15 Commodity Taxation

15.1 Introduction

Commodity taxes are levied on transactions involving the purchase of goods. The necessity for keeping accounts ensures that such transactions are generally public information. This makes them a good target for taxation. The drawback, however, is that commodity taxation distorts consumer choices and causes inefficiency. Some striking historical examples can be found in the United Kingdom where there have been window taxes and hearth taxes. The window tax was introduced in 1696 in the reign of William III and lasted until 1851. The tax was paid on any house with more than six windows (increased to eight in 1825), which gave an incentive to brick up any windows in excess of the allowable six. Even today, old houses can be found with windows still bricked up. The hearth tax was levied between 1662 and 1689 at the rate of two shillings (two days' wages for a ploughman) per annum on each hearth in a building. This induced people to brick up their chimneys and shiver through the winter. In the marketplace, commodity taxes drive a wedge between the price producers receive and the price consumers pay. This leads to inefficiency and reduces the attainable level of welfare compared to what could be achieved using lump-sum taxes. This is the price that has to be paid for implementable taxation.

The effects of commodity taxes are quite easily understood—the imposition of a tax raises the price of a good. On the consumer side of the market, the standard analysis of income and substitution effects predicts what will happen to demand. For producers, the tax is a cost increase, and they respond accordingly. What is more interesting is the choice of the best set of taxes for the government. There are several interesting settings for this question. The simplest version can be described as follows. There is a given level of government revenue to be raised that must be financed solely by taxes on commodities. How must the taxes be set so as to minimize the cost to society of raising the required revenue? This is the Ramsey problem of efficient taxation, first addressed in the 1920s. The insights its study gives are still at the heart of the understanding of setting optimal commodity taxes. More general problems introduce equity issues in addition to those of efficiency.

The chapter begins by discussing the deadweight loss that is caused by the introduction of a commodity tax. A diagrammatic analysis of optimal commodity taxation is then presented. This diagram is also used to demonstrate the Diamond–Mirrlees Production Efficiency result. Following this, the Ramsey rule is derived and an interpretation of this is provided. The extension to many consumers is then made and the resolution of the equity–efficiency trade-off is emphasized. This is followed by a review of some numerical calculations of optimal taxes based on empirical data.

15.2 Deadweight Loss

Lump-sum taxation was described as the perfect tax instrument because it does not cause any distortions. The absence of distortions is due to the fact that a lump-sum tax is defined by the condition that no change in behavior can affect the level of the tax. Commodity taxation does not satisfy this definition. It is always possible to change a consumption plan if commodity taxation is introduced. Demand can shift from goods subject to high taxes to goods with low taxes, and total consumption can be reduced by earning less or saving more. It is these changes at the margin, which we call *substitution effects*, that are the tax-induced distortions.

The introduction of a commodity tax raises tax revenue but causes consumer welfare to be reduced. The *deadweight loss* of the tax is the extent to which the reduction in welfare exceeds the revenue raised. This concept is illustrated in figure 15.1. Before the tax is introduced, the price of the good is p and the quantity consumed is X^0. At this price the level of consumer surplus is given by the triangle abc. A specific tax of amount t is then levied on the good, so the price rises to $q = p + t$ and quantity consumed falls to X^1. This fall in consumption together with the price increase

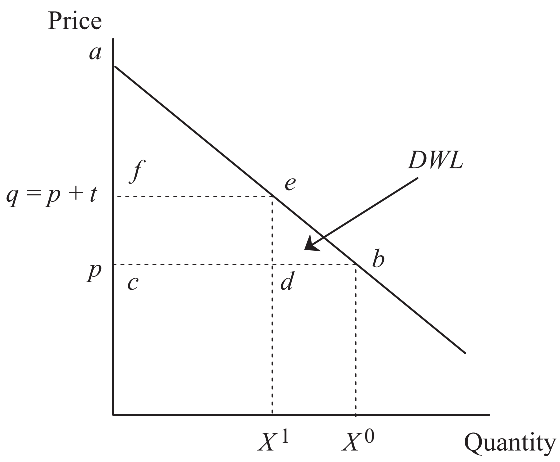

Figure 15.1
Deadweight loss

reduces consumer surplus to *aef*. The tax raises revenue equal to tX^1, which is given by the area *cdef*. The part of the original consumer surplus that is not turned into tax revenue is the deadweight loss, *DWL*, given by the triangle *bde*.

It is possible to provide a simple expression that approximates the deadweight loss. The triangle *bde* is equal to $\frac{1}{2}tdX$, where dX is the change in demand $X^0 - X^1$. This formula could be used directly, but it is unusual to have knowledge of the level of demand before and after the tax is imposed. Accepting this, it is possible to provide an alternative form for the formula. This can be done by noting that the elasticity of demand is defined by $\varepsilon^d = \frac{p}{X}\frac{dX}{dp}$, so it implies that $dX = \varepsilon^d \frac{X^0}{p} dp$. Substituting this into deadweight loss gives

$$DWL = \frac{1}{2}\left|\varepsilon^d\right| \frac{X^0}{p} t^2, \tag{15.1}$$

since the change in price is $dp = t$. The measure in (15.1) is approximate because it assumes that the elasticity is constant over the full change in price from p to $q = p + t$.

The formula for deadweight loss reveals two important observations. First, deadweight loss is proportional to the square of the tax rate. The deadweight loss will therefore rise rapidly as the tax rate is increased. Second, the deadweight loss is proportional to the elasticity of demand. For a given tax change the deadweight loss will be larger the more elastic is demand for the commodity.

An alternative perspective on commodity taxation is provided in figure 15.2. Point a is the initial position in the absence of taxation. Now consider the contrast between a

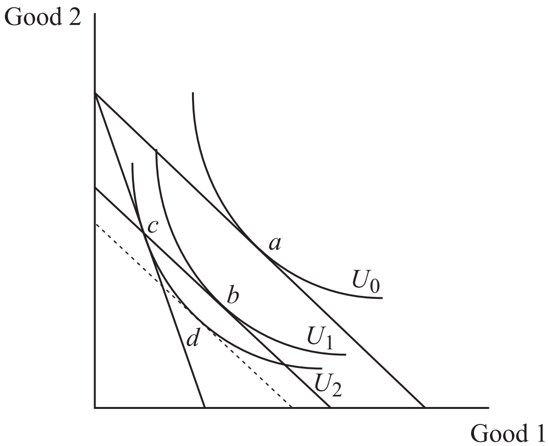

Figure 15.2
Income and substitution effects

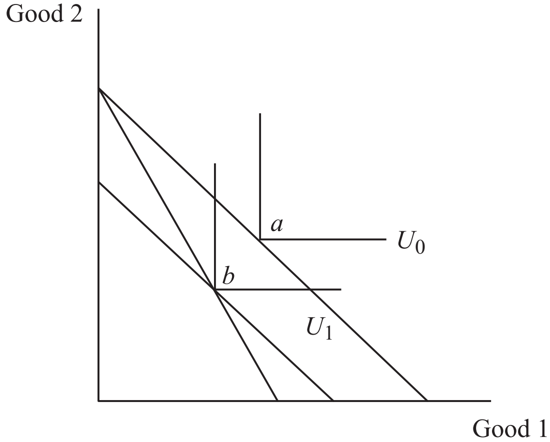

Figure 15.3
Absence of deadweight loss

lump-sum tax and a commodity tax on good 1 when the two tax instruments raise the same level of revenue. In the figure the lump-sum tax is represented by the move from point a to point b. The budget constraint shifts inward, but its gradient does not change. Utility falls from U_0 to U_1. A commodity tax on good 1 increases the price of good 1 relative to the price of good 2 and causes the budget constraint to become steeper. At point c the commodity tax raises the same level of revenue as the lump-sum tax. This is because the value of consumption at c is the same as that at b, so the same amount must have been taken off the consumer by the government in both cases. The commodity tax causes utility to fall to U_2, which is less than U_1. The difference $U_1 - U_2$ is the deadweight loss measured directly in utility terms.

Figure 15.2 illustrates two further points to which it is worth drawing attention. Notice that commodity taxation produces the same utility level as a lump-sum tax that would move the consumer to point d. This is clearly a larger lump-sum tax than that which achieved point a. The difference in the size of the two lump-sum taxes provides a monetary measure of the deadweight loss. The effect of the commodity tax can now be broken down into two separate components. First, there is the move from the original point a to point d. In line with the standard terminology of consumer theory, this is called an *income effect*. Second, there is a *substitution effect* due to the increase in the price of good 1 relative to good 2 represented by a move around an indifference curve. This shifts the consumer's choice from point d to point c.

This argument can be extended to show that it is the substitution effect that is responsible for the deadweight loss. To do this, note that if the consumer's indifference curves are all L-shaped so that the two commodities are perfect complements, then there is no substitution effect in demand—a relative price change with utility held constant just pivots the budget constraint around the corner of the indifference curve. As shown in figure 15.3, the lump-sum tax and the commodity tax result in exactly the same outcome, so the deadweight loss of the commodity tax is zero. The initial position without taxation is at *a* and both tax instruments lead to the final equilibrium at *b*. Hence the deadweight loss is caused by substitution between commodities.

15.3 Optimal Taxation

The purpose of optimal tax analysis is to find the set of taxes that gives the highest level of welfare while raising the revenue required by the government. The set of taxes that do this are termed *optimal*. In determining these taxes, consumers must be left free to choose their most preferred consumption plans at the resulting prices and firms to continue to maximize profits. The taxes must also lead to prices that equate supply to demand. This section will consider the problem for the case of a single consumer. This restriction ensures that only efficiency considerations arise. The more complex problem involving equity, as well as efficiency, will be addressed in section 15.6.

To introduce a number of important aspects of commodity taxation in a simple way, it is best to begin with a diagrammatic approach. Among the features that this makes clear are the second-best nature of commodity taxes relative to lump-sum taxes. In other words, the use of commodity taxes leads to a lower level of welfare compared to the optimal set of lump-sum taxes. Despite this effect, the observability of transactions makes commodity taxes feasible, whereas optimal lump-sum taxes are generally not, for the reasons explored in chapter 13.

Consider a two-good economy with a single consumer and a single firm (the Robinson Crusoe economy of chapter 2). One of the goods, labor, is used as an input (so it is supplied by the consumer to the firm), and the output is sold by the firm to the consumer. In figure 15.4 the horizontal axis measures labor use and the vertical axis output. The firm's production set, marked Y in the figure, is also the production set for the economy. This is displaced from the origin by a distance R that equals the tax revenue requirement of the government. The interpretation is that the government takes out of the economy R units of labor for its own purposes. After the revenue requirement has been met, the economy then has constant returns to scale in turning labor into output. The commodity taxes have to be chosen to attain this level of revenue. Normalizing the wage rate to 1, the only output price for the firm that leads to zero profit is shown by p. This is the only level of profit consistent with the assumption of competitive behavior, and p must

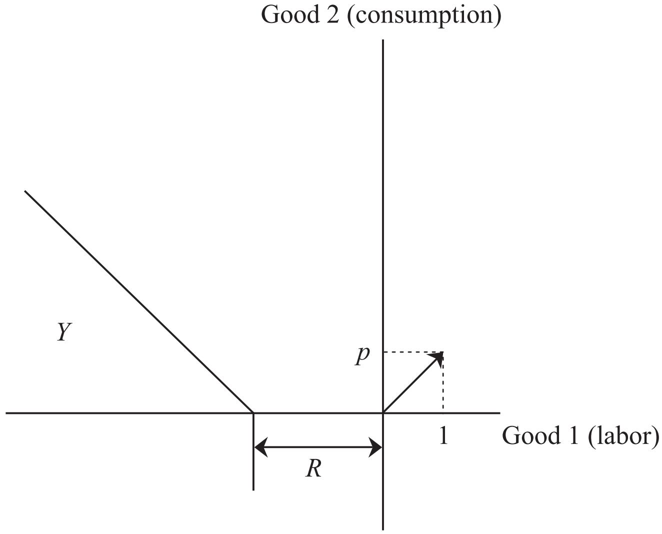

Figure 15.4
Revenue and production possibilities

be the equilibrium price for the firm. Given this price, the firm is indifferent to where it produces on the frontier of its production set.

Figure 15.5 shows the budget constraint and the preferences of the consumer. With the wage rate of 1, the budget constraint for the consumer is constructed by setting the consumer's price for the output to q. The difference between q and p is the tax on the consumption good. It should be noticed that labor is not taxed. As will become clear, this is not a restriction on the set of possible taxes. With these prices the consumer's budget constraint can be written $qx = \ell$, where x denotes units of the output and ℓ units of labor. The important properties of this budget constraint are that it is upward sloping and must pass through the origin. The preferences of the consumer are represented by indifference curves. The form of these follows from noting that the supply of labor causes the consumer disutility, so an increase in labor supply must be compensated for by further consumption of output in order to keep utility constant. The indifference curves are therefore downward sloping. Given these preferences, the optimal choice is found by the tangency of the budget constraint and the highest attainable indifference curve. Varying the price, q, faced by the consumer gives a series of budget constraints whose slopes increase as q falls. Forming the locus of optimal choices determined by these budget constraints traces out the consumer's offer curve. Each point on this offer curve can be associated with a budget constraint that runs through the origin and

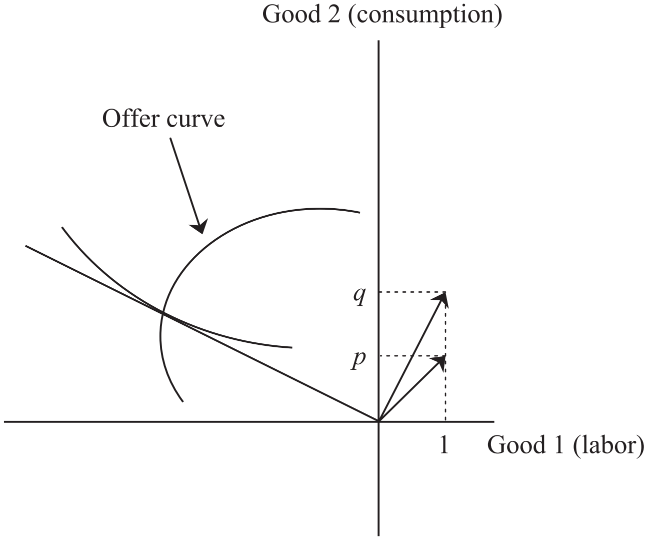

Figure 15.5
Consumer choice

an indifference curve tangential to that budget constraint. The interpretation given to the offer curve is that the points on the curve are the only ones consistent with utility maximization by the consumer in the absence of lump-sum taxation. It should also be noted that the consumer's utility rises as the move is made up the offer curve.

Figures 15.4 and 15.5 can be superimposed to represent the production and consumption decisions simultaneously. This is done in figure 15.6, which can be used to find the optimal tax rate on the consumption good. The only points that are consistent with choice by the consumer are those on the offer curve. The maximal level of utility achievable on the offer curve is at the point where it intersects the production frontier. Any level higher than this is not feasible. This optimum is denoted by point e, and here the consumer is on indifference curve I_0. At this optimum the difference between the consumer price and the producer price for the output, $t^* = q - p$, is the optimal tax rate. That is, it is the tax that ensures that the consumer chooses point e. By construction, this tax rate must also ensure that the government raises its required revenue so that $t^* x^* = R$, where x^* is the level of consumption at point e.

This discussion has shown how the optimal commodity tax is determined at the highest point of the offer curve in the production set. This is the solution to the problem of finding the optimal commodity taxes for this economy. The diagram also shows why labor can remained untaxed without affecting the outcome. The choices of the consumer and the firm are determined by the ratio of prices they face or the direction of the price vector (which is orthogonal to the budget constraint). By changing the length (but not the direction) of either p or q, one can introduce a tax on labor, but it does not alter the fact that e is the optimum. This reasoning can be expressed by saying that the zero tax on labor is a normalization, not a real restriction on the system.

Figure 15.6 also illustrates the second-best nature of commodity taxation relative to lump-sum taxation. It can be seen that there are points above the indifference curve I_0 (the best achievable by commodity taxation) that are preferred to e and that are also productively feasible. The highest attainable indifference curve for the consumer given the production set is I_1 with utility maximized at point e^*. This point would be chosen by the consumer if they faced a budget constraint that is coincident with the production frontier. A budget constraint of this form would cross the horizontal axis to the left of the origin and would have equation $qx = \ell - R$, where R represents a lump-sum tax equal to the revenue requirement. This lump-sum tax would decentralize the first-best outcome at e^*. Commodity taxation can only achieve the second-best at e.

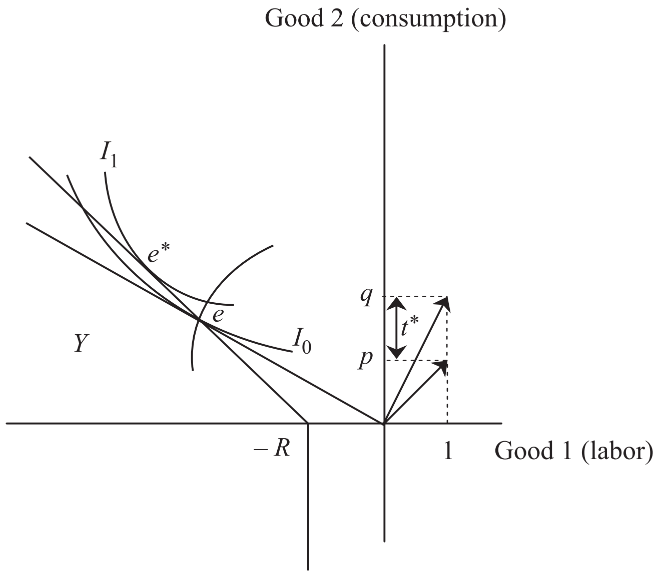

Figure 15.6
Optimal commodity taxation

15.4 Production Efficiency

The diagrammatic illustration of optimal taxation in the one-consumer economy also shows another important result. This result, known as the *Diamond–Mirrlees Production Efficiency Lemma*, states that the optimal commodity tax system should not disrupt production efficiency. In other words, the optimum with commodity taxation must be on the boundary of the production set and all distortions are focused on consumer choice. This section provides a demonstration of the efficiency lemma and discusses its implications.

Production efficiency occurs when an economy is maximizing the output attainable from its given set of resources. This can only happen when the economy is on the boundary of its production possibility set. Starting at a boundary point, no reallocation of inputs among firms can increase the output of one good without reducing that of another (compare this with the conditions for Pareto-efficiency in chapter 2). In the special case where each firm employs some of all the available inputs, a necessary condition for production efficiency is that the marginal rate of substitution (MRS) between any two inputs be the same for all firms. Such a position of equality is attained, in the absence of taxation, by the profit maximization of firms in competitive markets. Each firm sets the marginal rate of substitution equal to the ratio of factor prices, and since factor prices are the same for all firms, this induces the necessary equality in the MRSs. The same is true when there is taxation, provided that all firms face the same after-tax prices for inputs, meaning inputs taxes are not differentiated among firms.

To see that the optimum with commodity taxation must be on the frontier of the production set, consider the interior point f in figure 15.7. If the equilibrium were at f, the consumer's utility could be raised by reducing the use of the input while keeping output constant. Since this is feasible, f cannot be an optimum. Since this reasoning can be applied to any point that is interior to the production set, the optimum must be on the boundary.

Although figure 15.7 was motivated by considering the input to be labor, a slight reinterpretation can introduce intermediate goods. Assume that there are several industries and that each industry has a production process that uses one unit of labor to produce one unit of an intermediate good. The intermediate goods are then combined by the final goods industry using a production process that has constant returns to scale. The intermediate good production is displayed indirectly in figure 15.7 as the link between the use of labor and the output of the final good. The production efficiency argument

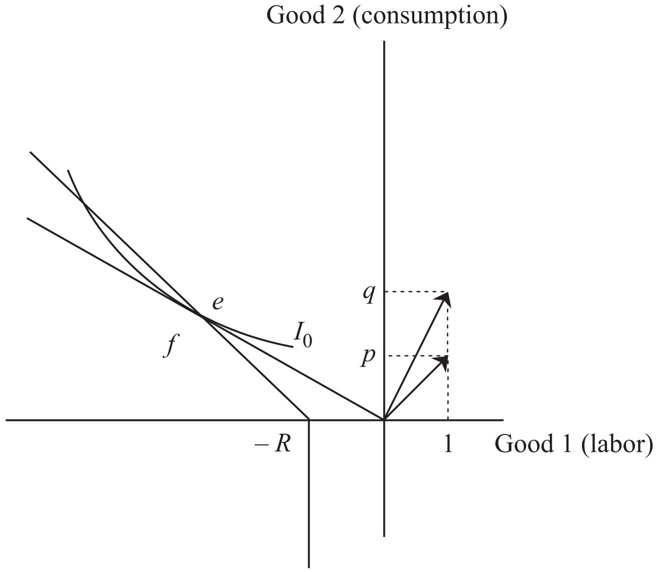

Figure 15.7
Production efficiency

then follows directly as before and now implies that intermediate goods should not be taxed, since this would violate the equalization of MRSs between firms.

The logic of the single-consumer economy can be adapted to show that the efficiency lemma still holds when there are many consumers. What makes the result so obvious in the single-consumer case is that a reduction in labor use or an increase in output raises the consumer's utility. With many consumers, such a change would have a similar effect if all consumers supply labor or prefer to have more, rather than less, of the consumption good. This will hold if there is some agreement in the tastes of the consumers. If this is so, a direction of movement can be found from an interior point in the production set to an exterior point that is unanimously welcomed. The optimum must then be on the boundary.

In summary, the Diamond–Mirrlees Production Efficiency Lemma provides a persuasive argument for the nontaxation of intermediate goods and the nondifferentiation of input taxes among firms. These are results of immediate practical importance, since they provide a basic property that an optimal tax system must possess. As will become clear, it is rather hard to make precise statements about the optimal levels of tax, but what the efficiency lemma provides is a clear and simple statement about the structure of taxation.

15.5 Tax Rules

The diagrammatic analysis has shown the general principle behind the determination of the optimal taxes. What is not shown is how the tax burden is allocated across different commodities. The optimal tax problem is to set the taxes on commodities to maximize social welfare subject to raising a required level of revenue. This section looks at tax rules that characterize the solution to this problem.

To derive the rules, it is first necessary to precisely specify a model of the economy. Let there be n goods, each produced with constant returns to scale by competitive firms. Since the firms are competitive, the price of the commodity they sell must be equal to the marginal cost of production. Under the assumption of constant returns, this marginal cost is also independent of the scale of production. Labor is assumed to be the only input into production.

With the wage rate as numéraire, these assumptions imply that the producer (or before-tax) price of good i is determined by

$$p_i = c_i, \qquad i = 1, \dots, n, \tag{15.2}$$

where c_i denotes the number of units of labor required to produce good i. The consumer (or after-tax) prices are equal to the before-tax prices plus the taxes. For good i the consumer price q_i is

$$q_i = p_i + t_i, \qquad i = 1, \dots, n. \tag{15.3}$$

Writing x_i for the consumption level of good i, the tax rates on the n consumption goods must be chosen to raise the required revenue. With the revenue requirement denoted by R, the revenue constraint can be written as

$$R = \sum_{i=1}^{n} t_i x_i. \tag{15.4}$$

In line with this numbering convention, labor is denoted as good 0, so x_0 is the supply of labor (labor is the untaxed good, so $t_0 = 0$).

This completes the description of the economy. The simplifying feature is that the assumption of constant returns to scale fixes the producer prices via (15.2) so that equilibrium prices are independent of the level of demand. Furthermore constant returns also implies that whatever demand is forthcoming at these prices will be met by the firms. If the budget constraints are satisfied (both government and consumer), any demand will be backed by sufficient labor supply to carry out the necessary production.

15.5.1 The Inverse Elasticity Rule

Figure 15.6 shows some of the features that the optimal set of commodity taxes will have. What the single-good formulation cannot do is give any insight into how that tax burden should be spread across different goods. For example, should all goods have the same rate of tax or should taxes be related to the characteristics of the goods? The first tax rule considers a simplified situation that delivers a very precise answer to this question. This answer, the *inverse elasticity rule*, provides a foundation for proceeding to the more general case. The simplifying assumption is that the goods are independent in demand so that there are no cross-price effects between the taxed goods. This independence of demands is a strong assumption, so it is not surprising that a clear result can be derived. The way the analysis works is to choose the optimal allocation and infer the tax rates from this. This was the argument used in the diagram when the intersection of the offer curve and the frontier of the production set was located and the tax rate derived from the implied budget constraint.

Consider a consumer who buys the two taxed goods and supplies labor. The consumer's preferences are described by the utility function $U(x_0, x_1, x_2)$, and his budget constraint is $q_1x_1 + q_2x_2 = x_0$. The utility-maximizing consumption levels of the two consumption goods are described by the first-order conditions $U_i' = \alpha q_i$, $i = 1, 2$, where U_i' is the marginal utility of good i and α is the marginal utility of income. The choice of labor supply satisfies the first-order condition $U_0' = -\alpha$.

With taxes t_1 and t_2 the government revenue constraint is $R = t_1x_1 + t_2x_2$. Since producer and consumer prices are related by $t_i = q_i - p_i$, this can be written as

$$q_1x_1 + q_2x_2 = R + p_1x_1 + p_2x_2. \tag{15.5}$$

The optimal tax rates are inferred from an optimization whereby the government chooses the consumption levels to maximize the consumer's utility while meeting the revenue constraint. This problem is summarized by the constrained maximization

$$\max_{\{x_1,x_2\}} L = U(x_0, x_1, x_2) + \lambda\,[q_1x_1 + q_2x_2 - R - p_1x_1 - p_2x_2]. \tag{15.6}$$

In this maximization the quantity of labor supply, x_0, is determined endogenously by x_1 and x_2 from the consumer's budget constraint, $x_0 = q_1x_1 + q_2x_2$.

The basic assumption that the demands are independent can be used to write the (inverse) demand function $q_i = q_i(x_i)$. Using these demand functions and the consumer's budget constraint to replace x_0, we write the first-order condition for the quantity of good i:

$$U_i' + U_0'\left[q_i + x_i \frac{\partial q_i}{\partial x_i}\right] + \lambda\left[q_i + x_i \frac{\partial q_i}{\partial x_i} - p_i\right] = 0. \tag{15.7}$$

The conditions $U_i' = \alpha q_i$ and $U_0' = -\alpha$ can be used to write this as

$$-\alpha x_i \frac{\partial q_i}{\partial x_i} + \lambda t_i + \lambda x_i \frac{\partial q_i}{\partial x_i} = 0. \tag{15.8}$$

where $t_i = q_i - p_i$. Now note that $\frac{x_i}{q_i}\frac{\partial q_i}{\partial x_i} = \frac{1}{\varepsilon_i^d}$, where ε_i^d is the elasticity of demand for good i. The first-order condition can then be solved to write

$$\frac{t_i}{p_i + t_i} = -\left[\frac{\lambda - \alpha}{\lambda}\right]\frac{1}{\varepsilon_i^d}. \tag{15.9}$$

Equation (15.9) is the inverse elasticity rule. This is interpreted by noting that α is the marginal utility of another unit of income for the consumer and λ is the utility cost of another unit of government revenue. Since taxes are distortionary, $\lambda > \alpha$. Since ε_i^d is negative, this makes the tax rate positive.

The inverse elasticity rule states that the proportional rate of tax on good i should be inversely related to its price elasticity of demand. Furthermore the constant of proportionality is the same for all goods. Recalling the discussion of the deadweight loss of taxation, it can be seen that this places more of the tax burden on goods where the deadweight loss is low. Its implication is clearly that necessities, which by definition have low elasticities of demand, should be highly taxed. It is this latter aspect that emphasizes the fact that the inverse elasticity rule describes an efficient way to tax commodities but not an equitable way. Placing relative high taxes on necessities will result in lower income consumers bearing relatively more of the commodity tax burden than high-income consumers.

15.5.2 The Ramsey Rule

The inverse elasticity rule is restricted by the fact that the demand for each good depends only on the price of that good. This rules out all cross-price effects in demand, meaning that the goods can be neither substitutes nor complements. When this restriction is relaxed, a more general tax rule is derived. The general result is called the *Ramsey rule*, and it is one of the oldest results in the theory of optimal taxation. It provides a description of the optimal taxes for an economy with a single consumer and with no equity considerations.

To derive the Ramsey rule, it is necessary to change from choosing the optimal quantities to choosing the taxes. Assume that there are just two consumption goods in order to simplify the notation, and let the demand function for good i be $x_i = x_i\,(q)$ where $q = q_1,\, q_2$. The fact that the prices of all the commodities enter this demand function shows that the full range of interactions between the demands and prices are allowed. Using these demand functions, the preferences of the consumer can be written as

$$U = U(x_0(q)\,,\, x_1(q)\,,\, x_2(q)). \tag{15.10}$$

The optimal commodity taxes are those that give the highest level of utility to the consumer, while ensuring that the government reaches its revenue target of $R > 0$. The government's problem in choosing the tax rates can then be summarized by the Lagrangean

$$\max_{\{t_1, t_2\}} L = U\,(x_0\,(q)\,,\, x_1\,(q)\,,\, x_2\,(q)) + \lambda\left[\sum_{i=1}^{2} t_i x_i\,(q) - R\right], \tag{15.11}$$

where it is recalled that $q_i = p_i + t_i$. Differentiating (15.11) with respect to the tax on good k, we have the first-order necessary condition

$$\frac{\partial L}{\partial t_k} \equiv \sum_{i=0}^{2} U_i' \frac{\partial x_i}{\partial q_k} + \lambda\left[x_k + \sum_{i=1}^{2} t_i \frac{\partial x_i}{\partial q_k}\right] = 0. \tag{15.12}$$

This first-order condition needs some manipulation to place it in the form we want. The first step is to note that the budget constraint of the consumer is

$$q_1 x_1\,(q) + q_2 x_2\,(q) = x_0\,(q). \tag{15.13}$$

Any change in price of good k must result in demands that still satisfy this constraint so that

$$q_1 \frac{\partial x_1}{\partial q_k} + q_2 \frac{\partial x_2}{\partial q_k} + x_k = \frac{\partial x_0}{\partial q_k}. \tag{15.14}$$

In addition the conditions for optimal consumer choice are $U_0' = -\alpha$ and $U_i' = \alpha q_i$. Using these optimality conditions and (15.14), we rewrite the first-order condition for the optimal tax, (15.12), as

$$\alpha x_k = \lambda \left[x_k + \sum_{i=1}^{2} t_i \frac{\partial x_i}{\partial q_k} \right]. \tag{15.15}$$

Notice how this first-order condition involves quantities rather than the prices that appeared in the inverse elasticity rule. After rearrangement, (15.15) becomes

$$\sum_{i=1}^{2} t_i \frac{\partial x_i}{\partial q_k} = - \left[\frac{\lambda - \alpha}{\lambda} \right] x_k. \tag{15.16}$$

The next step in the derivation is to employ the Slutsky equation, which breaks the change in demand into the income and substitution effects. The effect of an increase in the price of good k upon the demand for good i is determined by the Slutsky equation as

$$\frac{\partial x_i}{\partial q_k} = S_{ik} - x_k \frac{\partial x_i}{\partial I}, \tag{15.17}$$

where S_{ik} is the substitution effect of the price change (the move around an indifference curve) and $-x_k \frac{\partial x_i}{\partial I}$ is the income effect of the price change (I denotes lump-sum income). Substituting from (15.17) into (15.16) gives

$$\sum_{i=1}^{2} t_i \left[S_{ik} - x_k \frac{\partial x_i}{\partial I} \right] = - \left[\frac{\lambda - \alpha}{\lambda} \right] x_k. \tag{15.18}$$

Equation (15.18) is now simplified by extracting the common factor x_k, which yields

$$\sum_{i=1}^{2} t_i S_{ik} = - \left[1 - \frac{\alpha}{\lambda} - \sum_{i=1}^{2} t_i \frac{\partial x_i}{\partial I} \right] x_k. \tag{15.19}$$

The substitution effect of a change in the price of good i on the demand for good k is exactly equal to the substitution effect of a change in the price of good k on the demand for good i because both are determined by movement around the same indifference curve. This symmetry property implies $S_{ki} = S_{ik}$, which can be used to rearrange (15.19) to give the expression

$$\sum_{i=1}^{2} t_i S_{ki} = -\theta x_k, \tag{15.20}$$

where $\theta = \left[1 - \frac{\alpha}{\lambda} - \sum_{i=1}^{2} t_i \frac{\partial x_i}{\partial I}\right]$ is a positive constant. Equation (15.20) is the Ramsey rule describing a system of optimal commodity taxes and an equation of this form must hold for all goods, $k = 1, \ldots, n$.

The optimal tax rule described by (15.20) can be used in two ways. If the details of the economy are specified (the utility function and production parameters), then the actual tax rates can be calculated. Naturally the precise values would be a function of the structure chosen. Although this is the direction that heads toward practical application of the theory (and more is said later), it is not the route that will be currently taken. The second use of the rule is to derive some general conclusions about the determinants of tax rates. This is done by analyzing and understanding the different components of (15.20).

To proceed with this, the focus on the typical good k is maintained. Recall that a substitution term measures the change in demand with utility held constant. Demand defined in this way is termed *compensated demand*. Now begin in an initial position with no taxes. From this point the tax t_i is the change in the tax rate on good i. Then $t_i S_{ki}$ is a first-order approximation to the change in compensated demand for good k due to the introduction of the tax t_i. If the taxes are small, this will be a good approximation to the actual change. Extending this argument to take account of the full set of taxes, it follows that $\sum_{i=1}^{2} t_i S_{ki}$ is an approximation to the total change in compensated demand for good k due to the introduction of the tax system from the initial no-tax position. In employing this approximation, the Ramsey rule can be interpreted as saying that the optimal tax system should be such that the *compensated demand for each good is reduced in the same proportion* relative to the before-tax position. This is the standard interpretation of the Ramsey rule.

The importance of this observation is reinforced when it is set against the alternative, but incorrect, argument that the optimal tax system should raise the prices of all goods by the same proportion in order to minimize the distortion caused by the tax system. This is shown by the Ramsey rule to be false. What the Ramsey rule says is that it is the distortion in terms of quantities, rather than prices, that should be minimized. Since it is the level of consumption that actually determines utility, it is not surprising that what happens to prices is secondary to what happens to quantities. Prices only matter so far as they determine demands.

Although the actual tax rates are only implicit in the Ramsey rule, some general comments can still be made. By the approximation interpretation, the rule suggests that as the proportional reduction in compensated demand must be the same for all goods, and those goods whose demand is unresponsive to price changes must bear higher taxes in order to achieve the same reduction. Although broadly correct, this statement can only be completely justified when all cross-price effects are accounted for. One simple case that overcomes this difficulty is that in which there are no cross-price effects among the taxed goods. This is the special case that led to the inverse elasticity rule.

Returning to the general case, goods that are unresponsive to price changes are typically necessities such as food and housing. Consequently using the Ramsey rule leads to a tax system that bears most heavily on necessities. In contrast, the lowest tax rates would fall on luxuries. If put into practice, such a tax structure would involve low-income consumers paying disproportionately larger fractions of their incomes in taxes relative to high-income consumers. The inequitable nature of this is simply a reflection of the single-consumer assumption: the optimization does not involve equity and the solution reflects only efficiency criteria.

The single-consumer framework is not accurate as a description of reality, and it leads to an outcome that is unacceptable on equity grounds. The value of the Ramsey rule therefore arises primarily through the framework and method of analysis it introduces. This can easily be generalized to more relevant settings. It shows how taxes are determined by efficiency considerations and hence gives a baseline from which to judge the effects of introducing equity.

15.6 Equity Considerations

The lack of equity in the tax structure determined by the Ramsey rule is inevitable given its single-consumer basis. The introduction of further consumers who differ in incomes and preferences makes it possible to see how equity can affect the conclusions. Although the method that is now discussed can cope with any number of consumers, it is sufficient to consider just two. Restricting the number in this way has the merit of making the analysis especially transparent.

Consider then an economy that consists of two consumers. Each consumer h, $h = 1$, 2, is described by their (indirect) utility function

$$U^h = U^h\Big(x_0^h(q), x_1^h(q), x_2^h(q)\Big). \tag{15.21}$$

These utility functions may vary between the consumers. Labor remains the untaxed numéraire, and all consumers supply only the single form of labor service.

The government revenue constraint is now given by

$$R = \sum_{i=1}^{2} t_i x_i^1(q) + \sum_{i=1}^{2} t_i x_i^2(q), \tag{15.22}$$

where the first term on the right-hand side is the total tax payment of consumer 1 and the second term is the total tax payment of consumer 2. The government's policy is guided by a social welfare function that aggregates the individual consumers' utilities. This social welfare function is denoted by

$$W = W\Big(U^1\left(x_0^1, x_1^1, x_2^1\right), \quad U^2\big(x_0^2, x_1^2, x_2^2\big)\Big). \tag{15.23}$$

Combining (15.22) and (15.23) into a Lagrangean expression (as in equation 15.11), we have the first-order condition for the choice of the tax on good k:

$$\frac{\partial L}{\partial t_k} \equiv -\frac{\partial W}{\partial U^1}\alpha^1 x_k^1 - \frac{\partial W}{\partial U^2}\alpha^2 x_k^2 + \lambda\left[\sum_{h=1}^{2}\left[x_k^h + \sum_{i=1}^{2} t_i \frac{\partial x_i^h}{\partial q_k}\right]\right] = 0. \tag{15.24}$$

The expression in (15.24) has been derived by using (15.13), (15.14), and the first-order conditions for consumer choice ($\frac{\partial U^h}{\partial x_1^h} = \alpha^h q_k$) to deduce that

$$\frac{\partial U^h}{\partial x_0^h}\frac{\partial x_0^h}{\partial q_k} + \frac{\partial U^h}{\partial x_1^h}\frac{\partial x_1^h}{\partial q_k} + \frac{\partial U^h}{\partial x_2^h}\frac{\partial x_2^h}{\partial q_k} = -\alpha^h x_k^h, \qquad h = 1, 2. \tag{15.25}$$

To obtain a result that is easily comparable to the Ramsey rule, define

$$\beta^h = \frac{\partial W}{\partial U^h}\alpha^h. \tag{15.26}$$

β^h is formed as the product of the effect of an increase in consumer h's utility on social welfare and their marginal utility of income. It measures the increase in social welfare that results from a marginal increase in the income of consumer h. Consequently β^h is termed the *social marginal utility of income* for consumer h. Employing the definition of β^h and the substitutions used to obtain the Ramsey rule, we write the first-order condition (15.24) as

$$\begin{aligned} \frac{\sum_{i=1}^{2} t_i S_{ki}^1 + \sum_{i=1}^{2} t_i S_{ki}^2}{x_k^1 + x_k^2} &= \frac{1}{\lambda}\frac{\beta^1 x_k^1 + \beta^2 x_k^2}{x_k^1 + x_k^2} - 1 \\ &\quad + \frac{\left[\sum_{i=1}^{2} t_i \frac{\partial x_i^1}{\partial I^1}\right] x_k^1 + \left[\sum_{i=1}^{2} t_i \frac{\partial x_i^2}{\partial I^2}\right] x_k^2}{x_k^1 + x_k^2}. \end{aligned} \tag{15.27}$$

The tax structure that is described by (15.27) can be interpreted in the same way as the Ramsey rule. The left-hand side is approximately the proportional change in aggregate compensated demand for good k caused by the introduction of the tax system from an initial position with no taxes. When a positive amount of revenue is to be raised (so that $R > 0$), the level of demand will be reduced by the tax system, so this term will be negative.

The first point to observe about the right-hand side is that unlike the Ramsey rule, the proportional reduction in compensated demand is not the same for all goods. It is therefore necessary to discuss the factors that influence the extent of the reduction, and it is by doing this that the consequences of equity can be seen. The essential component in this regard is the first term on the right-hand side. The proportional reduction in demand for good k will be smaller the larger is the value of $\beta^1 \frac{x_k^1}{x_k^1+x_k^2} + \beta^2 \frac{x_k^2}{x_k^1+x_k^2}$. The value of this will be large if a high β^h is correlated with $\frac{x_k^h}{x_k^1+x_k^2}$. The meaning of this is clear, since a consumer will have a high value of β^h when their personal marginal utility of income, α^h, is large and when $\frac{\partial W}{\partial U^h}$ is also large so that the social planner gives them a high weight in social welfare. If the social welfare function is concave, both of these will be satisfied by low-utility consumers with low incomes. The term $\frac{x_k^h}{x_k^1+x_k^2}$ will be large when good k is consumed primarily by consumer h. Putting these points together, we have that the proportional reduction in the compensated demand for a good will be smaller if it is consumed primarily by the poor consumer. This is the natural reflection of equity considerations.

The second term on the right-hand side shows that the proportional reduction in demand for good k will be smaller if its demand comes mainly from the consumer whose tax payments change most as income changes. This term is related to the efficiency aspects of the tax system. If taxation were to be concentrated on goods consumed by those whose tax payments fell rapidly with reductions in income, then increased taxation, and consequently greater distortion, would be required to meet the revenue target.

This has shown how the introduction of equity modifies the conclusions of the Ramsey rule. Rather than all goods having their compensated demand reduced in the same proportion, equity results in the goods consumed primarily by the poor facing less of a reduction. In simple terms, this should translate into lower rates of tax on the goods consumed by the poor relative to those determined solely by efficiency. Equity therefore succeeds in moderating the hard edge of the efficient tax structure.

15.7 Applications

At this point in the discussion it should be recalled that the fundamental motive for the analysis is to provide practical policy recommendations. The results that have been derived do give some valuable insights: the need for production efficiency and the non-uniformity of taxes being foremost among them. Accepting this, the analysis is only of real merit if the tax rules are capable of being applied to data and the actual values of the resulting optimal taxes calculated. The numerical studies that have been undertaken represent the development of a technology for achieving this aim and also provide further insights into the structure of taxation.

Referring back to (15.27), it can be seen that two basic pieces of information are needed in order to calculate the tax rates. The first is knowledge of the demand functions of the consumers. This provides the levels of demand x_k^h and the demand derivatives $\frac{\partial x_k^h}{\partial q_i}$. The second piece of information is the social marginal utilities of income, β^h. Ideally these should be calculated from a specified social welfare function and individual utility functions for the consumers. The problem here is the same as that raised in previous chapters: the construction of some meaningful utility concept. The difficulties are further compounded in the present case by the requirement that the demand functions also be consistent with the utility functions.

In practice, the difficulties are circumvented rather than solved. The approach that has been adopted is first to ignore the link between demand and utility and to impose a procedure to obtain the social marginal utilities of income. The demand functions are then estimated using standard econometric techniques. One common procedure is to combine the utility function defined by (14.24) with a utilitarian social welfare function. Hence assume that the social welfare function is utilitarian ($W = \sum_h U^h$) and that the individual utility functions are isoelastic ($U^h = K \frac{M^{h\,1-\varepsilon}}{1-\varepsilon}$). The social marginal utility for h is then given by

$$\beta^h = K\left[M^h\right]^{-\varepsilon}. \tag{15.28}$$

The value of the parameter K can be fixed by, for instance, setting the value of β^h equal to 1 for the lowest income consumer. With $\varepsilon > 0$ the social marginal utility declines as income rises. It decreases faster as ε rises, so relatively more weight is given to low-income consumers. This way the value of ε can be treated as a measure of the concern for equity.

15.7.1 Reform

The first application of the analysis is to consider marginal reforms of tax rates. By a marginal reform it is meant a small change from the existing set of tax rates that moves the system closer to optimality. This should be distinguished from an optimization of tax rates that might imply a very significant change from the initial set of taxes.

Marginal reforms are much easier to compute than optimal taxes, since it is only necessary to evaluate effect of changes not of the whole move. An analogy can be drawn with hill-climbing: to climb higher, you only need to know which direction leads upward and do not need to know where the top is. Essentially studying marginal reforms reduces the informational requirement.

Return to the analysis of the optimal taxes in the economy with two consumers. The effect on welfare of a marginal increase in the tax on good k is

$$\frac{\partial W}{\partial t_k} = -\sum_{i=1}^{2} \beta^i x_k^i, \tag{15.29}$$

and the effect on revenue is

$$\frac{\partial R}{\partial t_k} = \sum_{h=1}^{2} \left[x_k^h + \sum_{i=1}^{2} t_i \frac{\partial x_i^h}{\partial q_k} \right] = X_k + \sum_{i=1}^{2} t_i \frac{\partial X_i}{\partial q_k}, \tag{15.30}$$

where X_i is the aggregate demand for good i. The marginal revenue benefit of taxation of good k is defined as the extra revenue generated relative to the welfare change of a marginal increase in a tax. This can be written as

$$MRB_k = -\frac{\frac{\partial R}{\partial t_k}}{\frac{\partial W}{\partial t_k}}. \tag{15.31}$$

At the optimum all goods should have the same marginal revenue benefit. If that was not the case, taxes could be raised on those with a high marginal revenue benefit and reduced for those with a low value. This is exactly the process we can use to deduce the direction of reform.

From the marginal revenue benefit the economy of information can be clearly seen. All that is needed to evaluate MRB_k are the social marginal utilities, β^h, the individual commodity demands, x_k^h, and the aggregate derivatives of demand $\frac{\partial X_i}{\partial q_k}$ (or, equally, the aggregate demand elasticities). The demands and the elasticities are easily obtainable from data sets on consumer demands.

Table 15.1 displays the result of a calculation of the MRB_k using Irish data for ten commodity categories in 1987. Two different values of ε are given, with $\varepsilon = 5$ representing a greater concern for equity. The interpretation of these figures is that the

Table 15.1
Tax reform

Good	$\varepsilon = 2$	$\varepsilon = 5$
Other goods	2.316	4.349
Services	2.258	5.064
Petrol	1.785	3.763
Food	1.633	3.291
Alcohol	1.566	3.153
Transport and equipment	1.509	3.291
Fuel and power	1.379	2.221
Clothing and footwear	1.341	2.837
Durables	1.234	2.514
Tobacco	0.420	0.683

Source: Madden (1995).

tax levied on the goods toward the top of the table should be raised and the tax should be lowered on the goods at the bottom. Hence services should be more highly taxed and the tax on tobacco should be reduced! The rankings are fairly consistent for both values of ε; there is some movement, but no good moves very far. Therefore a reform based on these data would be fairly robust to changes in the concern for equity.

15.7.2 Optimality

The most developed implementation of the optimal tax rule for an economy with many consumers uses data from the Indian National Sample Survey. Defining θ to be the wage as a proportion of expenditure, a selection of these results are given in table 15.2 for $\varepsilon = 2$. The table shows that these tax rates achieve some redistribution, since cereals and milk products, both basic foodstuffs, are subsidized. Such redistribution results from the concern for equity embodied in a value of ε of 2. Interesting as they are, these results are limited, as are other similar analyses, by the degree of commodity aggregation that leads to the excessively general other nonfood category.

The same dataset has been used to analyze the redistributive impact of Indian commodity taxes. The redistributive impact is found by calculating the total payment of commodity tax, T^h, by consumer h relative to the expenditure, μ^h, of the consumer. The net gain from the tax system for h is then defined by $-\frac{T^h}{\mu^h}$. The consumer gains from the tax system if $-\frac{T^h}{\mu^h}$ is positive, since this implies that a net subsidy is being received. Contrasting the gain of a consumer from the existing tax system with the gain under the

Table 15.2
Optimal tax rates

Item	$\theta = 0.05$	$\theta = 0.1$
Cereals	−0.015	−0.089
Milk and milk products	−0.042	−0.011
Edible oils	0.359	0.342
Meat, fish, and eggs	0.071	0.083
Sugar and tea	0.013	0.003
Other food	0.226	0.231
Clothing	0.038	0.014
Fuel and light	0.038	0.014
Other nonfood	0.083	0.126

Source: Ray (1986a).

Table 15.3
Redistribution of Indian commodity taxes

	Rural	Urban
Expenditure level	$-\frac{T^h}{\mu^h}$	$-\frac{T^h}{\mu^h}$
Rs. 20	0.105	0.220
Rs. 50	0.004	0.037

Source: Ray (1986b).

optimal system provides an indication of both the success of the existing system and the potential gains from the optimal system. The calculations for the existing Indian tax system give the gains shown in table 15.3. The expenditure levels of Rs. 20 and Rs. 50 place consumers with these incomes in the lower 30 percent of the income distribution. The table shows a net gain to consumers at both income levels from the tax system, with the lower expenditure consumer making a proportionately greater gain.

The same calculations can be used to find the redistributive impact of the optimal tax system for a consumer with expenditure level $\mu = 0.5\overline{\mu}$, where $\overline{\mu}$ is mean expenditure, is given in table 15.4. For $\varepsilon = 1$ or more, it can be seen that the potential gains from the tax system, relative to the outcome that would occur in the absence of taxation, are substantial. This shows that with sufficient weight given to equity considerations, the optimal set of commodity taxes can effect significant redistribution and that the existing Indian tax system does not attain these gains.

Table 15.4
Optimal redistribution

	$\nu = 0.1$	$\nu = 1.5$	$\nu = 5$
$-\frac{T}{\mu}$	0.07	0.343	0.447

Source: Ray (1986b).

This section has discussed a method for calculating the taxes implied by the optimal tax rule. The only difficulty in doing this is the specification of the social welfare weights. To determine these, it is necessary to know both the private utility functions and the social welfare function. In the absence of this information, a method for deriving the weights is employed that can embody equity criteria in a flexible way. Although these weights are easily calculated, they are not entirely consistent with the other components of the model. The numbers derived demonstrate clearly that when equity is embodied in the optimization, commodity taxes can secure a significant degree of redistribution. This is very much in contrast to what occurs with efficiency alone.

15.8 Efficient Taxation

The tax rules in the previous section have only considered the competitive case. When there is imperfect competition, additional issues have to be taken into account. The basic fact is that imperfectly competitive firms produce less than the efficient output level, so the equilibrium without intervention is not Pareto-efficient. This gives a reason to use commodity taxes to subsidize the output of imperfectly competitive firms relative to that of competitive firms. However, the strength of this argument depends on the degree of tax-shifting, as identified in chapter 9.

The issues involved in tax design can be understood by determining the direction of welfare-improving tax reform starting from an initial position with no commodity taxation. This is undertaken for an economy with a single consumer and a zero-revenue requirement. The fact that no revenue is raised implies that the taxes are used merely to correct for the distortion introduced by the imperfect competition. There are two consumption goods, each produced using labor alone. Good 1 is produced with constant returns to scale by a competitive industry with after-tax price $q_1 = p_1 + t_1$. There is a single household in the economy whose (indirect) utility function is

$$U = U\left(x_0\left(q_1, q_2\right), x_1\left(q_1, q_2\right), x_2\left(q_1, q_2\right)\right). \tag{15.32}$$

Tax revenue, R, is defined by

$$R = t_1 x_1 + t_2 x_2. \tag{15.33}$$

Good 2 is produced by a monopolist who chooses its output to maximize profit

$$\pi_2 = [q_2 - c - t_2]\, x_2\,(q_1, q_2)\,, \tag{15.34}$$

where c is the constant marginal cost. The profit-maximizing price depends on the tax, t_2, and the price of good 1, q_1. This relationship is denoted $q_2 = q_2\,(q_1, t_2)$. The derivative $\frac{\partial q_2}{\partial t_2}$ measures the rate of shifting of the tax. In the terminology of chapter 9, there is undershifting if $\frac{\partial q_2}{\partial t_2} < 1$ and overshifting if $\frac{\partial q_2}{\partial t_2} > 1$. The dependence of the demand for good 2 on the price of good 1 is reflected in the profit-maximizing price. The derivative $\frac{\partial q_2}{\partial q_1}$ is the cross-price effect of taxation. It can be positive or negative, and since $q_1 = p_1 + t_1$, it follows that $\frac{\partial q_2}{\partial q_1} = \frac{\partial q_2}{\partial t_1}$.

The tax reform problem searches for a pair of tax changes that raises welfare while collecting zero revenue. The initial position is taken to be one where both commodity taxes are zero initially, so the intention is to find a pair of tax changes dt_1, dt_2, starting from an initial position with $t_1 = t_2 = 0$, such that $dU > 0$ and $dR = 0$. The formulation ensures that one of the taxes will be negative and the other positive. The aim is to provide a simple characterization of the determination of the relative rates. It should be noted that if both industries were competitive the initial equilibrium would be Pareto-efficient and the solution to the tax problem would have $dt_1 = dt_2 = 0$. So nonzero tax rates will be a consequence of the distortion caused by the imperfect competition.

Totally differentiating the utility function and using the first-order conditions for consumer choice, the effect of the tax changes on utility is

$$dU = -\alpha x_1 \frac{\partial q_1}{\partial t_1} dt_1 - \alpha x_2 \frac{\partial q_2}{\partial t_1} dt_1 - \alpha x_2 \frac{\partial q_2}{\partial t_2} dt_2, \tag{15.35}$$

where α is the consumer's marginal utility of income. Totally differentiating the revenue constraint and using the fact that the initial values of the taxes are $t_1 = t_2 = 0$ gives

$$dR = 0 = x_1 dt_1 + x_2 dt_2, \tag{15.36}$$

Solving (15.36) for dt_1 and substituting into the derivative of utility determines the utility change as dependent on dt_2 alone:

$$dU = \left[-\alpha x_2 \frac{\partial q_2}{\partial t_2} + \alpha x_2 + \alpha \frac{x_2^2}{x_1} \frac{\partial q_2}{\partial t_1} \right] dt_2. \tag{15.37}$$

It is condition (15.37) that provides the key to understanding the determination of the relative tax rates. Since we wish to choose the tax change dt_2 to ensure that $dU > 0$, it follows that the sign of the tax change must be the same as the bracketed term in (15.37). From this observation follows the conclusion that

$$x_1\left[1 - \frac{\partial q_2}{\partial t_2}\right] + x_2 \frac{\partial q_2}{\partial t_1} < 0 \Rightarrow dt_2 < 0. \tag{15.38}$$

From (15.38) the output of the imperfectly competitive industry should be subsidized and the competitive industry taxed when $\frac{\partial q_2}{\partial t_2}$ is large, so that overshifting is occurring and $\frac{\partial q_2}{\partial t_1}$ is negative. These are, of course, sufficient conditions. In general, the greater the degree of tax shifting, the more likely is subsidization. The explanation for this result is that if firms overshift taxes, they will also do the same for any subsidy. Hence a negative dt_2 will be reflected by an even greater reduction in price. If $\frac{\partial q_2}{\partial t_1}$ is also negative, the tax on the competitive industry secures a further reduction in the price of good 2.

The conclusion of this analysis is that the rate of tax-shifting is important in the determination of relative rates of taxation. Although the economy is simplified, it does demonstrate that with imperfect competition commodity taxation can be motivated on efficiency grounds alone to mitigate the inefficiency cost of market power.

15.9 Public Sector Pricing

The theory that was developed in the previous sections also has a second application. This arises because there are close connections between the theory of commodity taxation and that of choosing optimal public sector prices. Firms operated by the public sector can be set the objective of choosing their pricing policy to maximize social welfare subject to a revenue target. If the firms have increasing returns to scale, which is often the reason they are operated by the public sector, then marginal cost pricing will lead to a deficit (because marginal cost is below the decreasing average cost). The government will then want to find the optimal deviation from marginal-cost pricing that ensures that the firms break even.

For both commodity taxation and public sector pricing, the government is choosing the set of consumer prices that maximizes welfare subject to a revenue constraint. Under the commodity taxation interpretation, these prices are achieved by setting the level of tax to be included in each consumer price, whereas with public sector pricing, the prices are chosen directly. However, the choice of tax rate is equivalent to the choice of consumer price.

In the context of public sector pricing, the optimal prices are generally known as *Ramsey prices*. The constraint on the optimization with commodity taxation requires the raising of a specified level of revenue. With public sector pricing this can be reinterpreted as the need to raise a given level of revenue in excess of marginal cost. The tax rates of the commodity taxation problem then translate into the markup over marginal cost in the public sector pricing interpretation. The rules for optimal taxation derived above then characterize the public sector prices.

15.10 Conclusions

This chapter has reviewed the determination of optimal commodity taxes. It has been shown how an efficient system places the burden of taxation primarily on necessities. If implemented, such a system would be very damaging to low-income consumers. When equity is introduced, this outcome is modified to reduce the extent to which goods consumed primarily by those with low incomes are affected by the tax system. These interpretations were borne out by the numerical calculations.

As well as providing these insights into the structure of taxes, the chapter has also been shown that the optimal tax system should ensure production efficiency. The implication of this finding is that there should be no taxes on intermediate goods. This is a very strong and clear prediction. It is also a property that actual value-added tax systems satisfy.

Further Reading

The theory of optimal commodity taxation was given its modern form in:

Diamond, P. A., and Mirrlees, J. A. 1971. Optimal taxation and public production 1: Production efficiency and 2: Tax rules. *American Economic Review* 61: 8–27 and 261–78.

A simplified version of the optimal tax rule for a many-consumer economy is developed in:

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Madden, D. 1995. An analysis of indirect tax reform in Ireland in the 1980s. *Fiscal Studies* 16: 18–37.

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The extension of efficient taxation to imperfect competition is described in:

Myles, G. D. 1987. Tax design in the presence of imperfect competition: An example. *Journal of Public Economics* 34: 367–78.

Public sector pricing is described in:

Dréze, J. H. 1964. Some postwar contributions of French economists to theory and public policy with special emphasis on problems of resource allocation. *American Economic Review* 54: 1–64.

Exercises

15.1 For the linear demand function $x = a - bp$, calculate the deadweight loss of introducing a commodity tax t when the marginal cost of production is constant at c. How is the deadweight loss affected by changes in a and b? How does a change in b affect the elasticity of demand at the equilibrium without taxation?

15.2 A good is traded in a competitive market. The demand function is given by $X = 75 - 5P$ and supply is perfectly elastic at the price $P = 10$.

a. A specific tax of value $t = 2$ is introduced. Determine the tax incidence.

b. An ad valorem tax at a rate of $t = 0.2$ is introduced. Determine the tax incidence.

c. How do the incidence of the specific tax and the ad valorem tax differ if supply is given by $Y = 2.5P$?

15.3 Assume that the demand function is given by $x = p^{-\varepsilon_d}$ and the supply function by $y = p^{\varepsilon_s}$. Find the equilibrium price. What is the effect on the equilibrium price of the introduction of a tax $t = \frac{1}{10}$ if $\varepsilon_d = p^{\varepsilon_s} = \frac{1}{2}$? Describe how the incidence of the tax is divided between consumers and suppliers.

15.4 The analysis of taxation in the single-consumer economy used labor as an untaxed numéraire. Show that the optimal allocation with commodity taxation is unchanged when the consumption good becomes the untaxed numéraire. Then establish that it does not matter which good is the numéraire and which is taxed.

15.5 The value-added tax system requires that all goods to be sold at a price that includes tax. Any firm purchasing a good to use as an input can reclaim the tax it has paid. Assess this tax structure using the Diamond-Mirrlees Production Efficiency Lemma.

15.6 Consider an economy with a single consumer whose preferences are given by $U = \log(x) - \ell$, where x is consumption and ℓ labor supply. Assume that the consumption good is produced using labor alone with a constant-returns-to-scale technology. Units of measurement are chosen so that the producer prices of both the consumption good and the wage rate are equal to 1.

a. Let the consumer's budget constraint be $qx = \ell$, where the consumer price is $q = 1 + t$, and t is the commodity tax. By maximizing utility, find the demand function and the labor supply function.

b. Assume the revenue requirement of the government is $\frac{1}{10}$ of a unit of labor. Draw the production possibilities for the economy and the consumer's offer curve.

c. By using the offer curve and the production possibilities, show that the optimal allocation with commodity taxation has $x = \frac{9}{10}$ and $\ell = 1$.

d. Calculate the optimal commodity tax.

e. By deriving the first-best allocation, show that the commodity tax optimum is second-best.

15.7 Two consumers A and B have an income of \$30,000 and \$100,000 respectively. A and B consume the same bundle of goods with a cost (including tax) of \$24,000. The only tax in the economy is a commodity tax levied uniformly on all goods at a rate of 20 percent.

a. What proportion of income is paid in tax by A and B?

b. What implications does such a tax have in terms of equity?

c. Is there any way the commodity tax can be restructured to improve its equity properties?

15.8 For an economy with one consumption good that is produced using only labor, show that at the optimal allocation with commodity taxation tax revenue, tx, is equal to the government use of labor, R.

15.9 Assume that the production technology is such that each unit of output requires one unit of labor and that the government has a revenue requirement of one unit of labor. Also assume that there is a single consumer.

a. Using a diagram, describe how the optimal tax on the consumption good is determined. Now assume that the consumer has preferences given by $U = \log(x) + \log(10 + \ell)$, where x is consumption and ℓ is labor supply (recall that ℓ is a supply, so is a negative number).

b. By maximizing utility subject to the budget constraint $qx + w\ell = 0$, construct the consumer's offer curve.

c. Treating the equations of the production frontier and the offer curve as a simultaneous system, determine the optimal tax rate.

15.10 An economy has a single consumption good produced using labor and a single consumer. The production process has decreasing returns to scale. Explain the derivation of the optimal commodity tax when profit is not taxed.

15.11 Consider the utility function $U = \alpha \log x_1 + \beta \log x_2 - \ell$ and budget constraint $w\ell = q_1x_1 + q_2x_2$.

a. Show that the price elasticity of demand for both commodities is equal to -1.

b. Setting producer prices at $p_1 = p_2 = 1$, show that the inverse elasticity rule implies $\frac{t_1}{t_2} = \frac{q_1}{q_2}$.

c. Letting $w = 100$ and $\alpha + \beta = 1$, calculate the tax rates required to achieve revenue of $R = 10$.

15.12 Let the consumer have the utility function $U = x_1^{\rho_1} + x_2^{\rho_2} - l$.

a. Show that the utility maximizing demands are $x_1 = \left[\frac{\rho_1 w}{q_1}\right]^{1/(1-\rho_1)}$ and $x_2 = \left[\frac{\rho_2 w}{q_2}\right]^{1/(1-\rho 2)}$.

b. Letting $p_1 = p_2 = 1$, use the inverse elasticity rule to show that the optimal tax rates are related by $\frac{1}{t_2} = \left[\frac{\rho_2-\rho_1}{1-\rho_2}\right] + \left[\frac{1-\rho_1}{1-\rho_2}\right]\frac{1}{t_1}$.

c. Setting $w = 100$, $\rho_1 = 0.75$, and $\rho_2 = 0.5$, find the tax rates required to achieve revenue of $R = 0.5$ and $R = 10$.

d. Calculate the proportional reduction in demand for the two goods comparing the no-tax position with the position after imposition of the optimal taxes for both revenue levels. Comment on the results.

15.13 "If all commodities are taxed at the same rate, the distortion in prices is minimized." Explain why this statement does not act as a guide for setting optimal commodity taxes.

15.14 Consider an economy with a single consumer whose preferences are given by $U = \log(x_1) + \log(x_2) + \ell$, where x_1 and x_2 are the consumption levels of goods 1 and 2 and ℓ is leisure. Assume that both goods are produced using labor alone, subject to a constant-returns-to-scale technology. Units of measurement are chosen so that the producer prices of both goods and the wage rate are equal to 1.

a. Using L to denote the consumer's endowment of time, explain the budget constraint

$$q_1 x_1 + q_2 x_2 + w\ell = wL.$$

b. Show that the consumer's demands satisfy the conditions required for the inverse elasticity rule to apply.

c. Use the inverse elasticity rule to conclude that both goods should be subject to the same level of tax.

d. Calculate the tax required to obtain a level of revenue of $R = 1$.

e. Show that the commodity taxes are second-best.

15.15 Show how the impact of a commodity tax upon a consumer's optimal demand can be separated into an "income effect" and a "substitution effect."

15.16 In the absence of taxation a consumer has the budget constraint $p_1 x_1 + p_2 x_2 - w\ell = 0$. Show that an ad valorem tax levied at rate t on both commodities and on labor raises no revenue. Explain this fact.

15.17 (Ramsey rule) Consider a three-good economy ($k = 1, 2, 3$) in which every consumer has preferences represented by the utility function $U = x_1 + g(x_2) + h(x_3)$, where the functions $g(\cdot)$ and $h(\cdot)$ are increasing and strictly concave. Suppose that each good is produced with constant returns to scale from good 1, using one unit of good 1 per unit of good $k \neq 1$. Let good 1 be the numéraire, and normalize the price of good 1 to equal 1. Let t_k denote the (specific) commodity tax on good k so that the consumer price is $q_k = (1 + t_k)$.

a. Consider two commodity tax schemes $t = (t_1, t_2, t_3)$ and $t' = (t_1', t_2', t_3'))$. Show that if $1 + t_k' = \phi[1 + t_k]$ for $k = 1, 2, 3$ for some scalar $\phi > 0$, then the two tax schemes raise the same amount of tax revenue.

b. Argue from part a that the government can without cost restrict tax schemes to leave one good untaxed.

c. Set $t_1 = 0$, and suppose that the government must raise revenue of R. What are the tax rates on goods 2 and 3 that minimize the welfare loss from taxation?

d. Show that the optimal taxes are inversely proportional to the elasticity of the demand for each good. Discuss this tax rule.

e. When should both goods be taxed equally? Which good should be taxed more?

15.18 Consider a three-good economy ($k = 1, 2, 3$) in which every consumer has preferences represented by the utility function $U = x_1 + g(x_2, x_3)$, where the function $g(\cdot)$ is increasing and strictly concave. Suppose that each good is produced with constant returns to scale from good 1, using one unit of good 1 per unit of good $k \neq 1$. Let good 1 be the numéraire and normalize the price of good 1 to equal 1. Let t_k denote the (specific) commodity tax on good k so that the consumer price is $q_k = 1 + t_k$. Suppose that a tax change is restricted to only good 2 so that $t_2' = t_2 + \Delta$ with $\Delta > 0$.

a. What is the correct measure of the welfare loss arising from this tax increase if $t_3 = 0$?

b. Show that if $t_3 > 0$, then the measure of welfare loss in part a overestimates the welfare loss if good 3 is a substitute for good 2. What is then the correct measure of the welfare change?

c. Show that if $t_3 > 0$, then the measure of welfare loss in part a underestimates the welfare loss if good 3 is a complement for good 2. What is the correct welfare change?

d. Show that if good 3 is subsidized, $t_3 < 0$, then the the measure of welfare loss in part a underestimates the welfare loss if good 3 is a substitute for good 2. How can you explain this result?

e. Show that if good 3 is subsidized, $t_3 < 0$, then the the measure of welfare loss in part a overestimates the welfare loss if good 3 is a complement for good 2.

15.19 The purpose of this exercise is to contrast the incidence of a commodity tax under different market structures. Consider an economy with identical households and identical firms. The representative household receives labor income for its labor supply ℓ and profit income π for its ownership of the firm. The utility function of the household is $U = 2\sqrt{x} - \ell$. The firm produces one unit of final consumption good x with one unit of labor input. Labor is the numéraire good: the price of labor is normalized to 1, and labor is the untaxed good. The producer price is p and the consumer price is $q = p + t$, where $t > 0$ is the (specific) commodity tax.

a. Describe the household's optimization program treating profit income and the consumer prices in the budget constraint as fixed. Find the demand for good x as a function of consumer price q.

b. Calculate the elasticity of the slope of the inverse demand function.

c. Suppose that the firms act in unison like a monopolist. Find the supply of the monopoly as a function of t.

d. What is the equilibrium price charged by the monopolist? What is the producer price? What is the division of the tax burden between the producer and the consumer?

e. Suppose that the firms act independently maximizing their own profit-taking prices as given. What is the equilibrium producer price? What is the division of the tax burden between producer and consumer? Compare with the result in part d.

15.20 Consider an economy with two representative households ($h = 1, 2$) that supply labor ℓ^h to the one representative firm and buy a consumption good x^h. Labor supply is inelastic (with $\ell^1 = 4$ and $\ell^2 = 2$) and perfectly substitutable in production. There is no disutility of labor. The utility function is $U = x^h$, and the firm produces one unit of x with one unit of labor. Labor is the numéraire good with its price normalized to 1. The producer price of x is p. The government can levy individualized commodity tax t^h on good x. Thus the consumer price facing household h is $q^h = p + t^h$. There is no revenue requirement so $R = t^1x^1 + t^2x^2 = 0$.

a. What is the equilibrium producer price?

b. What is the demand for good x as a function of the tax rate for each household?

c. Use the demand function to express the utility of each household as a function of the price of the consumption good.

d. Show that government budget balance implies that the taxes are related by $t^2 = -\frac{2t^1}{3t^1+1}$.

e. Use the budget balance condition in part d to find the tax rates maximizing the Rawlsian social welfare function $W = \min\{U^1, U^2\}$.

f. Why individualized commodity taxes are not used in practice?

15.21 Are the following statements true or false?

a. The theory of optimal commodity taxation argues that equal tax rates should be set across all commodities so as to maximize efficiency by "smoothing taxes."

b. In the United States prescription drugs and CDs are taxed at the same rate of 10 percent. The Ramsey rule suggests that this is the optimal tax policy.

c. Some economists have proposed replacing the income tax with a consumption tax to avoid taxing savings twice. This is a good policy both in terms of efficiency and equity.

15.22 Consider two consumers with preferences

$$U^1 = \alpha \log(x_1^1) + (1 - \alpha)\log(x_2^1),$$

$$U^2 = \beta \log(x_1^2) + (1 - \beta)\log(x_2^2),$$

and incomes $M^1 < M^2$. What is the maximum amount of redistribution that can be obtained by levying commodity taxes on goods 1 and 2? Why is it zero if $\alpha = \beta$?

15.23 A public sector firm has cost function $c(x) = F + cx$. It faces the inverse demand curve $p = a - bx$.

a. What is the first-best price for the firm to charge? What quantity is sold? Does the firm make a loss at this price?

b. What is the break-even price for the firm? What quantity is sold?

c. Compare the difference in consumer surplus between the outcomes in parts a and b. Is there scope for the government to subsidise the firm when it sets the first-best price?