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# LECTURE 9

## TAX POLICY

### Optimal Income Taxation: Part II

The hardest thing in the world to understand is the ***income tax***. ...

The hardest thing in the world to understand is the *income tax*. ...

**Albert Einstein**

# A Model of Income Taxation

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- The optimal income tax trades off efficiency and equity to maximise welfare
- A model that can provide an interesting analysis of this question must have the following attributes:
  - there must be an unequal distribution of income in order for there to be equity motivations for taxation
  - the income tax must affect the labour supply decisions of the consumers so that it has efficiency effects
  - there must be no prior restrictions placed upon the optimal tax function
- The “Mirrlees model” of income taxation is the simplest that has these attributes

# The Optimal (Income) Tax Problem

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Choose a tax schedule

- to raise revenue required for government spending
- to redistribute to low-income individuals
- to minimise distortions to the economy (maximise a SWF)

Combining equity and efficiency considerations:  
willingness to distort will depend on desire for redistribution  
(or public spending)

# A Model of Income Taxation

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- All consumers have identical preferences but differ in their level of skill in employment
- The hourly wage received by each consumer is determined by the level of skill
- Income is the product of skill and hours worked
- The level of skill is private information and cannot be observed by the government
  - this makes it impossible to tax it directly.
  - a tax levied on skill would be the first-best policy but this not feasible.
- The government employs an income tax as a second-best policy

# A Model of Income Taxation

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- The government is subject to two constraints when it chooses the tax function
  - the first constraint is that the income tax must achieve the government's revenue requirement
  - the second constraint is that the tax function must be incentive compatible
- To understand incentive compatibility view the government as assigning to each consumer an allocation of labor and consumption
- Incentive compatibility requires that each consumer must find it utility maximizing to choose the allocation the government intends for them rather than an allocation assigned to a different consumer

# The fundamental problem

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- The objective is to tax high income individuals to finance subsidy to low income individuals.
- Income potential is related to ability
  - High income earners are “very able”
  - Low income earners are “less able”
- Ability is private information and you don't want to reveal through your choices that you are really able if it means paying very high taxes!



# Three issues to discuss

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- The tax schedule and its relationship to the consumption function.
- Ability and preferences over gross income and consumption.
- Incentive compatible tax schedules.

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# The tax schedule and the consumption function

# The tax schedule and the consumption function

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- Denote a consumer's labour supply by  $\ell$  and consumption by  $x$
- If a consumer of skill level  $s$  supplies  $\ell$  hours of labour they earn income of  $s\ell$  before tax
- Denote the income of a consumer with skill  $s$  by  $z(s)$
- For a consumer with income  $z$  the income tax paid is given by  $T(z)$
- $T(z)$  is the tax function the analysis aims to determine
- The **tax schedule** shows the relationship between gross income and the total tax payment:  $T(z)$ .

# The tax schedule and the consumption function

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A consumer who earns income  $z(s)$  can consume

$$x(s) = c(z(s)) = z(s) - T(z(s))$$

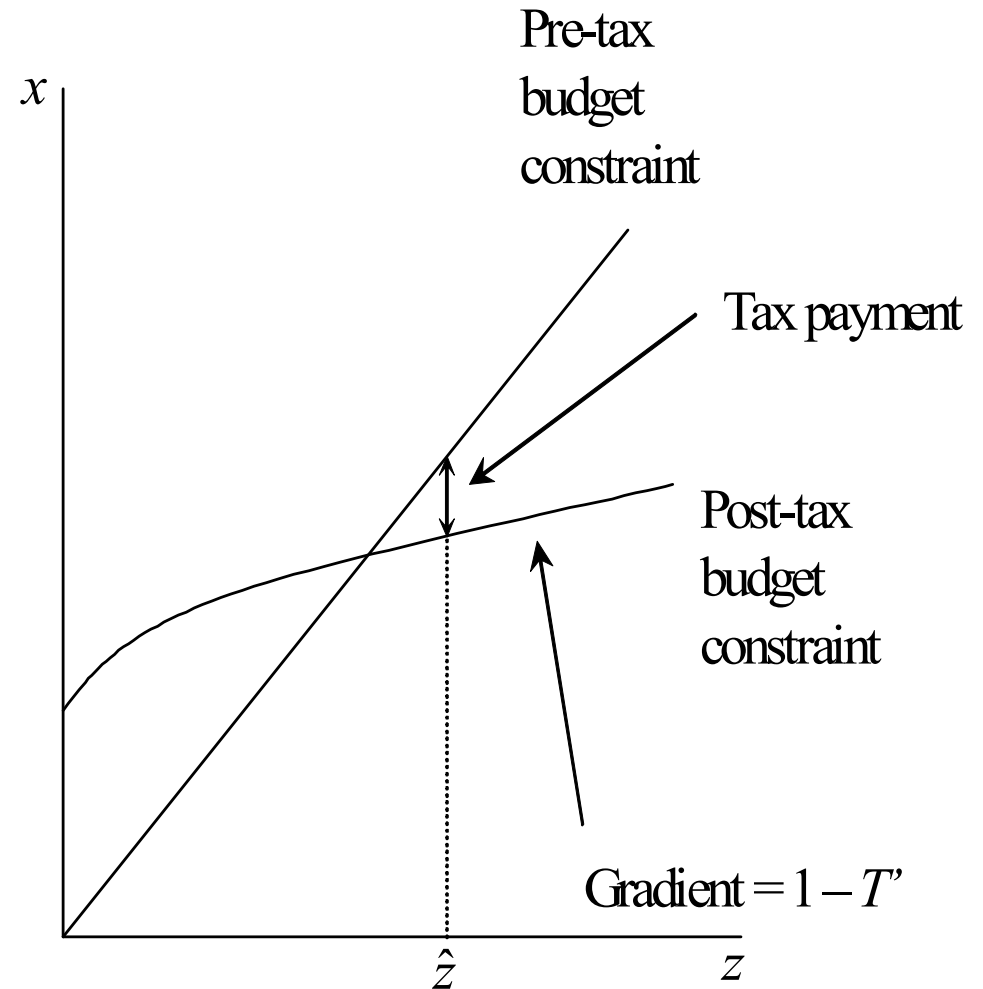
The **consumption function** shows the relationship between gross income and consumption:  $c(z) = z - T(z)$

# A Model of Income Taxation

- Without taxation income the budget constraint is the 45° line
- Where the consumption function lies above the line the tax payment is negative
- It is positive when the consumption function is below the line
- E.g. a consumer with income  $z$  that pays positive tax
- The gradient of the consumption function is equal to 1 minus the marginal rate of tax

$$x = c(z) = z - T(z)$$

$$\frac{\partial c}{\partial z} = 1 - \frac{\partial T}{\partial z}$$



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*Ability and preferences  
over gross income and  
consumption*

# A Model of Income Taxation

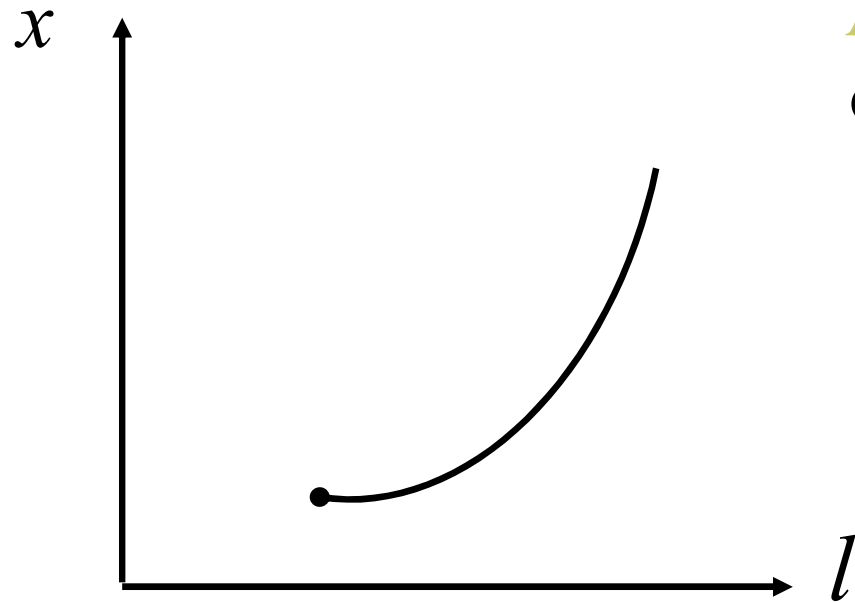
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- A consumer of ability  $s$  chooses  $x$  and  $z$  to

$$\max u\left(x, \frac{z}{s}\right) \text{ subject to } x = c(z) = z - T(z)$$

- A structure is placed on this problem by restricting preferences
- All consumers have the same utility function (the possibility of workers displaying different aversion to work is ruled out).
- Agent monotonicity relates the gradient of the indifference curves for consumers of different abilities through each consumption-income point
  - at any point in  $(z, x)$  space the indifference curve of a consumer of skill  $s_1$  passing through that point is steeper than the curve of a consumer of skill  $s_2$  if  $s_2 > s_1$
  - this makes consumers of lower skill less willing to supply labor

# Individual Preferences



**Assumption I: Identical preferences over consumption ( $x$ ) and labour ( $l$ )**

$$u(x, z / s)$$

**Pre - tax (gross) income :  $z(s) = sl(s)$**

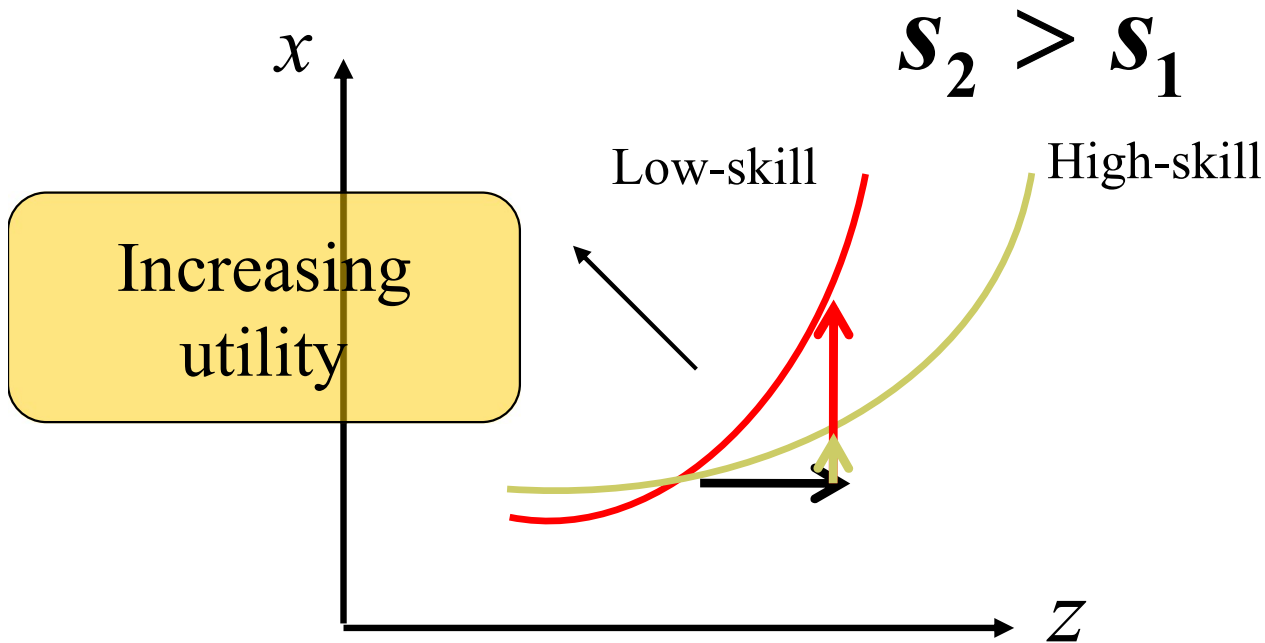
Pre-tax (gross) wage reflects ability:  $w_i = s_i \hat{w}$  (set  $\hat{w} = 1$ )

So, utility of person  $i$  can be written,  $u(x, \frac{z}{s_i})$



**Identical preferences over consumption and labour, but different abilities.**

➔ **Different preferences over consumption and gross income.**



Slope of indifference curve is  $MRS_{xz}^i = \frac{u_z}{s_i u_x}$

Flatter for high ability individuals than for low ability individuals.

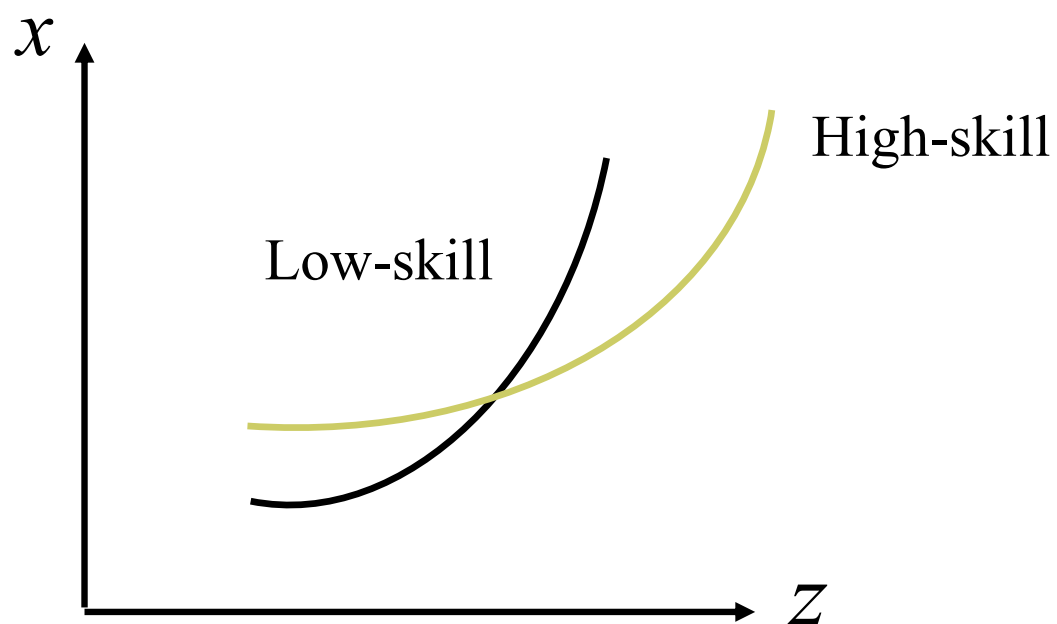
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# Incentive Compatibility or Self-selection

Ability is private information.

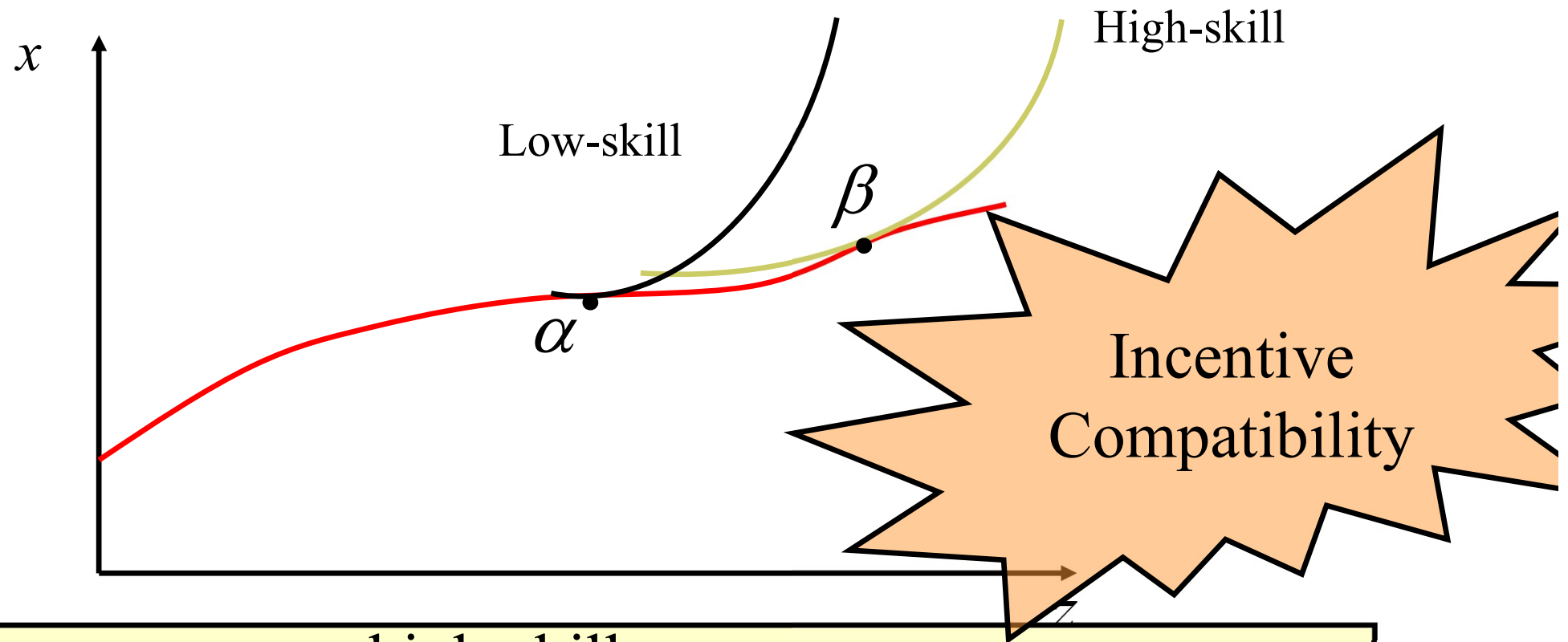
# Single-Crossing Property

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Indifference curves of high-skill individual will cross the indifference curves of low-skill individual *once*

# Self-Selection I

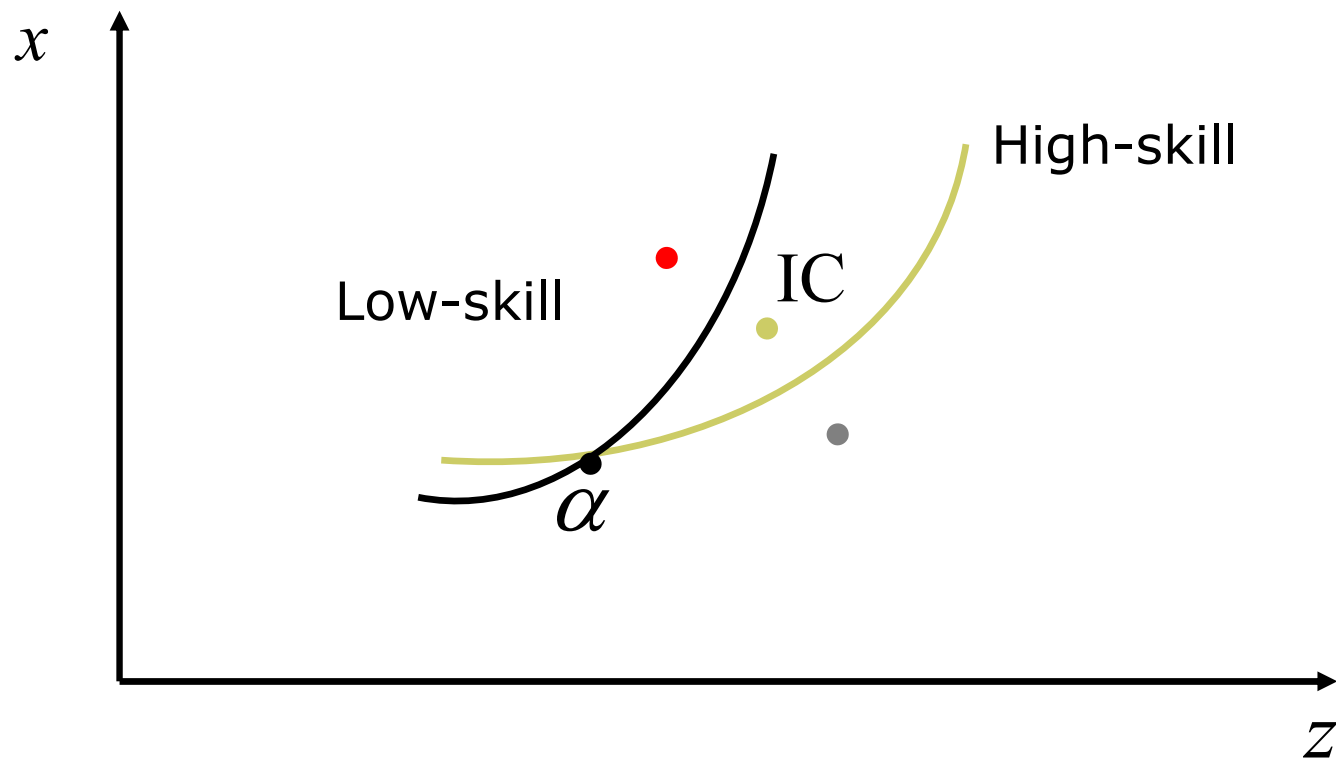


high-skill consumers  
will never earn less income than low-skill.

*IC Tax Schedule*

It must be in the individuals' interests to choose the  
income-consumption pair that the government  
intends.

# Self-Selection II

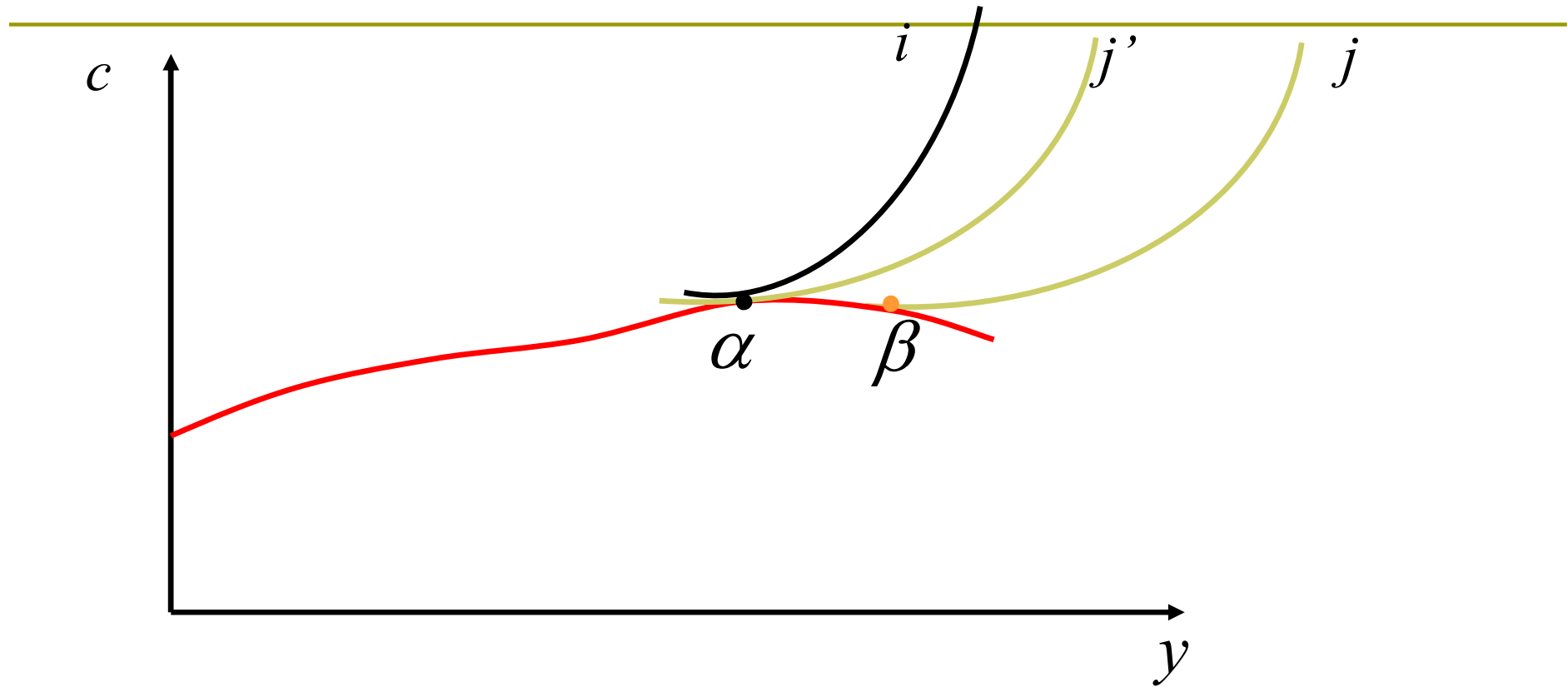


## *IC Tax Schedule*

*Intended*  $z$  and  $x$  must increase with ability.

If not: better to pretend to be low ability.

# The consumption function must be upwards sloping



... person  $j$  would prefer to be like person  $i$ .



$\beta$  is not incentive compatible

# Summary so far

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- Can analyse tax as mapping from gross income to consumption
  - consumption function  $\Leftrightarrow$  tax schedule.
- Identical preferences over consumption and leisure (work), but different preferences over consumption and gross income because of ability differences.
- Incentive compatibility tax schedules:
  - IC consumption function must be **increasing** in gross income.

# What is next?

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- Choosing marginal tax rates
- Shape of the optimal tax schedule



# Three general properties of the optimal income tax schedule

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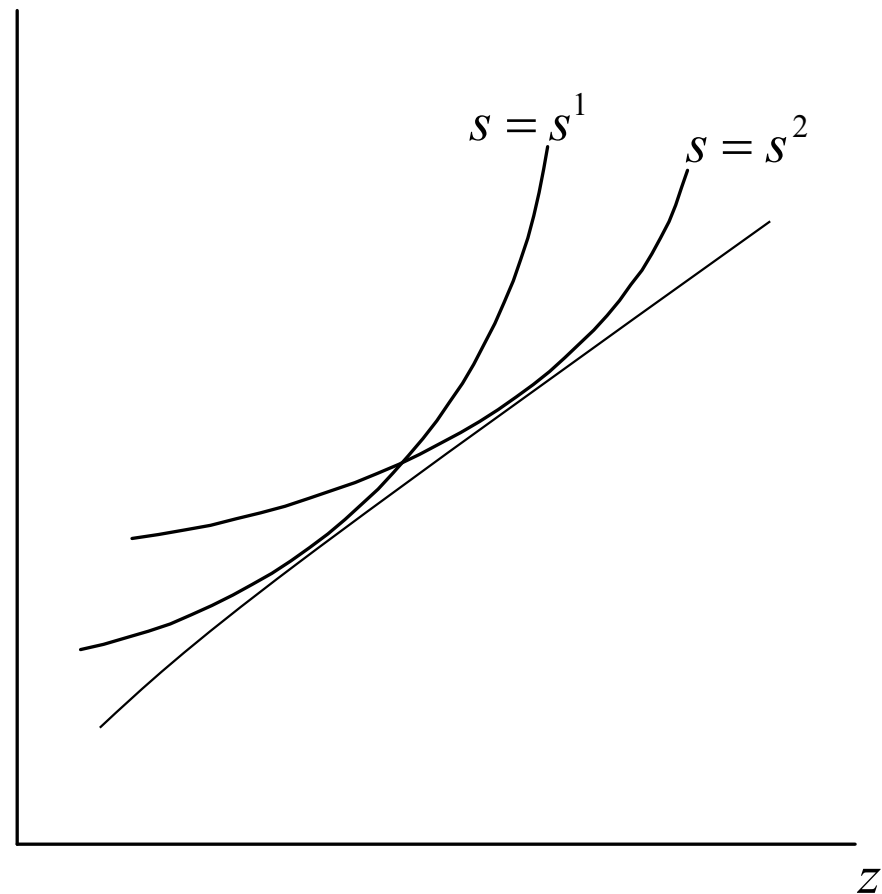
1. Marginal tax rate is less than 100 percent.
2. Marginal tax rate is non-negative.
3. Marginal tax rate is zero for the most able individual.

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Result 1:  
Optimal marginal tax rate is less than 100  
percent

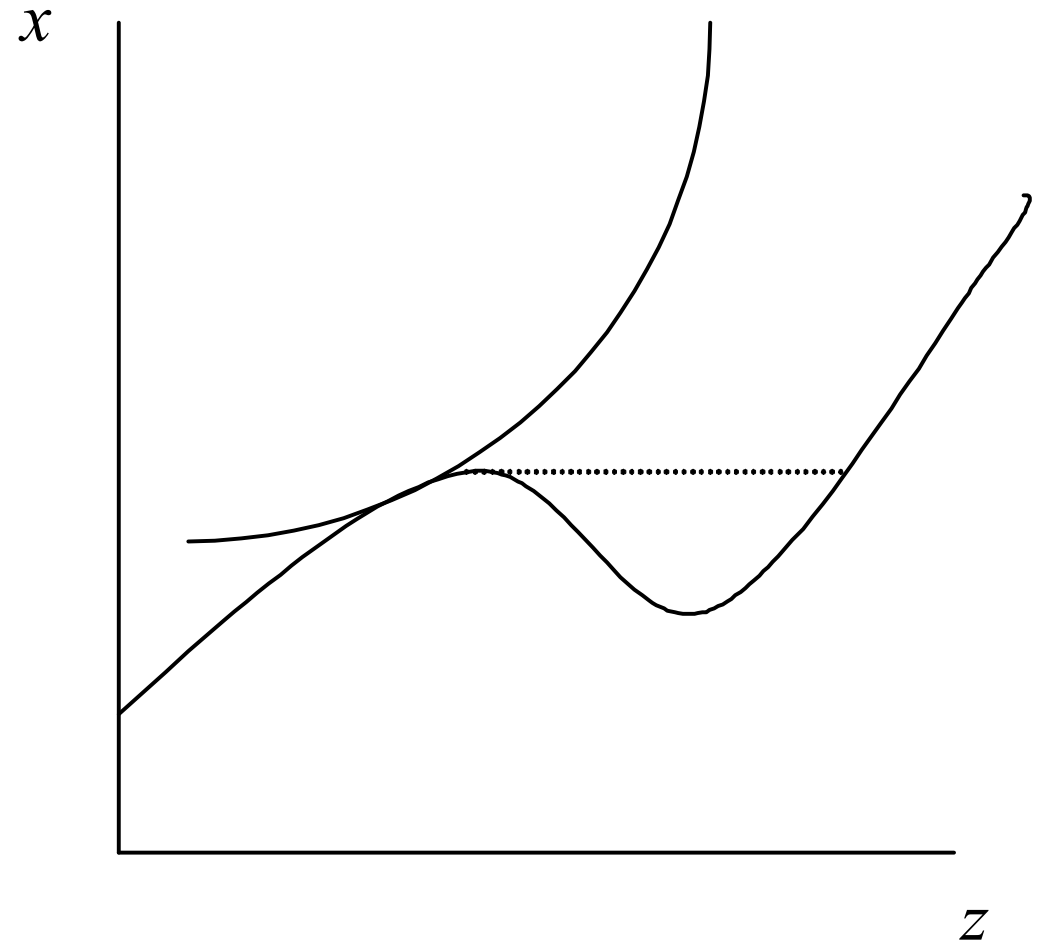
# Result I: Marginal Tax rate $< 100\%$

- The first result follows from agent  $x$  monotonicity: high-skill consumers will never earn less income than low skill
- At the point where the indifference curve of the low-skill consumer is tangential to the consumption function that of the high skill is flatter and so cannot be at a tangency
- The choice for the high skill must then be further to the right
- Income is increasing with skill



# Result I: Marginal Tax rate $< 100\%$

- ❑ This result relates to the maximum tax rate that will be charged
- ❑ If the consumption function slopes downward, the shape of the indifference curves ensures that no consumer will choose to locate on the downward sloping section
- ❑ Economically, along the downward-sloping section increased work-effort is met with lower consumption
- ❑ This part of the consumption function is therefore redundant and could be replaced by the flat dashed section
- ❑ This shows  $c'(z) > 0$  so  $1 - T'(z) > 0$  and the marginal tax rate is less than 100 per cent.

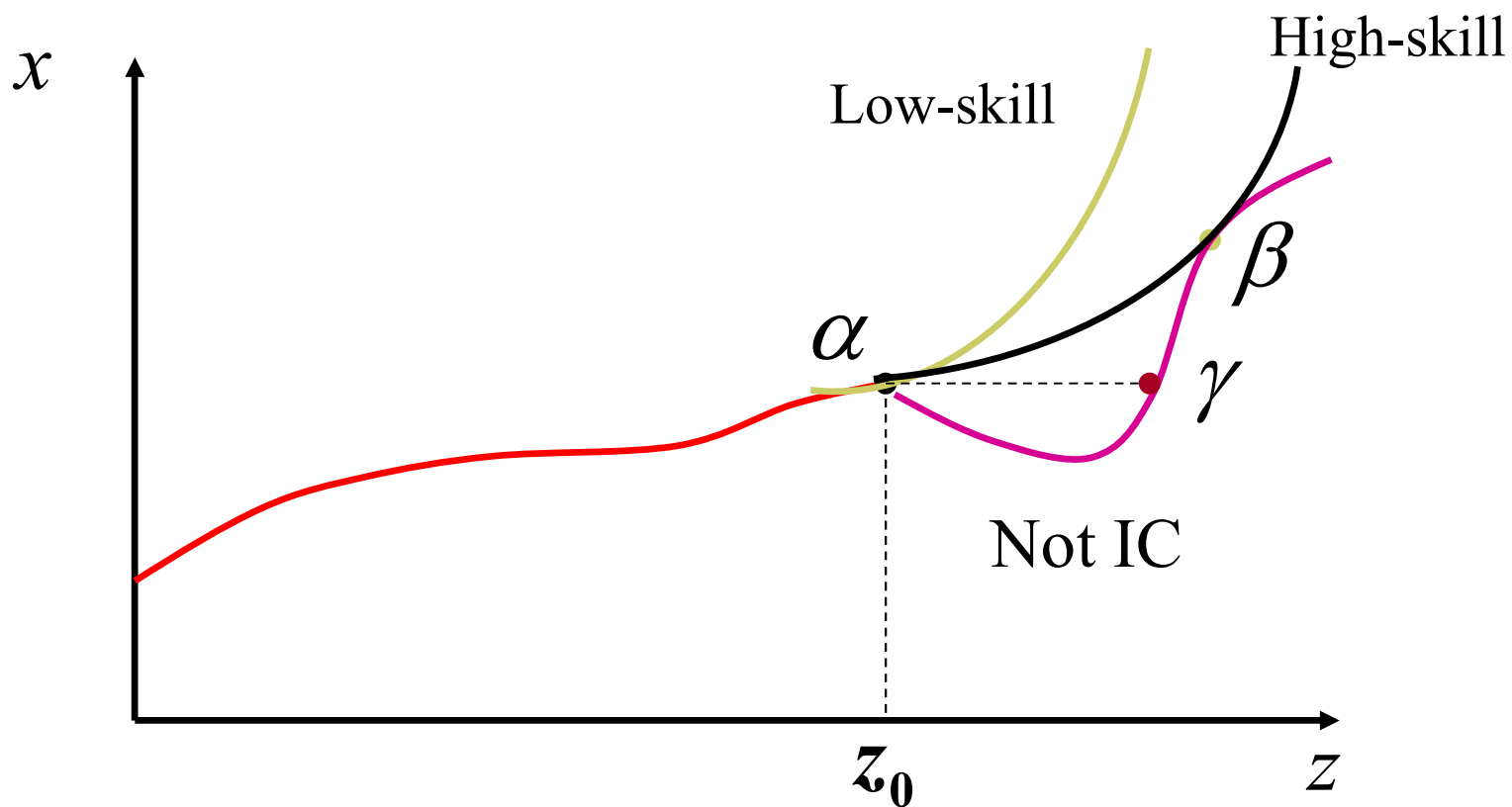


# Result I: Marginal Tax rate < 100%

$$c(z) = z - T(z)$$

$$\frac{\partial c(z)}{\partial z} = 1 - \frac{\partial T(z)}{\partial z} > 0$$

SO  $\frac{\partial T(z)}{\partial z} < 1$



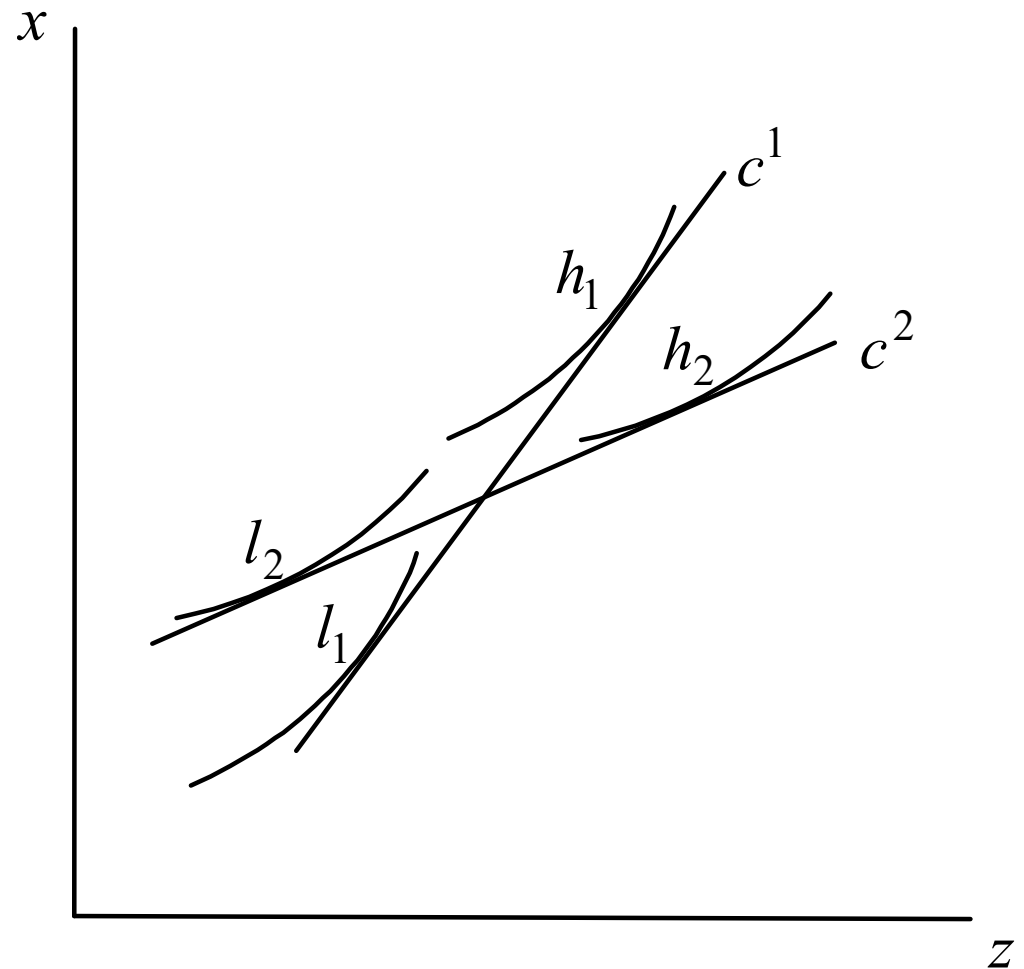
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## Result 2: Optimal marginal Tax rate $\geq 0\%$

. but the average tax rate can be negative

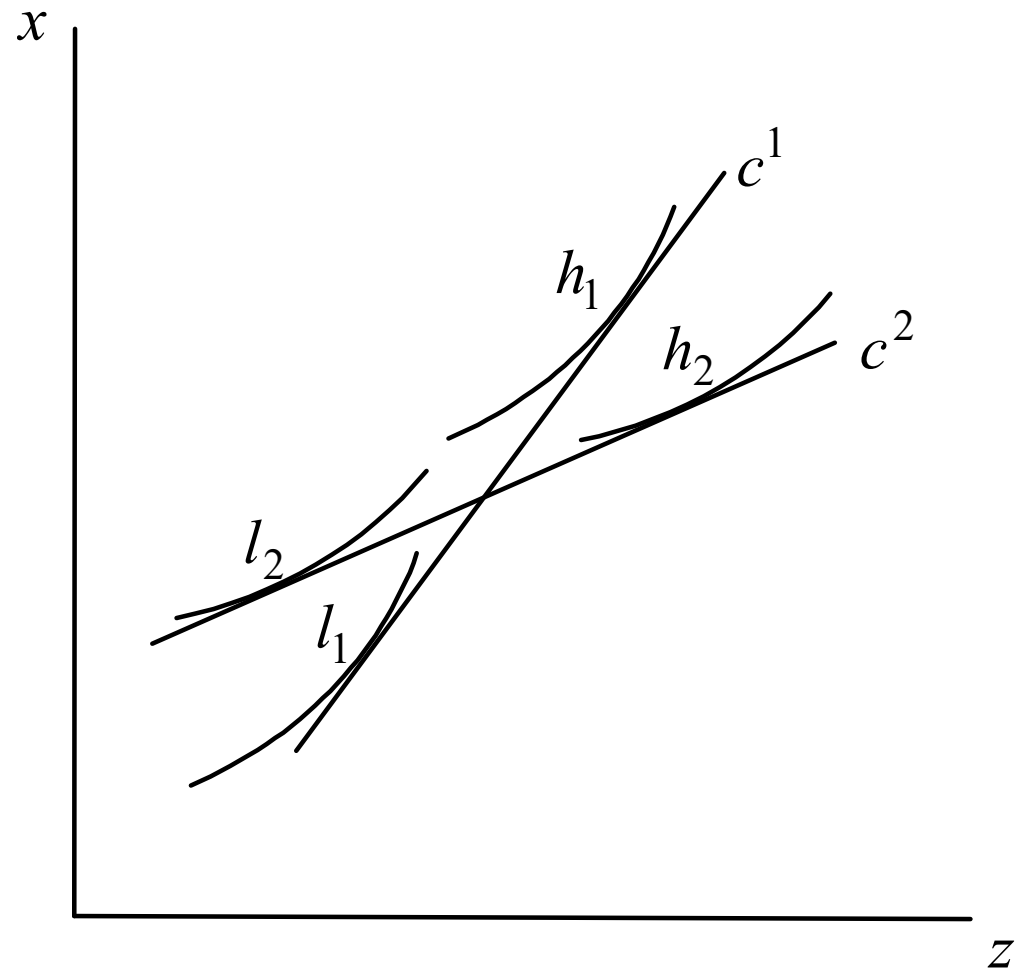
# Lower limit on the marginal tax rate

- The marginal tax rate must be positive
- Consumption function  $c_1$  has a gradient  $> 1$  (so  $T' < 0$ ). A negative marginal tax rate represents a marginal subsidy to the tax payer from the tax system. The after-tax wage for additional work is greater than the before-tax wage.
- This can never be optimal. Why?



# Lower limit on the marginal tax rate

- Move to consumption function  $c_2$  with gradient  $< 1$  (so  $T' > 0$ )
- High skill moves from  $h_1$  to  $h_2$ , low skill from  $l_1$  to  $l_2$
- $c_2$  chosen so that total income and total consumption are unchanged, i.e. the extra before-tax income earned by the high-skill is exactly equal to the reduction in earnings by the low skill and the consumption of the low-skill rises by exactly the amount that of the high-skill falls.

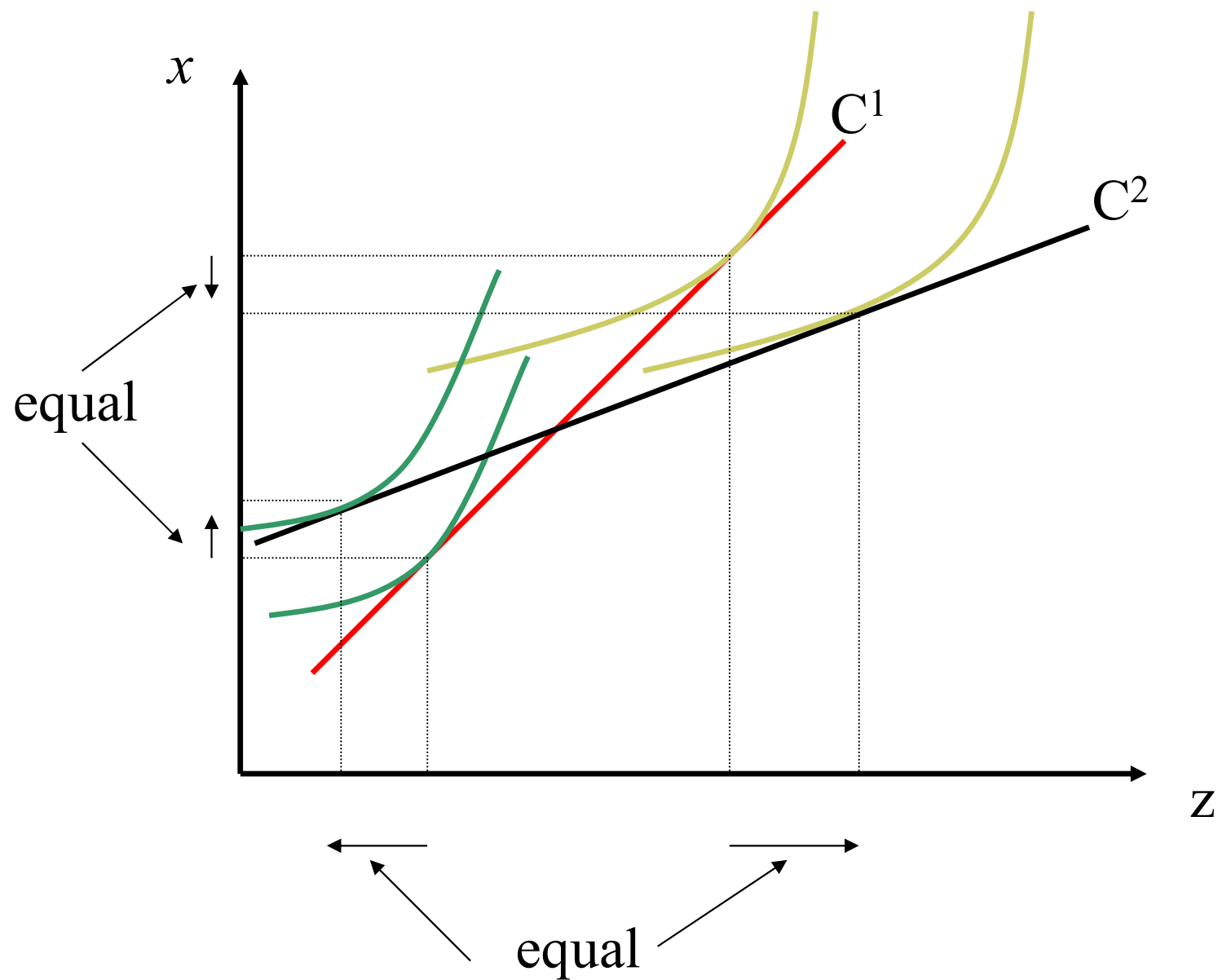




# Result II: Marginal Tax rate $\geq 0\%$

$C_1$ : marginal subsidy

$C_2$ : marginal tax



# Net effect of the changes:

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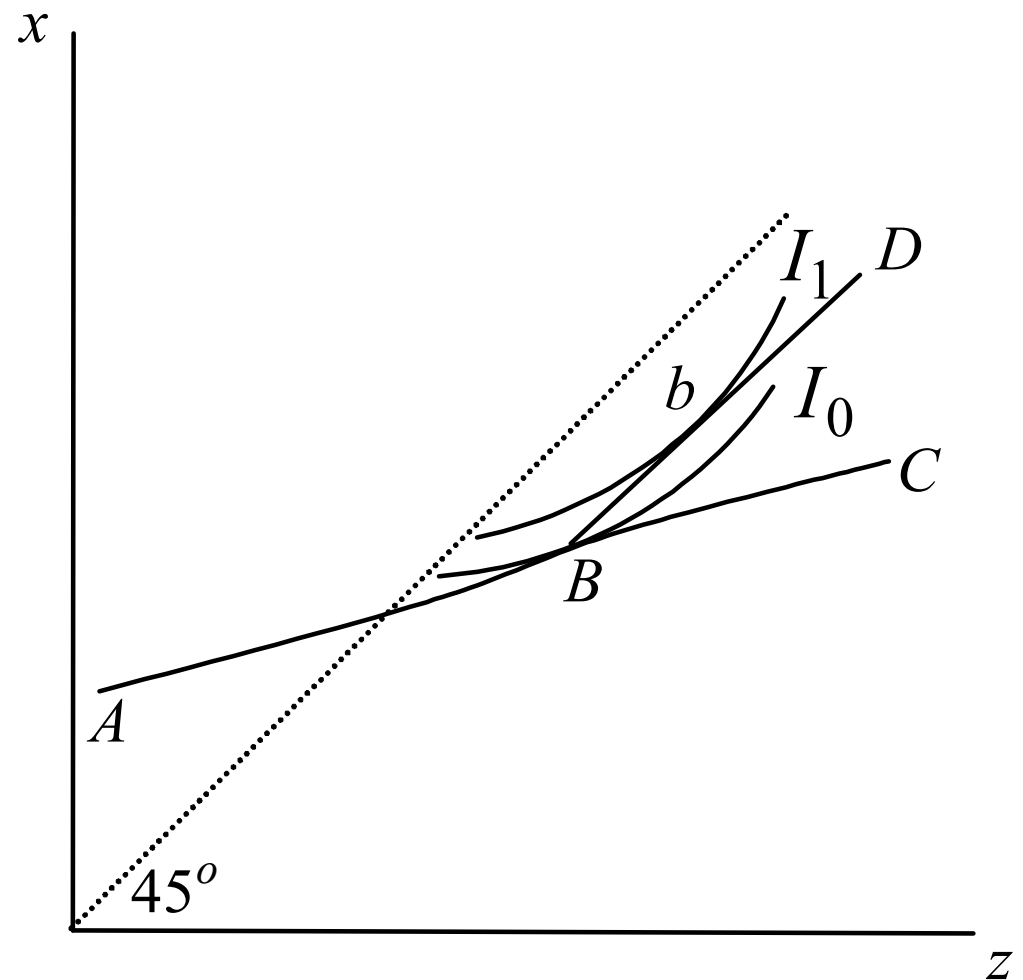
- ❑ Equal change in consumption
  - Large utility gain for low-skilled (MU high)
  - Small utility loss for high-skilled (MU low)
- ❑ Equal change in gross income
  - large utility gain for low ability person (who reduces hours a lot)
  - Small utility loss for high ability person (who only has to increase hours a little)
- ❑ I have transferred consumption to the low-skill and work effort to the high-skill
- ❑ Net gain in utilitarian social welfare.
- ❑ So,  $C^1$  could not have been optimal.

### Result 3:

No distortion at the top: the optimal  
marginal tax rate  
is 0% for the top earner

# Marginal Tax rate on top earner is zero...

- The highest-skill consumer should face a zero marginal rate of tax
- Assume  $ABC$  does not have this property
- Replace with  $ABD$  where section  $BD$  has gradient of 1 (so  $T' = 0$ )
- Highest-skill consumer moves to  $b$
- Utility rises but tax payment (vertical distance from consumption point to the  $45^\circ$  line) unchanged
- Replacing  $ABC$  with  $ABD$ 
  - leaves tax revenue unchanged
  - makes one person better-off
  - makes no-one worse-off
- $ABC$  cannot be optimal



# Summary

- Marginal tax rates must lie between 0 and 1
- Marginal tax rate on highest ability (highest income) person should be 0

# Evaluation of the results of the optimal income taxation model

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- In observed tax systems the marginal rate of tax rises with income
  - the highest income consumers face the highest marginal tax rate
  - such tax systems cannot be optimal
- A tax system is progressive if the marginal rate of tax increases with income
  - a zero rate at the top shows optimal tax system cannot be progressive
- This result has caused debate, partly due to its contrast with what is observed in practice.
  - result is valid only for the highest-skill consumer. No prediction about the tax rate facing the consumer with the second-highest skill.
  - limited implications for those close to the top of the skill range
  - observed tax systems may only be ‘wrong’ at the very top
  - result relies on identification of highest-skill person
- Result questions preconceptions about the structure of taxes
  - limited immediate policy relevance

# Designing an optimal income tax

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Optimal tax theory suggests that the marginal rate should be between zero and one, should be zero at the upper end point, equity considerations are expected to raise the marginal tax rate, but apparently concrete answers on the optimal income tax schedule can be given only on the basis of numerical analysis combining information on:

- A specific social welfare function
- Individual behaviour (with regard to labour supply)
- the distribution of abilities in the population

(assuming that pre-tax wages reflect ability)

# Numerical Results: Mirrlees, 1971

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- Numerical results use a social welfare function

$$W = \int_0^S \frac{1}{\varepsilon} e^{-\varepsilon U} \gamma(s) ds, \varepsilon > 0$$
$$= \int_0^S U \gamma(s) ds, \varepsilon = 0$$

- Where  $S$  is the maximum level of skill in the population and 0 is the lowest
  - The concern for equity is given by  $e$
  - Higher  $e$  represent greater concern for redistribution
  - If  $e = 0$  social welfare is utilitarian
- The individual utility function is Cobb-Douglas,  
 $U = \log(x) + \log(1-l)$ .
  - The skill distribution is log-normal, with standard deviation 0.39 (approximated from the income distribution)



# Numerical Results: Mirrlees, 1971

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**Table 15.2**  
Utilitarian case ( $\varepsilon = 0$ )

Income	Consumption	Average tax (%)	Marginal tax (%)
0	0.03	—	23
0.05	0.07	-34	26
0.10	0.10	-5	24
0.20	0.18	9	21
0.30	0.26	13	19
0.40	0.34	14	18
0.50	0.43	15	16

# Numerical Results: Mirrlees, 1971

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**Table 15.3**

Some equity considerations ( $\varepsilon = 1$ )

Income	Consumption	Average tax (%)	Marginal tax (%)
0	0.05	—	30
0.05	0.08	-66	34
0.10	0.12	-34	32
0.20	0.19	7	28
0.30	0.26	13	25
0.40	0.34	16	22
0.50	0.41	17	20

# Numerical Results

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- The average rate of tax for low-skill consumers is negative
  - this is a negative income tax where income is supplemented by the government through the tax system
- The average rate of tax then increases with skill
  - the maximum average rate of tax is actually quite low
- The marginal tax rate first rises with skill and then falls.
  - the maximum rate is around the median of the skill distribution
- **The optimal tax systems have a basically constant marginal rate of tax**
  - the consumption function is close to being a straight line
- The zero tax for the highest-skill consumer is reflected in the fall of the marginal rate at high skills
  - not significant until close to the top of the skill distribution.

# Intuition behind Mirrlees (1971)

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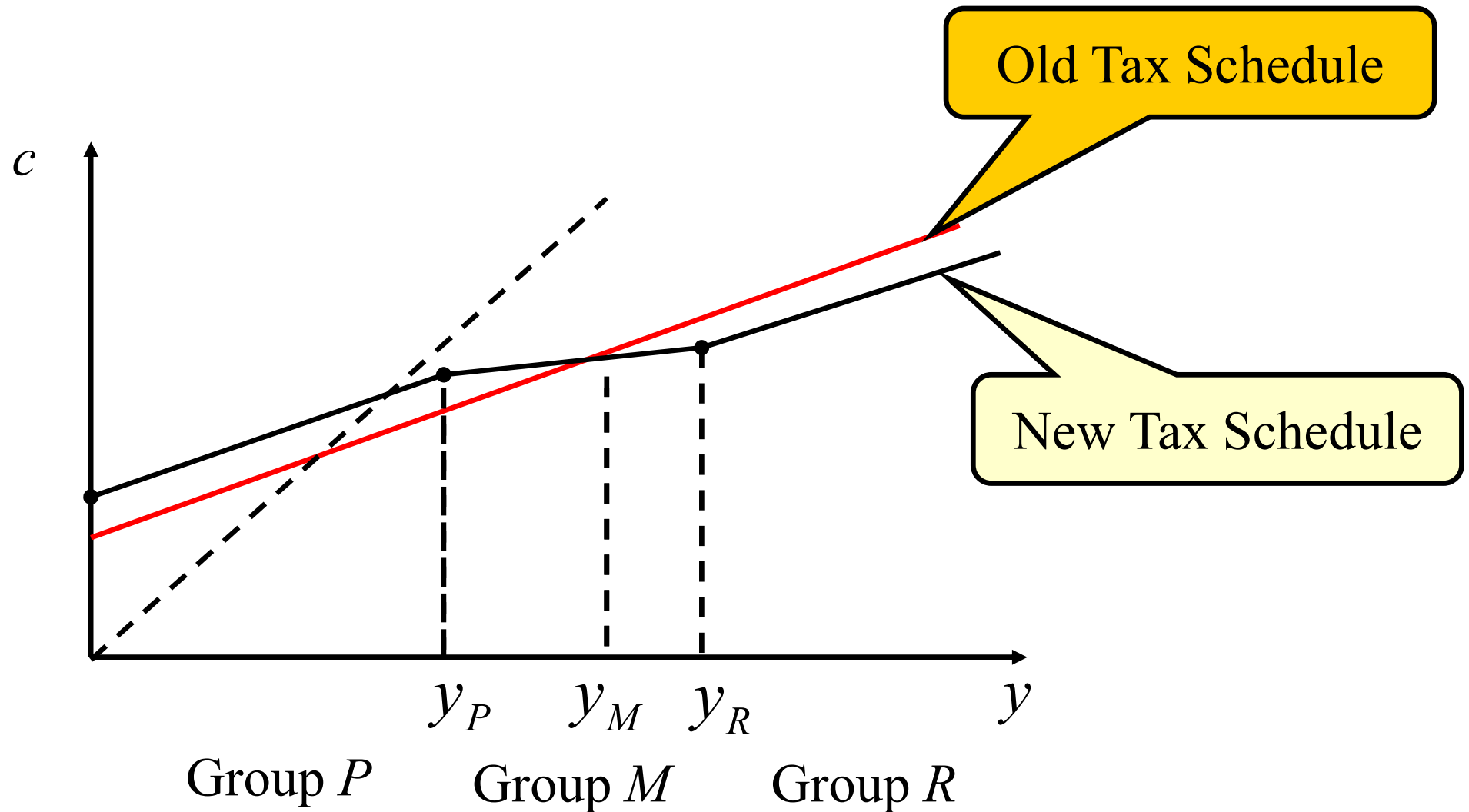
- Fundamental policy issue:

Is it a good idea to increase the rate of the income tax and use the proceeds to fund the poor? (Use of income taxation in order to reduce after-tax income inequality)

- Suppose that we start from a linear income tax and consider a reform in tax rates. What factors should we take into account?

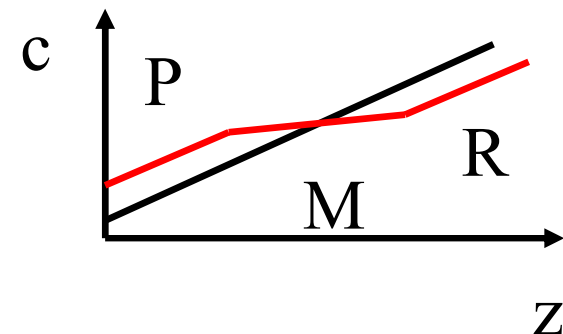
# Non-linear income taxation

*Increase marginal tax rate on Group M*



# Effects of the tax reform

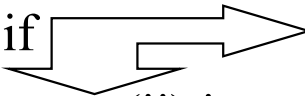
- Group M (middle class): Higher marginal and average tax
  - Deadweight cost
  - Uncertain revenue effect
  - Welfare reduction
- Group R (the rich): Higher **average** tax
  - No additional deadweight cost (no substitution effect)
  - Increase in revenue
  - Welfare reduction
- Group P (the poor): Lower average tax (or higher subsidy)
  - Welfare increase.



# Effects of the tax reform

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- ❑ (i) the tax payments of people with the increased marginal rate (M) will probably fall, if the substitution effect dominates.
- ❑ (ii) the tax payments of people with income above the range of increase will rise
- ❑ (iii) the utility levels of (M) and (R) will fall, while those of (P) will rise.
  
- ❑ If the net effect of (i) and (ii) is negative → no extra revenue available to fund redistribution to the poor (reform not desirable).

This is likely if  (i) is large (high compensated elast. of labour supply)  
(ii) is small, because (R) is a small number of people.

- ❑ If the net effect of (i) and (ii) is positive, the utility gain for lower-income people (P) must be weighted against the utility loss of middle- and higher- income people (M) and (R).

# What factors matter?

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- The net effect on social welfare will depend on the following factors:
  - **The compensated elasticity of labour supply** (high elasticity → lower or negative revenue gain). If compensated elasticity of labour supply for the middle class is high then high welfare cost of marginal tax increase so need even more rich people to compensate (through the principle of tax interaction). High elasticity implies that a tax increase is less likely to increase social welfare.
  - (1) **Inequality aversion** ( $\varepsilon$ ). Higher  $\varepsilon$  implies that a higher weight is placed on the utility of lower-income people, so the tax increase is more likely to increase social welfare



# What factors matter?

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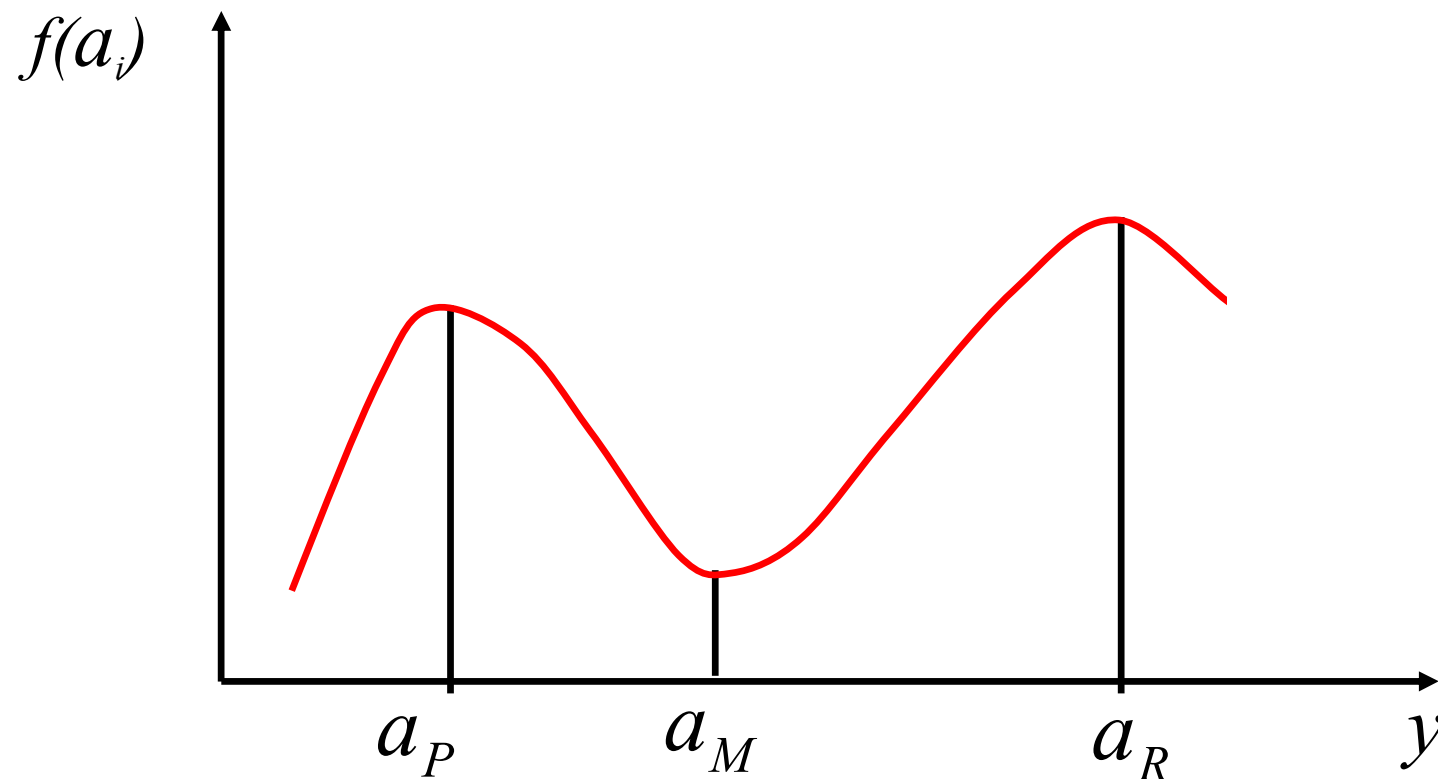
- The net effect on social welfare will depend on the following factors:
  - (3) **Degree of inequality** A high level of inequality implies that a greater income difference between the low-income and the high-income people translates into a greater relative weight attached to the gains, and the tax increase is more likely to increase social welfare.
  - (4) **The proportion of the population above the range of the tax increase.** The higher this proportion, the greater the tax revenue increase, so the tax increase is more likely to increase social welfare.
  
- (3) and (4) depend on the distribution of ability (income)

# What factors matter?

## Distribution of population by gross income (ability)

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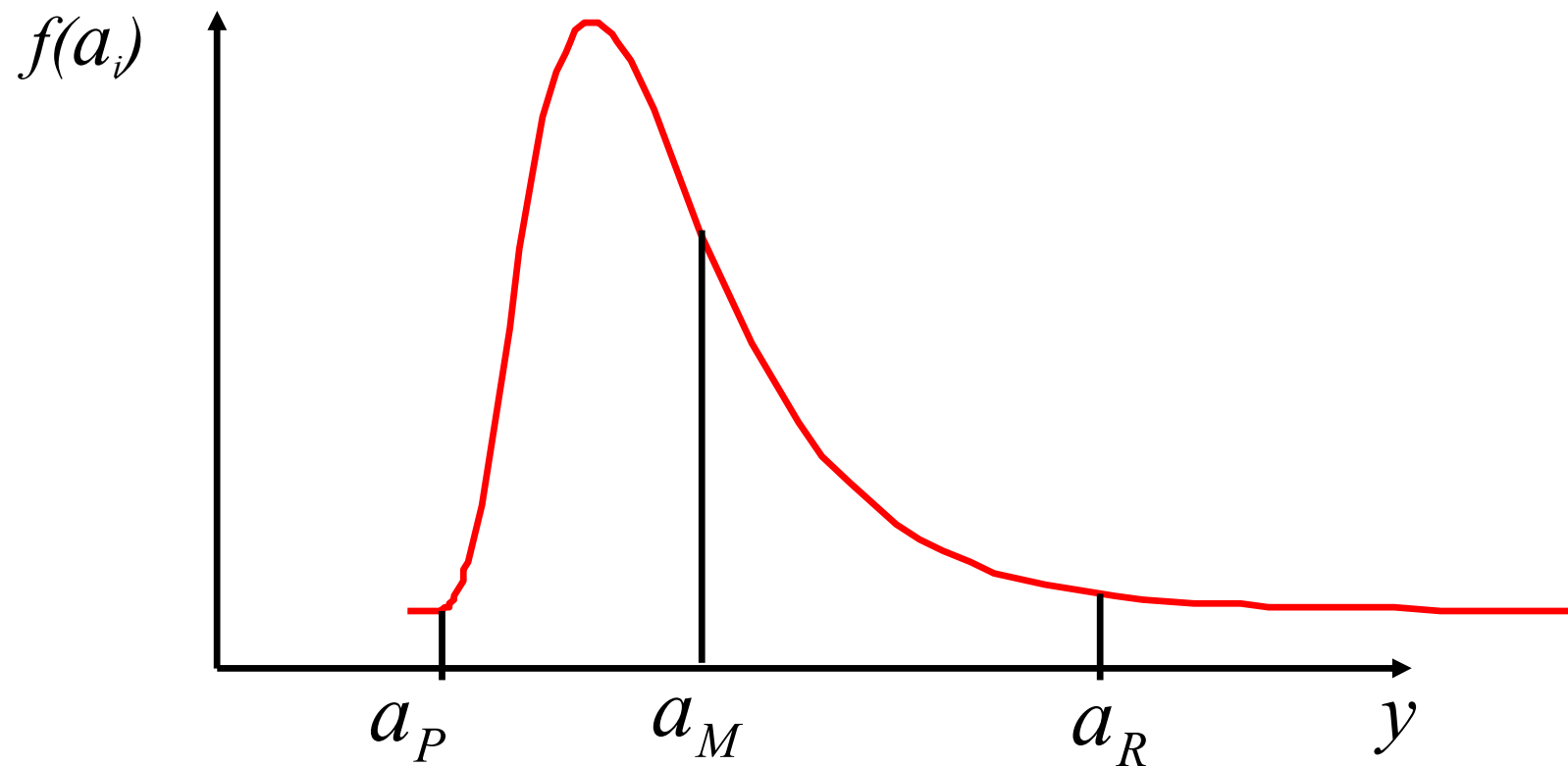
If large number of rich people, but few in the middle class, then increase marginal tax rate on group on the middle class good idea. An increase in the marginal tax rate is more attractive when few individuals would be affected at the margin and many would be affected inframarginally.



## Distribution of population by gross income (ability)

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If the middle class is large, but there are few poor or rich, then costly to increase marginal tax rate on the middle class. In general, the marginal tax rate schedule must be tailored to the shape of the ability distribution.



# What happens in practice?

- Over the recent past, tax policy has moved toward lower marginal tax rates on high earners

## Marginal Tax Rates on High Incomes

Country	<i>Marginal tax rate at 250% of average employee compensation</i>		Change
	1981–1982	2005–2006	
Australia	53.0	47.0	–6.0
Austria	55.0	50.0	–5.0
Belgium	55.0	50.0	–5.0
Canada	31.0	26.0	–5.0
Denmark	39.8	26.5	–13.3
France	62.5	48.1	–14.4
Greece	38.0	40.0	2.0
Italy	37.0	39.0	2.0
Netherlands	64.4	52.0	–12.4
Norway	38.0	23.8	–14.2
Spain	25.3	29.2	3.8
Sweden	58.0	25.0	–33.0
United Kingdom	42.5	40.0	–2.5
United States	50.0	28.0	–22.0

*Note:* Central government income taxes only; excludes payroll taxes.

# What happens in practice?

- Flatter is better...

## 250–67 Spread as a Measure of Flattening Marginal Tax Schedules

<i>Country</i>	<i>250–67 spread</i>		<i>Change</i>
	<i>1981–1982</i>	<i>2005–2006</i>	
Australia	21.7	17.0	–4.7
Austria	22.0	11.7	–10.3
Belgium	13.9	5.0	–8.9
Canada	12.0	10.9	–1.1
Denmark	25.3	21.0	–4.3
France	22.5	15.3	–7.2
Greece	25.5	32.5	7.0
Italy	13.5	16.0	2.5
Netherlands	32.0	42.5	10.5
Norway	32.0	12.0	–20.0
Spain	8.2	13.3	5.1
Sweden	44.0	25.0	–19.0
United Kingdom	12.5	18.0	5.5
United States	27.6	13.0	–14.6
<b>All 14 countries</b>			<b>–4.3</b>

*Notes:* The 250–67 spread is the marginal tax rate at 250 percent of average employee compensation minus the marginal tax rate at 67 percent.

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