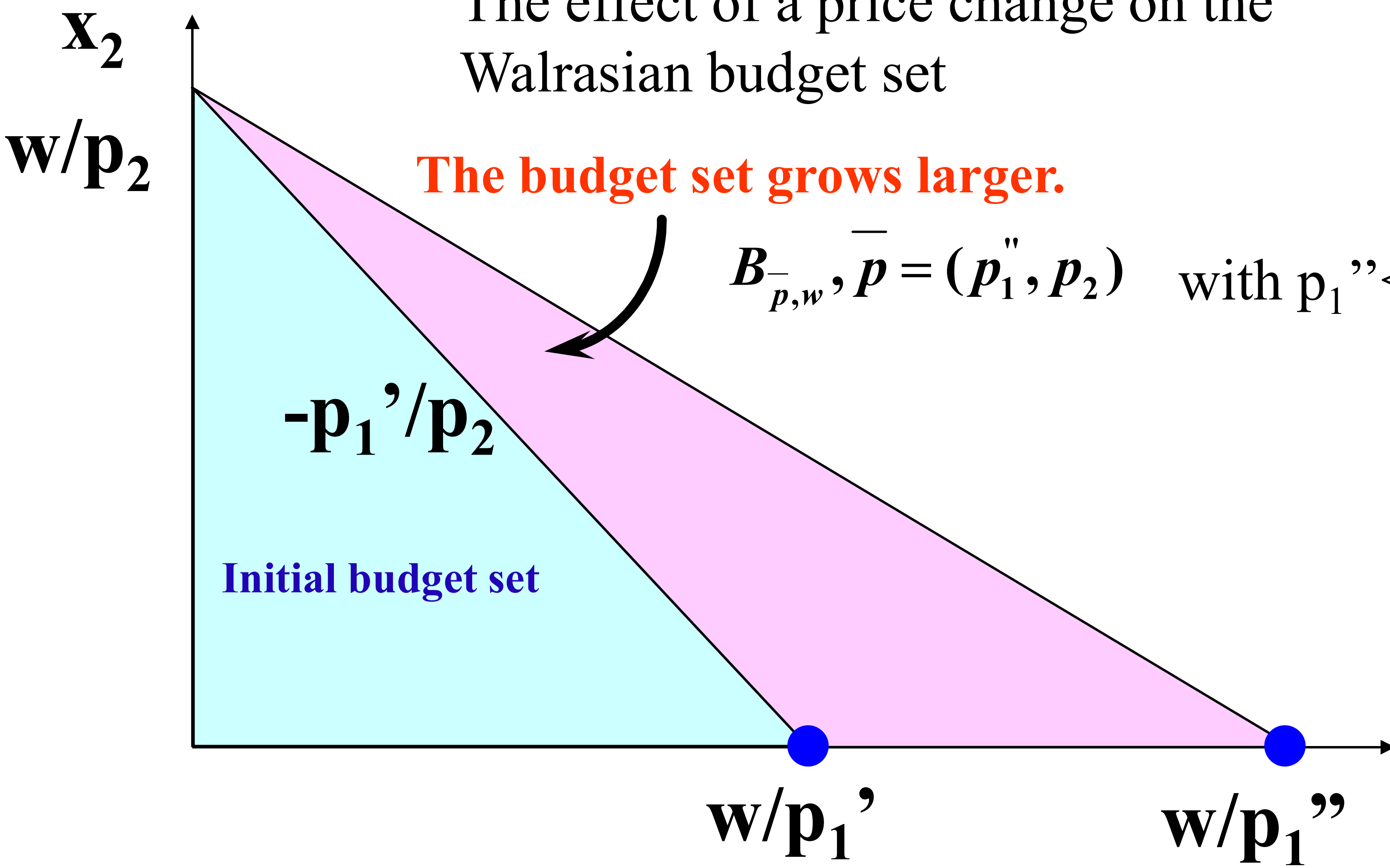
- *Budget constraint is*

$$\sum_{i=1}^{n} p_i x_i \leq w$$

Consumer choice: basic concepts example

The effect of a price change on the Walrasian budget set

$B_{\bar{p},w}, \bar{p} = (p_1'', p_2)$ with $p_1'' < p_1'$



BUDGET CONSTRAINT: KEY POINTS

- Slope of the budget constraint given by price ratio.
- There is more than one way of specifying "income":
  - Determined exogenously as an amount y.
  - Determined endogenously from resources.
- The exact specification can affect behaviour when prices change.
  - Take care when income is endogenous.
  - Value of income is determined by prices.

Consumer choice: basic concepts
Budget set

The Walrasian budget set is convex.

Let x" = ax+(1-a)x'. If x and x' are elements of the budget set (i.e. if $x \cdot p \leq w$ and $x' \cdot p \leq w$), then for a in [0,1]

$p \cdot x'' = a(p \cdot x) + (1 - a)(p \cdot x') \leq w$ and x" is also element of the budget set, i.e. $x'' \in B$.

Proof?

Intuition: linear combinations of consumption bundles that belong to the budget set, are also affordable.

Consumer choice: basic concepts
Demand function

In the neoclassical model of consumer behavior, the demand function maps prices (p) and income (or wealth, w) into a commodity-choice vector x(p,w).

- In general, demand is a correspondence; $x(p,w) \subset X$, but we usually assume that x(p,w) contains only one point (singleton) so that demand is a function.

Price and wealth determine budget set, nothing more.

Consumer choice: basic concepts Demand function

- What kind of information about the consumer does the demand function contain?
  - How much a consumer will buy?
  - How much a consumer will buy when prices are at the market equilibrium?
  - How much a consumer would want to buy at every reasonable combination of income and prices?

Consumer choice: basic concepts
Demand function

- 2 important properties of the demand correspondence
  - Homogeneity of degree 0
  - Walras' law is satisfied.

Definition (Homogeneity of degree 0)

A demand function $x(p, w)$ is homogeneous of degree 0 if

$$x(\alpha p, \alpha w) = x(p, w) \text{ for any } (p, w) \text{ and } \alpha > 0$$

  - Homogeneity of degree zero means that the absolute level of prices and wealth doesn't matter. No money illusion.
  - Only the relative values have an effect

Consumer choice: basic concepts
Demand function

- Implication of homogeneity of degree 0: we can choose some price p_i and fix it to be equal to 1. We express all other prices relative to the price of this good:

$$\frac{p_1}{p_i}, \frac{p_2}{p_i}, \ldots, \underset{\substack{\uparrow \\ i\text{-th good (reference good)}}}{1}, \ldots, \frac{p_l}{p_i}, \frac{w}{p_i}$$

Consumer choice: basic concepts
Demand function

- Walras' law: the consumer fully expends his wealth (reasonable as long as there is some good that is clearly desirable and we consider a lifetime perspective)

Definition (Satisfaction of Walras' law)

A demand function $x(p,w)$ satisfies Walras' law if

$$\textit{for every } p \gg 0) \textit{ and } w > 0, \; \textit{we have } p \cdot x(p,w) = w$$

i.e. the budget constraint is binding.

We spend all our wealth

Consumer choice: comparative statics

- What happens to the consumer's choice if his wealth or prices change (comparative statics)?
- We assume $\underline{x}(\underline{p},w)$ is a function (as opposed to a correspondence)

Consumer choice: comparative statics

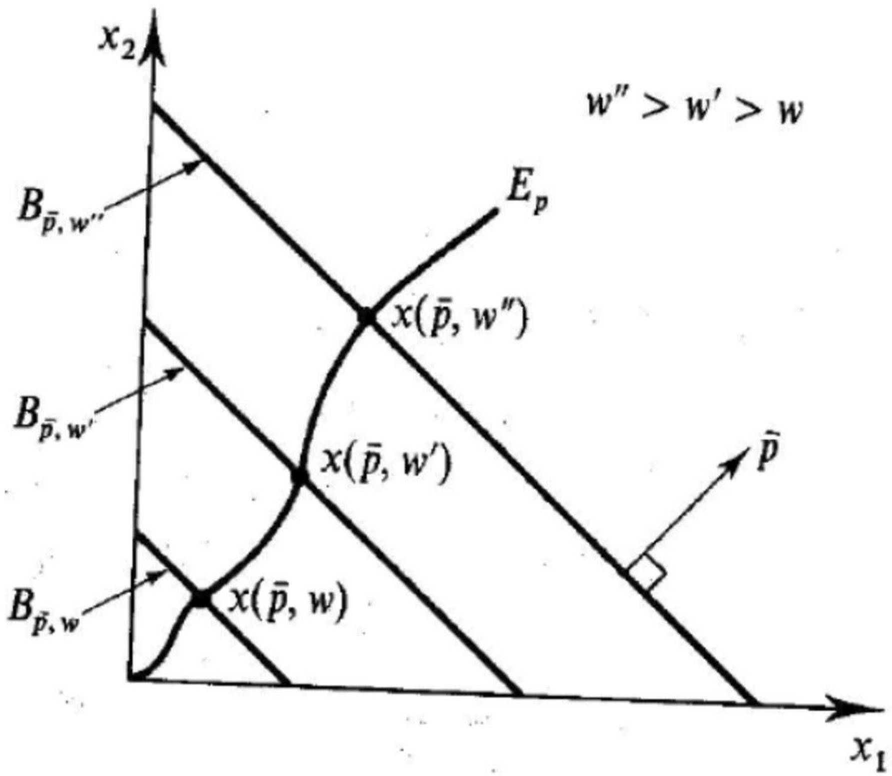
Comparative Statics

- Wealth (income) effect
  - the consumer's Engel function: demand as a function of wealth for given prices $x(\underline{p},w)$
  - Notation: due to the limitations of powerpoint I use underline instead of overline to denote fixed variables
  - wealth/income expansion path: its image in the commodity space R_L^+ (see figure on next slide)
  - $\partial x_l(p,w)/\partial w$: wealth/income effect for the l-th commodity
  - commodity l is normal if $\partial x_l(p,w)/\partial w \geq 0$
  - commodity l is inferior if $\partial x_l(p,w)/\partial w < 0$
  - we say that demand is normal if every commodity is normal at all (p,w)

Consumer choice: comparative statics

- wealth effects in matrix notation:

$$D_w x(p,x) = \begin{bmatrix} \dfrac{\partial x_1(p,w)}{\partial w} \\ . \\ . \\ \dfrac{\partial x_L(p,w)}{\partial w} \end{bmatrix}$$



The assumption of normal demand makes sense at a high degree of aggregation (e.g. “food”, “shelter”, as opposed to e.g. “camper shoes”, “kellogg’s cornflakes”)

Consumer choice: comparative statics

- (Ordinary) price effect: $\partial x_i(p,w)/\partial p_k$: the effect of a change in p_k on the demand of good *i*
- price effects in matrix form:

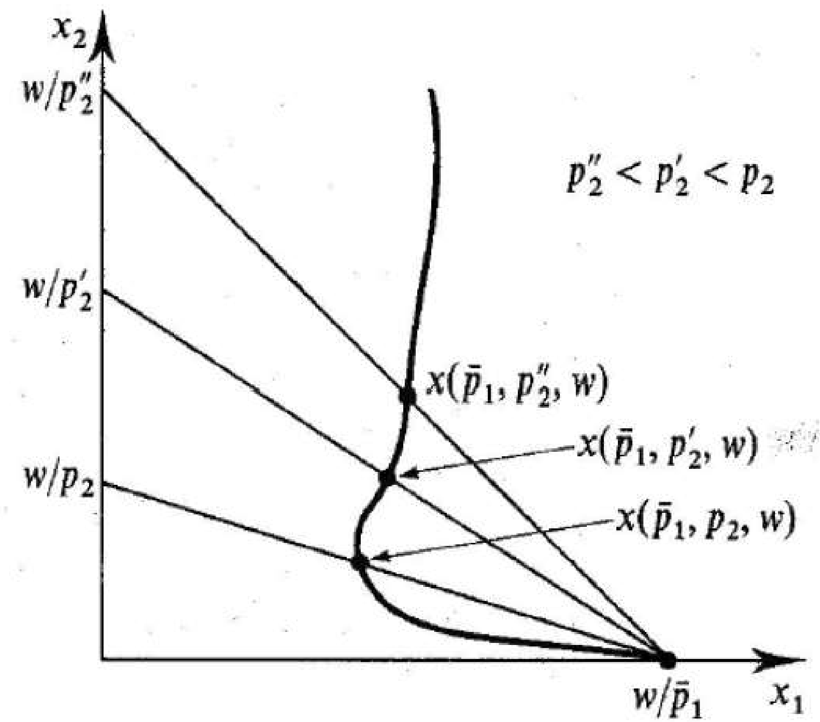
$$D_p x(p,x) = \begin{bmatrix} \frac{\partial x_1(p,w)}{\partial p_1} & \cdots & \frac{\partial x_1(p,w)}{\partial p_L} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \frac{\partial x_L(p,w)}{\partial p_1} & \cdots & \frac{\partial x_L(p,w)}{\partial p_L} \end{bmatrix}$$

With L goods, this is an LxL matrix

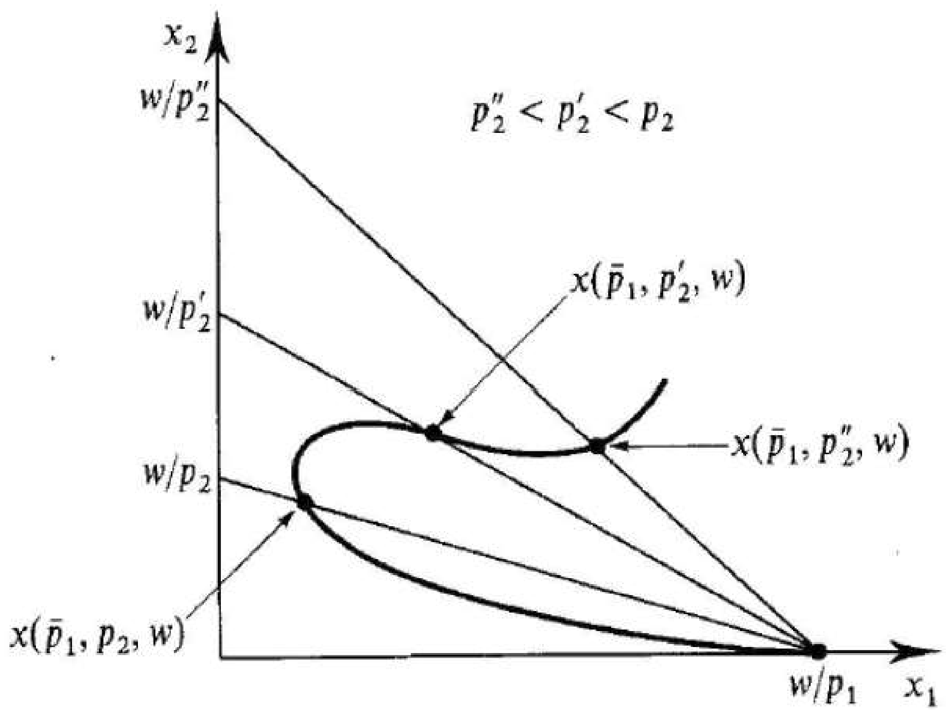
- offer curve: demand in R_+^2 as we range over all possible values of p_2 (see figures on next slide)
- Commodity i is a Giffen good at (p,w) if $\partial x_i(p,w)/\partial p_i > 0$.

Consumer choice: comparative statics

offer curve



Offer curve when good 2 is Giffen



Consumer choice: comparative statics

- Examples of Giffen goods: low quality goods consumed by consumers with low wealth levels.
- A poor consumer fulfills much of his dietary requirements with potatoes (low cost, filling food).
- Price of potatoes falls. Now he can afford to buy other, more desirable foods, and his consumption of potatoes may fall as a result.
- Wealth consideration involved (when the price of potatoes falls, the consumer is effectively wealthier)

Consumer choice: comparative statics

- Some implications of Walras' law for demand

1. By Walras' Law, p·x(p,w) = w. Differentiation w.r.t. the price of good k yields:

$$\sum_{\ell=1}^{L} p_\ell \cdot \partial x_\ell(p,w)/\partial p_k + x_k(p,w) = 0$$

indirect effects due to demand changes of all goods

direct effect of price increase on expenditures at given demand or good *k*

- intuition: total expenditures cannot change in response to a change in prices

Consumer choice: comparative statics

2. By Walras' Law, p·x(p,w) = w. Differentiation w.r.t. wealth w yields:

$$\sum_{\ell=1}^{L} p_\ell \cdot \partial x_\ell(p,w) / \partial w = 1$$

- Intuition: Total expenditure must change by an amount equal to the wealth change.

WARP and the law of demand

- We assume the following:
  (i) Weak Axiom of Revealed Preferences (Chapter 1)
  (ii) Homogeneity of degree 0
  (iii) Walras' law

i.e. we impose more consistency on choices. In fact, these three assumptions will be satisfied when we derive the consumer's demand from the classical demand theory (see the preference-based approach, next chapter).

What are the implications?

WARP and the law of demand

Definition (Weak Axiom (comparing two situations))

The demand function $x(p, w)$ satisfies the WA if $\forall(p, w)$ and $\forall(p', w')$ we have the following property:

$$\text{If } p \cdot x(p', w') \leq w \text{ and } x(p', w') \neq x(p, w)$$
$$\Rightarrow \; p' \cdot x(p, w) > w'$$

Intuition: If the bundle $x(p', w')$ is feasible when the agent faces price-wealth (p, w) and (by definition) the agent chooses $x(p, w)$, this **reveals** a preference of the agent for $x(p, w)$ over $x(p', w')$. **Then**, since the agent chooses $x(p', w')$ when facing price-wealth (p', w'), it **must be** that he cannot afford $x(p, w)$.

WARP and the law of demand

- Take two different consumption bundles $x(p,w)$ and $x(p',w')$, both being affordable (p,w), i.e.,

$$p \cdot x(p,w) \leq w \text{ and } p \cdot x(p',w') \leq w$$

- When prices and wealth are (p,w), the consumer chooses $x(p,w)$ despite $x(p',w')$ is also affordable.
- Then he "reveals" a preference for $x(p,w)$ over $x(p',w')$ when both are affordable.
- Hence, we should expect him to choose $x(p,w)$ over $x(p',w')$ when both are affordable. (Consistency)
- Therefore, bundle $x(p,w)$ must not be affordable at (p',w') because the consumer chooses $x(p',w')$. That is, $p' \cdot x(p,w) > w'$.

WARP and the law of demand

- In summary, Walrasian demand satisfies WARP, if, for two different consumption bundles, $x(p,w) \neq x(p',w')$,
$$p \cdot x(p',w') \leq w \;\; \Rightarrow \;\; p' \cdot x(p,w) > w'$$

- *Intuition*: if $x(p',w')$ is affordable under budget set $B_{p,w}$, then $x(p,w)$ cannot be affordable under $B_{p',w'}$.

Checking for WARP

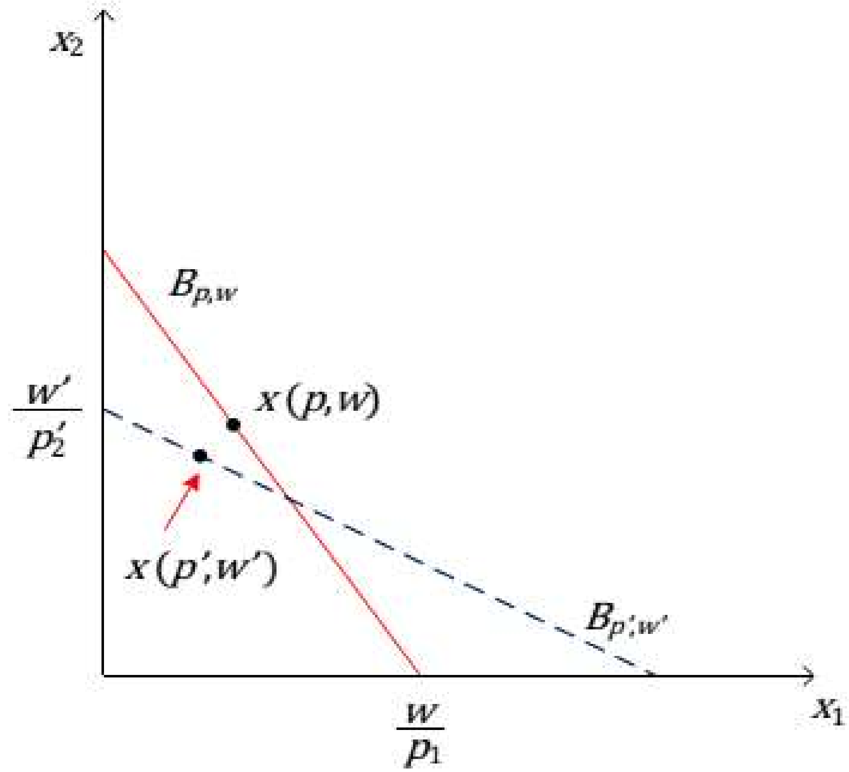
- A systematic procedure to check if Walrasian demand satisfies WARP:
  - ***Step 1:*** Check if bundles $x(p, w)$ and $x(p', w')$ are both affordable under $B_{p,w}$.
    - That is, graphically $x(p, w)$ and $x(p', w')$ have to lie on or below budget line $B_{p,w}$.
    - If step 1 is satisfied, then move to step 2.
    - Otherwise, the premise of WARP does not hold, which does not allow us to continue checking if WARP is violated or not. In this case, we can only say that "*WARP is not violated*".

Checking for WARP

- ***Step 2:*** Check if bundles $x(p, w)$ is affordable under $B_{p',w'}$.
  - That is, graphically $x(p, w)$ must lie on or below budge line $B_{p',w'}$.
  - If step 2 is satisfied, then this Walrasian demand violates WARP.
  - Otherwise, the Walrasian demand satisfies WARP.

Checking for WARP: Example 1

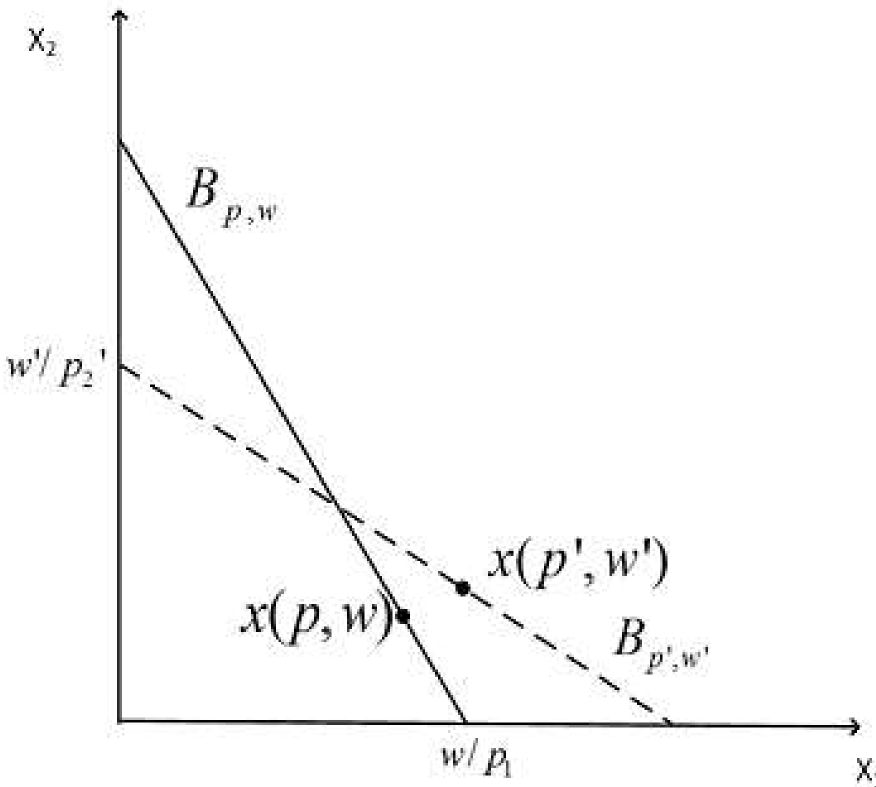
- First, $x(p,w)$ and $x(p',w')$ are both affordable under $B_{p,w}$.
- Second, $x(p,w)$ is not affordable under $B_{p',w'}$.
- Hence, WARP is *satisfied*!



Checking for WARP: Example 2

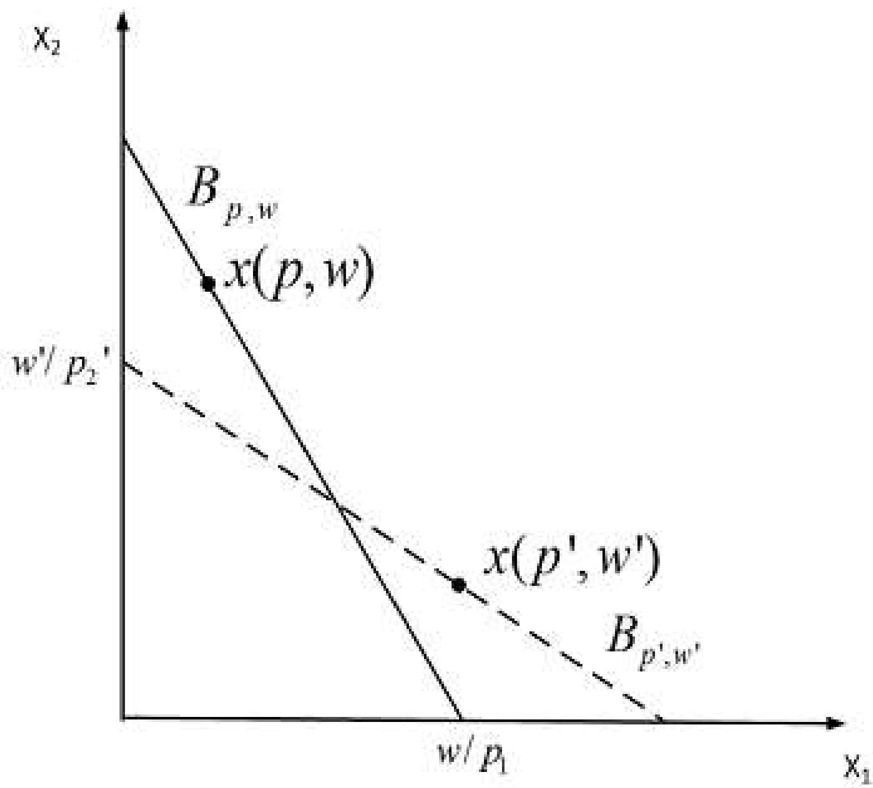
The demand $x(p', w')$ under final prices and wealth is not affordable under initial prices and wealth, i.e., $p \cdot x(p', w') > w$.

- The premise of WARP does not hold.
- Violation of Step 1!
- WARP is *not violated*.



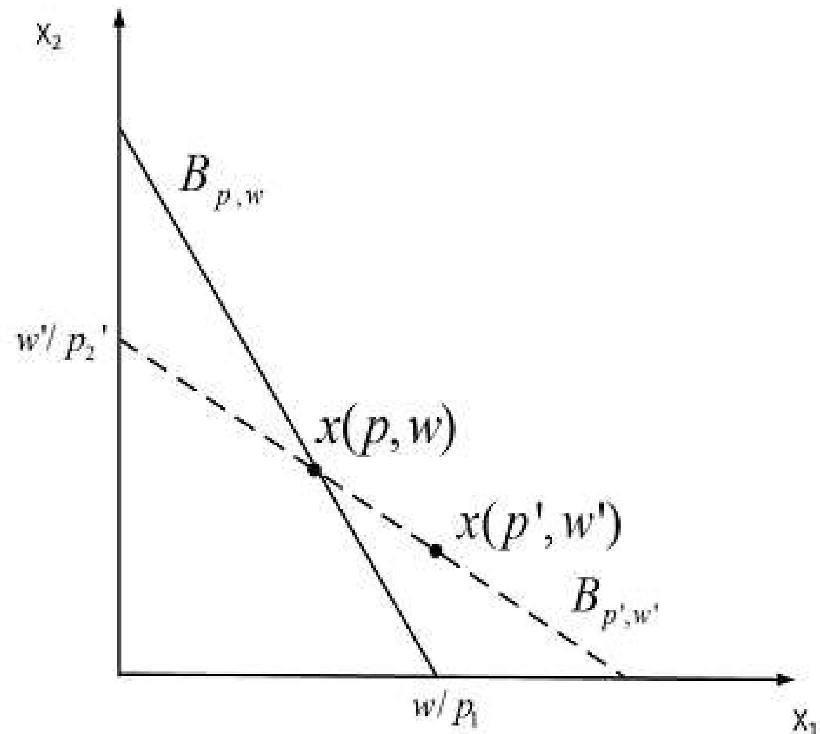
Checking for WARP: Example 3

- The demand $x(p', w')$ under final prices and wealth is not affordable under initial prices and wealth, i.e., $p \cdot x(p', w') > w$.
  - The premise of WARP does not hold.
  - Violation of Step 1!
  - WARP is *not violated*.



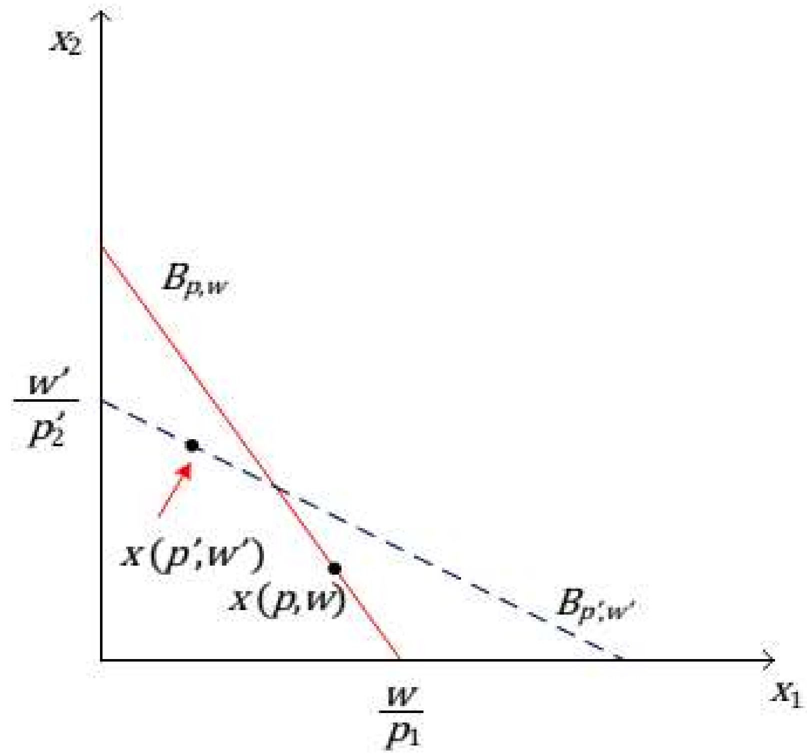
Checking for WARP: Example 4

- The demand $x(p', w')$ under final prices and wealth is not affordable under initial prices and wealth, i.e., $p \cdot x(p', w') > w$.
  - The premise of WARP does not hold.
  - Violation of Step 1!
  - WARP is *not violated*.



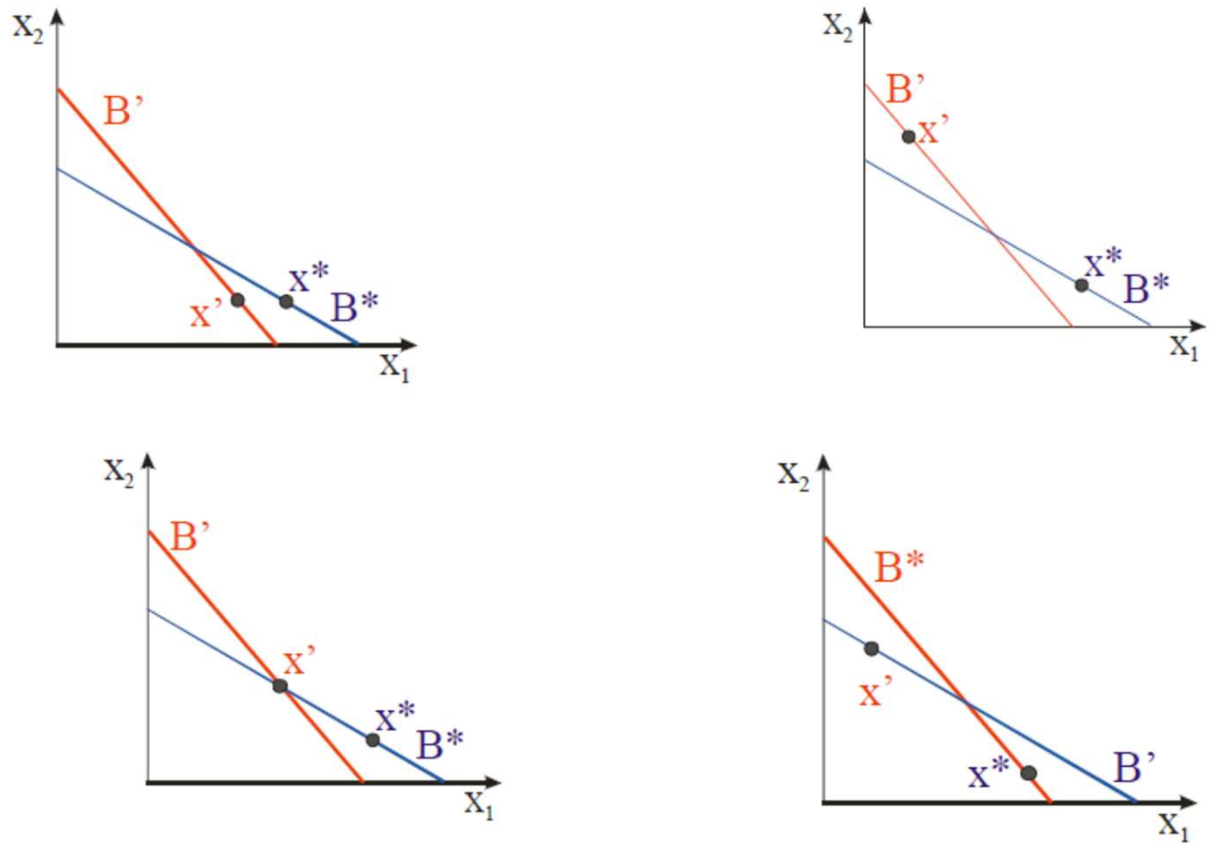
Checking for WARP: Example 5

- First, $x(p,w)$ and $x(p',w')$ are both affordable under $B_{p,w}$.
- Second, $x(p,w)$ is affordable under $B_{p',w'}$, i.e., $p' \cdot x(p,w) < w'$
- Hence, WARP is *NOT satisfied*!



WARP and the law of demand

Compatible with the weak axiom of revealed preferences?



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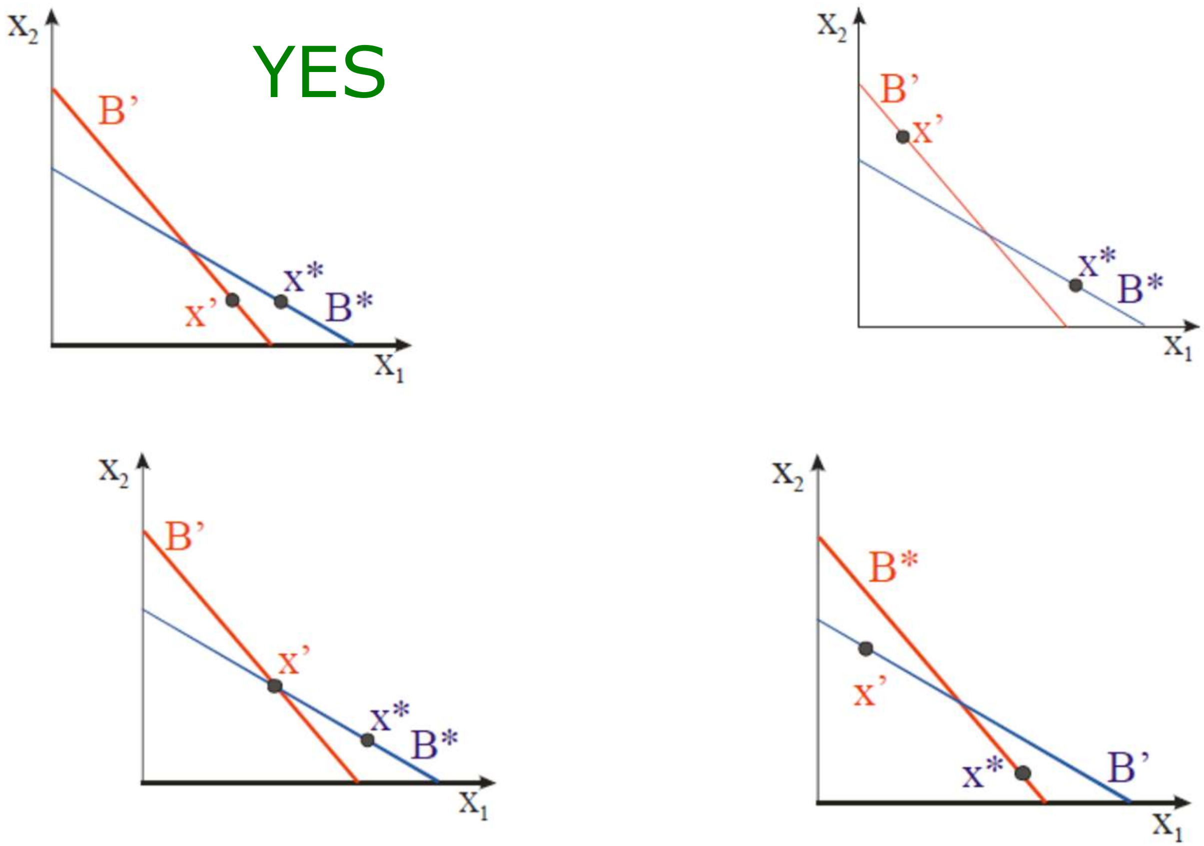

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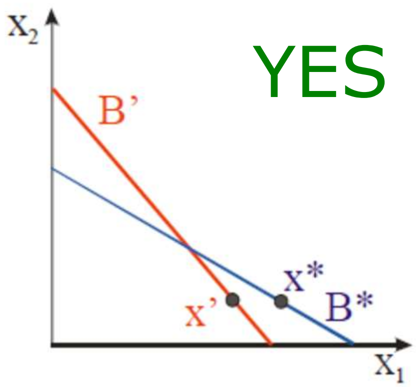

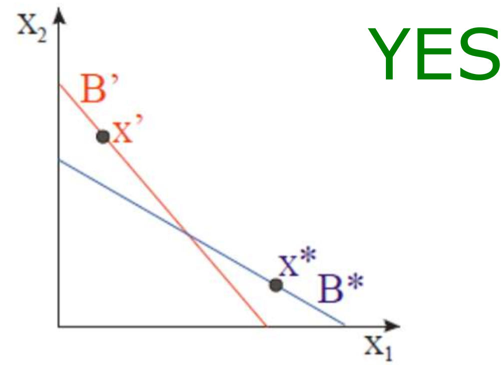

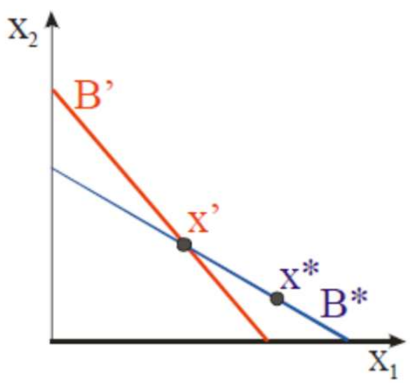

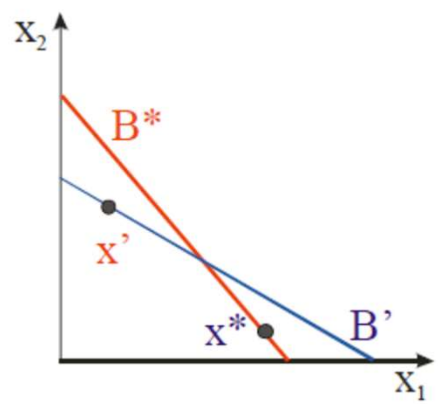

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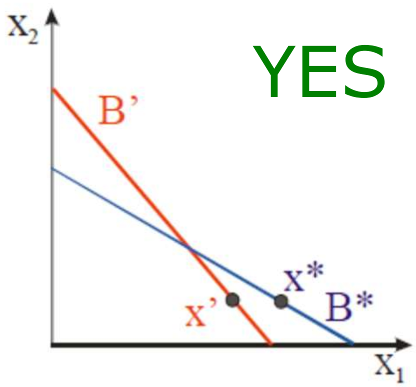

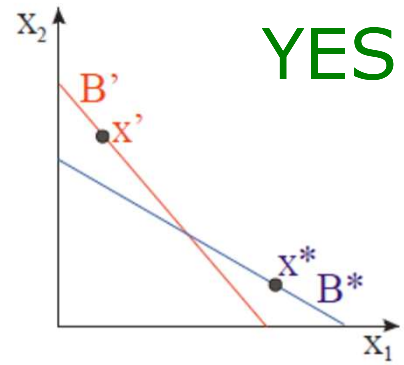

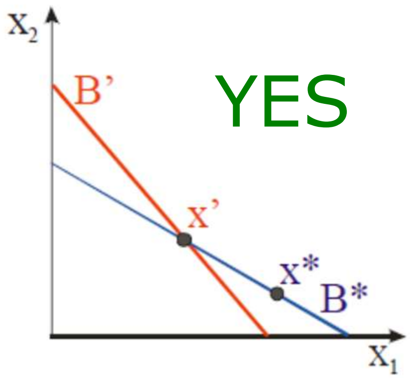

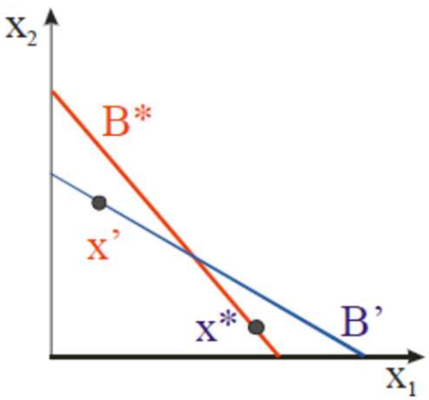

WARP and the law of demand

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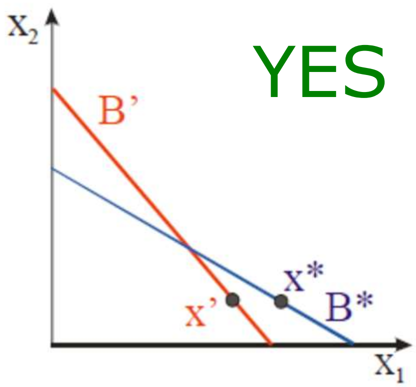

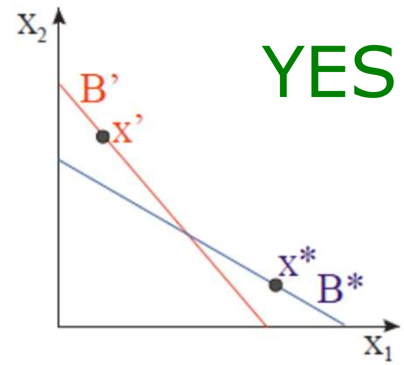

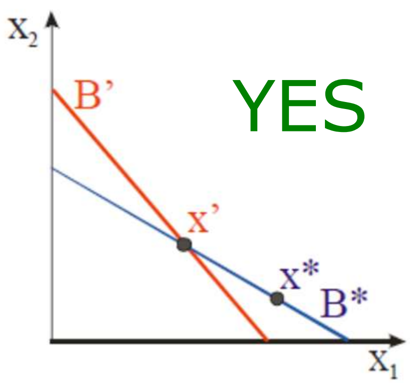

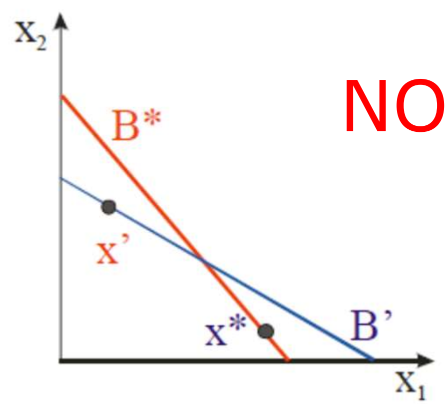

Implications of the WARP

- Before we elaborate on the law of demand, an additional concept shall be introduced.
- When the price of a commodity changes (e.g. increases) the consumer is affected in two ways:
  - The commodity whose price has increased has become more expensive relative to other commodities.
  - The consumer is impoverished (the purchasing power of his wealth has decreased).

Slutsky wealth compensation

• Price changes affect relative prices and the real value of wealth (income).

• Slutsky compensated price changes combine a price change with a (hypothetical) adjustment of wealth such that the previously demanded consumption bundle again is just affordable.[1]

• Let $x^*=x(p^*,w^*)$. If the price vector changes to p', then the wealth compensation is defined as $\Delta w = \Delta p \cdot x^*$, where $\Delta p = (p' - p^*)$.

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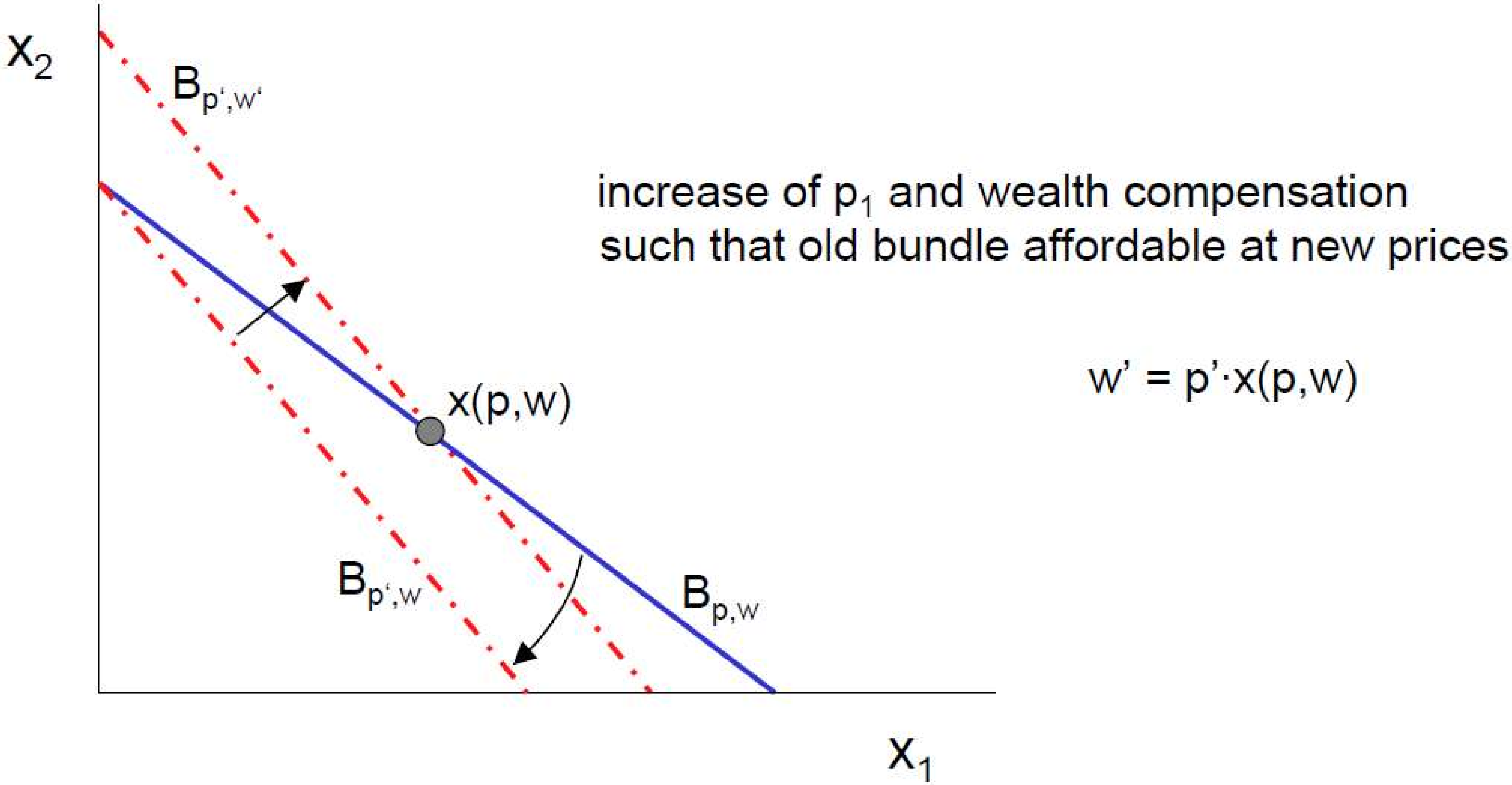
[1] Conversely, a Hicks compensation would adjust wealth such that the old utility level can just be reached despite the price change.

Implications of the WARP

- Graphical illustration.



Law of (compensated) demand

The law of compensated demand

Assume that $x(p, w)$ *is homogeneous of degree* 0*,*
and satisfies Walras' law:

$x(p, w)$ *satisfies the WA*

$$\Leftrightarrow$$

For any (Slutsky) compensated variation of prices
(i.e. from (p, w) *to* (p', w') *with* $w' - w$ *that compensates the price variation), we have:*

$$(p' - p) \cdot [x(p', w') - x(p, w)] \leq 0 \qquad (2.1)$$

Law of (compensated) demand

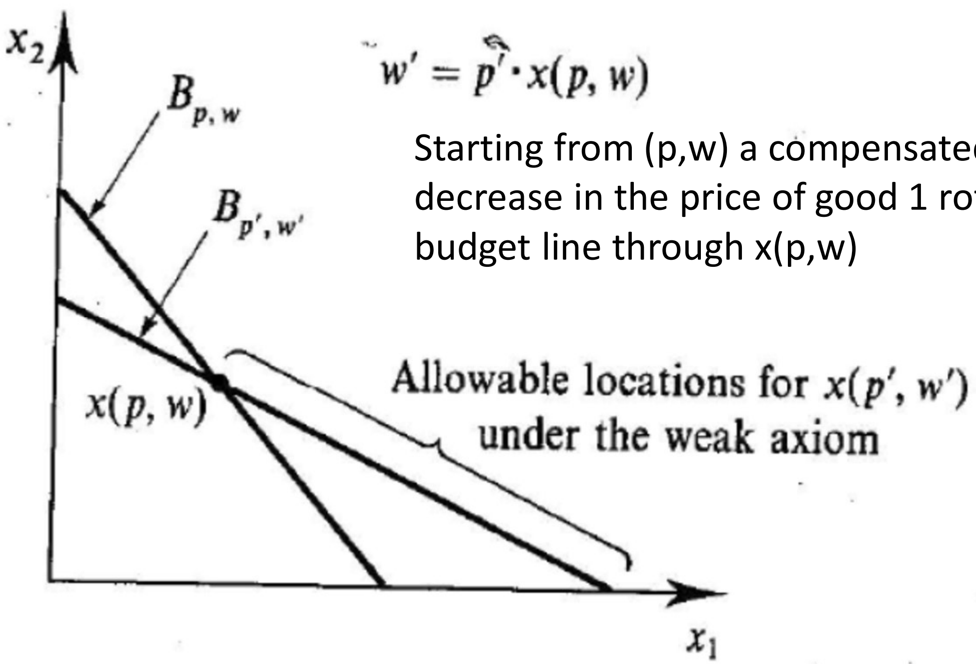
- short-hand notation of (2.1): for $x(p,w) \neq x(p',w')$ we have $\Delta x \cdot \Delta p < 0$
  - the *law of demand* says that *demand and price move in opposite direction*
  - proposition MWG 2.F.1 shows that it holds for *compensated* price changes. Hence we call it the *compensated law of demand*
- if only the price of good i changes, we get

$$\begin{bmatrix} \Delta x_1 \\ \cdot \\ \Delta x_i \\ \cdot \\ \Delta x_L \end{bmatrix} \cdot \begin{bmatrix} 0 \\ \cdot \\ \Delta p_i \\ \cdot \\ 0 \end{bmatrix} = \Delta x_i \Delta p_i$$

  - Hence, if the price of good *i* increases ($\Delta p_i > 0$), then compensated demand of *i* must go down,
  - i.e. the own price effect is always negative.

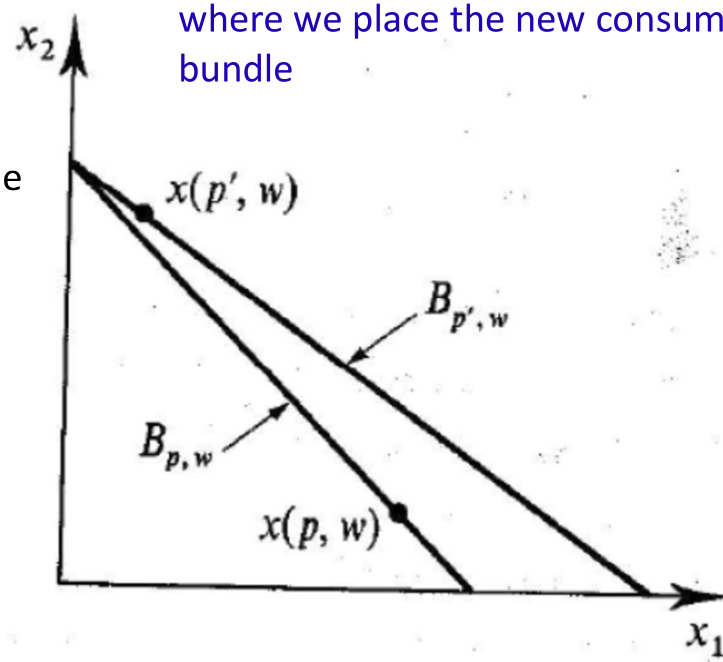
Law of (compensated) demand

- fig 1: compensated decrease of p_1. By the weak axiom, demand must be nonincreasing in own price for a compensated price change
- fig 2: the weak axiom is not sufficient to yield the law of demand for price changes that are *not* compensated
  - e.g., demand for good 1 can fall despite a lower price



Starting from (p,w) a compensated decrease in the price of good 1 rotates the budget line through x(p,w)

The WARP imposes no restriction on where we place the new consumption bundle



Law of (compensated) demand

- The proof of MWG 2.F.1 implies two steps. (i) First, that the weak axiom implies (2.1). (ii) Second, show that (2.1) implies the weak axiom (to justify the phrase "*if and only if*").

(i) For $x(p',w') = x(p,w)$, (2.1) holds with equality. So suppose $x(p',w') \neq x(p,w)$. Lhs of ineq. (2.1) may be written as

(2) $$(p'-p)[x(p',w')-x(p,w)] = p' [x(p',w')-x(p,w)] - p [x(p',w')-x(p,w)]$$

The first term is zero: by Walras law, $p'x(p',w') = w'$ and $p'x(p,w) = w'$ because the price change is compensated.

Second term: Compensation makes sure that $x(p,w)$ is affordable under price-wealth situation (p',w'). Hence by the weak axiom $x(p',w')$ must not be affordable at (p,w). Hence $px(p',w') > w$.
By Walras' law, $px(p,w) = w$. Hence the second term is strictly positive for $x(p,w) \neq x(p',w')$.

(ii) Omitted.

Law of (compensated) demand: implications

- If consumer demand $x(p,w)$ is a differentiable factor of prices and wealth, the law of *compensated* demand can be written as:

$$dp \bullet dx \leq 0$$

What are the implications of this relation?

Law of (compensated) demand: implications

- What does it mean to give the consumer a compensated price change?
- Let the initial consumption bundle be $\hat{x} = x(p,w)$, where p and w are the original prices and wealth.
- A compensated price change means that at any price, p, the original bundle is still available. Hence after the price change, wealth is changed to $\hat{w} = p \bullet \hat{x}$.

Law of (compensated) demand: implications

- Consider the consumer's demand for good *i*:

$$x_i^c = x_i\left(p, p \cdot \hat{x}\right)$$

following a compensated change in the price of good *j*:

$$\begin{aligned} \frac{d}{dp_j}\left(x_i\left(p, p \cdot \hat{x}\right)\right) &= \frac{\partial x_i}{\partial p_j} + \frac{\partial x_i}{\partial w}\frac{\partial\left(p \cdot \hat{x}\right)}{\partial p_j} \\ \frac{dx_i^c}{dp_j} &= \frac{\partial x_i}{\partial p_j} + \frac{\partial x_i}{\partial w}\hat{x}_j. \end{aligned}$$

Law of (compensated) demand: implications

- Writing this as a differential:

$$dx_i^c = \left(\frac{\partial x_i}{\partial p_j} + \frac{\partial x_i}{\partial w} x_j \right) dp_j = s_{ij} dp_j$$

where $s_{ij} = \left(\frac{dx_i}{dp_j} + \frac{dx_i}{dw} x_j \right)$

If we change more than one price, the change in demand for x_i will be the sum of changes due to differential price changes:

$$dx_i^c = \sum_{j=1}^{L} \left(\frac{\partial x_i}{\partial p_j} + \frac{\partial x_i}{\partial w} x_j \right) dp_j = s_i \cdot dp$$

Where $s_i = (s_{i1}, ..., s_{ij}, ..., s_{iL})$ and $dp = (dp_1, ..., dp_L)$ is the vector of price changes

Law of (compensated) demand: implications

$$d\underline{x} = \begin{bmatrix} dx_1(\underline{p}, w) \\ dx_2(\underline{p}, w) \\ . \\ . \\ dx_i(\underline{p}, w) \\ . \\ . \\ dx_L(\underline{p}, w) \end{bmatrix}$$

We can arrange the dx_i^c into a vector by stacking the equations of the previous slide vertically. We get:

$$dx^c = S\ dp$$

where S is an L x L matrix with the element in the ith and jth column being s_{ij}.

Law of (compensated) demand: implications

Returning to the statement of the WARP:

$$dp \bullet dx^c \leq 0$$

Substituting in $dx^c = S\ dp$ we get:

$$dp \cdot S \cdot dp^T \leq 0$$

$$(1\text{x}L) \cdot (L\text{x}L) \cdot (L\text{x}1) = (1\text{x}1)$$

This has a mathematical significance: it implies that matrix S, which we call the substitution matrix, is negative semi-definite (i.e. if you pre- and post- multiply it by the same vector, the result is always a non-positive number).

Law of (compensated) demand: implications

- One nice mathematical property of negative semi-definite matrices:
  - The diagonal elements s_{ii} are non-positive (generally there will be strictly negative)
- This means that the change in demand for a good in response to a compensated price increase is negative (compensated law of demand)
- Why so much fuss about something so obvious?
- We derived this based only on the WARP and Walras' law.

Law of (compensated) demand: implications

- How does s_{ii} help us explain the existence of Giffen goods?
- Ordinarily if the price of a good increases, holding wealth constant, the demand of that good will decrease (law of demand). But not in the case of Giffen goods.
- Example: A consumer spends all of her money on two things: food and trips to Hawaii. Suppose the price of food increases. It may be that after the increase, the consumer can no longer afford the trip to Hawaii and therefore spends all of her money on food. The result is that the consumer actually buys more food than she did before the price increase.

Law of (compensated) demand: implications

- How does this story fit into the framework we developed before? We know

$$s_{ii} = \left(\frac{\partial x_i}{\partial p_i} + \frac{\partial x_i}{\partial w} x_i \right)$$

By rearranging:

$$\frac{\partial x_i\left(p, w\right)}{\partial p_i} = s_{ii} - \frac{\partial x_i}{\partial w} x_i$$

Law of (compensated) demand: implications

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positive

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positive

negative

???

Law of (compensated) demand: implications

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$$s_{ii} = \left(\frac{\partial x_i}{\partial p_i} + \frac{\partial x_i}{\partial w} x_i \right)$$

By rearranging:

$$\frac{\partial x_i\,(p,w)}{\partial p_i} = s_{ii} - \frac{\partial x_i}{\partial w} x_i$$

positive

Law of demand holds

negative

If positive (i.e. normal good)

Law of (compensated) demand: implications

- How does this story fit into the framework we developed before? We know

$$s_{ii} = \left(\frac{\partial x_i}{\partial p_i} + \frac{\partial x_i}{\partial w} x_i \right)$$

By rearranging:

$$\frac{\partial x_i(p, w)}{\partial p_i} = s_{ii} - \frac{\partial x_i}{\partial w} x_i$$

positive

Giffen good

negative

If negative and $-\frac{\partial x_i}{\partial w} x_i > s_{ii}$
(i.e. strongly inferior good)

Law of (compensated) demand: implications

- Result
  - In order for a good to be a Giffen good, it must be a strongly inferior good.
  - A normal good can never be a Giffen good.

Substitution and Income Effects following a reduction in price

	SE	IE	TE
Normal Good	+	+	+
Inferior Good	+	-	+
Giffen Good	+	-	-

- NG: Demand curve is negatively sloped (as usual)
- GG: Demand curve is positively sloped

Substitution and Income Effects

- Summary:
  1) SE is negative (since $\downarrow p_1 \Rightarrow \uparrow x_1$)
     - SE < 0 does not imply $\downarrow x_1$
  2) If good is inferior, IE < 0. Then,

$$\text{TE} = \underbrace{\underbrace{\text{SE}}_{-} - \underbrace{\text{IE}}_{-}}_{+} \quad \Rightarrow \quad \text{if } |\text{IE}| \begin{Bmatrix} > \\ < \end{Bmatrix} |\text{SE}|, \text{ then } \begin{Bmatrix} \text{TE}(-) \\ \text{TE}(+) \end{Bmatrix}$$

For a price decrease, this implies

$$\begin{Bmatrix} \text{TE}(-) \\ \text{TE}(+) \end{Bmatrix} \Rightarrow \begin{Bmatrix} \downarrow x_1 \\ \uparrow x_1 \end{Bmatrix} \quad \begin{matrix} \text{Giffen good} \\ \text{Non–Giffen good} \end{matrix}$$

  3) Hence,
     a) A good can be inferior, but not necessarily be Giffen
     b) But all Giffen goods must be inferior.

Law of (compensated) demand: implications

- Take an example of a tax which increases the price of a good.
- How can we measure the impact of the price change on consumers?
- s_{ii} is unobservable! We only observe uncompensated price changes!
- But we can recover s_{ii} from

$$s_{ii} = \left(\frac{\partial x_i}{\partial p_i} + \frac{\partial x_i}{\partial w} x_i \right)$$

Main points

- The *consumer* is the decision maker
- In the market economy prices are given (the consumer is a price-taker)
- *Commodities* are the objects of choice.
- The *consumption set* describes the *physical constraints* that limit the consumer's choices
- The *Walrasian budget set* describes the *economic constraints* that limit the consumer's choices.

Main points

- The *Walrasian demand function* describes the consumer's choices (decision) subject to the above constraints.
- We studied the ways in which consumer demand changes when economic constraints vary (*comparative statics*)
- We studied the implication of the WARP for the consumer's demand function
- The WARP is essentially equivalent to the *law of compensated demand* (i.e. prices and demanded quantities move in opposite directions for price changes that leave real wealth unchanged).
- We studied several implications of the law of compensated demand.