Prerequisites

Almost essential
Game Theory: Basics

Game Theory: Strategy and Equilibrium

MICROECONOMICS

Principles and Analysis

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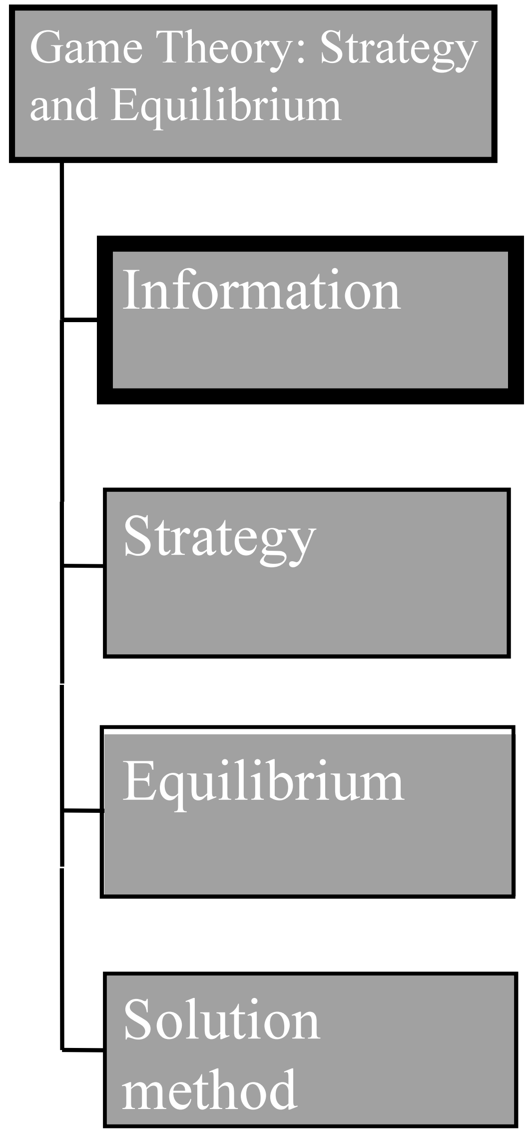
Introduction

- This presentation builds on *Game Theory: Basics* .
- We'll be re-examining some of the games introduced there, but…
  - We move from a focus on *actions* to a focus on *strategies*.
  - We move from *intuiting* an answer to *defining* an equilibrium
  - We will refine the solution method.
- First we need to introduce the topic of information in games.

Overview...



The underlying structure of games.

Information

- Consider the path through the tree of an extensive-form game.
- Which node is a player at?
  - At the beginning of the game this is obvious.
  - Elsewhere there may be ambiguity.
  - The player may not have observed some previous move.
- At a point after the beginning of the game he may know that he is at one of a number of nodes.
- Collection of these nodes is formally known as the *information set*.

Working with information sets

- The information set is crucial to characterising games.
- Focus on the number of nodes in the information set.
- Useful to distinguish two types of game
  - If each information set in the game contains just one node then it is a game of *perfect information*.
  - Otherwise it is a game of *imperfect information*.
- Can be used to clarify issues concerning timing in games.
- Let’s illustrate this…

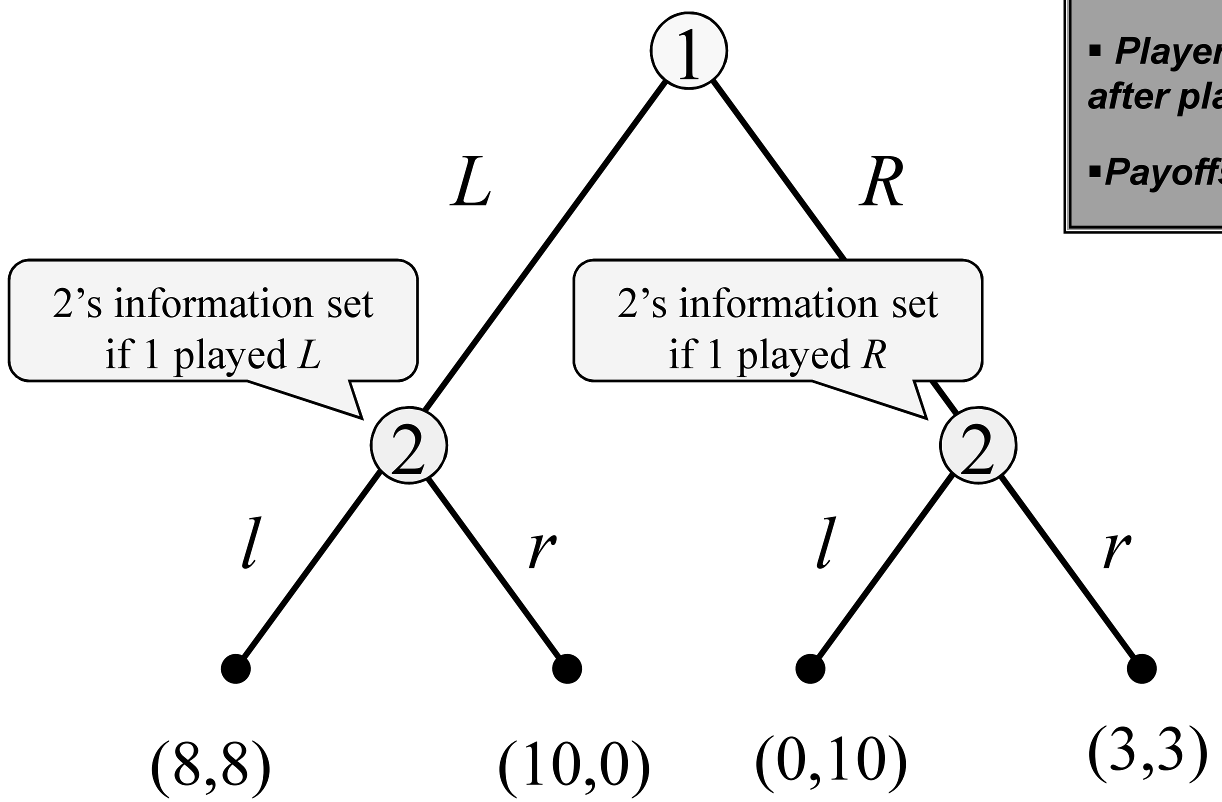
A pair of examples

- Reuse a pair of games from the basic presentation.
- Each game has:
  - Two players.
  - Player 1 chooses to move *L*eft or *R*ight
  - Player 2 chooses to move *l*eft or *r*ight
  - Payoffs according to the choices made by both players.
- The two games differ as to timing
  - First version: ("sequential") player 1 moves on Monday and player 2 moves on Tuesday.
  - Second version: ("simultaneous") both move Monday.
- But let's reinterpret the two games slightly…

The examples: reinterpretation

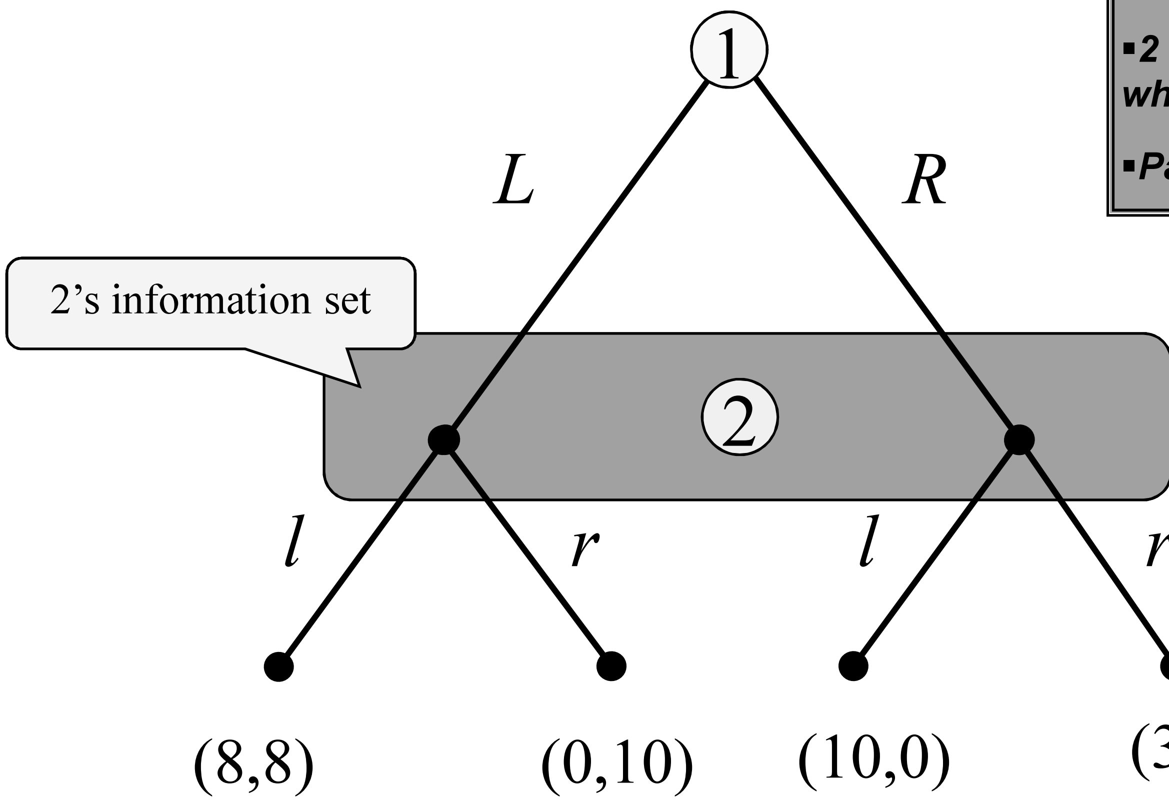
- Reconsider the sequential game we've considered earlier
  - Two periods.
  - Player 1 moves on Monday
  - Player 2 moves on Tuesday
  - But 1's move is not revealed until Tuesday evening.
- This is equivalent to the game where 1 and 2 move simultaneously on Monday.
- Now check the games in extensive form…
- …note how person 2's information set differs for the two examples

Information set (1)



- ***Player 1 chooses L or R***
- ***Player 2 chooses l or r after player 1's choice***
- ***Payoffs, given as (u_1, u_2)***

Information set (2)

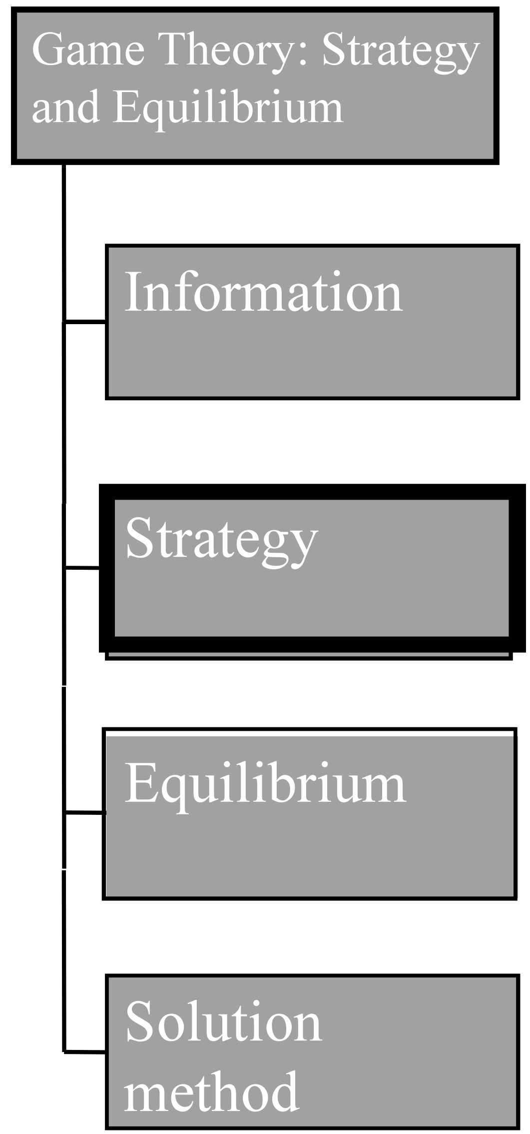


- *Player 1 chooses L or R*
- *2 doesn't know 1's choice when choosing l or r*
- *Payoffs as before*

Using information sets

- Case 1 (perfect information):
  - Two possibilities for person 2.
  - In each situation person 2 has a "singleton" information set.
- Case 2 (imperfect information)
  - Just one information set for person 2.
  - But this set contains multiple nodes.
- The information set captures the essential information for a specified player at a specified stage of the game.
- It is also useful in defining a key concept:

Overview...



Essential building block of game theory.

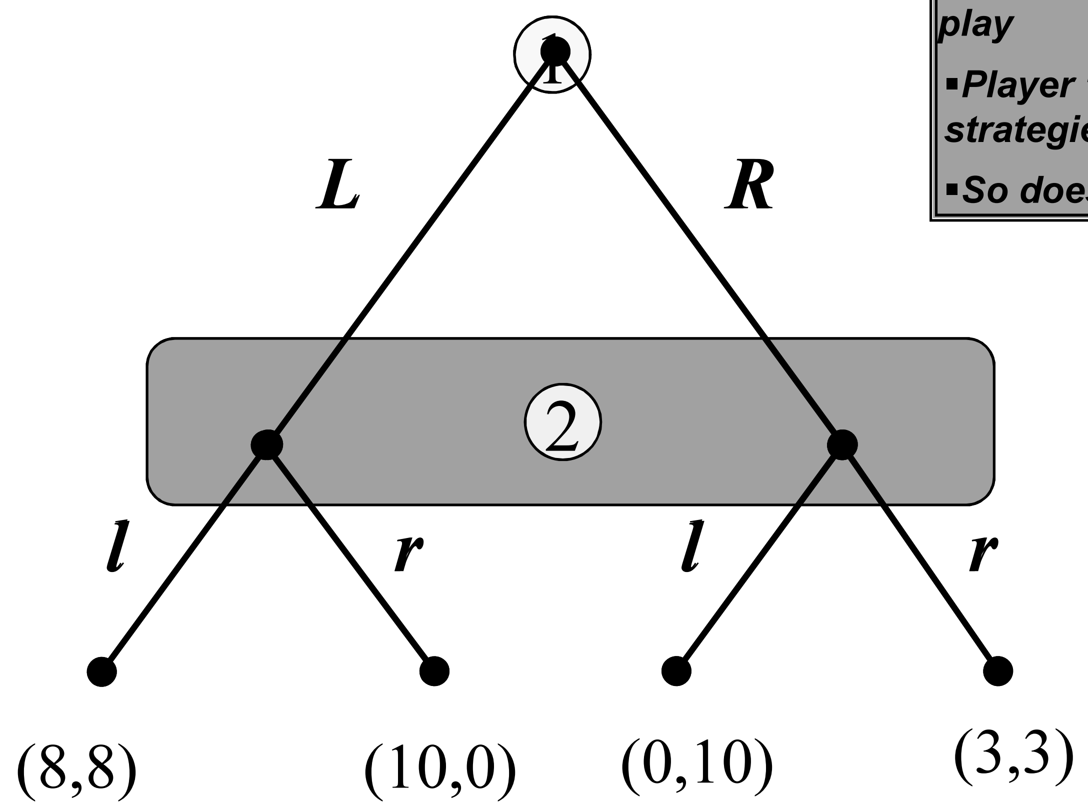
Strategy: a plan of action

- How do I move at each information set?
- I need a collection of rules for action.
- A *strategy s* is a comprehensive contingent plan of actions.
- "Contingent" – takes into account others' actions for example.
- "Comprehensive" – means that every possible node of the game is considered:
  - Not just those that "seem likely".
  - Includes those that may not be reached.

Strategy: representation

- Using the extensive-form representation it is easy to visualise a strategy.
- But we can also use normal form.
- Take the two games just illustrated.
- Consider differences between:
  - Information set (1) – sequential play.
  - Information set (2) – simultaneous play.
- Same number of *actions*; but how many *strategies*?
- We'll deal with the examples in reverse order…

Imperfect information (Extensive form)



- *Captures simultaneous play*
- *Player 1 has just two strategies*
- *So does player 2*

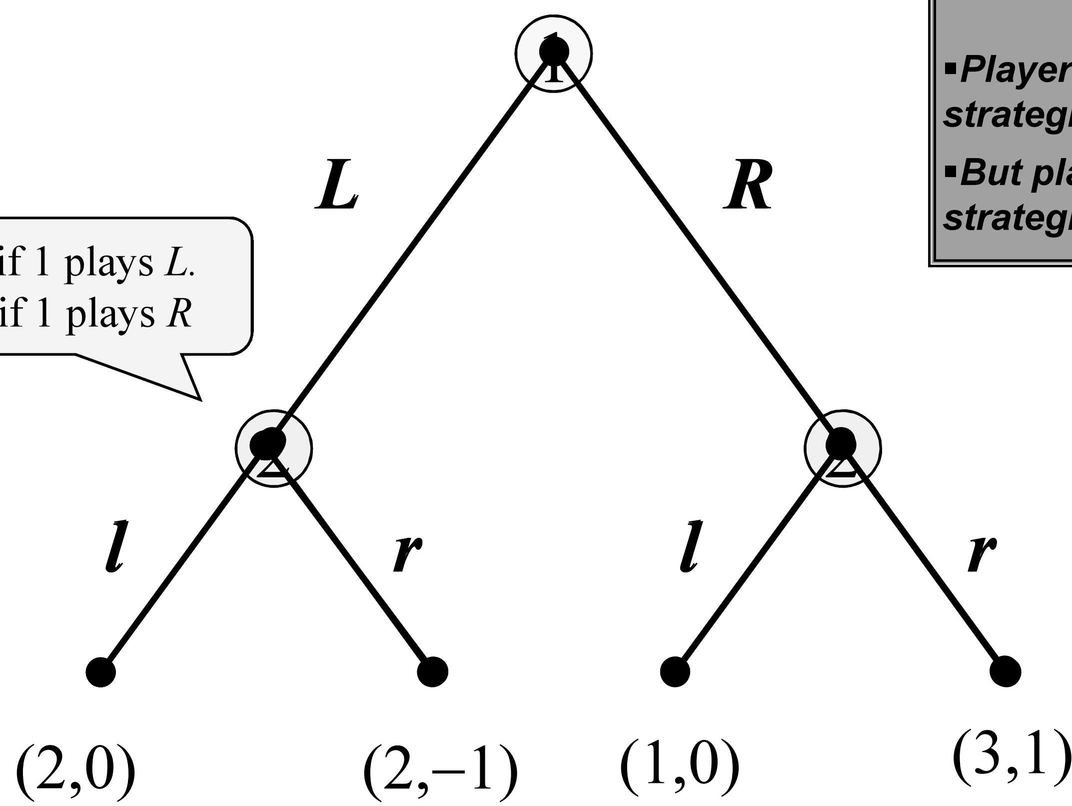
Imperfect information (Normal form)

- *Player 1's two strategies*
- *Player 2s two strategies*

Player 1	Player 2: *l*	Player 2: *r*
L	8,8	0,10
R	10,0	3,3

Perfect information (Extensive form)

- *Captures sequential play*
- *Player 1 has just two strategies*
- *But player 2 has <u>four</u> strategies…*



Play *r* if 1 plays *L*.
Play *l* if 1 plays *R*

Perfect information (Normal form)

- *Player 1's two strategies*
- *Player 2's four strategies*

Player 1 \ Player 2	*ll*	*lr*	*rl*	*rr*
L	8,8	8,8	10,0	10,0
R	0,10	3,3	0,10	3,3

ll: Always play *l* whatever 1 chooses

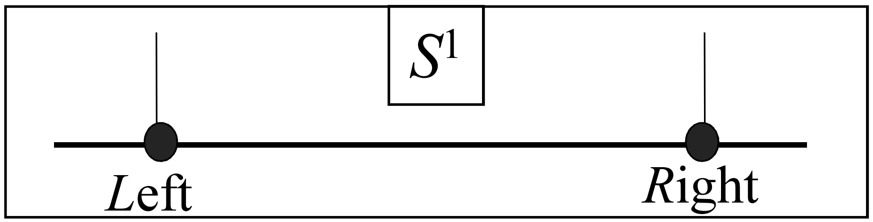
lr: Play *l* if 1 plays *L*. Play *r* if 1 plays *R*

rl: Play *r* if 1 plays *L*. Play *l* if 1 plays *R*

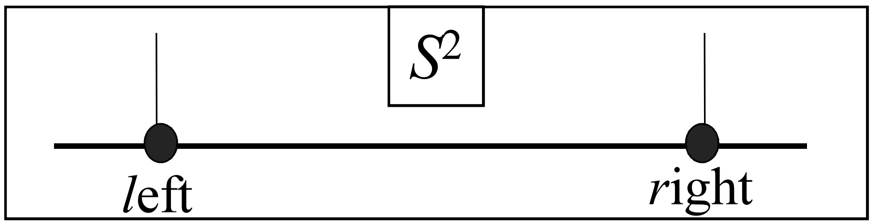
rr: Always play *r* whatever 1 chooses

Strategy space

- It's useful to describe the space of all strategies available to player h.
- Call it S^h.
- For player 1 in our examples S^1 is just two blobs

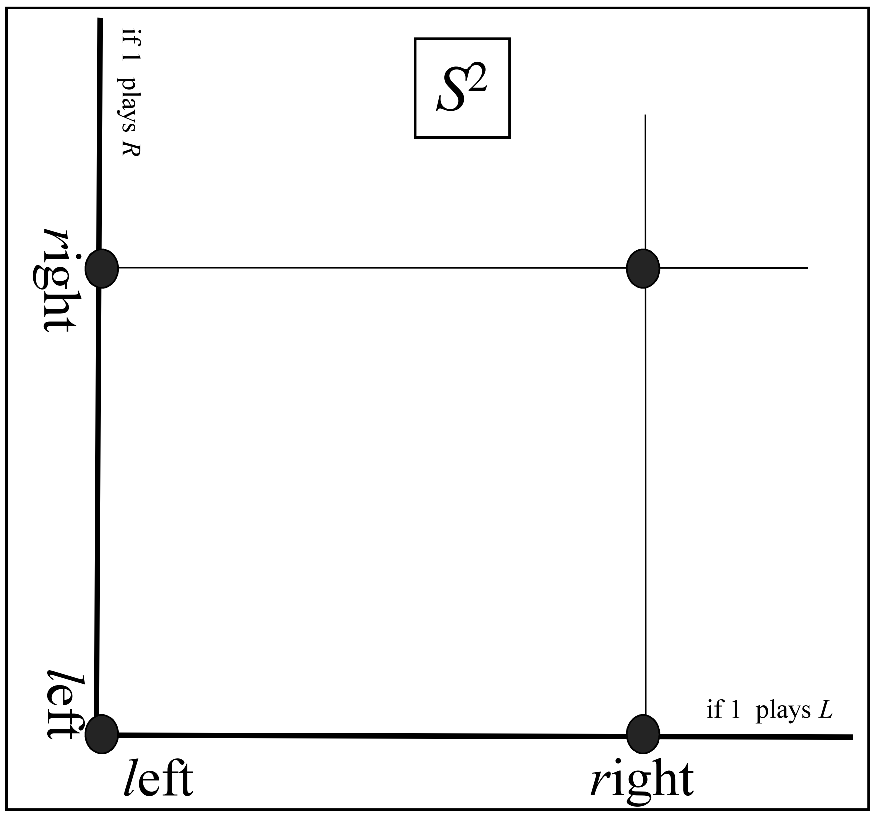


- Likewise for player 2 in the simultaneous move (imperfect information) example:



Strategy space (2)

- But S^2 in the sequential-move (perfect information) case is a little more complicated:



Building block for a solution

- The *strategy* is the basic object of choice in the economic problem represented by a game.
- How to choose a strategy?
- Let's re-examine the idea of optimisation.
  - The environment in which the player optimises is not self-evident.
  - Unlike the situations modelled in perfect markets.
- We are looking for the "best a person can do" in the light of the circumstances he faces in the game.
- Specifying the circumstances requires care:
  - What technological and or budget constraints?
  - What beliefs about others' strategies?
- But if we do this carefully then…

Best response

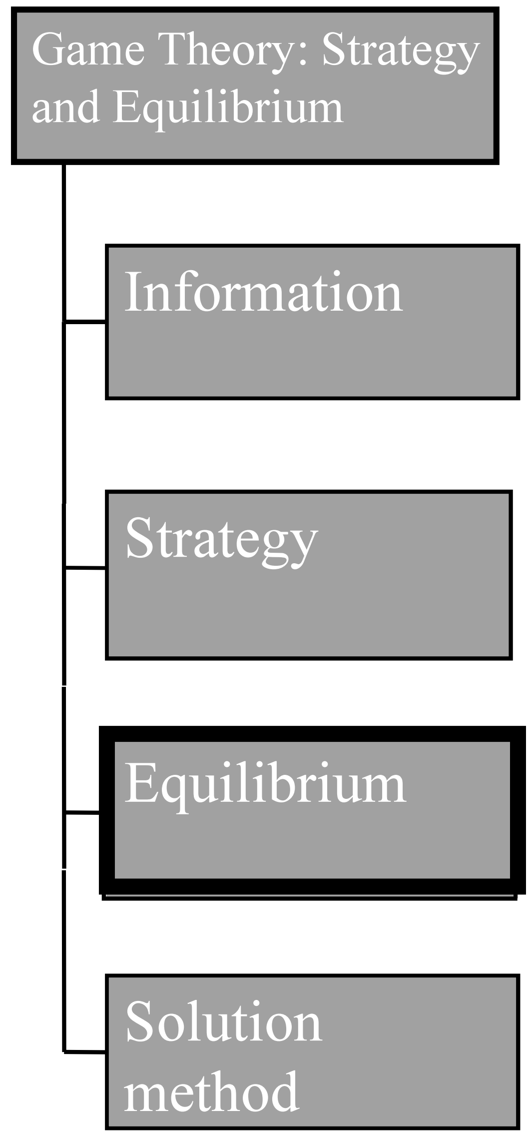
- Take one particular player *h*
- Specify a strategy for each player other than *h*…
- … and call the collection of these $[s]^{-h}$
- Find the strategy that maximises *i*'s payoff, given $[s]^{-h}$
- Is this maximising strategy unique?
  - If so, then it is the *strongly best response* to $[s]^{-h}$
  - Otherwise it is a *weakly best response* to $[s]^{-h}$
- Yields a precise definition for a particular set of beliefs about what others' plans may be
- It also provides a basis for defining a solution to the game

Dominance

- Consider the set of strategies available to all players other than *h*.
- Work out player *h*'s best response for each $[s]^{-h}$ in this set
- Suppose that in each case the *same* strategy $\hat{s}^h$ emerges for as player *h*'s best response
- Then we say that $\hat{s}^h$ is a *dominant* strategy for *h*
- Could use this to define a concept of equilibrium

Overview...

A fundamental concept and its application.



Dominance and equilibrium

- Use the idea of a *profile* of strategies
  - a list $[s^1, s^2, s^3,\ldots]$, one for each player in the game
  - for shorthand, write profile as $[s]$
  - so $[s]^{-h}$ is just a profile with the h^{th} element deleted
- An equilibrium is characterised in terms of a profile with specific properties
- So a dominant-strategy equilibrium is a profile $[\hat{s}]$ where
  - $[\hat{s}] = [\hat{s}^1, \hat{s}^2, \hat{s}^3,\ldots]$ and
  - for each player h, $\hat{s}^h$ is a dominant strategy for h
- Clearly appealing
  - everyone is maximising
- But this is a special case
  - dominance requirement is very demanding
  - we need a more general concept

Solution concept

- Again use the idea of h's best response to $[s]^{-h}$
- Is there a profile $[s^{*1}, s^{*2}, s^{*3},\ldots]$ such that, for every h, strategy s^{*h} is the best response to $[s^{*}]^{-h}$?
- If so, then $[s^{*1}, s^{*2}, s^{*3},\ldots]$ is a *Nash Equilibrium*
- More general than dominant-strategy equilibrium
  - DSE requires that for all h the $\hat{s}^{h}$ is the best response to *any* strategy played by other agents
  - NE just requires that for all h the s^{*h} is the best response to the strategy played by other agents in equilibrium
- Look at the NE solution for three classic games…
  - payoffs are in terms of utilities 0 (worst), 1, 2, 3 (best)
  - utility is ordinal

“Prisoner’s dilemma”

Player 1 \ Player 2	[+]	[–]
[+]	*2,2*	*0,3*
[–]	*3,0*	*1,1*

- ***Start with the point we found by elimination***
- ***If 1 plays* [–] *then 2’s best response is* [–].**
- ***If 2 plays* [–] *then 1’s best response is* [–].**
- ***A Nash equilibrium***

"Battle of the sexes"

Player 1 \ Player 2	*West*	*East*
West	*2,1*	*0,0*
East	*0,0*	*1,2*

- ***If 1 plays W then 2's best response is W.***
- ***If 2 plays W then 1's best response is W.***
- ***A Nash equilibrium***
- ***By symmetry, another Nash equilibrium***

"Chicken"

Player 1 \ Player 2	[+]	[–]
[+]	*2,2*	*1,3*
[–]	*3,1*	*0,0*

▪*If 1 plays* [–] *then 2's best response is* [+].

▪*If 2 plays* [+] *then 1's best response is* [–].

▪*A Nash equilibrium*

▪*By symmetry, another Nash equilibrium*

▪*But there's more to the Nash-equilibrium story here*

▪*(to be continued)*

▪*Now for a game we haven't seen before…*

Story

Discoordination

This game may seem no more than a frustrating chase round the payoff matrix. The two players' interests are always opposed (unlike Chicken or the Battle of the Sexes). But it is an elementary representation of class of important economic models. An example is the tax-audit game where Player 1 is the tax authority (“audit”, “no-audit”) and Player 2 is the potentially cheating taxpayer (“cheat”, “no-cheat”). More on this later.

rdination”

Player 1	[+]	[–]
	3,0	*1,2*
[–]	*0,3*	*2,1*

Player 2

- ***If 1 plays* [–] *then 2's best response is* [+].**
- ***If 2 plays* [+] *then 1's best response is* [+].**
- **If 1 plays [+] then 2's best response is [–].**
- **If 2 plays [–] then 1's best response is [–].**
- ***Apparently, no Nash equilibrium!***

- *Again there's more to the Nash-equilibrium story here*
- *(to be continued)*

Nash Equilibrium

- NE builds on the idea of "Best Response".
- Everyone is adopting the best-response rule and so…
- …no-one can unilaterally do better for himself.
- Suggests an equilibrium outcome even if there is no dominant strategy.
- Nash equilibrium can be seen as:
  - A focal point.
  - Social convention.
- How do we find the Nash equilibrium?

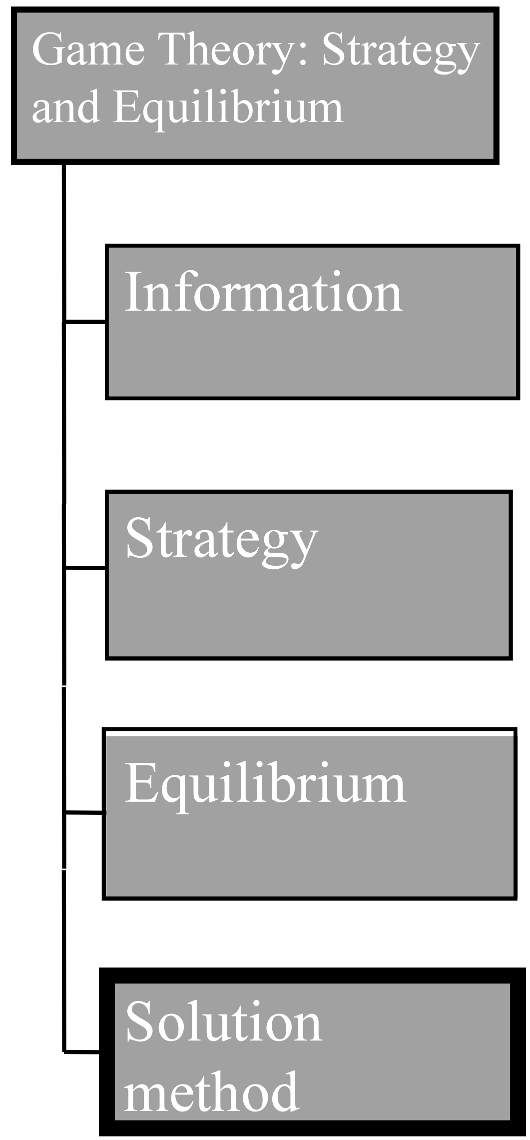
More on dominance

- The idea of a dominant strategy is demanding.
- It requires a strategy to come out as the best response to *any* strategy played by others in the game.
- But we may be able to use the concept of dominance in a more subtle fashion.
- What if player 1 could ignore some strategies for players 2,3,… because he knows they would be irrelevant?
- We need a basis for arguing which strategies could be dismissed in this way.

“Rationalisability”

- It seems illogical for any player to play a “dominated” strategy.
  - s^h is dominated for player h if there is some other strategy $s' \in S^i$ such that s' gives a higher payoff than s^h.
- So perhaps player 1 should eliminate from consideration any strategy that is dominated for some other player 2,3,...
- Could develop this into a rule:
  - Rational player only uses strategies that are best responses to some beliefs about strategies of other players
  - But, if he knows that *they* are rational, he should not have arbitrary beliefs about others’ strategies.
- This concept – *rationalisability* – helps to narrow down candidates for a solution.

Overview...



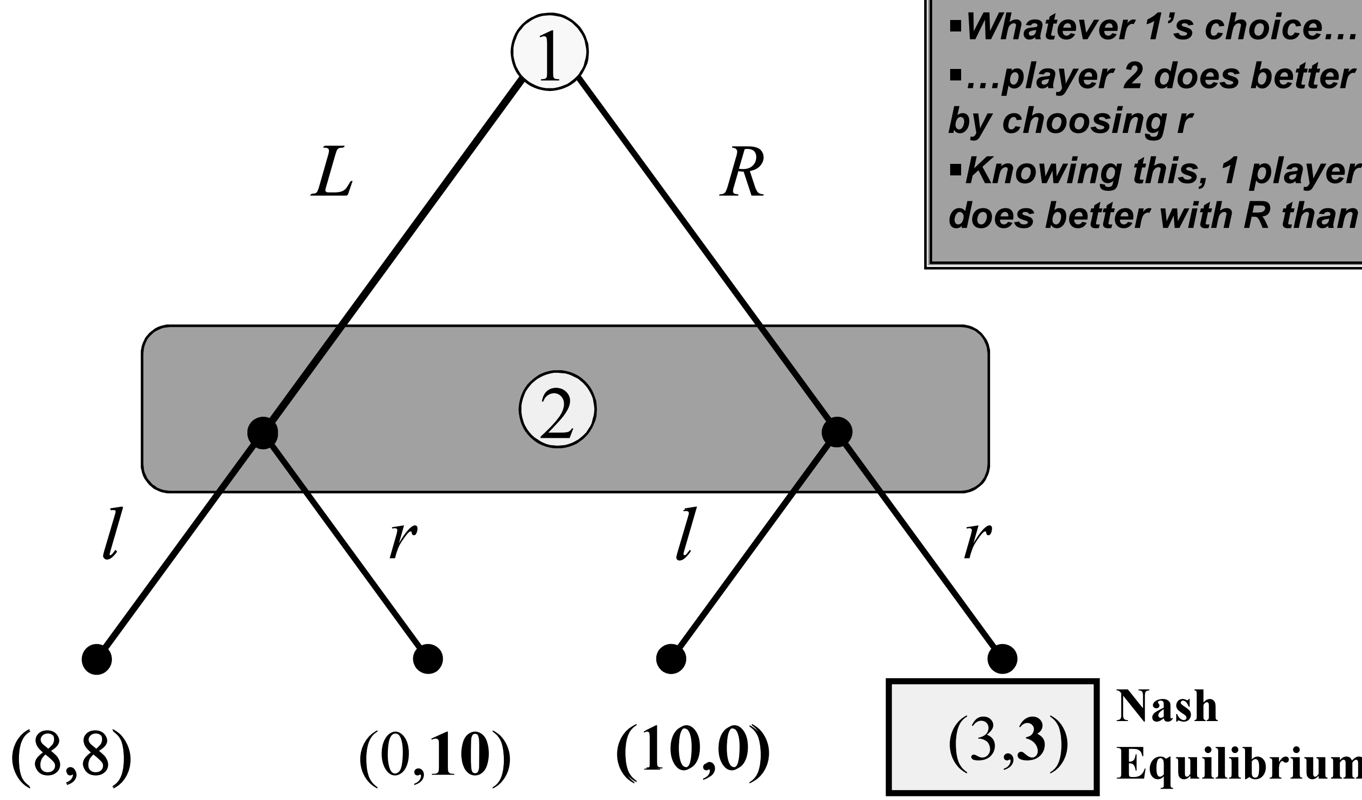
Implementing the Nash equilibrium concept?

Solution method?

- We can apply this to give us a step-by-step approach to a solution for the game
- If there are dominated strategies, the solution *may* be simple.
  - Start at "final stage" of game (player *n*, let's say)
  - Eliminate any dominated strategies
  - Now consider the set of strategies after the dominated strategies for player *n* have been eliminated.
  - Are there strategies that can now be eliminated for player *n*-1?
  - And then for player *n*-2…?
- Repeated elimination seems to do the job
- Here's how it works in our earlier example…

Eliminate dominated strategies

- ***The game tree***
- ***Whatever 1's choice…***
- ***…player 2 does better by choosing r***
- ***Knowing this, 1 player does better with R than L.***



Applying dominance again

- However, in using the repeated deletion method, we assume it's *common knowledge* that everyone acts rationally.
- "Common knowledge" is a strong assumption.
- It means more than "what I know to be generally true".
- It includes what I know that *others also* know to be true.
  - (ad infinitum).
- A small relaxation of this assumption may lead to big changes in equilibria.

Review: basic concepts

Review

- Information set:
  - What a player knows at any specified point in the game.
  - A way of introducing uncertainty.
  - A way of characterising order of play.

Review

- Strategy:
  - The basic tool for analysing how games are played.
  - Distinguish carefully from simple actions.

Review

- Best response:
  - An obvious way of characterising optimisation in models of conflict.

Review

- Nash equilibrium:
  - Based on the concept of best response.
  - Precise definition of equilibrium in terms of strategy.

Review

- Repeated deletion:
  - A possible solution method?

What next?

- Extend the concept of strategy:
  - See *Game Theory: Mixed Strategies*.
- Introduce time:
  - See *Game Theory: Dynamic*.
- Both of these enable us to get more out of the Nash-Equilibrium concept