The Firm: Basics

MICROECONOMICS

Principles and Analysis

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Overview...

The environment for the basic model of the firm.

- The Firm: Basics
  - **The setting**
  - Input require-ment sets
  - Isoquants
  - Returns to scale
  - Marginal products

The basics of production...

- We set out some of the elements needed for an analysis of the firm.
  - Technical efficiency
  - Returns to scale
  - Convexity
  - Substitutability
  - Marginal products
- This is in the context of a single-output firm...
- ...and assuming a competitive environment.
- First we need the building blocks of a model...

Notation

- **Quantities**

z_i — •amount of input i

$\mathbf{z} = (z_1, z_2, \ldots, z_m)$ — •input vector

q — •amount of output

For next presentation

- **Prices**

w_i — •price of input i

$\mathbf{w} = (w_1, w_2, \ldots, w_m)$ — •Input-price vector

p — •price of output

Feasible production

- The basic relationship between output and in

The production function

$$q \le \phi(z_1, z_2, \ldots, z_m)$$

- This can be written more compactly as:

Vector of inputs

$$q \le \phi(\mathbf{z})$$

- ϕ gives the *maximum* amount of output that can be produced from a given list of inputs

•single-output, multiple-input production relation

•Note that we use "≤" and not "=" in the relation. Why?

•Consider the meaning of ϕ

distinguish two important cases...

Technical efficiency

- Case 1:

$$q = \phi(\mathbf{z})$$

•The case where production is *technically efficient*

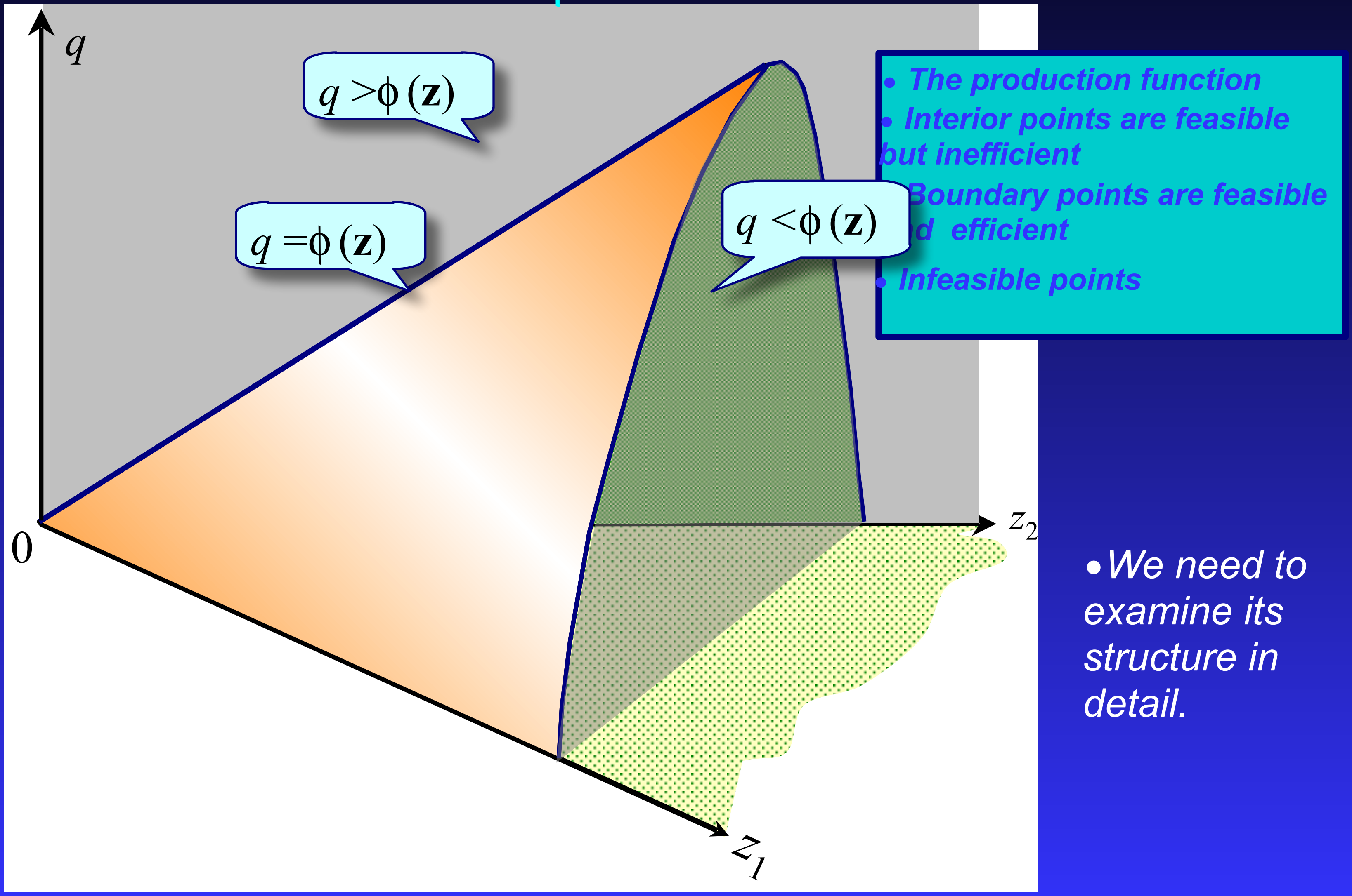
- Case 2:

$$q < \phi(\mathbf{z})$$

•The case where production is (technically) inefficient

Intuition: if the combination $(\mathbf{z}, q)$ *is inefficient you can throw away some inputs and still produce the same output*

The function ϕ



- *We need to examine its structure in detail.*

Overview...

The structure of the production function.

- The Firm: Basics
  - The setting
  - Input require-ment sets
  - Isoquants
  - Returns to scale
  - Marginal products

The input requirement set

- Pick a particular output level q
- Find a feasible input vector $\mathbf{z}$
- Repeat to find all such vectors
- Yields the input-requirement set

$$Z(q) := \{\mathbf{z}: \phi(\mathbf{z}) \geq q\}$$

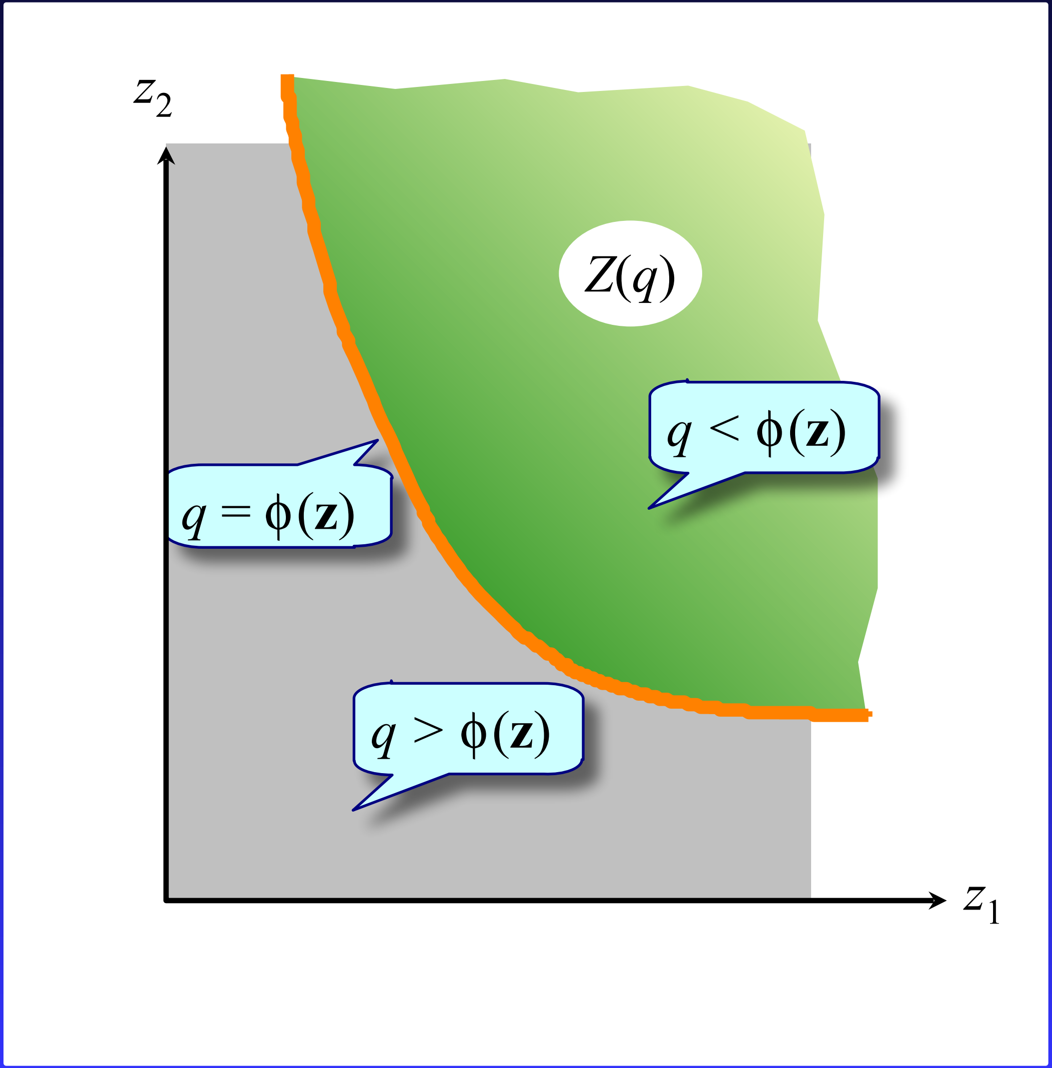
- The shape of Z depends on the assumptions made about production...
- We will look at four cases.

- remember, we must have $q \leq \phi(\mathbf{z})$

- The set of input vector that meet the technical feasibility condition for output q...

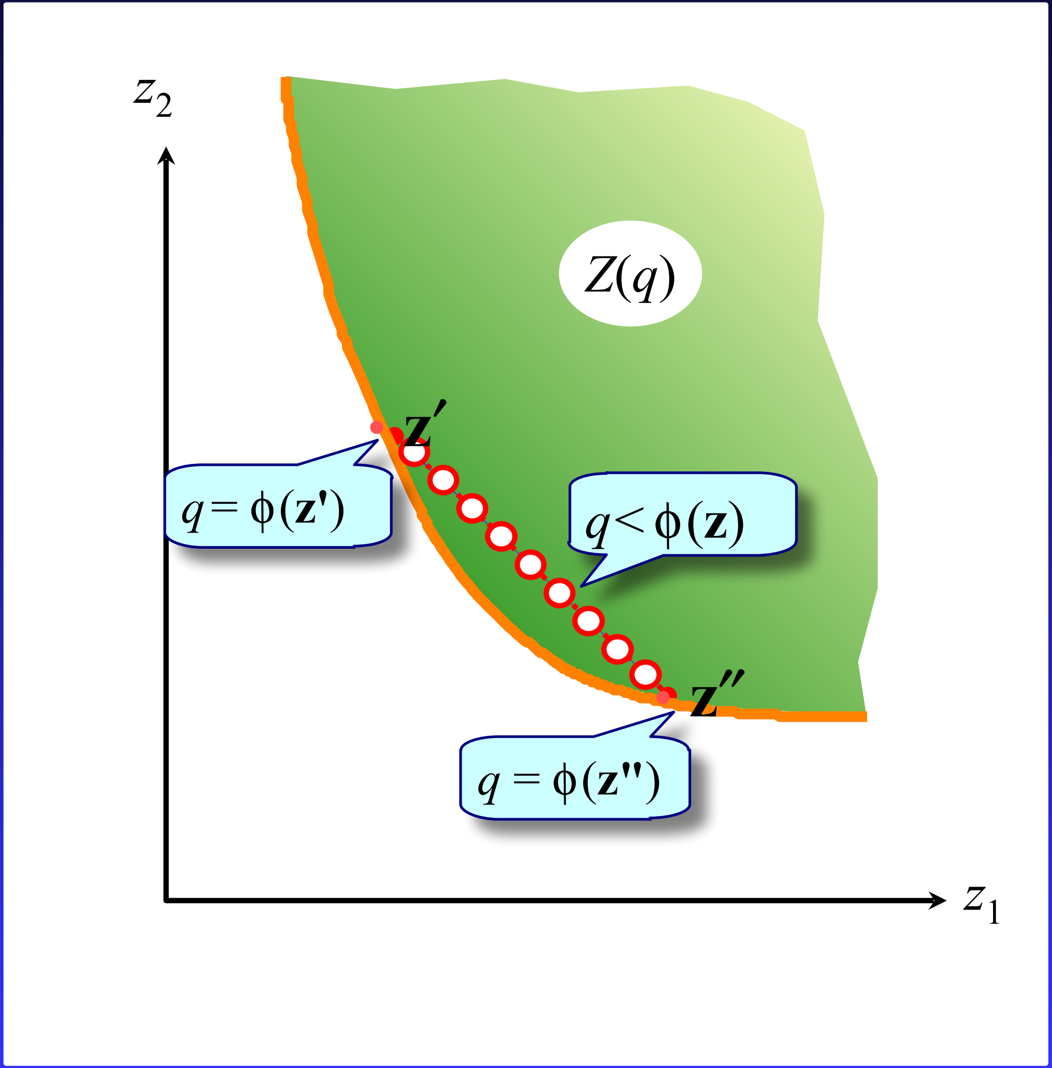
First, the "standard" case.

The input requirement set



- *Feasible but inefficient*
- *Feasible and technically efficient*
- *Infeasible points.*

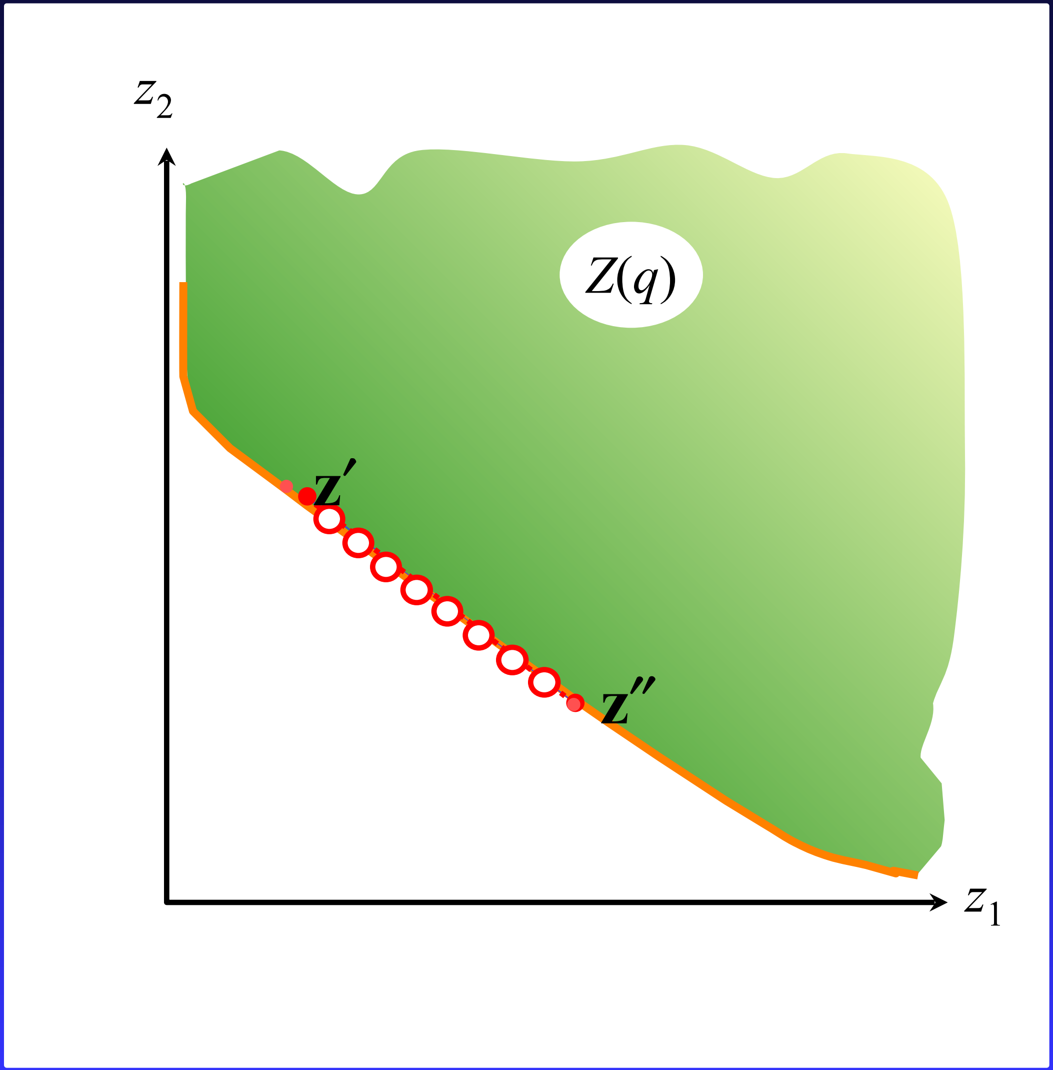
Case 1: Z smooth, strictly convex



- *Pick two boundary points*
- *Draw the line between them*
- *Intermediate points lie in the interior of Z.*

- *Note important role of convexity.*
- *A combination of two techniques may produce more output.*
- *What if we changed some of the assumptions?*

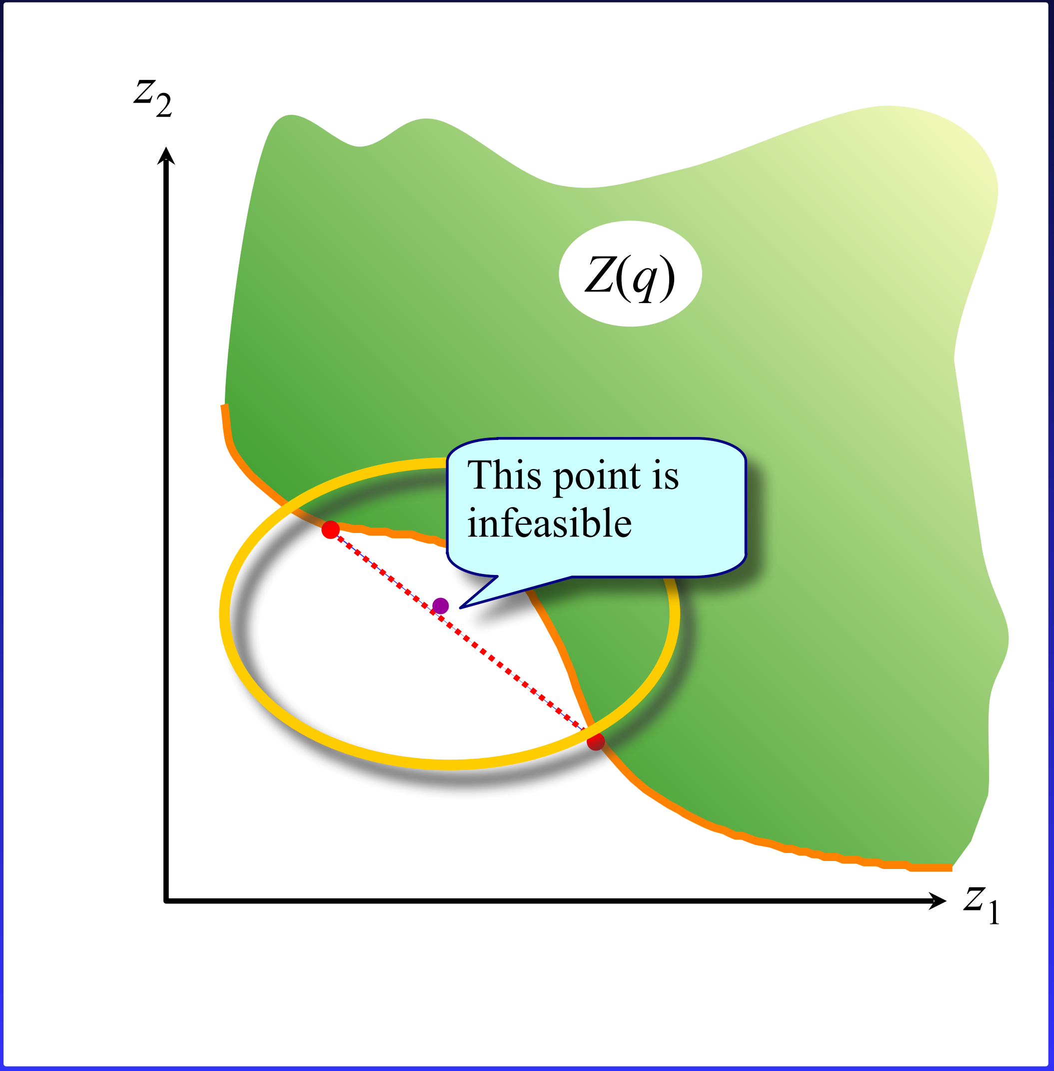
Case 2: *Z* Convex (but not strictly)



- *Pick two boundary points*
- *Draw the line between them*
- *Intermediate points lie in Z (perhaps on the boundary).*

- *A combination of feasible techniques is also feasible*

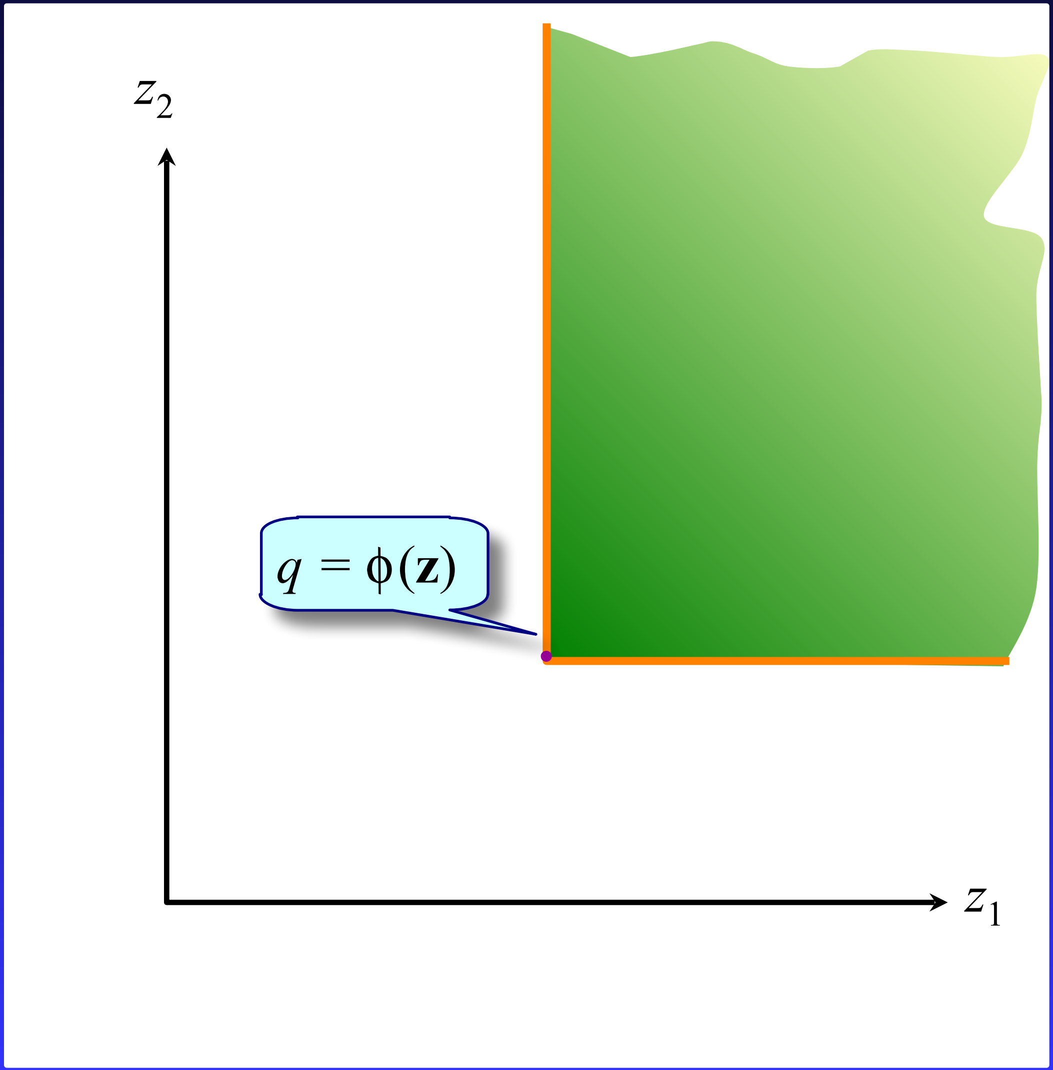
Case 3: Z smooth but *not* convex



- ***Join two points across the "dent"***
- ***Take an intermediate point***
- ***Highlight zone where this can occur.***

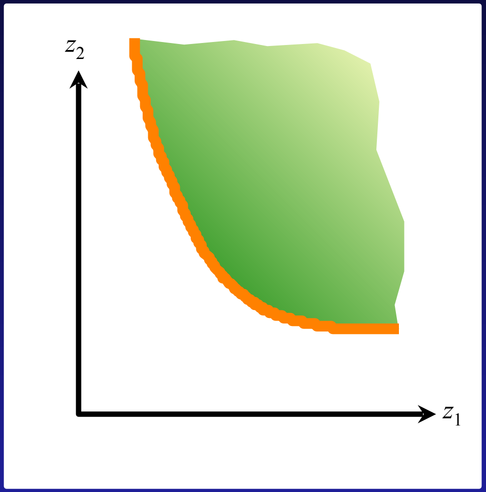
- *in this region there is an indivisibility*

Case 4: Z convex but not smooth

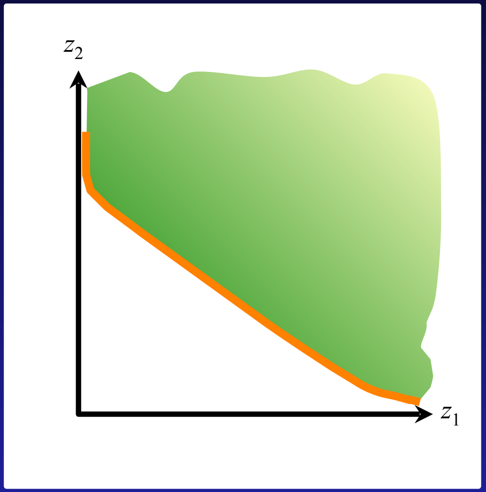


- *Slope of the boundary is undefined at this point.*

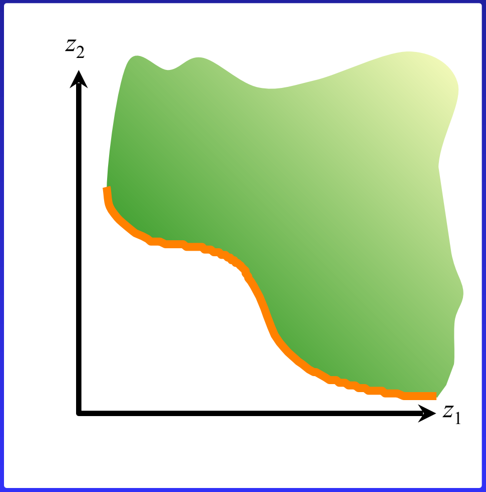
Summary: 4 possibilities for *Z*



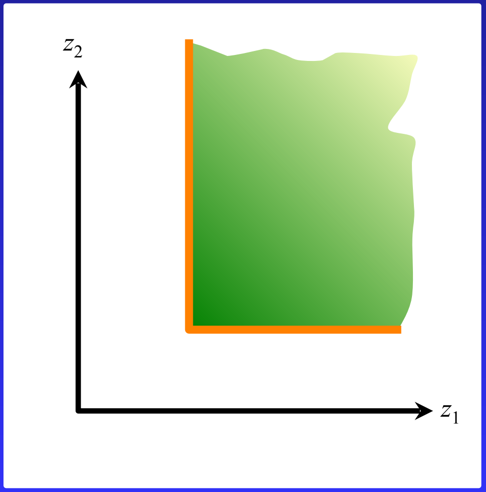
Standard case, but strong assumptions about divisibility and smoothness



Almost conventional: mixtures may be just as good as single techniques



Problems: the "dent" represents an indivisibility



Only one efficient point and not smooth. But not perverse.

Overview...

Contours of the production function.

- The Firm: Basics
  - The setting
  - Input requirement sets
  - **Isoquants**
  - Returns to scale
  - Marginal products

Isoquants

- Pick a particular output level q
- Find the input requirement set $Z(q)$
- The *isoquant* is the boundary of Z:

$$\{ \mathbf{z} : \phi(\mathbf{z}) = q \}$$

- If the function ϕ is differentiable at $\mathbf{z}$ then the *marginal rate of technical substitution* is the slope at $\mathbf{z}$:

$$\frac{\phi_j(\mathbf{z})}{\phi_i(\mathbf{z})}$$

- Gives the rate at which you can trade off one output against another along the isoquant – to maintain a constant q.

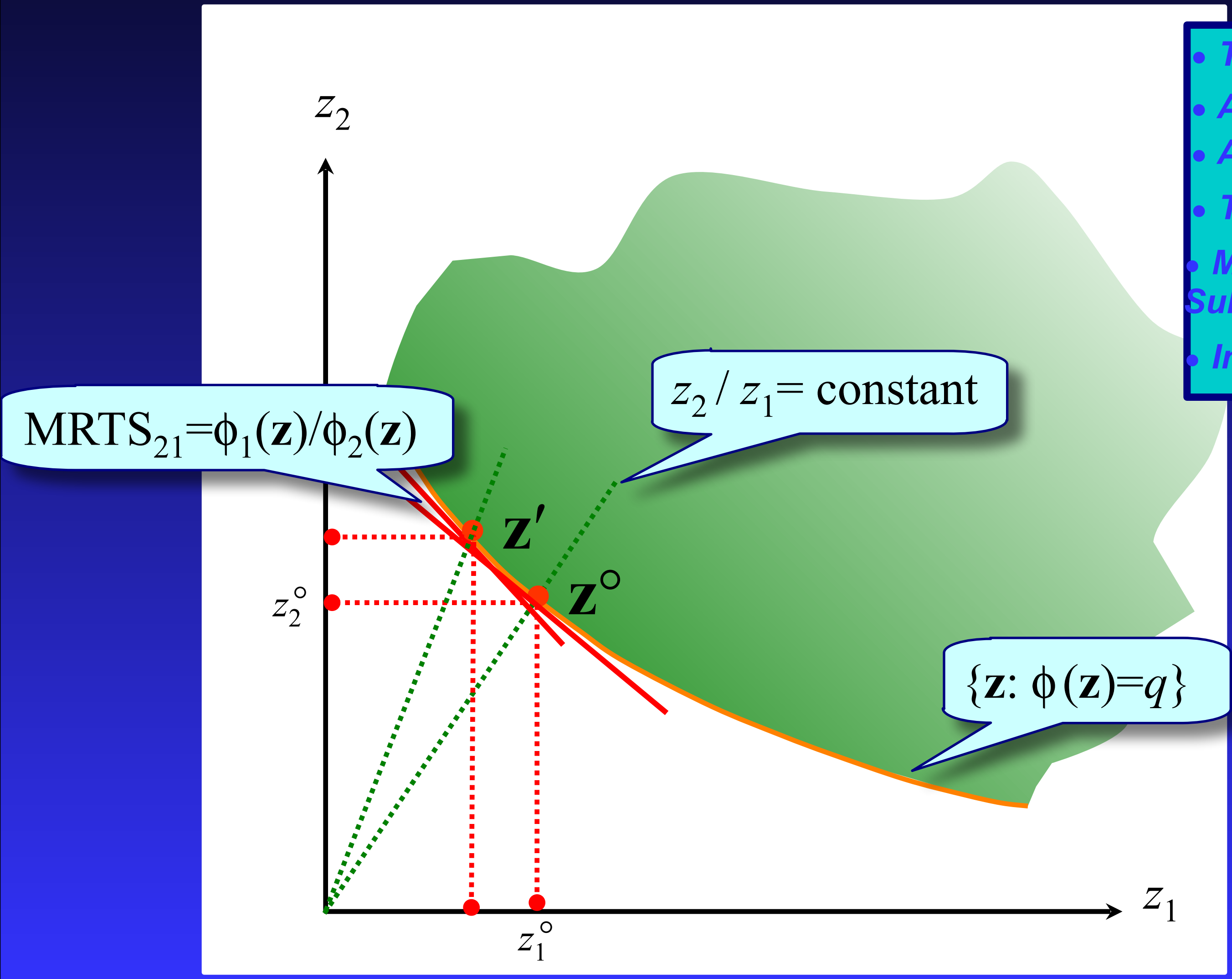
- Think of the isoquant as an integral part of the set $Z(q)$...

- Where appropriate, use subscript to denote partial derivatives. So

$$\phi_i(\mathbf{z}) := \frac{\partial\phi(\mathbf{z})}{\partial z_i}.$$

Let's look at its shape

Isoquant, input ratio, MRTS



- *The set Z(q).*
- *A contour of the function ϕ.*
- *An efficient point.*
- *The input ratio*
- *Marginal Rate of Technical Substitution*
- *Increase the MRTS*

- *The isoquant is the boundary of Z.*
- *Input ratio describes one particular production technique.*
- *Higher input ratio associated with higher MRTS..*

Input ratio and MRTS

- $MRTS_{21}$ is the implicit "price" of input 1 in terms of input 2.
- The higher is this "price", the smaller is the relative usage of input 1.
- Responsiveness of input ratio to the MRTS is a key property of ϕ.
- Given by the *elasticity of substitution*

$$-\frac{\partial \log(z_1/z_2)}{\partial \log(\phi_1/\phi_2)}$$

- Can think of it as measuring the isoquant's "curvature" or "bendiness"

A simple derivation of the logarithmic form of elasticity of substitution

See also A.4.6

$$\sigma_{21} = \frac{\dfrac{d(z_1 / z_2)}{z_j / z_i}}{\dfrac{d(\phi_1 / \phi_2)}{\phi_1 / \phi_2}}$$

$$d \ln y = \frac{1}{y} dy \quad d \ln x = \frac{1}{x} dx$$

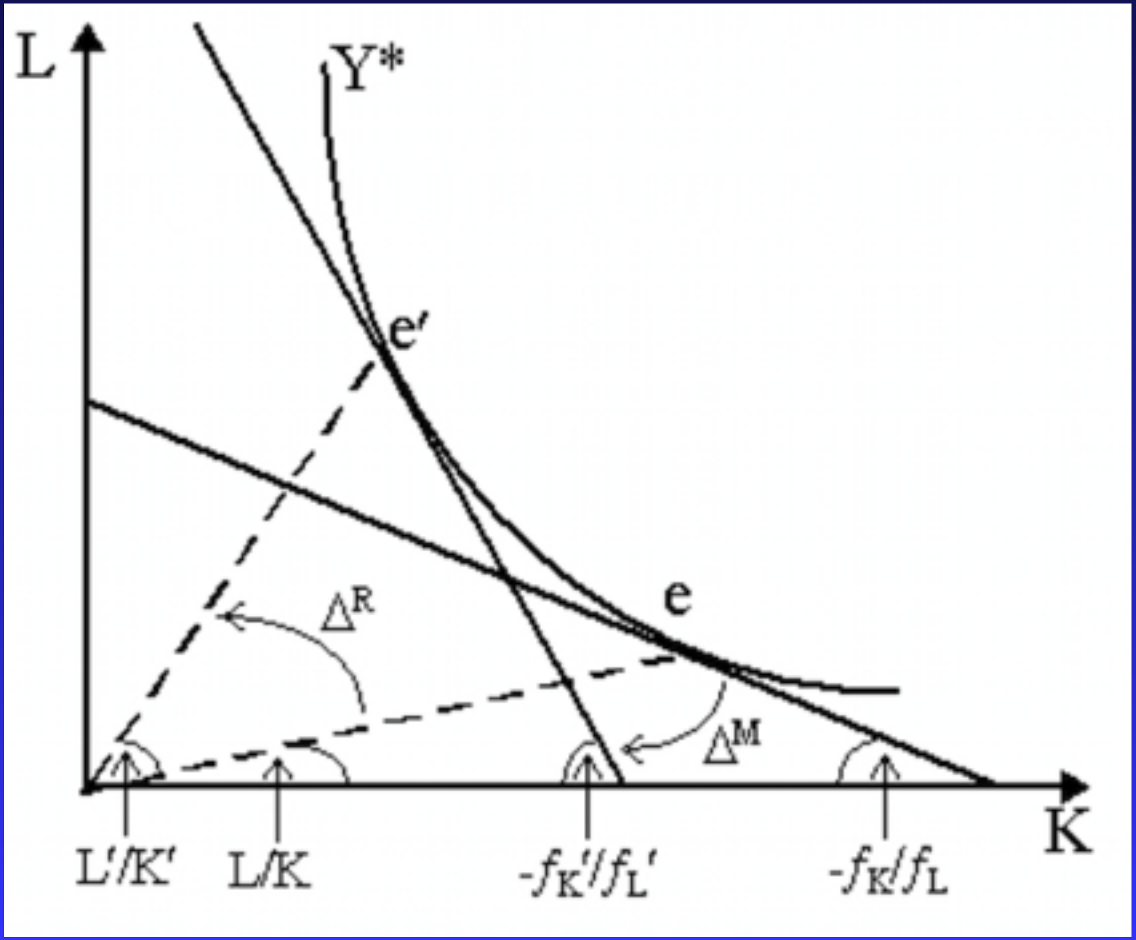
$$\varepsilon = \frac{d \ln y}{d \ln x} = \frac{dy}{dx}\frac{x}{y} = \frac{\dfrac{dy}{y}}{\dfrac{dx}{x}},$$

$$let\, y = z_1 / z_2$$

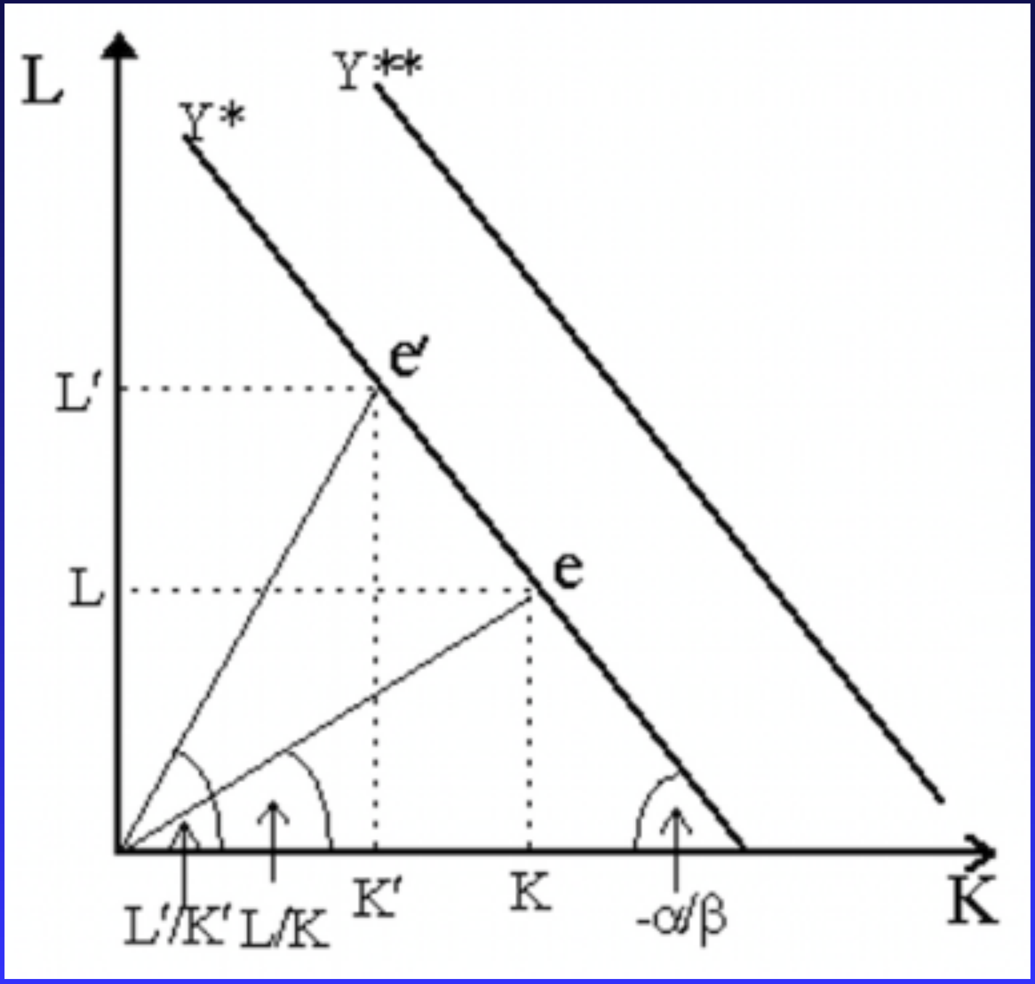
$$let\, x = \phi_1 / \phi_2$$

$$\sigma_{21} = \frac{d \ln(z_1 / z_2)}{d \ln |(\phi_1 / \phi_2)|}$$

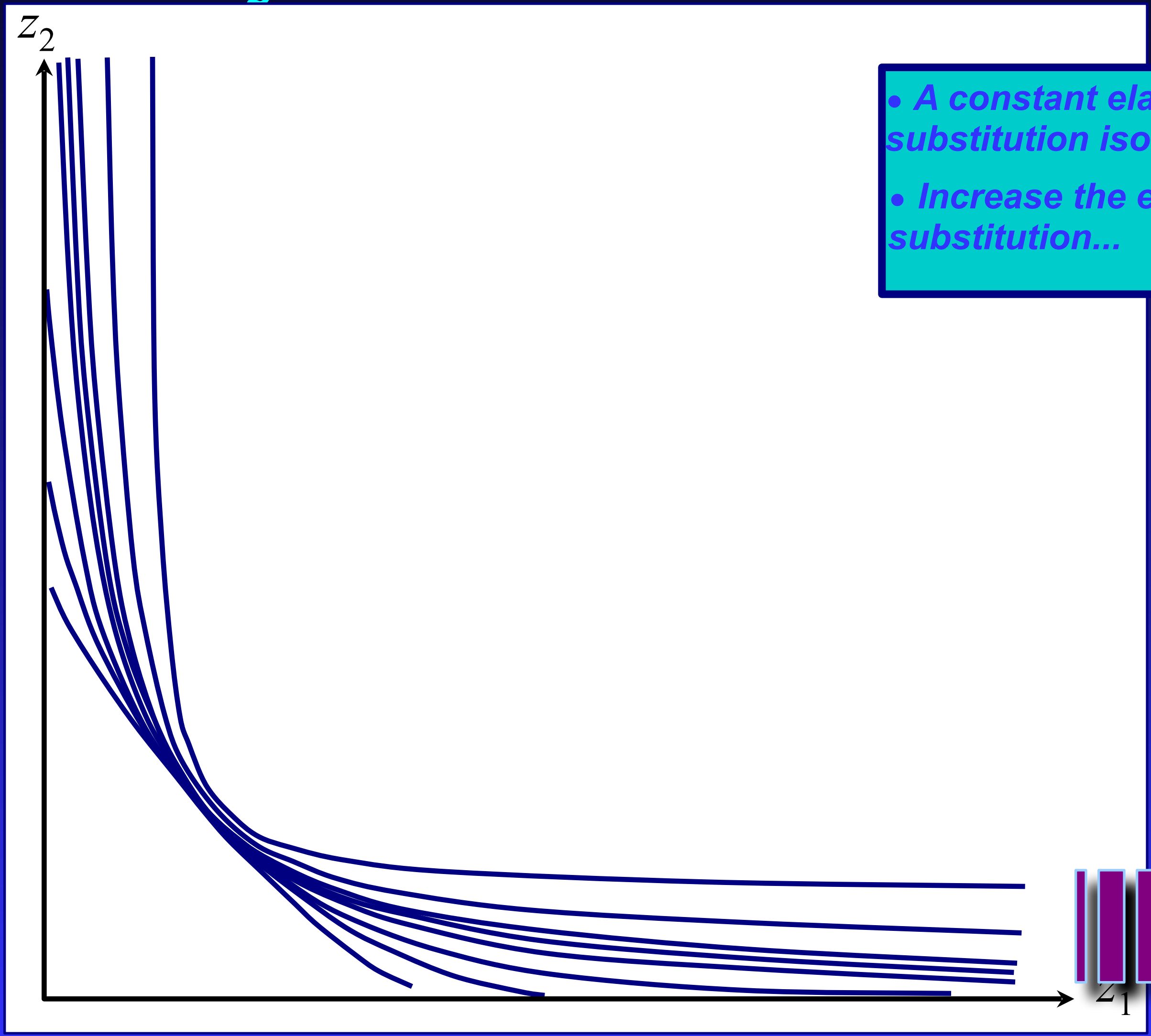
Elasticity: diagrammatic explanation



Elasticity: perfect substitute isoquants



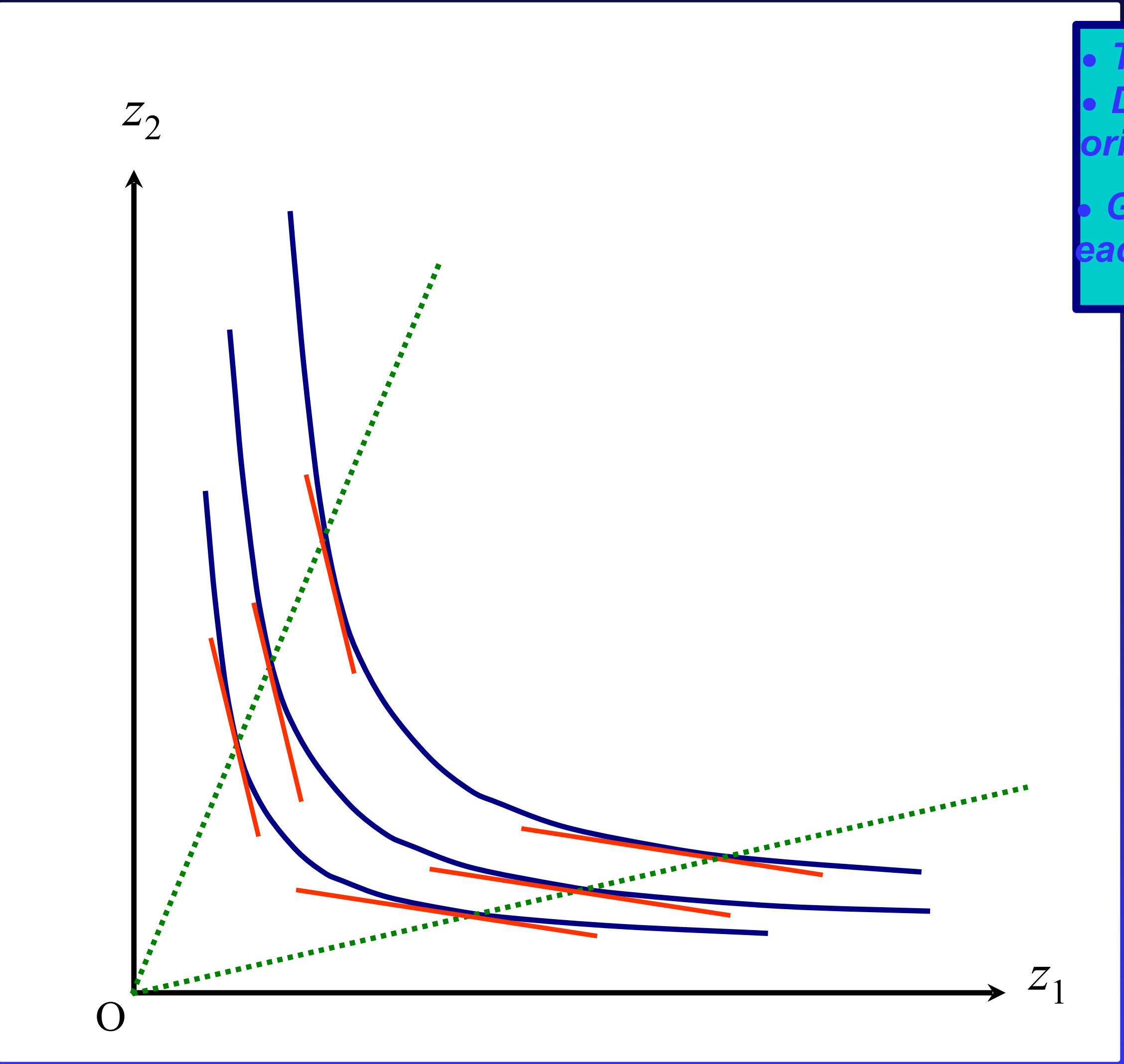
Elasticity of substitution



- *A constant elasticity of substitution isoquant*
- *Increase the elasticity of substitution...*

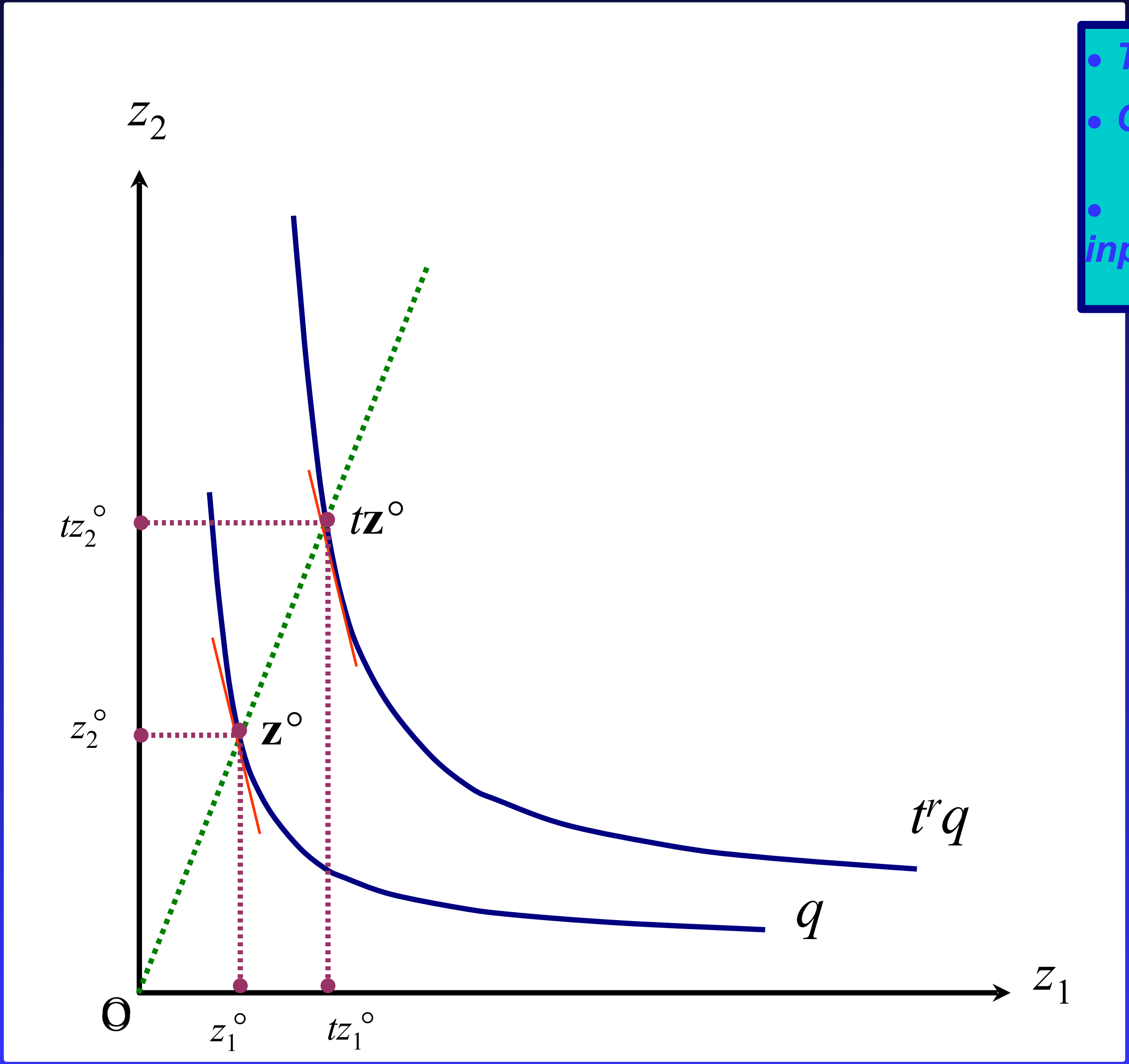
structure of the contour map...

Homothetic contours



- *The isoquants*
- *Draw any ray through the origin…*
- *Get same MRTS as it cuts each isoquant.*

Contours of a homogeneous function



- *The isoquants*
- *Coordinates of input z°*
- *Coordinates of "scaled up" input tz°*

$$\phi(t\mathbf{z}) = t^r \phi(\mathbf{z})$$

Overview...

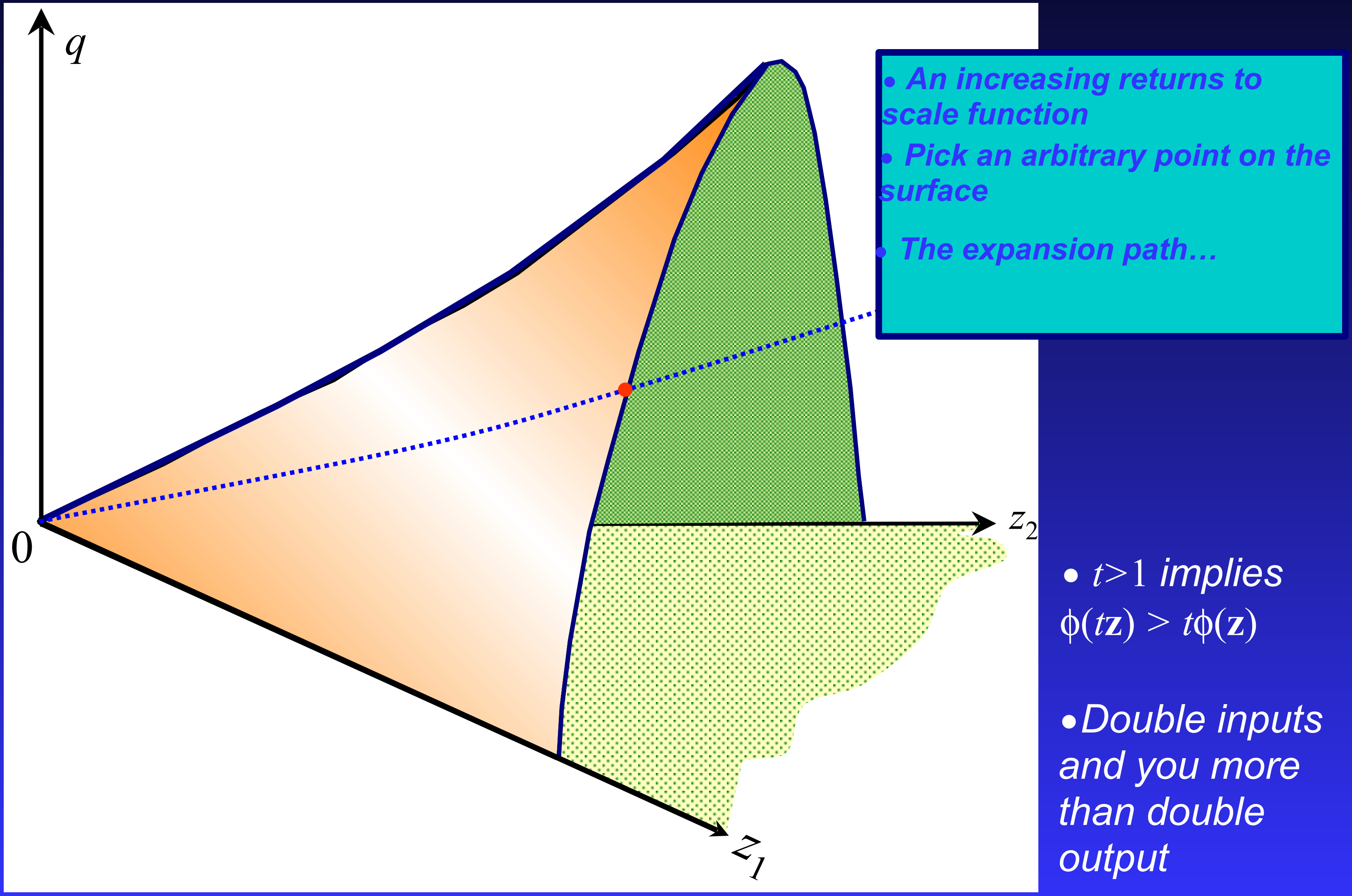
Changing all inputs together.

- The Firm: Basics
  - The setting
  - Input requirement sets
  - Isoquants
  - **Returns to scale**
  - Marginal products

Let's rebuild from the isoquants

- The isoquants form a contour map.
- If we looked at the "parent" diagram, what would we see?
- Consider *returns to scale* of the production function.
- Examine effect of varying all inputs together:
  - Focus on the expansion path.
  - q plotted against proportionate increases in $\mathbf{z}$.
- Take three standard cases:
  - Increasing Returns to Scale
  - Decreasing Returns to Scale
  - Constant Returns to Scale
- Let's do this for 2 inputs, one output…

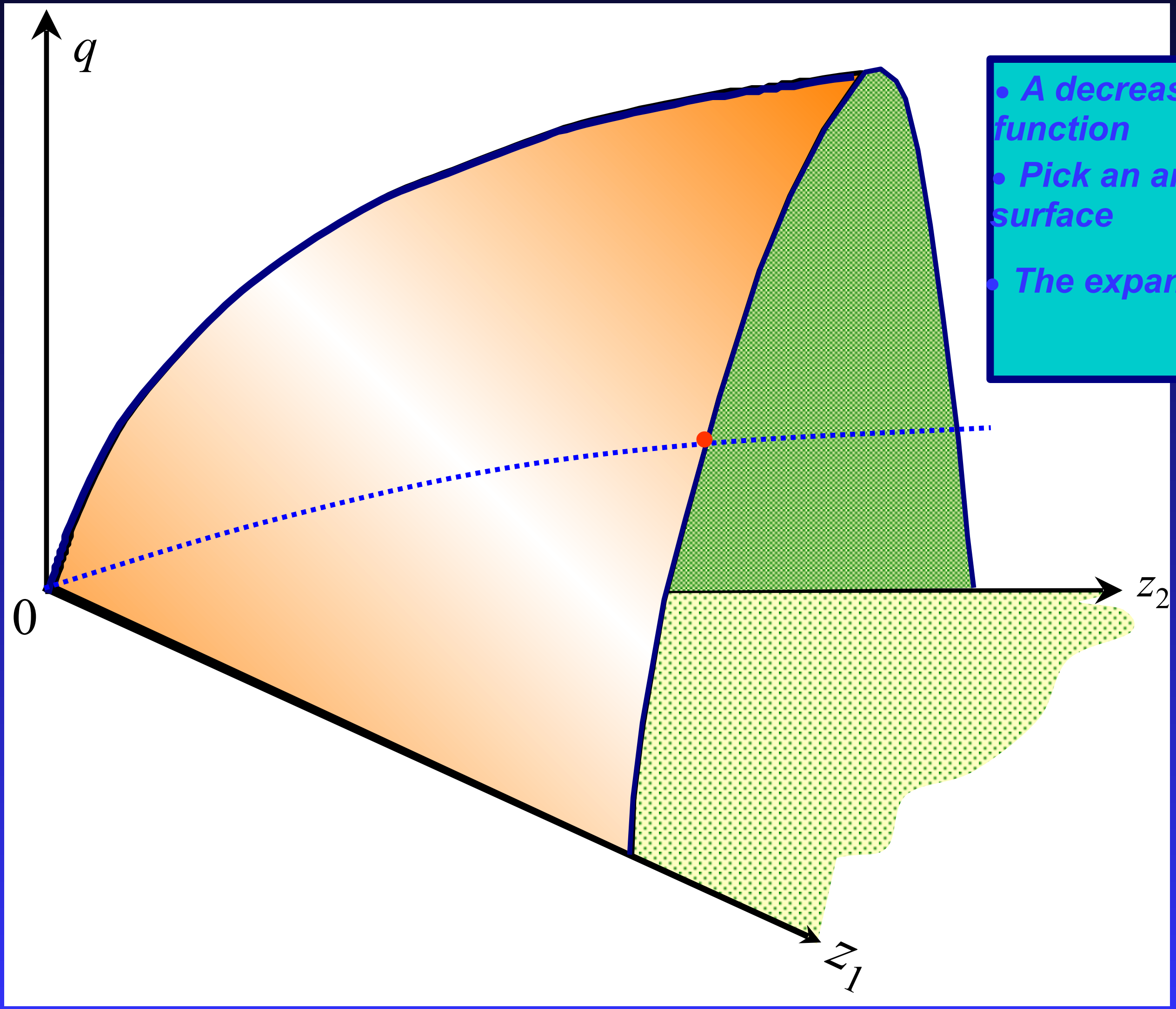
Case 1: IRTS



- $t>1$ *implies* $\phi(t\mathbf{z}) > t\phi(\mathbf{z})$

- *Double inputs and you more than double output*

Case 2: DRTS

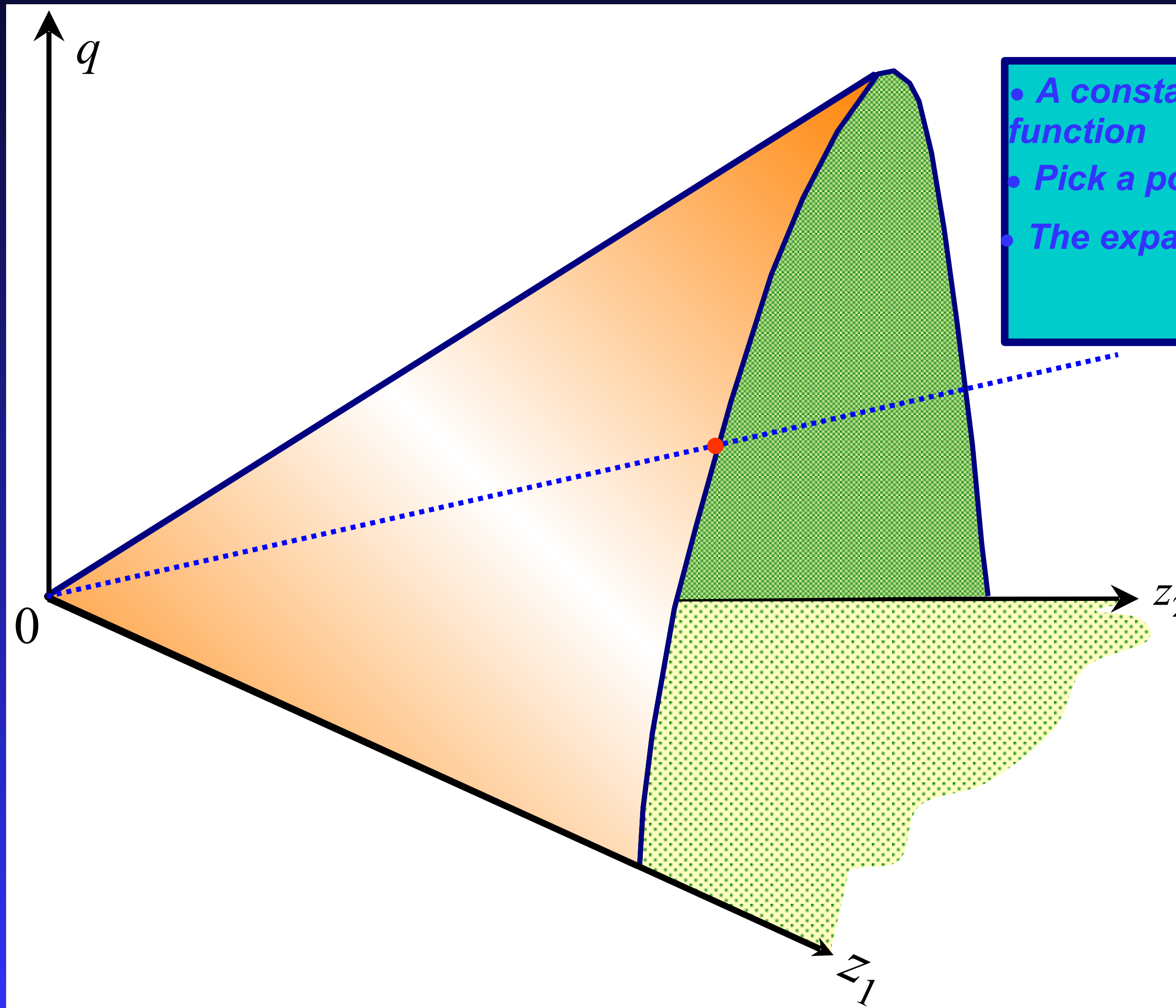


- A decreasing returns to scale function
- Pick an arbitrary point on the surface
- The expansion path…

- $t>1$ implies $\phi(t\mathbf{z}) < t\phi(\mathbf{z})$

- Double inputs and output increases by less than double

Case 3: CRTS

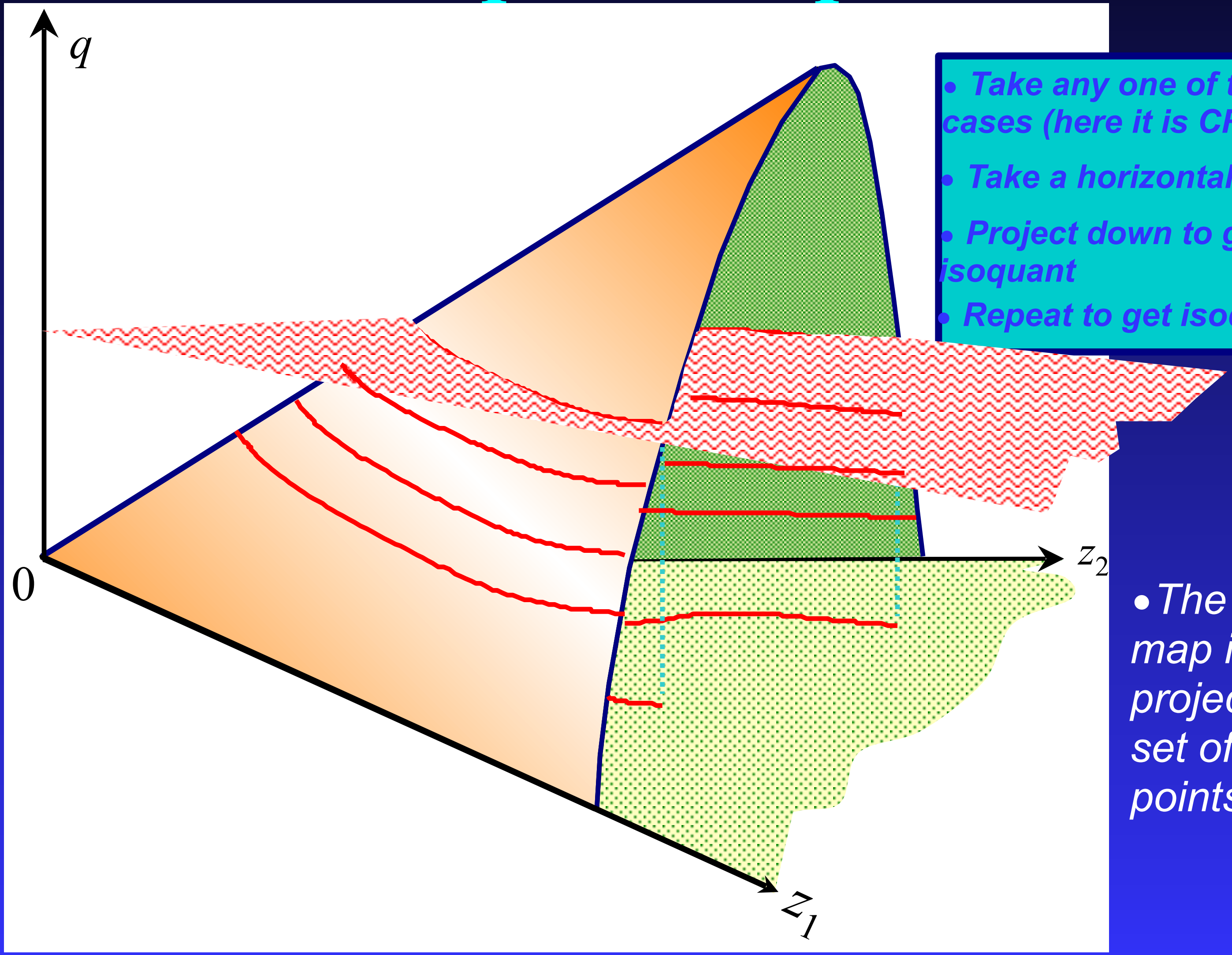


- A constant returns to scale function
- Pick a point on the surface
- The expansion path is a ray

- $\phi(t\mathbf{z}) = t\phi(\mathbf{z})$

- *Double inputs and output exactly doubles*

Relationship to isoquants



- Take any one of the three cases (here it is CRTS)
- Take a horizontal "slice"
- Project down to get the isoquant
- Repeat to get isoquant map

- *The isoquant map is the projection of the set of feasible points*

Overview...

Changing one input at time.

- The Firm: Basics
  - The setting
  - Input require-ment sets
  - Isoquants
  - Returns to scale
  - **Marginal products**

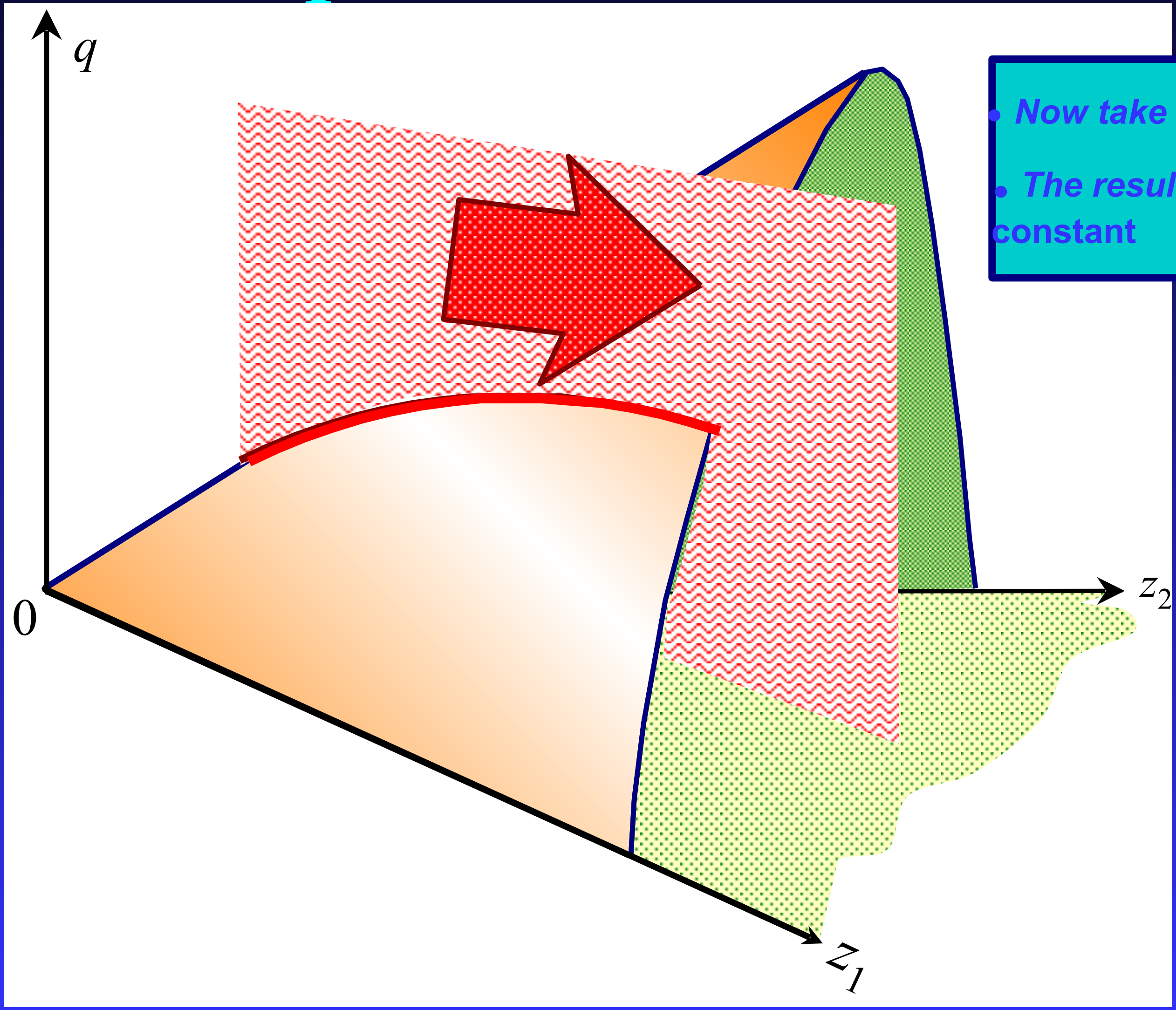
Marginal products

- Pick a technically efficient input vector
- Keep all but one input constant
- Measure the marginal change in output w.r.t. this input

$$\text{MP}_i = \phi_i(\mathbf{z}) = \frac{\partial \phi(\mathbf{z})}{\partial z_i}$$

- Remember, this means a $\mathbf{z}$ such that $q = \phi(\mathbf{z})$
- The marginal product

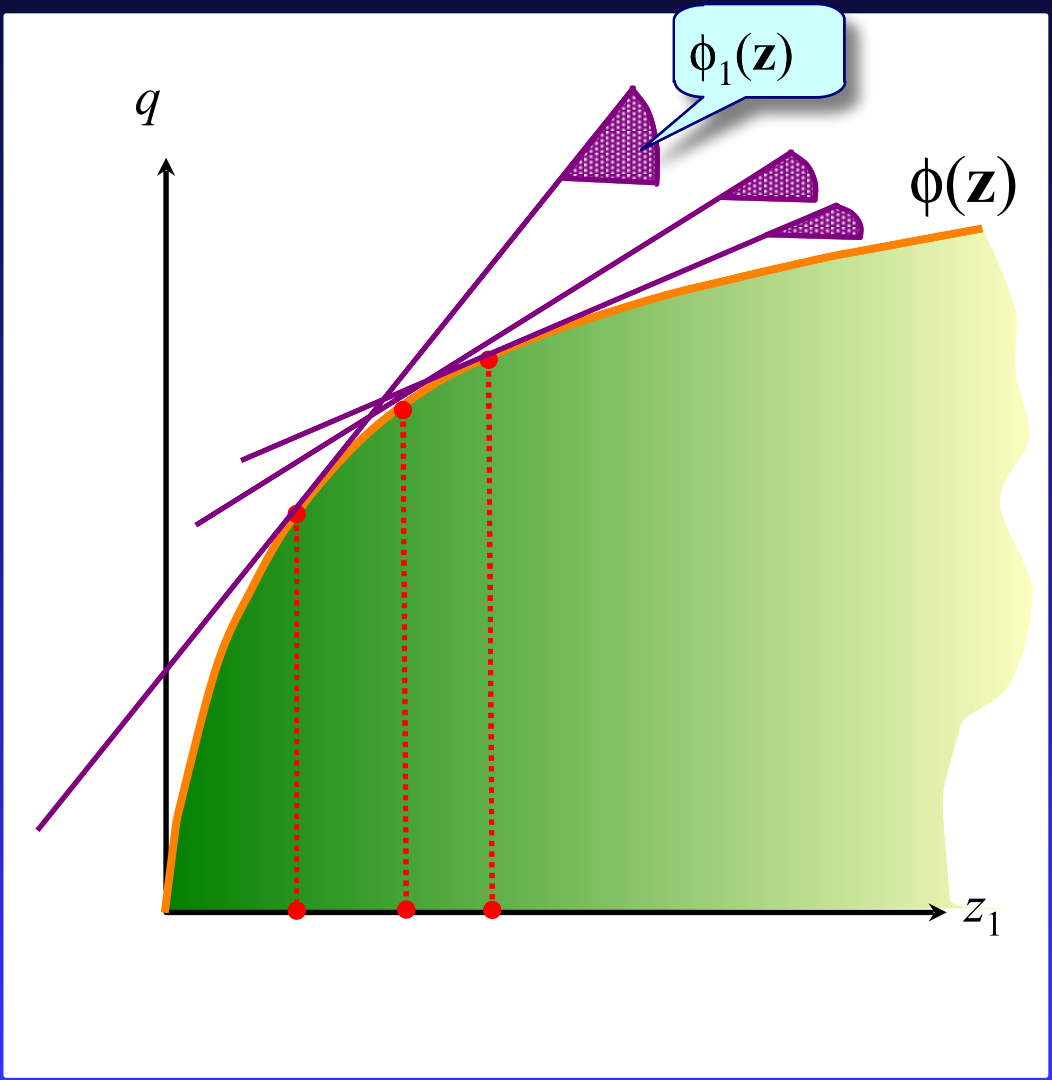
CRTS production function again



- Now take a vertical "slice"
- The resulting path for z_2 = constant

Let's look at its shape

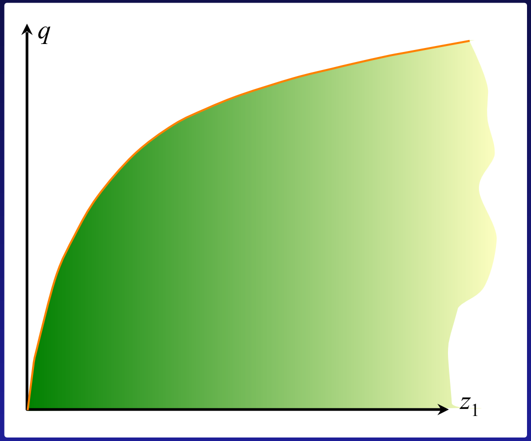
MP for the CRTS function



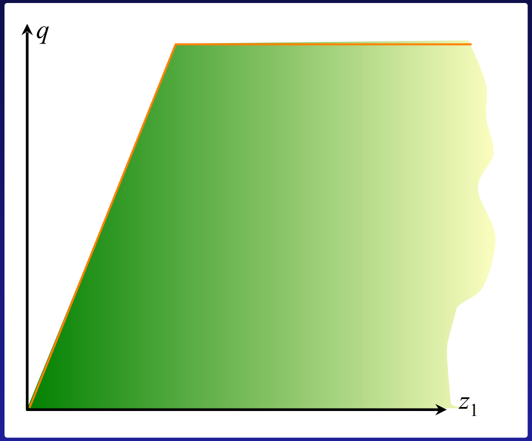
- *The feasible set*
- *Technically efficient points*
- *Slope of tangent is the marginal product of input 1*
- *Increase z_1…*

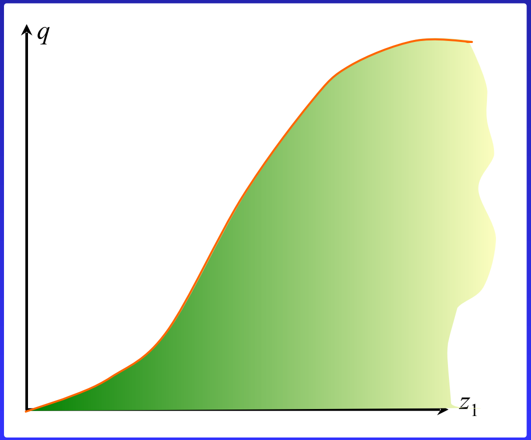
- *A section of the production function*
- *Input 1 is essential: If $z_1 = 0$ then $q = 0$*
- $\phi_1(\mathbf{z})$ *falls with z_1 (or stays constant) if ϕ is concave*

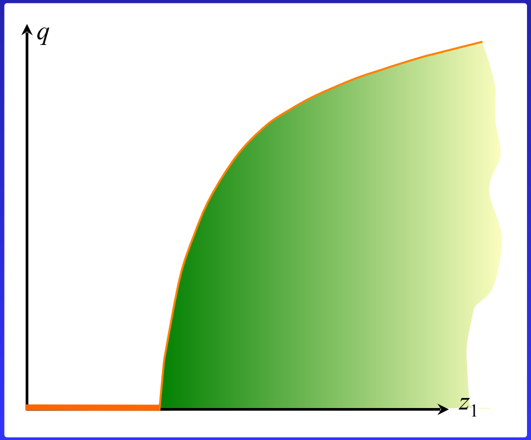
Relationship between q and z_1



- *We've just taken the conventional case*
- *But in general this curve depends on the shape of ϕ.*
- *Some other possibilities for the relation between output and one input…*







Key concepts

Review
- Technical efficiency

Review
- Returns to scale

Review
- Convexity

Review
- MRTS

Review
- Marginal product

What next?

- Introduce the market
- Optimisation problem of the firm
- Method of solution
- Solution concepts.