MSc in Applied Economics and Finance
Microeconomic Theory
Theory of Production – January 2017

Answer all three questions. Good luck!

Question 1 (35%)
For the production function $q = z_1^{1/4} z_2^{1/4}$.
(a) Find the conditional demand functions for z_1 and z_2.

Answer: $z_1^*(w_1, w_2, q) = q^2 \left(\frac{w_2}{w_1}\right)^{1/2}$ and $z_2^*(w_1, w_2, q) = q^2 \left(\frac{w_1}{w_2}\right)^{1/2}$

(b) Find the cost function.

Answer: $w_1 z_1^* + w_2 z_2^* = 2q^2 w_1^{\frac{1}{2}} w_2^{\frac{1}{2}}$

(c) Find the supply function.

Answer: From FOC of profit maximization problem $pq - c(w_1, w_2, q)$ we get $q(p, w_1, w_2) = \frac{p}{4w_1^{\frac{1}{2}} w_2^{\frac{1}{2}}}$

(d) Find the input demand (Marshallian) function for z_1. Briefly explain other ways of deriving the demand function.

Answer: We can find this by simply substituting the supply function into the conditional demand $z_1^*(p, w_1, w_2) = \left(\frac{p}{4w_1^{\frac{1}{2}} w_2^{\frac{1}{2}}}\right)^2 \left(\frac{w_2}{w_1}\right)^{1/2} = \frac{p^2}{16 w_1^{3/2} w_2^{1/2}}$. We could find the Marshallian demand by maximizing profits and solving for input demands. Also, if we were given the profit function we could use Hotelling's lemma to find the Marshallian demands ($z_1^* = -\frac{\partial \pi}{\partial w_1}$).

(e) Find the short run supply function when $\bar{z}_2 = 16$ (note $z_2^{1/4} = 2$). Will this firm always supply at a positive price? Explain.

should be 1/4

Answer:Need to max with respect to z_1 $pz_1^{\cancel{1/2}}2 - w_1 z_1 - w_2 16$, take FOC to solve for short run demand for z_1 $z_1(p, w_1, w_2; \bar{z}_2) = \left(\frac{p}{2w_1}\right)^{4/3}$, then put this into the production function to get the short run supply of $q(p, w_1, w_2; \bar{z}_2) = 2\left(\frac{p}{2w_1}\right)^{1/3}$. We can observe that the average variable cost will have a minimum at zero so that the firm will always supply a positive amount at a positive price.

Question 2 (30%)
A firm has a fixed cost of €400 and a total variable costs $= 20q + 0.25q^2$ where q is output.
(a) If the firm were a price-taker, what is the lowest price at which it would be prepared to produce a positive amount of output? How much output q* would it produce at this price? What is the perfectly competitive firm's supply curve?

1

$\min \; w_1 z_1 + w_2 \cdot z_2 \quad \text{s.t.} \; q = z_1^{1/4} \cdot z_2^{1/4}$

$L = w_1 z_1 + w_2 z_2 + \lambda(q - z_1^{1/4} \cdot z_2^{1/4})$

$\frac{\partial L}{\partial z_1}: \; w_1 - \lambda z_1^{-3/4} \cdot z_2^{1/4} = 0$

$\frac{\partial L}{\partial z_2}: \; w_2 - \lambda z_1^{1/4} \cdot z_2^{-3/4} = 0$

$\frac{w_1}{w_2} = \frac{z_2}{z_1} \Rightarrow z_2 = \frac{w_1}{w_2} \cdot z_1$

$\frac{\partial L}{\partial \lambda}: \; q - z_1^{1/4} \cdot z_2^{1/4} = 0 \qquad q = z_1^{1/4}\left(\frac{w_1}{w_2} \cdot z_1\right)^{1/4}$

$\Rightarrow z_1^{1/2} \cdot \left(\frac{w_1}{w_2}\right)^{1/4} = q \Rightarrow z_1 = \left(\frac{w_2}{w_1}\right)^{\frac{1}{2}} q^2$

2

$w_1 \cdot \left(\frac{w_2}{w_1}\right)^{\frac{1}{2}} \cdot q^2 + w_2 \left(\frac{w_1}{w_2}\right)^{\frac{1}{2}} q^2 =$

3

$p \cdot q - 2q^2 w_1^{1/2} \cdot w_2^{1/2}$

$\frac{\partial \Pi}{\partial q}: \; p - 4q w_1^{1/2} \cdot w_2^{1/2} = 0 \Rightarrow$

4

$\frac{1}{4} p z_1^{-3/4} \cdot 2 - w_1 = 0 \Rightarrow \frac{1}{2} p z_1^{-3/4} = w_1$

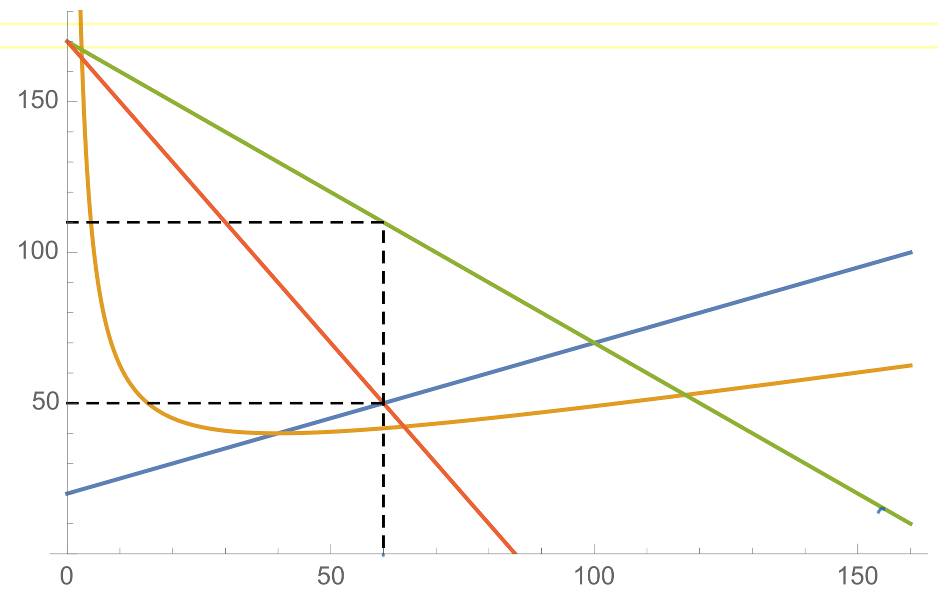
$\Rightarrow \frac{p}{2w_1} = z_1^{3/4} \Rightarrow z_1 = \left(\frac{p}{2w_1}\right)^{4/3}$

$q = z_1^{1/4} \cdot \bar{z}_2^{1/4} \Rightarrow q = z_1^{1/4} \cdot 2 \Rightarrow q = \left(\frac{p}{2w_1}\right)^{1/3} \cdot 2$

Answer: Total costs are $400 + 20q + 0.25q^2$, so average costs are $\frac{400}{q} + 20 + 0.25q$ which are a minimum at $q = 40$ where average costs are 40. For a price above the minimum AC p=20+0.5q so that $q^* = 2p - 20$.

(b) If the firm is actually a monopolist and the inverse demand function is p = 170 – q. What is the price charged p** and the marginal cost c** at this ouput. Illustrate the monopoly optimum in a diagram.

Answer: p** = 110 and c** = 50.



(c) The government decides to regulate the monopoly. The government can set a ceiling of pmax. In a separate duplicate graph of b plot the average and marginal revenue curves that would face the monopolist, explaining how output will react to different price ceilings relative to c** and p**.

(d) Linking to diagram in (b) provide a diagramatic exposition of monopolistic competition and explain.

Question 3 (35%)

You are given the following payoffs associated with two pure strategies of each of two players (a,b) in a simultaneous move game.

		Player b	
Player a		s_1^b	s_2^b
	s_1^a	3,5	10,0
	s_2^a	6,2	6,4

(a) Are there any dominant strategies in this game? Explain.

Answer: A dominant strategy is arises when a player has a best-response strategy whatever the actions of the other player. There are no dominant strategies in this game. In every instance a player's best response depends on what action the other player is taking.

(b) Are there any Nash equilibria in this game? Explain.

Answer: There are no Nash equilibria. This would require that there are actions that are simultaneously best responses to each other. There are no such pairs of actions.

(c) How would you describe this game? Can you think of any real world examples?

I could describe it as a "discoordination" game. A real world example might be tax authorities trying to decide whether to audit and tax payers trying to decide whether to report income.

(d) Find the mixed-strategy equilibrium.

1

$400q^{-1} + 20 + 0{,}25q$

$\frac{\partial}{\partial q} \quad -\frac{400}{q^2} + 0{,}25 = 0 \Rightarrow 0{,}25q^2 = 400$

$\Rightarrow q^2 = 1600 \Rightarrow \underline{q = 40}$

AC: $\frac{400}{40} + 20 + 0.25 \cdot 40 = 10 + 20 + 10 = 40$

$p = MC \Rightarrow p = 20 + 0{,}5q \Rightarrow q = 2p - 20$

2

$p \cdot q - 20q - 0{,}25q^2$

$(170-q)q - 20q - 0{,}25q^2$

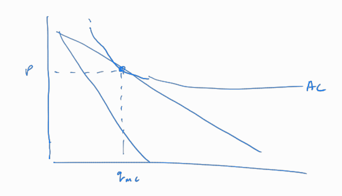
$\frac{\partial \pi}{\partial q}: \quad 170 - 2q - 20 - 0{,}5q = 0$

$\Rightarrow 150 = 2{,}5q \Rightarrow q^{**} = \frac{150}{2{,}5} = 60$

$p^{**} = 170 - 60 = 110$

$MC = 20 + 0{,}5q \Rightarrow 20 + 0{,}5 \cdot 60 = 50$

3



Answer below based on double table values.

$$v^a = \pi^a[\pi^b 6 + (1-\pi^b)20] + (1-\pi^a)[\pi^b 12 + (1-\pi^b)12] = 12 + 8\pi^a - 14\pi^a\pi^b$$

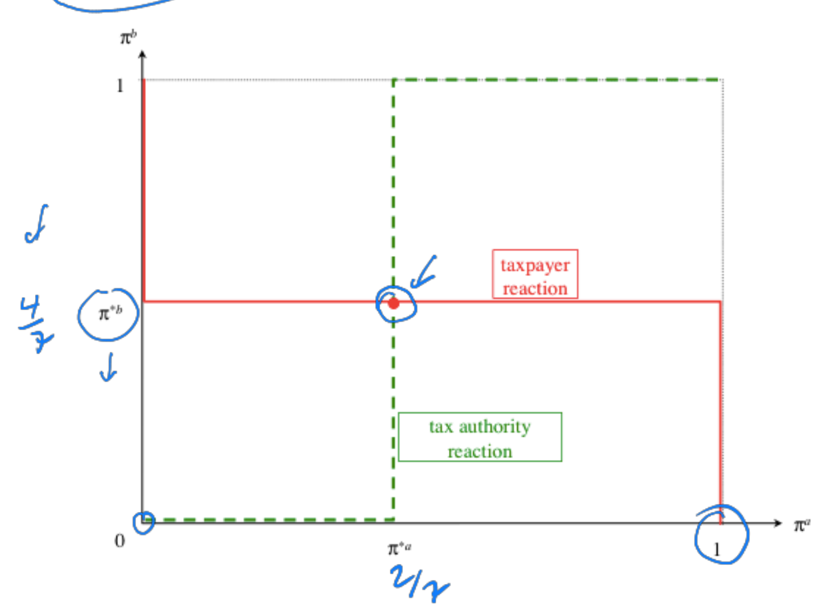
$$\frac{dv}{d\pi^a} = 8 - 14\pi^b \Rightarrow \frac{dv}{d\pi^a} \frac{>}{<} 0 \; as \; \pi^b \frac{<4}{>7}$$

$$v^b = \pi^b[\pi^a 10 + (1-\pi^a)4] + (1-\pi^b)[\pi^a 0 + (1-\pi^a)8] = 8 - 8\pi^a - 4\pi^b + 14\pi^a\pi^b$$

$$\frac{dv}{d\pi^b} = 14\pi^a - 4 \Rightarrow \frac{dv}{d\pi^a} \frac{>}{<} 0 \; as \; \pi^a \frac{<2}{>7}$$

(e) Show the mixed-strategy equilibrium in the space of probabilities. Explain.

Answer: Same as the taxpayer/tax authority reaction curves only we have specific numerical values for the mixed strategy Nash equilibrium. Player a will want to play strategy 1 for sure as long as the probability of player 2 playing strategy 1 is lower than 4/7. If player 2's probability of playing strategy 1 is higher than 4/7 than player 1 will prefer to play strategy 2 for sure. The only point where each player's best response corresponds to the other player's best response is when they each play strategy 1 with probability (4/7,2/7).



(f) Show an extensive form of this simultaneous move game. Explain.

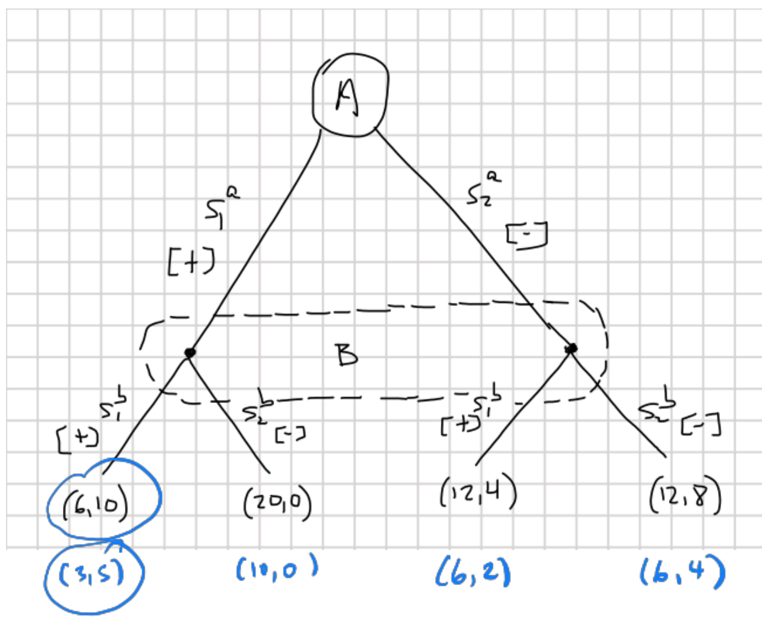
Answer: Diagram below. Since this is a simultaneous move game the choice of which player to be at the top node is arbitrary. Notice the information set that surrounds the two middle nodes. This indicates that player B does not not know which of the two nodes she is at which in this case just means that the move is simultaneous. The plus minus signs is just to give a sense of alternative actions (not needed), or in the case of the tax authority game it could be cheat/report, audit/no audit.

		Player b	
Player a		s_1^b	s_2^b
	s_1^a	3,5	10,0
	s_2^a	6,2	6,4

$\hat{V}_a = \pi^a(\pi^b \cdot 3 + (1-\pi^b)\cdot 10] + (1-\pi^a)[\pi^b \cdot 6 + (1-\pi^b)]\cdot 6 =$

$= 6 + 4\pi^a - 7\pi^a \cdot \pi^b$

$\frac{dv}{d\pi^a} = \boxed{4 - 7\pi^b} \Rightarrow \frac{dv}{d\pi} \frac{\geq}{<} \; as \; \pi^b \frac{\leq}{>} \frac{4}{7}$


A
s_1^a
[+]
s_2^a
[-]
B
s_1^b
[+]
s_2^b
[-]
s_1^b
[+]
s_2^b
[-]
(6,10)
(20,0)
(12,4)
(12,8)
(3,5)
(10,0)
(6,2)
(6,4)