Solutions to Problem Set 3

1)

i) When they act independently each will maximize its own profit:

$$max\Pi_0 = p_0 q_0 - \frac{q_0^2}{100} + q_h$$

From the FOC we get that $q_0 = 200$

Similarly, $max\Pi_h$ and we get $q_h = 100$

ii) When they merge, they act as one firm:

$$max\Pi = p_0 q_0 + p_h q_h - \frac{q_0^2}{100} + q_h - \frac{q_h^2}{100}$$

From the FOCS we have that:

$$\frac{\partial \Pi}{\partial q_o} = 0 \Rightarrow q_0 = 200$$

And $\frac{\partial \Pi}{\partial q_h} = 0 \Rightarrow q_h = 150$

2)

i) Indirect utility function:

$maxu_1$ subject to: $p_1 x_1 + m_1 \leq w_1$

$$L = m_1 + 10\ln(1 + x_1) + 5h - h^2 + \lambda(p_1 x_1 + m_1 - w_1)$$

From the FOCS we get that:

$$x_1 = \frac{10}{p_1} - 1$$

And $m_1 = w_1 - 10 + p$

So substituting into u_1 we get:

$$v_1 = w_1 - 10 + p + 10\ln\left(\frac{10}{p}\right) + 5h - h^2$$

In a similar manner, $v_2 = w_2 - 5 + p + 5\ln\left(\frac{5}{p}\right) - h^2$

ii) $v_1 = w_1 - 10 + p + 10\ln\left(\frac{10}{p}\right) + 5h - h^2 \Rightarrow \Phi_1(h) = 5h - h^2$

$$v_2 = w_2 - 5 + p + 5\ln\left(\frac{5}{p}\right) - h^2 \Rightarrow \Phi_2(h) = -h^2$$

From $max\Phi_1(h)$ we have that $h = 5/2$

iii) $max\Phi_1(h) + \Phi_2(h) = 5h - 2h^2$

From the FOC we get that $h^* = 5/4$

iv) Programs of the two agents:

Agent 1:

$max 5h - h^2 - p_h h$ and from the FOC we get $5 - 2h = p_h$

Agent 2:

$\max(-h^2) + p_h h$ and from the FOC: $p_h = 2h$

Solving the system we have that $h = 5/4$ and $p_h = 5/2$

3) MWG chapter 11 exercise 11.D.5 (see solution manual you can find it if you google it)

4)

i) A's maximization problem:

$$maxlogx_A + logG$$

s.t. $g_A + g_B = G \; and \; g_A + x_A = w$

or: $maxlog(w - g_A) + \log{(g_A + g_B)}$

From the FOC we get $g_A(g_B) = \frac{w-g_B}{2}$

In a similar manner we have that $g_B(g_A) = \frac{w-g_A}{2}$

(we have assumed that they hold the same amount of wealth as it simplifies the expressions)

ii) Because of symmetry it must be that $g_A = g_B$. So by the above equations, $g_A = g_B = w/3$

iii) $\max \log(w - g_A) = \log(g_A + g_B) + \log(w - g_B) + \log{(g_A + g_B)}$

due to symmetry: $g_A = g_B = g = G/2$

so from the FOC we get: $g = w/2$

In this case $G = w$ while in the previous case, $G = \frac{2w}{3}$ (underprovision)

5) From MWG chapter 11 exercise 11.D.4

6) From MWG chapter 10 exercise 10.C.1

7) From MWG chapter 10 exercise 10.C.2