$$f(l,K) = 20 l^{3/5} \cdot k^{2/5}$$

$\max_{q,l,k} \Pi = pq - w\cdot l - rk$ υπο το περιορισμο $q = 20 l^{3/5} \cdot K^{2/5}$

$$\Rightarrow \max_{l,k} \Pi = p\cdot(20 l^{3/5} k^{2/5}) - w\cdot l - rk$$

Συν Τ $\begin{cases} \frac{\partial \Pi}{\partial l} = 8p l^{-\frac{2}{5}} \cdot k^{\frac{2}{5}} - w = 0 \\ \frac{\partial \Pi}{\partial k} = 8p \cdot l^{\frac{3}{5}} k^{-\frac{3}{5}} - r = 0 \end{cases} \Rightarrow$ $w = 8p l^{-\frac{3}{5}} \cdot k^{\frac{2}{5}}$ (1)

$r = 8p l^{\frac{3}{5}} \cdot k^{-\frac{3}{5}} \Rightarrow k = \left(\frac{8pl^{2/5}}{r}\right)^{5/3}$ (2)

$(1) \overset{(2)}{\Rightarrow}$ $w = 8p \cdot l^{-\frac{3}{5}} \left[\left(\frac{8pl^{2/5}}{r}\right)^{\frac{5}{3}}\right]^{2/5} = 8p \cdot l^{-\frac{3}{5}} \left(\frac{8pl^{2/5}}{r}\right)^{2/3} = \frac{8^{5/3}\cdot p^{5/3}}{r^{2/3}} \cdot l^{-\frac{1}{3}} \left[l^{-\frac{9}{15}} l^{\frac{4}{15}} = l^{-\frac{1}{3}}\right]$

$\Rightarrow l^{-\frac{1}{3}} = \frac{w\cdot r^{2/3}}{8^{2/3}\cdot p^{5/3}} \Rightarrow l = \left(\frac{8^{5/3}\cdot p^{5/3}}{w\cdot r^{2/3}}\right)^3 = \frac{8^5 \cdot p^5}{w^3 \cdot r^2}$

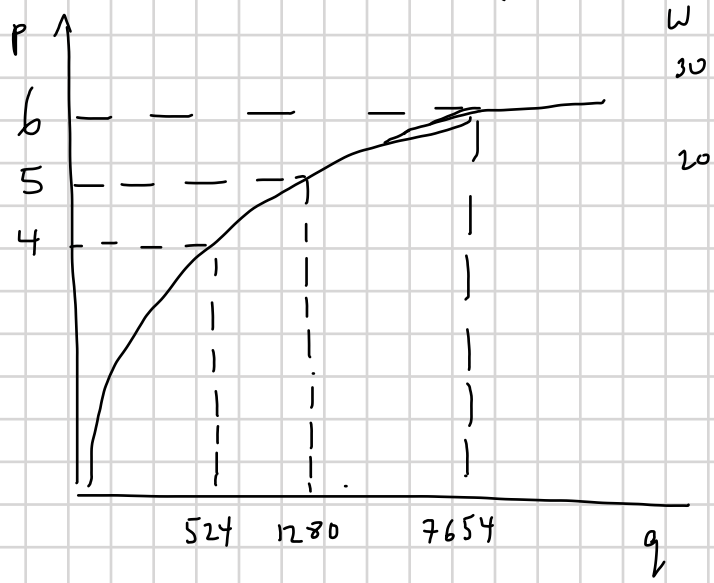
$\Rightarrow l(p,w,r) = \frac{(8p)^5}{w^3 r^2}$ (3)

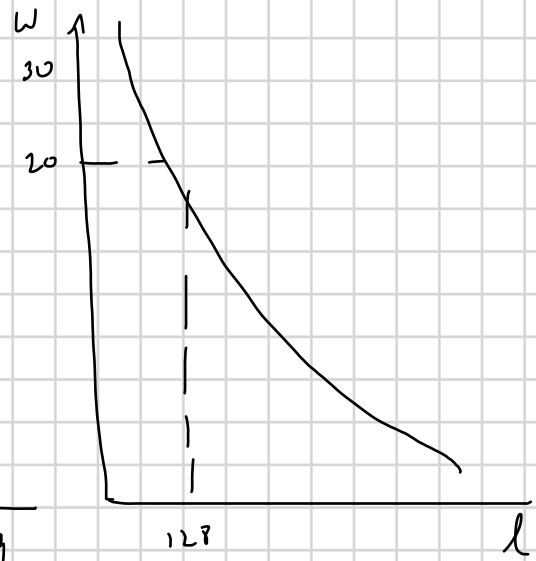
(2) $\overset{3}{\Rightarrow}$ $k(p,w,r) = \frac{(8p)^5}{w^2 r^3}$ (4)

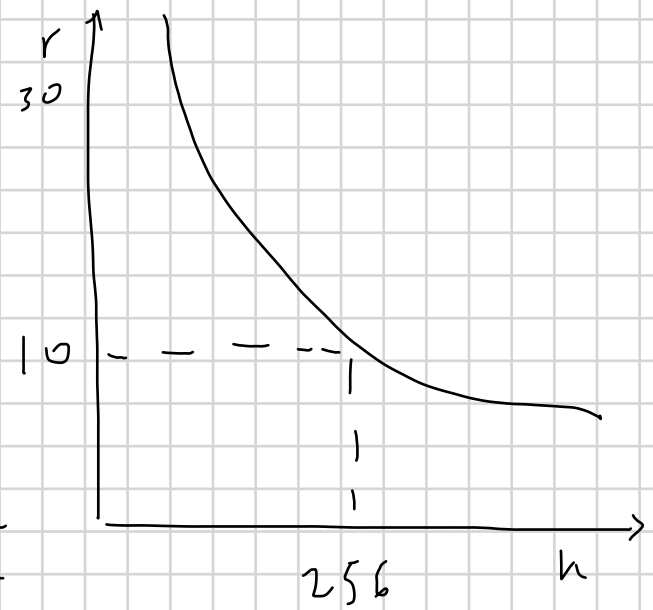
ΣΥΝΑΡΤΗΣΗ ΖΗΤΗΣΗΣ

$q = f(l,k) = 20 l^{\frac{2}{5}} \cdot k^{\frac{2}{5}} \overset{(3),(4)}{\Longrightarrow} 20\left(\frac{(8p)^5}{w^3 r^2}\right)^{\frac{2}{5}} \cdot \left(\frac{(8p)^5}{w^2 r^3}\right)^{\frac{2}{5}} = \frac{20\,(8p)^4}{w^2 \cdot r^2} = \frac{81920\, p^4}{(w\cdot r)^2}$

$\Rightarrow q(p,w,r) = \frac{81920\, p^4}{(w\cdot r)^2}$







Αντίστροφες συναρτήσεις: $(p=5, w=20, r=10)$

$q(p,20,10)$ $\quad l(5,w,10)$ $\quad k(5,20,r)$

$$\Pi(p,w,r) = p\cdot\frac{20(8p)^4}{w^2 r^2} - \frac{w(8p)^5}{w^3 r^2} - \frac{r(8p)^5}{w^2 r^3} = \frac{1}{2}\frac{(8p)^5}{w^2 r^2}$$

ΕΛΑΧΙΣΤΟΠΟΙΗΣΗ ΚΟΣΤΟΥΣ

$$\min_{l,k} C = w\cdot l + r\cdot k \quad \text{υ.τ.π.} \quad q = f(l,k)$$

$$\mathcal{L}(l,k,\lambda) = w\cdot l + rk + \lambda\left(q - 20\, l^{2/5} k^{2/5}\right)$$

Σ.Π.Τ.

$$\left.\begin{aligned} \frac{\partial \mathcal{L}}{\partial l} &= w - 8\lambda\, l^{-\frac{3}{5}} k^{\frac{2}{5}} = 0 \\ \frac{\partial \mathcal{L}}{\partial k} &= r - 8\lambda\, l^{\frac{2}{5}} k^{-\frac{3}{5}} = 0 \end{aligned}\right\} \Rightarrow \frac{w}{r} = \frac{k}{l} \Rightarrow k = \frac{w}{r}\cdot l \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = q - 20\, l^{\frac{2}{5}} k^{\frac{2}{5}} = 0 \quad (1)$$

$$(1) \overset{(2)}{\Rightarrow} q = 20\, l^{\frac{2}{5}} \left(\frac{w}{r} l\right)^{\frac{2}{5}} \Rightarrow l^{\frac{4}{5}} = \frac{q}{20}\left(\frac{r}{w}\right)^{\frac{2}{5}} \Rightarrow l = \left(\frac{r}{w}\right)^{\frac{1}{2}} \left(\frac{q}{20}\right)^{\frac{5}{4}} \quad (3)$$

$$(2) \overset{3}{\Rightarrow} k = \frac{w}{r}\cdot\left(\frac{r}{w}\right)^{\frac{1}{2}}\left(\frac{q}{20}\right)^{\frac{5}{4}} \Rightarrow k = \left(\frac{w}{r}\right)^{\frac{1}{2}} \left(\frac{q}{20}\right)^{\frac{5}{4}}$$

$$\Rightarrow \quad l(w,r,q) = \left(\frac{r}{w}\right)^{\frac{1}{2}} \cdot \left(\frac{q}{20}\right)^{\frac{5}{4}}$$

$$k(w,r,q) = \left(\frac{w}{r}\right)^{\frac{1}{2}} \left(\frac{q}{20}\right)^{\frac{5}{4}}$$

ΠΑΡΑΓΩΓΙΚΕΣ ΣΥΝΑΡΤΗΣΕΙΣ ΖΗΤΗΣΗΣ

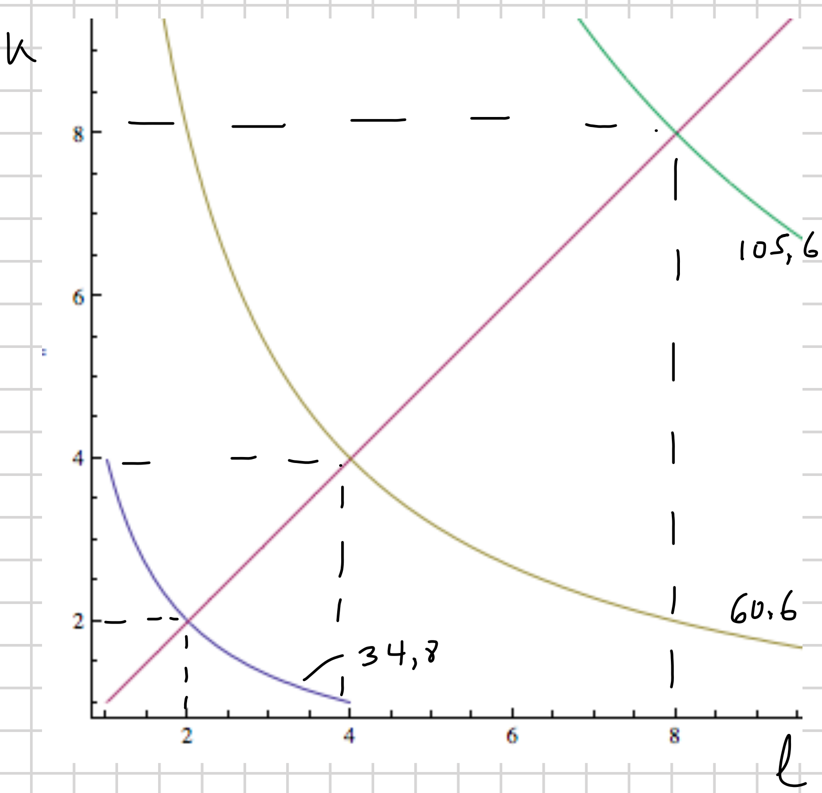
ΣΥΝΑΡΤΗΣΗ ΚΟΣΤΟΥΣ $\Rightarrow$ $C(w,r,q) = w\cdot l(w,r,q) + r k(w,r,q)$

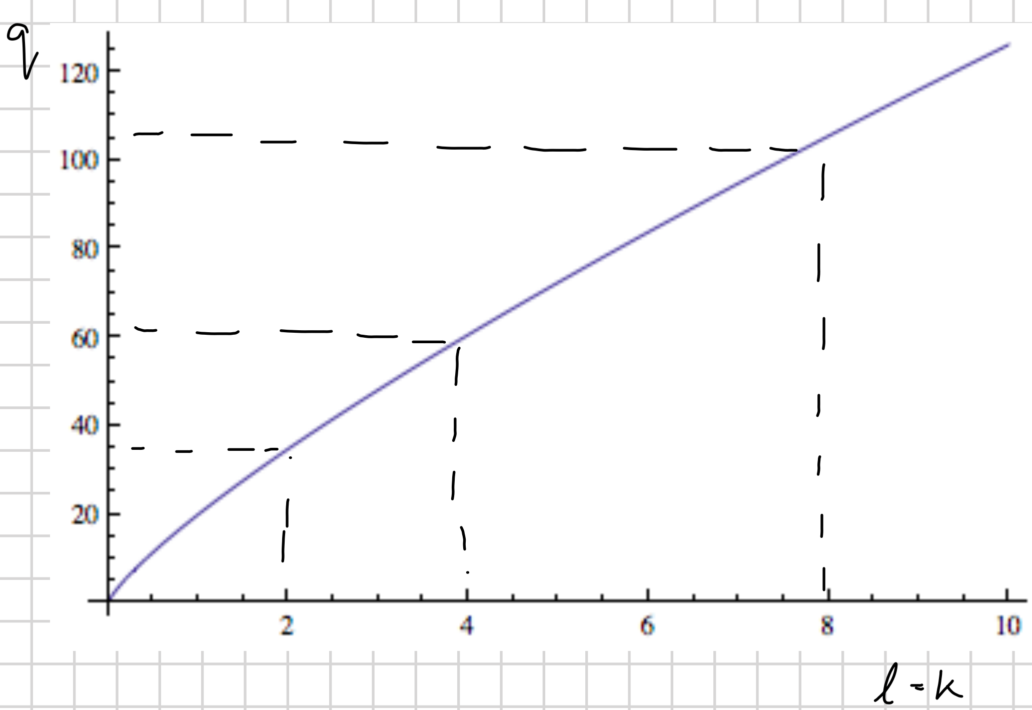
$$= w\cdot\left(\frac{r}{w}\right)^{\frac{1}{2}}\left(\frac{q}{20}\right)^{\frac{5}{4}} + r\left(\frac{w}{r}\right)^{\frac{1}{2}}\left(\frac{q}{20}\right)^{\frac{5}{4}}$$

$$= 2(w\cdot r)^{\frac{1}{2}}\left(\frac{q}{20}\right)^{\frac{5}{4}}$$

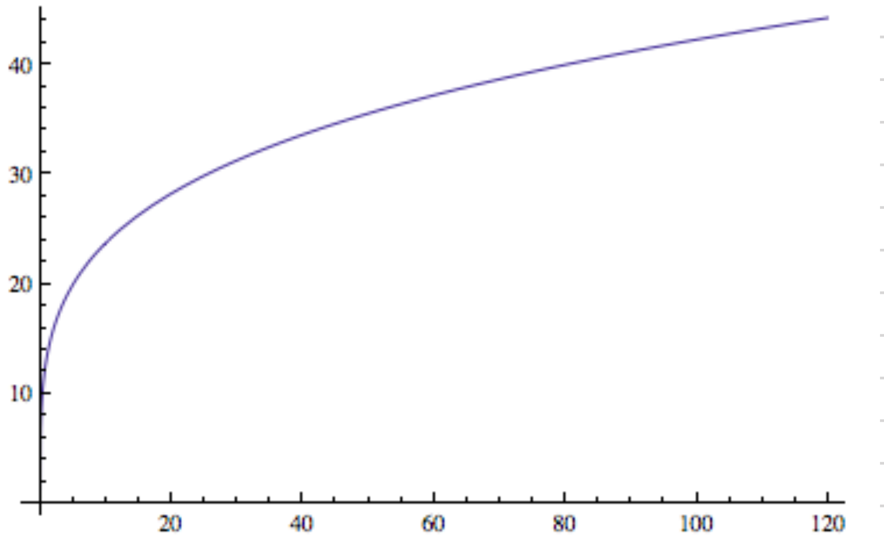
$$MC = \frac{\partial C(w,r,q)}{\partial q} = \frac{5}{4}\cdot 2\,(wr)^{\frac{1}{2}}\left(\frac{q}{20}\right)^{\frac{1}{4}}\cdot\frac{1}{20} = \frac{(w\cdot r)^{\frac{1}{2}}}{8}\cdot\left(\frac{q}{20}\right)^{\frac{1}{4}}$$

$$AC = \frac{C(w,r,q)}{q} = 2\cdot(w\cdot r)^{\frac{1}{2}}\left(\frac{q}{20}\right)^{\frac{1}{4}}\cdot\frac{1}{2} = \frac{(w\cdot r)^{\frac{1}{2}}}{10}\cdot\left(\frac{q}{20}\right)^{\frac{1}{4}}$$



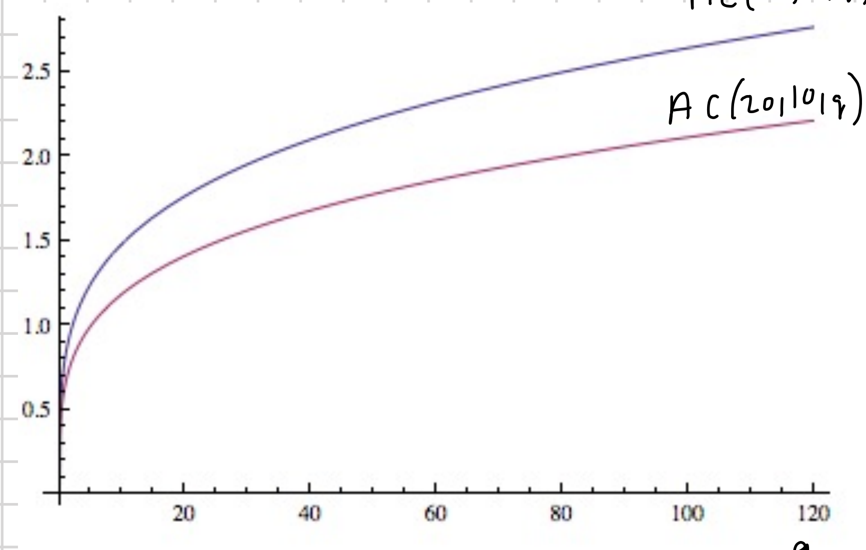


$C(20,10,q)$



$MC(20,10,q)$

$AC(20,10,q)$



ΜΕΓΙΣΤΟΠΟΙΗΣΗ ΚΕΡΔΟΥΣ ΣΕ ΔΕΥΤΕΡΟ ΣΤΑΔΙΟ

$$\max_{q} \pi = p \cdot q - C(w,r,q)$$

$$\pi = p\cdot q - 2(wr)^{\frac{1}{2}}\left(\frac{q}{20}\right)^{\frac{5}{4}}$$

$$\frac{\partial \pi}{\partial q} = 0 \Rightarrow \quad p - \frac{(w\cdot r)^{\frac{1}{2}}}{8}\cdot\left(\frac{q}{20}\right)^{\frac{1}{4}} = 0 \Rightarrow \quad p = \frac{(w\cdot r)^{\frac{1}{2}}}{8}\cdot\left(\frac{q}{20}\right)^{\frac{1}{4}} \Rightarrow \left(\frac{q}{20}\right)^{\frac{1}{4}} = \frac{8p}{(wr)^{\frac{1}{2}}}$$

$$\Rightarrow \quad q = 20\cdot\frac{(8p)^4}{(wr)^2} = \frac{81920\,p^4}{(w\cdot r)^2}$$ (ΟΠΟΙΟ ΒΡΗΚΑΜΕ ΑΠΟ ΜΕΓΙΣΤΟΠ. ΚΕΡΔΟΥΣ)

ΜΠΟΡΟΥΜΕ ΝΑ ΒΡΟΥΜΕ ΤΙΣ ΚΑΝΟΝΙΚΕΣ ΣΥΝΑΡΤΗΣΕΙΣ ΖΗΤΗΣΗΣ k, l ΒΑΖΟΝΤΑΣ ΤΟ

q ΠΟΥ ΒΡΗΚΑΜΕ ΣΤΗ ΠΡΟΗΓΟΥΜΕΝΗ ΣΥΝΑΡΤΗΣΗ ΖΗΤΗΣΗΣ

$$\left.\begin{aligned} l(w,r,q) &= \left(\frac{r}{w}\right)^{\frac{1}{2}}\cdot\left(\frac{q}{20}\right)^{\frac{5}{4}} \\ k(w,r,q) &= \left(\frac{w}{r}\right)^{\frac{1}{2}}\left(\frac{q}{20}\right)^{\frac{5}{4}} \end{aligned}\right] \Rightarrow \quad l(p,w,r) = \left(\frac{r}{w}\right)^{\frac{1}{2}}\left(\frac{\frac{20\cdot(8p)^4}{(w\cdot r)^2}}{20}\right)^{\frac{5}{4}}$$

ΠΡΕΠΕΙ ΝΑ ΒΓΕΙ ΙΔΙΟ ΜΕ ΜΕΓΙΣΤ. ΚΕΡΔΟΥΣ...

ESTO $k = \bar{k}$ (SHORT RUN)

$$\max_{l} \quad p \cdot q - w \cdot l - r \cdot \bar{k}$$

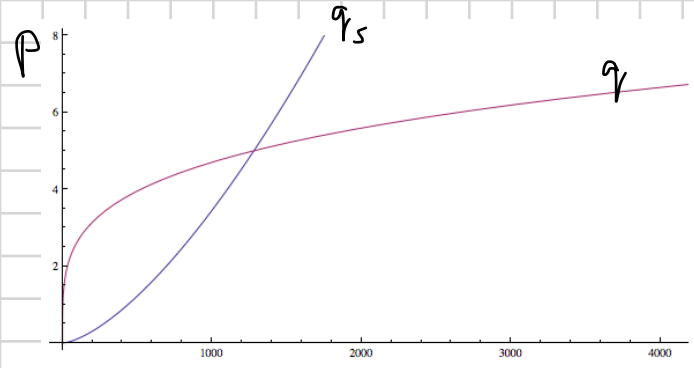
$$p\left(20 l^{\frac{2}{5}} \cdot \bar{k}^{\frac{2}{5}}\right) - w \cdot l - r \cdot \bar{k}$$

$$\frac{\partial \Pi}{\partial l} = 8p l^{-\frac{3}{5}} \cdot \bar{k}^{\frac{2}{5}} - w \cdot l - r\bar{k} = 0 \qquad [\, p \cdot MP_L = w \,]$$

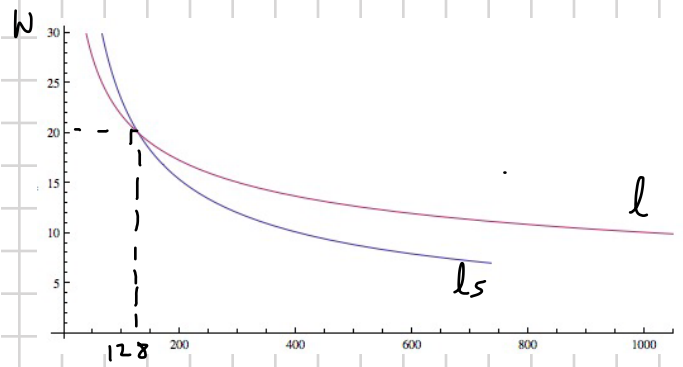
$$\Rightarrow l_s^{-\frac{3}{5}} = \frac{w}{8p\bar{k}^{\frac{2}{5}}} \Rightarrow l_s(p,w,r,\bar{k}) = \left(\frac{8p\bar{k}^{\frac{2}{5}}}{w}\right)^{\frac{5}{3}}$$

$$q_s = 20 l^{\frac{2}{5}} \cdot \bar{k}^{\frac{2}{5}} = 20\left(\left(\frac{8p\bar{k}^{\frac{2}{5}}}{w}\right)^{\frac{5}{3}}\right)^{\frac{2}{5}} \cdot \bar{k}^{\frac{2}{5}} = 20\left(\frac{8p\bar{k}^{\frac{2}{5}}}{w}\right)^{\frac{2}{3}} \cdot \bar{k}^{\frac{2}{5}} = 20\left(\frac{8p\bar{k}}{w}\right)^{\frac{2}{3}}$$

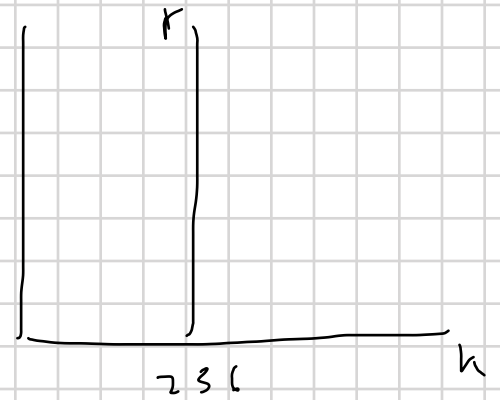
$$\Rightarrow q_s(p,w,r,\bar{k}) = 20\left(\frac{8p\bar{k}}{w}\right)^{\frac{2}{3}} \qquad \text{BMAX. PROFIT}$$



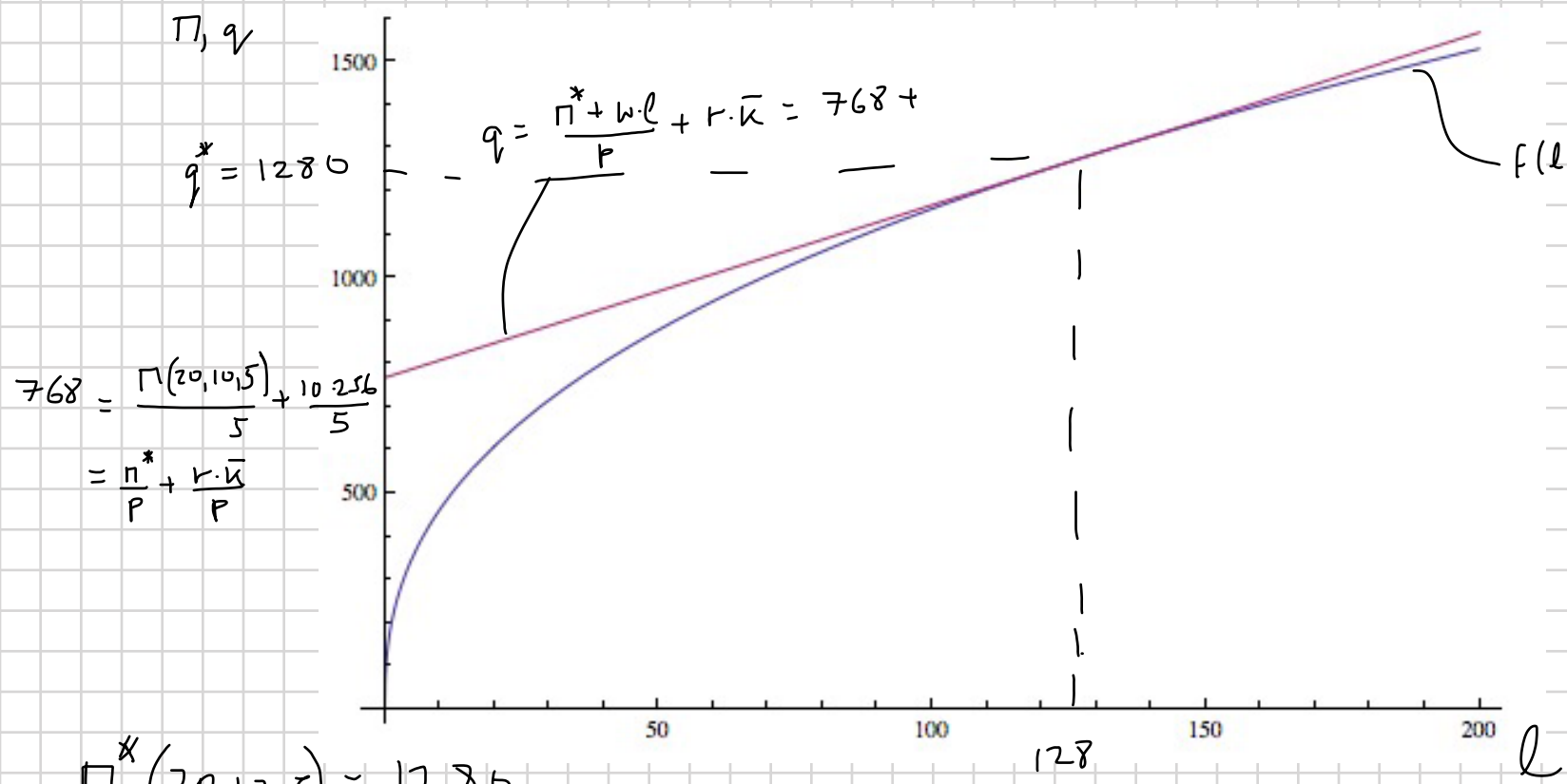
q



l



$$\Pi = p \cdot q - w \cdot l - r\bar{k} \Rightarrow p \cdot q = \Pi + w \cdot l + r \cdot \bar{k} \Rightarrow q = \frac{\Pi + w \cdot l}{p} + \frac{r \cdot \bar{k}}{p}$$



$$768 = \frac{\Pi(20,10,5)}{5} + \frac{10\,256}{5}$$

$$= \frac{\Pi^*}{p} + \frac{r \cdot \bar{k}}{p}$$

$$\Pi^*(20,10,5) = 1280$$

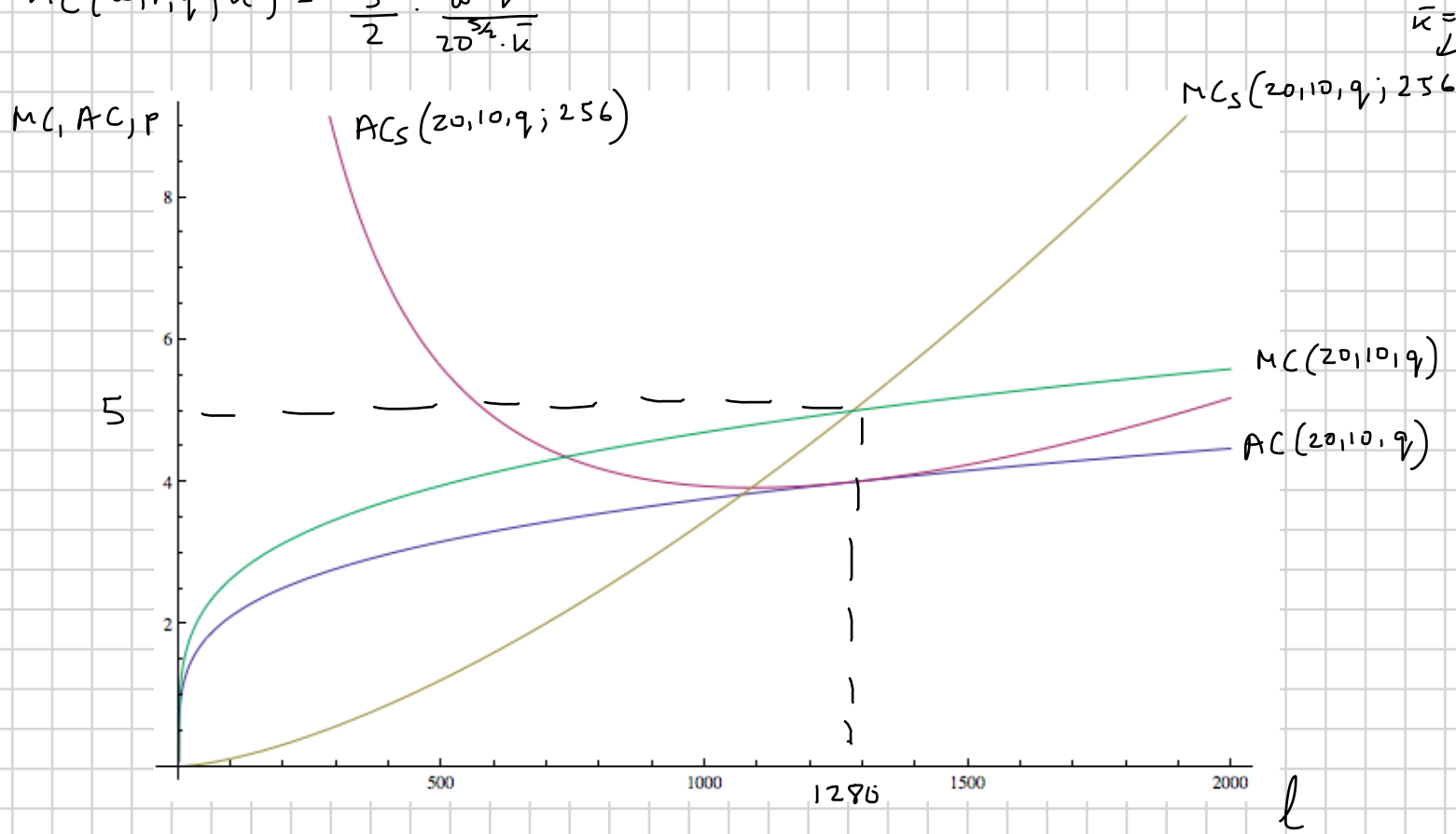
$$\min C_s = \overbrace{w \cdot l + r \cdot \bar{k}}^{F} \quad \text{u.t.n.} \quad q = f(l, \bar{k}) = 20 l^{2/5} \cdot \bar{k}^{2/5}$$

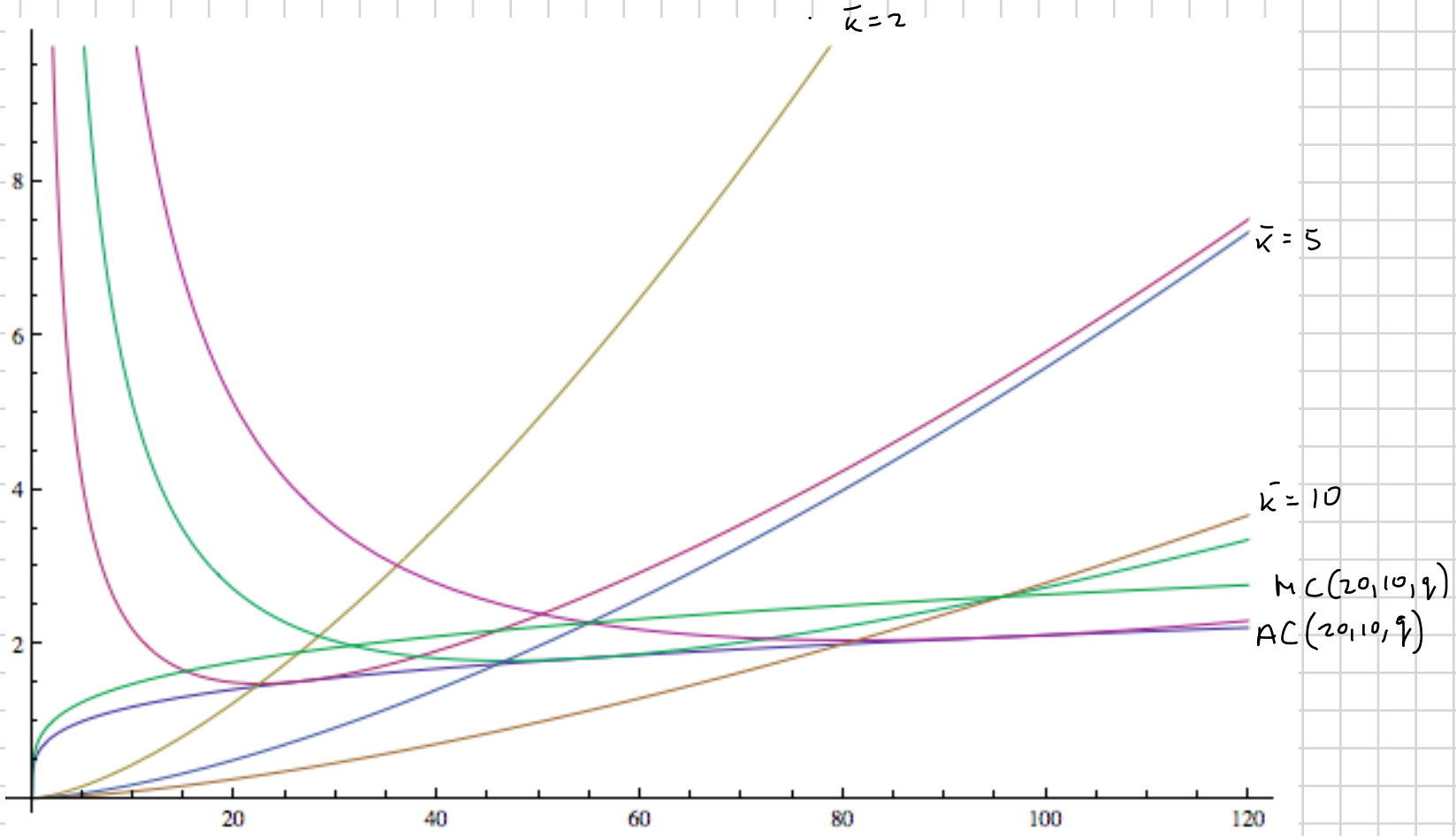
$$\Rightarrow l_s^{2/5} = \frac{q}{20 \bar{k}^{2/5}} = \frac{q^{5/2}}{20^{5/2} \cdot \bar{k}}$$

$$\Rightarrow C_s(w, r, q; \bar{k}) = \frac{w \cdot q^{5/2}}{20^{5/2} \cdot \bar{k}} + r \cdot \bar{k}$$

$$\Rightarrow AC(w, r, q; k) = \frac{w\, q^{3/2}}{20^{5/2} \cdot \bar{k}} + \frac{r \cdot \bar{k}}{q}$$

$$MC(w, r, q; k) = \frac{5}{2} \cdot \frac{w\, q^{3/2}}{20^{5/2} \cdot \bar{k}}$$





$$\Pi(w,r,p;k) = p\cdot q_s^* - w\cdot l_s^* - r\cdot\bar{k} = p\,20\left(\frac{8pk}{w}\right)^{\frac{2}{3}} - w\left(\frac{8pk}{w}\right)^{\frac{5}{3}} - r\bar{k}$$

$$= p\cdot 20\left(\frac{8pk}{w}\right)^{\frac{2}{3}} - w\left(\frac{8p}{w}\right)^{\frac{5}{3}}k^{\frac{2}{3}} - r\bar{k} = p\cdot 20\left(\frac{8pk}{w}\right)^{\frac{2}{3}} - \cancel{w}\cdot\frac{8p}{\cancel{w}}\cdot\left(\frac{8p}{w}\right)^{\frac{2}{3}}\cdot k^{\frac{2}{3}} - r\bar{k} = 12p\left(\frac{8p}{w}\right)^{\frac{2}{3}}\cdot k^{\frac{2}{3}} - r\bar{k}$$

$$\frac{\partial\Pi(w,r,p;k)}{\partial k} = 0 \Rightarrow \frac{2}{3}\cdot 12p\left(\frac{8p}{w}\right)^{\frac{2}{3}}\cdot k^{-\frac{1}{3}} - r = 0 \Rightarrow k^{-\frac{1}{3}} = \frac{r}{8p\left(\frac{8p}{w}\right)^{\frac{2}{3}}} \Rightarrow k = \left[\frac{8p\left(\frac{8p}{w}\right)^{\frac{2}{3}}}{r^3}\right]^3 \Rightarrow k = \frac{8^3\cdot 8^2\cdot p^5}{w^2\cdot r^3}$$

$$\Rightarrow \bar{k} = k^* = \frac{8^5 p^5}{w^2 r^3} = k(w,r,p)$$

$$\Pi(w,r,p;\bar{k}) = 12p\left(\frac{8p}{w}\right)^{\frac{2}{3}}\cdot k^{\frac{2}{3}} - r\bar{k} = 12p\left(\frac{8p}{w}\right)^{\frac{2}{3}}\cdot\left(\frac{8^5 p^5}{w^2 r^3}\right)^{\frac{2}{3}} - r\cdot\frac{8^5 p^5}{w^2 r^3} = \frac{1}{2}\,\frac{(8p)^5}{w^2 r^2}$$

$$= \Pi(w,r,p)$$

ΕΠΙΣΗΣ ΑΝ ΑΝΤΙΚΑΤΑΣΤΗΣΟΥΜΕ ΤΟ $k(p,w,r)$ ΣΤΗΝ ΑΡΧΙΚΗ ΒΡΑΧΥΧΡΟΝΙΑ ΣΥΝΑΡΤΗΣΗ

ΕΡΓΑΣΙΑΣ Ε ΠΡΟΣΦΟΡΑΣ ΘΑ ΒΡΟΥΜΕ ΤΙΣ ΜΑΚΡΟΧΡΟΝΙΕΣ ΣΥΝΑΡΤΗΣΕΙΣ

ΦΘΙΝΟΥΣΑ ΑΝΟ. ΚΗΙΜ.

ΣΤΑΘΕΡΗ ΑΝΟΔ ΚΗΙΜ.

ΑΥΞΟΥΣΑ ΑΝΟΔ ΚΗΙΜ.