$$U(x_1, x_2) = x_1 \cdot x_2$$

$$\mathcal{L} = x_1 \cdot x_2 + \lambda (m - p_1 x_1 - p_2 x_2)$$

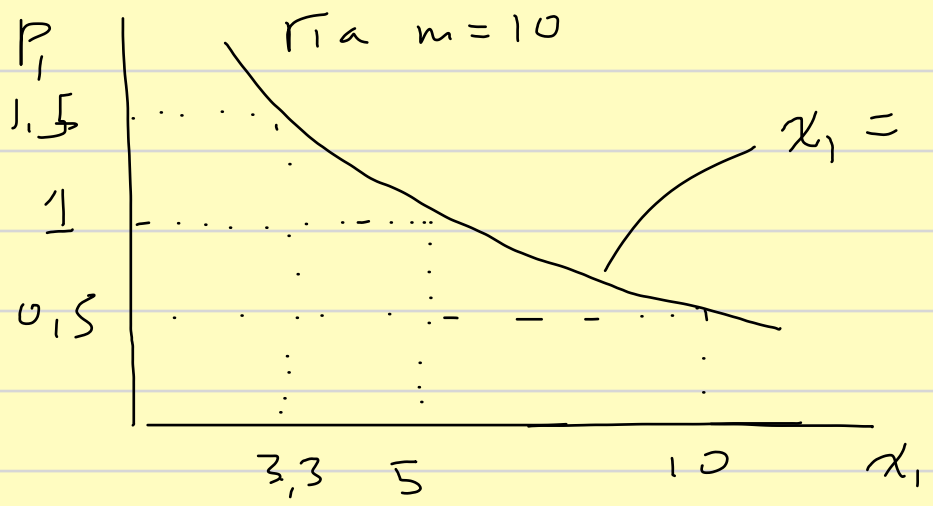
ΣΠΤ.
$$\left.\begin{array}{ll} \frac{\partial \mathcal{L}}{\partial x_1} : & x_2 - \lambda p_1 = 0 \\ \frac{\partial \mathcal{L}}{\partial x_2} : & x_1 - \lambda p_2 = 0 \end{array}\right\} \Rightarrow \frac{x_2}{x_1} = \frac{p_1}{p_2} \Rightarrow p_1 x_1 = p_2 x_2$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} : \quad m - p_1 x_1 - p_2 x_2 = 0 \quad \Rightarrow \quad m - 2 p_1 x_1 = 0$$

$$\Rightarrow x_1 = \frac{m}{2 p_1} \quad \Rightarrow \quad x_1(p_1, p_2, m) = \frac{m}{2 p_1}$$

$$x_2 = \frac{m}{2 p_2} \quad \Rightarrow \quad p_1(x_1) = \frac{m}{2 x_1}$$

ΣΥΝΑΡΤΗΣΗ ΖΗΤΗΣΗΣ
ΚΑΝΟΝΙΚΗ (ΜΑΡΣΑΛΙΑΝΗ)



ΕΜΜΕΣΗ ΣΥΝΑΡΤΗΣΗ ΧΡΗΣΙΜΟΤΗΤΑΣ

$$V(p_1, p_2, m) = u(x_1^*(p_1, p_2, m), x_2^*(p_1, p_2, m))$$

$$\Rightarrow V(p_1, p_2, m) = \frac{m}{2 p_1} \cdot \frac{m}{2 p_2} = \frac{m^2}{4 p_1 \cdot p_2}$$

ΛΥΝΟΥΜΕ ΓΙΑ m ⇒
$$m^2 = 4 \cdot V \cdot p_1 \cdot p_2$$
$$\Rightarrow m = 2\sqrt{p_1 \cdot p_2 \cdot u}$$

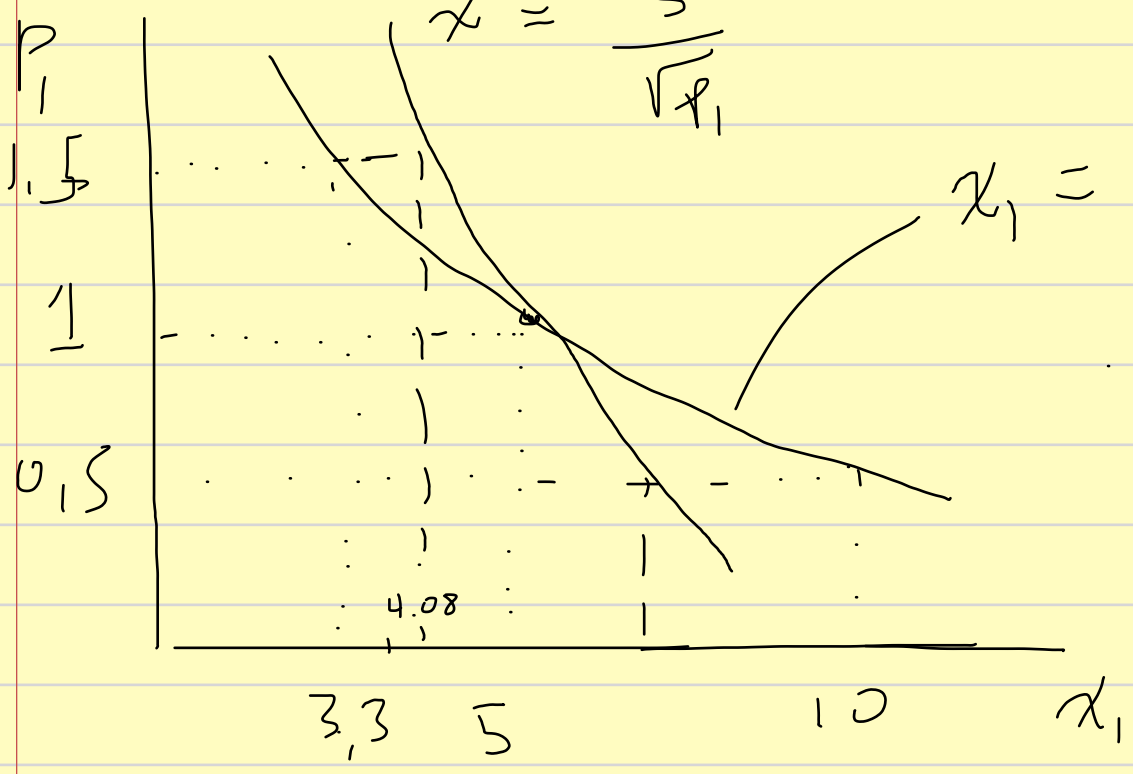
ΣΥΝΑΡΤΗΣΗ ΔΑΠΑΝΗΣ

$$x_1 = \frac{m}{2 p_1} \Rightarrow x_1^c = \frac{2\sqrt{p_1 \cdot p_2 \cdot u}}{2 p_1} = \sqrt{\frac{p_2 \cdot u}{p_1}}$$

$$x_2^c = \frac{2\sqrt{p_1 \cdot p_2 u}}{2 p_2} = \sqrt{\frac{p_1 \cdot u}{p_2}}$$

ΑΝΤΙΣΤΑΘΜΙΣΤΙΚΕΣ (ΧΙΚΣΙΑΝΕΣ)
ΣΥΝΑΡΤΗΣΕΙΣ ΖΗΤΗΣΗΣ

$m = 10$



$$x_1 = \frac{m}{2p_1} = \frac{10}{2p_1} = \frac{5}{p_1}$$

$$x_1^c(p_1, p_2, u) = x_1(p_1, p_2, E(p_1, p_2, u))$$

$$x_1^c(1, 1, 25) = x_1(1, 1, 10)$$

$$m = 10,\ p_1 = 1,\ p_2 = 1$$

$$V(p_1, p_2, m) = \frac{m^2}{4p_1 \cdot p_2} = \frac{10^2}{4 \cdot 1 \cdot 1}$$

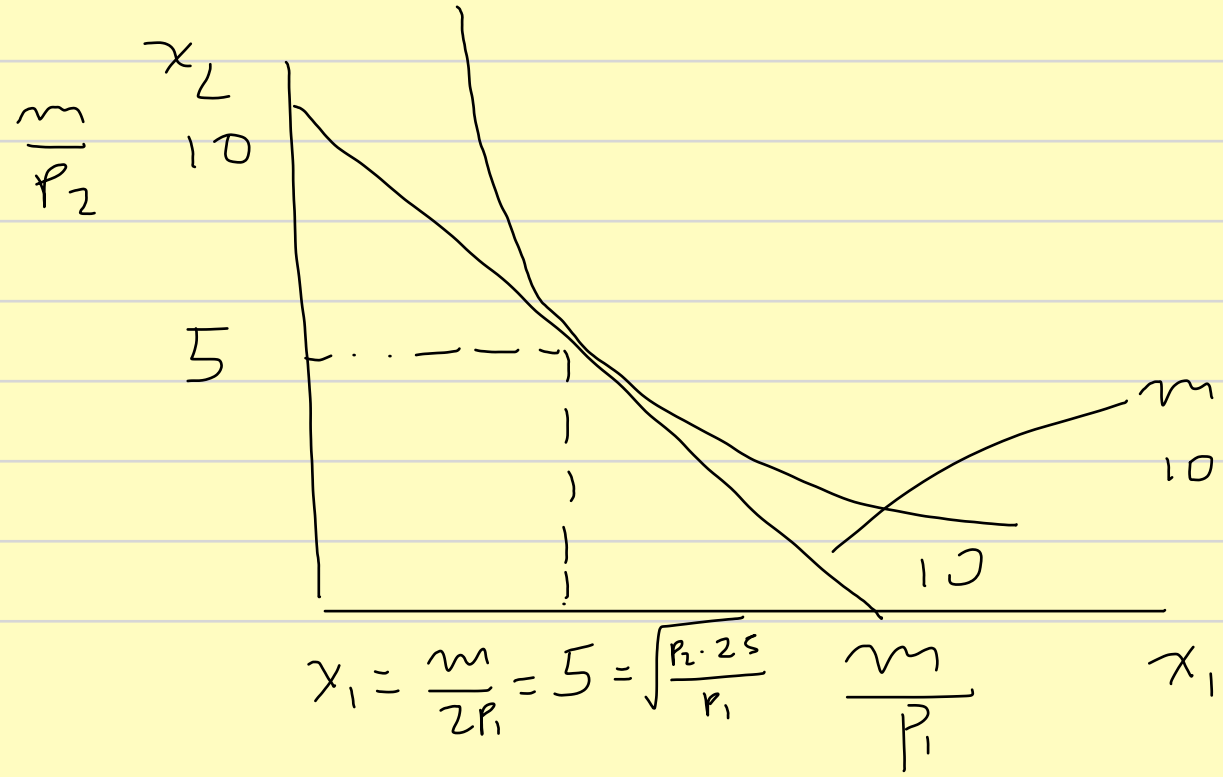
$$= \frac{100}{4} = 25$$

$$E(p_1, p_2, u) = 2\sqrt{p_1 \cdot p_2 \cdot u} = 2\sqrt{1 \cdot 1 \cdot 25} = 10$$

$$x_1^c = \sqrt{\frac{p_2 u}{p_1}} = \sqrt{\frac{1 \cdot 25}{1}} = x_1 = \frac{m}{2p_1} = \frac{10}{2 \cdot 1} = 5$$

$$x_1^c = \sqrt{\frac{p_2 \cdot u}{p_1}} = \sqrt{\frac{1 \cdot 25}{p_1}} = \frac{5}{\sqrt{p_1}}$$

$u(x_1^*, x_2^*) = V(p_1, p_2, m) = 25$



$$= E(p_1, p_2, u)$$

$$m = p_1 x_1 + p_2 x_2$$

$$10 = x_1 + x_2$$

x_2

$u = V(1,1,10) = \frac{10^2}{4 \cdot 1 \cdot 1} = 25$

$= x_1^* \cdot x_2^* = 5 \cdot 5$

$E(p_1, p_2, u) = 2\sqrt{1 \cdot 1 \cdot 25}$

$= 10$

$= 1.5 + 1.5$

$E(p_1', p_2, u) = 2\sqrt{2 \cdot 1 \cdot 25}$

$= 14.14$

5

x_1

$x_1' = \frac{10}{2 \cdot 2} = 2.5$

$E(p_1', p_2, u) - E(p_1, p_2, u) = 14.14 - 10$

$= 4,14$

ANTITHETIC HICKS

p_1

$p_1' = 2$

$p_1 = 1$

$x_1(p_1, p_1, m)$

2.5 3.53 5

x_1

$x_c' = \frac{5}{\sqrt{p_1}} = \frac{5}{\sqrt{2}} = 3,53$

Δυαδικό πρόβλημα

$u(x,y) = x_1 \cdot x_2$

$K = p_1 x_1 + p_2 x_2 + \lambda(u - x_1 \cdot x_2)$

ΣΠΤ

$$\left.\begin{array}{ll} \frac{\partial K}{\partial x_1}: & p_1 - \lambda x_2 = 0 \\ \frac{\partial K}{\partial x_2}: & p_2 - \lambda x_1 = 0 \\ \frac{\partial K}{\partial \lambda}: & u - x_1 \cdot x_2 = 0 \Rightarrow x_2 = \frac{u}{x_1} \end{array}\right\} \Rightarrow \frac{p_1}{p_2} = \frac{x_2}{x_1} \Rightarrow x_1 = \frac{p_2}{p_1} \cdot x_2 = \frac{p_2}{p_1} \cdot \frac{u}{x_1} \Rightarrow x_1^2 = \frac{p_2}{p_1} \cdot u$$

$$\Rightarrow x_1^c = \sqrt{\frac{p_2}{p_1} \cdot u}$$

ΑΝΤ. ΣΥΝ. ΣΗΤΗΣ

$$x_2^c = \sqrt{\frac{p_1}{p_2} u}$$

ΣΥΝΑΡΤΗΣΗ ΔΑΠΑΝΗΣ

$$p_1 x_1^* + p_2 \cdot x_2^* = p_1 \cdot \sqrt{\frac{p_2}{p_1} \cdot u} + p_2 \sqrt{\frac{p_1}{p_2} \cdot u} = 2\sqrt{p_1 \cdot p_2 \cdot u}$$

ΕΞΙΣΩΣΗ HOTELLING SLUTSKY

$$x^c(p_1, p_2, u) = x[p_1, p_2, E(p_1, p_2, u)] \Rightarrow \frac{\partial x^c}{\partial p_1} = \frac{\partial x}{\partial p_1} + \frac{\partial x}{\partial E} \cdot \frac{\partial E}{\partial p_1} \Rightarrow \frac{\partial x}{\partial p_1} = \frac{\partial x^c}{\partial p_1} - x \cdot \frac{\partial x}{\partial m}$$

Π.Χ. $x_1^c = \sqrt{\frac{p_2}{p_1} \cdot u}$, $x_1 = \frac{m}{2p_1}$, $E(p_1, p_2, u) = 2\sqrt{p_1 \cdot p_2 \cdot u}$

$$v(p_1, p_1, m) = \frac{m^2}{4 p_1 \cdot p_2}$$

$$\Rightarrow \frac{\partial x}{\partial p_1} = -\frac{m}{2p_1^2}$$

$$\frac{\partial x^c}{\partial p_1} = -\frac{1}{2} \frac{\sqrt{p_2}}{p_1^{3/2}} \cdot \sqrt{u} = -\frac{1}{2} \frac{\sqrt{p_2}}{p_1^{3/2}} \sqrt{\frac{m^2}{4 p_1 \cdot p_2}} = -\frac{1}{2} \frac{m}{2 p_1^2}$$

$$-x \frac{\partial x}{\partial m} = -x \cdot \frac{1}{2p_1} = -\frac{m}{2p_1} \cdot \frac{1}{2p_1}$$

$$\Rightarrow -\frac{m}{2p_1^2} = -\frac{1}{2} \frac{m}{2p_1^2} - \frac{m}{4 p_1^2}$$

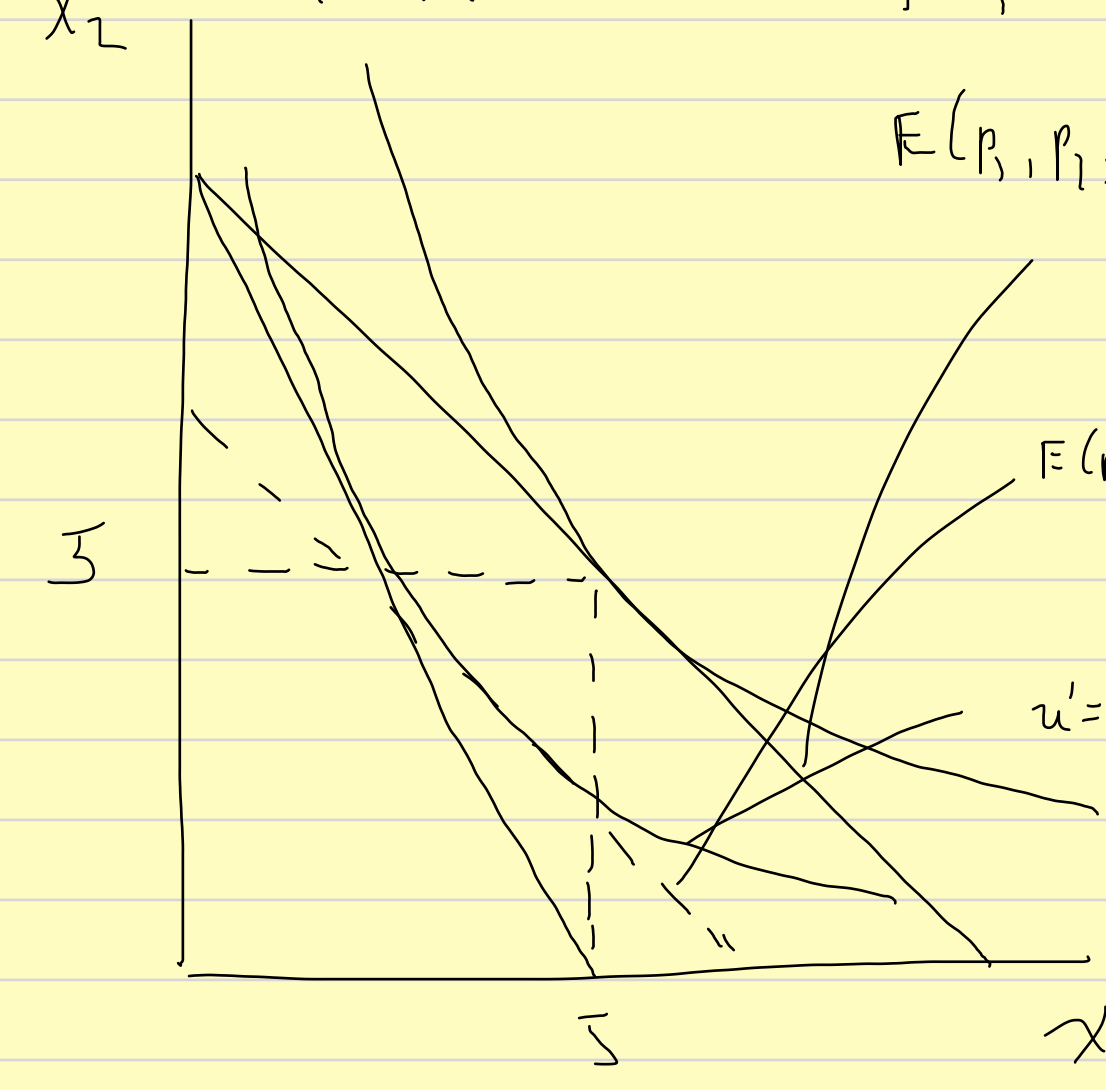
$$= -\frac{1}{4} \frac{m}{p_1^2} - \frac{1}{4} \frac{m}{p_1^2} \quad \text{QED.}$$

$u = V(1,1,10) = 25$

$1 \cdot x_1 + 1 \cdot x_2 = 10$

$E(p_1, p_2, u) = 2 \cdot \sqrt{1 \cdot 1 \cdot 25}$

$= 10$



$E(p_1, p_2, u') = 2\sqrt{1 \cdot 1 \cdot 12,5}$

$= 7,07$

$u' = V(2,1,10) = \frac{10^2}{4 \cdot 2 \cdot 1} = 12,5$

$E(p_1, p_2, u) - E(p_1, p_2, u') = 10 - 7,07 = 2,93$ ΙΣΟΔΥΝΑΜΗ ΜΕΤΑΒΟΛΗ