

ΠΜΣ «Οικονομική Επιστήμη»

ΕΚΠΑ

Τμήμα Οικονομικών Επιστημών

**Microeconomic Theory – Consumer Theory**

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### Problem Set 2

1. For each of the following questions, briefly explain whether they are true, false or uncertain.

- In equilibrium, the marginal rate of substitution (MRS) is always equal to  $p_1/p_2$ .
- The solution of the utility maximization problem must satisfy Walras' law and hence the constraint  $\mathbf{p}\mathbf{x} \leq w$  is binding.

2. In an economy with two goods only, assume that the indirect utility function of some individual is  $v(\mathbf{p}, w) = w/p_1 + \ln(p_1/p_2)$ ,  $p_1 > 0$  and  $p_2 > 0$ .

- Briefly explain what is expressed by the indirect utility function and the expenditure function of some individual.
- Find the Walrasian demand for both goods and the expenditure function of this individual.
- Without finding the Hicksian demand function, calculate the level of Hicksian demand for both goods if  $p_1 = 1$ ,  $p_2 = 1$  and the utility target level is  $u = 5$ .
- Find the Hicksian demand function. Use it in order to calculate the Slutsky matrix. Does it have the properties you expected?

3. In an economy with 3 goods, assume that in period 1, when prices are  $p_1=2$ ,  $p_2=2$  and  $p_3 = 3$ , and consumer  $i$ 's wealth is  $w = 21$ , the consumer chooses the consumption bundle  $x_1 = 3$ ,  $x_2 = 2$  and  $x_3 = 3$ .

- What modification would you make if you wanted to make good 3 the numeraire commodity (i.e.  $p_3 = 1$ )? What assumption enables you to make these changes?
- Suppose that in period 2, the prices change to are  $p_1'=3$ ,  $p_2'=1$  and  $p_3' = 2$ , and  $i$ 's wealth is  $w' = 22$ . In period 2,  $i$  chooses the consumption bundle  $x_1' = 4$ ,  $x_2' = 3$  and  $x_3' = 2$ . Is this consumer's demand consistent with Walras' law? Are this consumer's choices consistent with the weak axiom of revealed preferences?
- What should this consumer's wealth be in period 2 if the change of prices was accompanied by a Slutsky wealth compensation?
- Briefly explain the difference between a Slutsky and a Hicksian wealth compensation and illustrate with a diagram, assuming there are only two goods.

4. Assume the indirect utility function  $v(p_1, p_2, w) = w/2p_1^{1/2}p_2^{1/2}$ .

- Find the Walrasian demand function and check if it satisfies Walras' law.
- Would it be allowed to take some strictly increasing transformation of  $v(\cdot)$ , if one found that this would simplify the calculations in (a)?

5. In an economy with two goods only, assume a consumer with utility function  $u(\mathbf{x})=x_1x_2$ . The prices of goods 1 and 2 are  $p_1 > 0$  and  $p_2 > 0$  respectively. The required level of utility for this consumer is  $u=1$ .

- a) Find the Hicksian demand function (Remember that minimization of a function  $f$  is the same as maximization of the function  $-f$ ).
- b) Calculate the Slutsky matrix using the Hicksian demand function you found in (a).
- c) Write down (but don't solve) the dual utility maximization problem.

**6.** In an economy with two goods only, the expenditure function of an individual is  $e(\mathbf{p}, u) = 2u^{1/2}p_1^{1/2}p_2^{1/2}$ .

- a) Find the Hicksian demand function and the indirect utility function.
- b) Calculate the Slutsky matrix and briefly explain what is expressed by its elements.
- c) If  $p_1=4$ ,  $p_2=1$  and  $w=4$ , what is the level of Walrasian demand for good 2?
- d) How would you find the Walrasian demand function? (skip the actual calculations)

**7.** Consider the utility function  $u = 2x_1^{1/2} + 4x_2^{1/2}$ .

- (a) Find the demand functions for goods 1 and 2 as they depend on prices and wealth. How do we call them?
- (b) Find the demand functions for goods 1 and 2 as they depend on prices and a given utility level? How do we call them?
- (c) Find the expenditure function and verify that  $h_i(\mathbf{p}, u) = e_{p_i}(\mathbf{p}, u)$ .
- (d) Find the indirect utility function and verify Roy's identity.