16 Income Taxation

16.1 Introduction

In 1799 an income tax was introduced for the first time in the United Kingdom to pay for the Napoleonic war. The tax was levied at a rate of 10 percent on income above £60 and survived until it was repealed in 1816 following major public opposition. Part of the opposition was due to concerns about privacy, and this was reflected in the decision of Parliament to pulp all documents relating to the income tax. The tax returned in 1842 as a temporary measure (imposed for three years with the possibility of a two-year extension) to cover a major budget deficit. It has remained in place ever since, although it is still temporary and Parliament has to re-apply it every year.

The income tax has remained controversial. As the discussion of chapter 4 showed, the taxation of income is a major source of government revenue. This fact, coupled with the direct observation by taxpayers of income tax payments, explains why the structure of income tax is the subject of much political discussion. The arguments that are aired in such debate reflect the two main perspectives on income taxation. The first views the tax primarily as a disincentive to effort and enterprise. On this ground, it follows that the rate of tax should be kept as low as possible in order to avoid such discouragement. This is essentially the expression of an efficiency argument. The competing perspective is that income taxation is well suited for the task of redistributing income. Hence notions of equity require that high earners should pay proportionately more tax on their incomes than low earners. The determination of the optimal structure of income taxation involves the resolution of these contrasting views.

The chapter begins by conducting an analysis of the interaction between income taxation and labor supply. A number of theoretical results are derived, and these are related to the empirical evidence. This evidence makes clear the extent of the difference between the responses of male and female labor supply to taxation. A model that permits the efficiency and equity aspects of taxation to be incorporated into the design of the optimal tax is then described. A series of results characterizing properties of the optimal tax function are derived using this model, and these properties are interpreted in terms of practical policy recommendations. Calculations of the optimal tax rates that emerge from the model are then reviewed. The chapter is completed by a discussion of political economy aspects of income taxation.

16.2 Equity and Efficiency

There are two major issues involved in the taxation of income. The first is the effect of taxation on the supply of labor. Taxation alters the choices that consumers make by affecting the trade-off between labor and leisure. In this respect a particularly important question is whether an increase in the rate of tax necessarily reduces the supply of labor. If this is the case, support would be provided for the argument that taxes should be kept low to meet the needs of efficiency. Both theoretical and empirical results addressing this question will be discussed. The second issue that has been studied is the determination of the optimal level of income taxation. For reasons that will become clear, this is a complex problem, since it can only be addressed in a model with a meaningful trade-off between efficiency and equity. Having said this, the search for the right trade-off has proved to be a fruitful avenue of investigation.

The essential idea we wish to convey in this chapter is that it is a major mistake to design the income tax structure to meet equity motives without taking into account the impact on work effort. To see why, consider the naïve solution of setting the marginal tax rate at 100 percent for all incomes above some threshold level z° and at a rate of zero for all incomes below this threshold. We might expect that such a tax structure will maximize the redistribution possible from the rich (those above the income threshold) to the poor (those below). However, this conclusion is incorrect when taxpayers respond to the tax structure. The confiscatory tax above the threshold removes the incentive to earn more than z°, and everyone previously above this level will choose to earn exactly that amount of income. This sets a vicious circle in motion. The government must lower the threshold, inducing everyone above the new level to lower their incomes again, and so forth, until no one chooses to work and income is zero.

It therefore stands to reason that we must analyze the equity of the tax structure in tandem with its effect on work incentives. The idea is to find the tax schedule that meets social objectives, as captured by the social welfare function (see chapter 13), given the adjustment in work effort and labor market participation by taxpayers. Such a tax scheme is said to be optimal conditional on the given objective. The results need to be interpreted with caution, however, because they are very sensitive to the distribution of abilities in the population and to the form of the utility function. More important, they depend on the equity objective as built into the social welfare function.

In this chapter we will only consider welfaristic equity criteria (of which the utilitarian and Rawlsian social welfare functions are noteworthy examples). Hence, insofar as the social objective is entirely based on individual welfare levels, we are not assessing the tax structure on the basis of its capacity for either redressing inequality or eliminating poverty. Neither do we consider egalitarian social objectives like equal sacrifice or equality of opportunities. There is indeed an interesting literature on "fair" income taxation that examines the distribution of taxes that impose the same loss of utility on everyone, either in absolute or relative terms. Such arguments are related to the ability-to-pay principle according to which \$1 of tax is less painful for a rich person than for a poor person (due to the decreasing marginal value of income). This equal sacrifice approach predicts that the resulting tax structure must be progressive (in the sense that everyone sacrifices equally if they pay an increasing percentage of their income in tax as their income rises). It was John Stuart Mill, in his *Principles of Political Economy*, who first pointed out this principle of equal sacrifice. He suggested that "Equality of taxation, therefore as a maxim of politics, means equality of sacrifice. It means apportioning the contribution of each person towards the expenses of government so that he shall feel neither more nor less inconvenience from his share of the payment than every other person experiences from his" (bk. V, ch. 2, p. 804).

16.3 Taxation and Labor Supply

The effect of income taxation on labor supply can be investigated using the standard model of consumer choice. The analysis will begin with the general question of labor supply and then move on to a series of specific analyses concerning the effect of variations in the tax system. The major insight this gives will be to highlight the importance of competing income and substitution effects.

As is standard, it is assumed that the consumer has a given set of preferences over allocations of consumption and leisure. The consumer also has a fixed stock of time available that can be divided between labor supply and time spent as leisure. The utility function representing the preferences can then be defined by

$$U = U(x, L - \ell) = U(x, \ell), \tag{16.1}$$

where L is the total time endowment, ℓ is labor supply, and x is consumption. Consequently leisure time is $L - \ell$. Labor is assumed to be unpleasant for the worker, so utility is reduced as more labor is supplied, implying that $\frac{\partial U}{\partial \ell} < 0$. Let each hour of labor earn the wage rate w so that income, in the absence of taxation, is $w\ell$. Letting the (constant) rate of tax be t, the budget constraint facing the consumer is $px = [1 - t]\, w\ell$, where p is the price of the consumption good.

The choice problem is shown in panel a of 1figure 16.1, which graphs consumption against leisure. The indifference curves and budget constraint are as standard for utility maximization. The optimal choice is at the tangency of the budget constraint and the highest attainable indifference curve. This results in consumption x^* and leisure $L - \ell^*$.

There is an alternative way to write the utility function. Let the before-tax income be denoted by z, so that $z = w\ell$. Since $\ell = \frac{z}{w}$, utility can then be written in terms of before-tax income as

$$U = U\left(x, \frac{z}{w}\right). \tag{16.2}$$

These preferences can be depicted on a graph of before-tax income against consumption. Expressed in terms of income, the budget constraint becomes $px = [1 - t]\, z$. This is shown in panel b of figure 16.1. The optimal choice occurs at the point of tangency between the highest attainable indifference curve and the budget constraint, with consumption x^* and before-tax income z^*. The important feature of this alternative representation is that the budget constraint is not affected as w changes, so it is the same whatever wage the consumer earns, but the indifference curves do change, since it is $\frac{z}{w}$ that enters the utility function. How they change is described below.

This standard model can now be used to understand the effects of variations in the wage rate or tax rate. Consider the effect of an increase in the wage rate, which is shown in panel a of figure 16.2 by the move to the higher budget line and the new

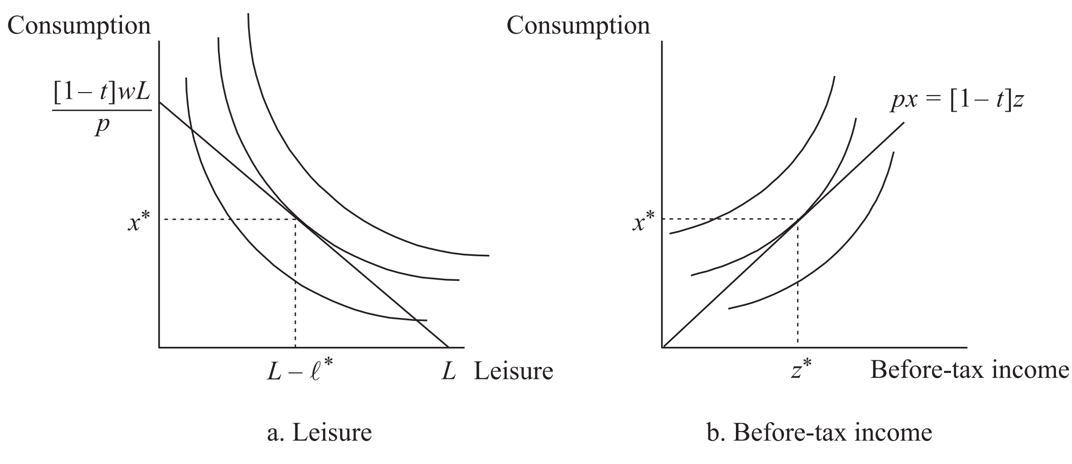
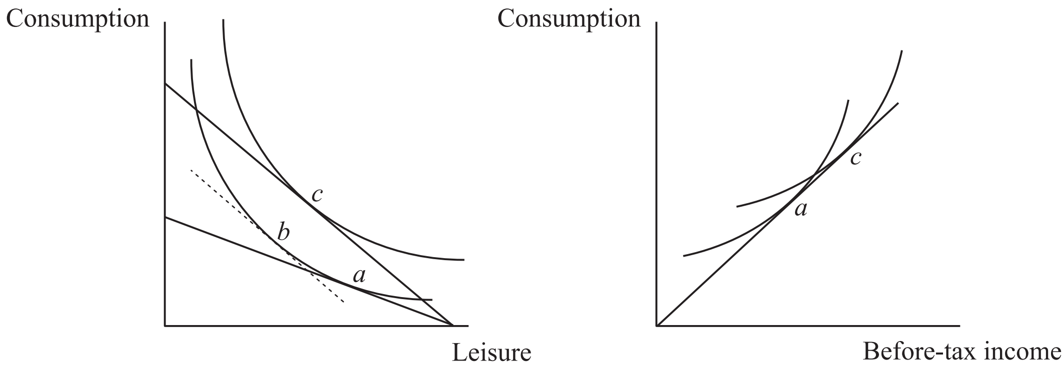


Figure 16.1
Labor supply decision



a. Leisure   b. Before-tax income

Figure 16.2
Effect of a wage increase

tangency at c. The move from a to c can be broken down into a substitution effect (a to b) and an income effect (b to c). The direction of the substitution effect can always be signed, since it is given by a move around the indifference curve. In contrast, the income effect cannot be signed: it may be positive or negative. Consequently the net effect is ambiguous: an increase in the wage rate can raise or lower labor supply. This is the basic ambiguity that runs throughout the analysis of labor supply. The effect of a wage increase when preferences are written as in (16.2) is shown in panel b of figure 16.2. An increase in the wage rate means that less additional labor is required to achieve any given increase in consumption. This change in the trade-off between labor and consumption causes the indifference curve through a point to pivot round and become flatter. This flattening of the indifference curves causes the optimal choice to move along the budget constraint. The level of before-tax income will rise, but the effect on hours worked is ambiguous.

The effect of a tax increase is now analyzed in the same way. In panel a of figure 16.3 the tax increase rotates the budget line down so that the optimal choice moves from a to c. The substitution effect of the tax increase is the move around the indifference curve from a to b and the income effect the move from b to c. Using the alternative form of preferences an increase in the tax rate rotates the budget constraint in panel b downward so that the chosen point moves from a to c. In neither diagram does the change in tax rate affect the indifference curves.

It is also helpful to consider more complex tax systems using this approach. A common feature of the income tax in many countries is that there is a threshold level of

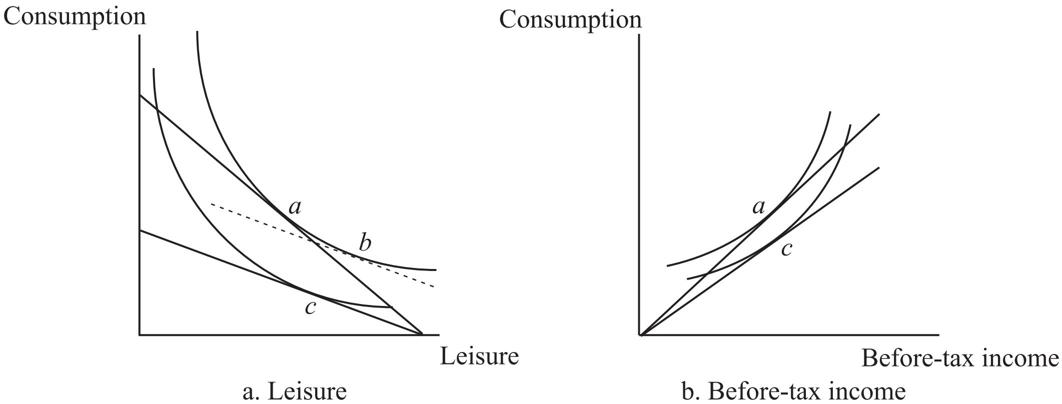


Figure 16.3
Effect of a tax increase

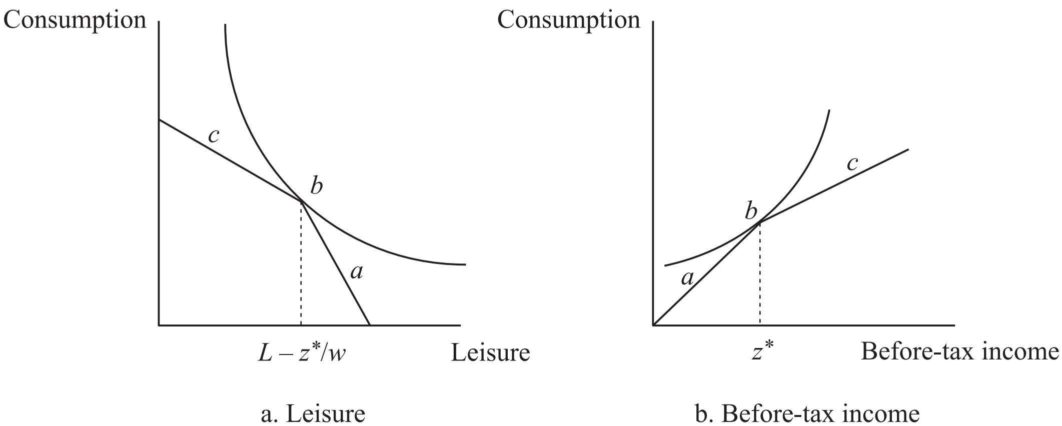


Figure 16.4
A tax threshold

income below which income is untaxed. This is shown in figures 16.4. The threshold level of income is z^* so at wage rate w this arises at $\frac{z^*}{w}$ hours of labor supply. The economic importance of this threshold is that it puts a kink into the budget constraint. If a set of consumers with differing preferences are considered, some may locate at points such as a and pay no tax, and some may locate at points like c. However, it can be expected that a number of consumers will cluster or "bunch" at the kink point b. The observation that consumers will bunch at a kink point is a common feature and reflects

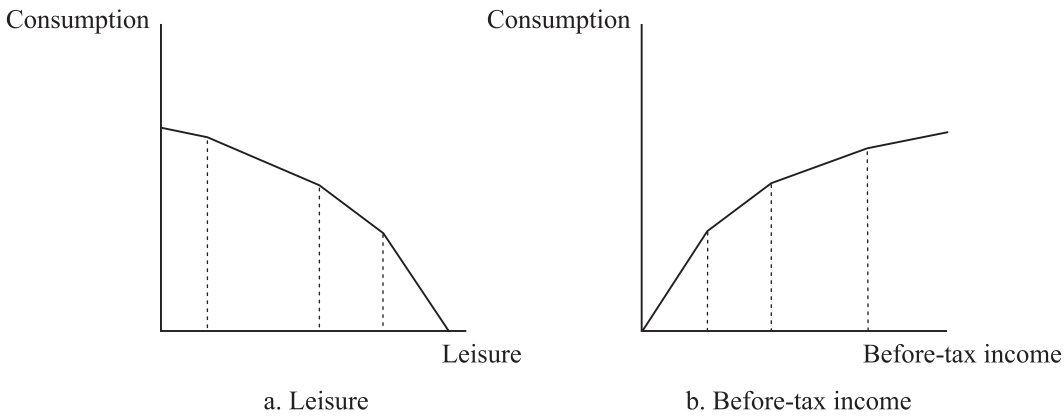


Figure 16.5
Several thresholds

the fact that an extra unit of labor will receive net pay $[1 - t]w$, whereas the previous unit received w. It is therefore helpful to distinguish between interior solutions, such as a and c, and corner solutions such as b. The consumer at an interior solution will respond to changes in the tax rate in the manner illustrated in figure 16.2. In contrast, a consumer at a corner solution may well be left unaffected by a marginal tax change. The consumer's choice will only be affected if the change is sufficient to allow the attainment of a utility level higher than at the kink.

More generally, an income tax system may have a number of thresholds with the marginal tax rate rising at each. Such a tax system appears as in figure 16.5. Again, with preferences varying across consumers, the expectation is that there will tend to be collections of consumers at each kink point.

The final issue that is worth investigating in this framework is that of participation in the labor force. The basic assumption so far has been that the worker can continuously vary the number of working hours in order to arrive at the most preferred outcome. In practice, it is often the case that either hours are fixed or else there is a minimum that must be undertaken with the possibility of more. Either case leads to a discontinuity in the budget constraint at the point of minimum hours. The choice for the consumer is then between undertaking no work and working at least the minimum. This is the participation decision: whether or not to join the workforce.

The participation decision and its relation to taxation is shown in figure 16.6 where ℓ^m denotes the minimum working time. The effect of an increase in taxation is to lower the budget constraint. A consumer who was previously indifferent between working

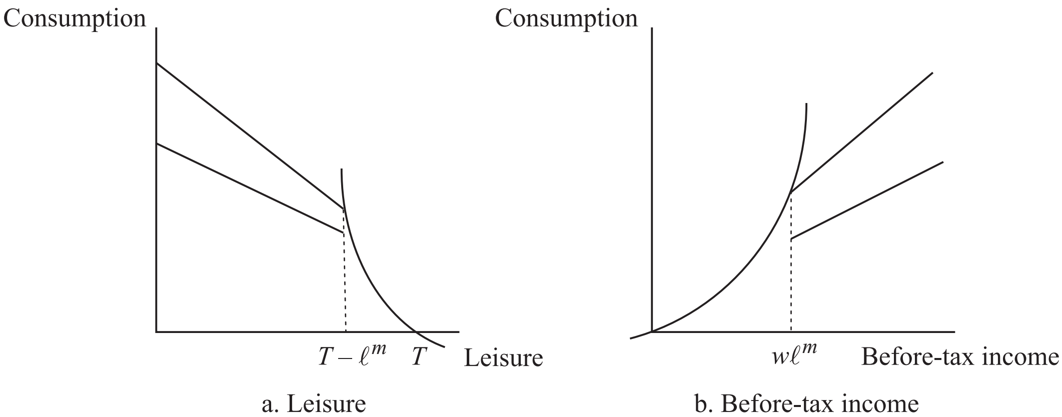


Figure 16.6
Taxation and the participation decision

and not (both points being on the same indifference curve) now strictly prefers not to do so. At this margin there is no conflict between income and substitution effects. An increase in taxation strictly reduces participation in the work force.

16.4 Empirical Evidence

The theoretical analysis of section 16.3 has identified the three major issues in the study of labor supply. These are the potential conflict between income and substitution effects that make it impossible to provide any clear-cut results for those consumers at an interior solution, kinks in the budget constraint that make behavior insensitive to taxes, and a participation decision that can be very sensitive to taxation. How important each of these factors is in determining the actual level of labor supply can only be discovered by reference to the empirical evidence.

Empirical evidence on labor supply and the effect of income taxes can be found in both the results of surveys and in econometric estimates of labor supply functions. In considering what evidence is useful, it is best to recall that the labor supply will be insensitive to taxation if working hours are determined by the firm or by a union and firm agreement. When this is the case, only the participation decision is of real interest. The effect of taxation at interior solutions can only be judged when the evidence relates to workers who have the freedom to vary their hours of labor. This is most commonly the position for those in self-employment rather than employment. For those in employment, variations in hours can sometimes be achieved by undertaking overtime, so this dimension of choice can be considered.

These comments also draw attention to the fact that the nature of labor supply may well be different between males and females, especially married females. It still remains a fact that males continue to be the dominant income earner in most families. This leaves the married female as typically a secondary income earner, and for them there is often no necessity to work. From this position it is the participation decision that is paramount. In contrast, most males consider work to be a necessity, so the participation decision is an irrelevance. It can therefore be expected that the labor supply of males and females will show different degrees of sensitivity to taxation.

Surveys on labor supply have normally arrived at the conclusion that changes in the tax rate have little effect on the labor-supply decision. For instance, a survey of the disincentive effect of high tax rates on solicitors and accountants in the United Kingdom, 63 percent of whom were subject to marginal tax rates above 50 percent, concluded that as many of the respondents were working harder because of the tax rates as were working less hard. Groups such as these are ideal candidates for study for the reasons outlined above: they can be expected to have flexibility in the choice of working hours and should be well informed about the tax system. A similar conclusion was also found in a survey of the effect of income taxation on the level of overtime worked by a sample of weekly paid workers: little net effect of taxation on working hours was found.

These results suggest the conclusion that labor supply does not vary significantly with the tax rate. If this were correct, the labor supply function would be approximately vertical. In terms of the theoretical analysis the survey results point to an income effect that almost entirely offsets the substitution effect. However, the discussion has already suggested that different groups in the population may have different reactions to changes in the tax system. This issue is now investigated by considering evidence from econometric analysis.

Table 16.1 presents some summary econometric estimates of labor-supply elasticities. These are divided into those for married men, married women, and single mothers. Each gives the overall elasticity and its breakdown into substitution and income effects. Estimates for both the United Kingdom and the United States are given.

Since the results in table 16.1 relate to the effect of a wage increase, theory would predict that the substitution effect should be positive. This is what is found in all cases. The income effect, which theoretically can be positive or negative, is found to be always negative. Consequently the negativity offsets the substitution effect, sometimes more than completely. While there are a range of estimates for each category, some general observations can be made. The estimated elasticity for married men is the lowest and is the only one that is ever estimated to be negative. This implies that the labor-supply curve for married men is close to vertical and may even slope backward. One explanation for this result could be that the working hours of this group are constrained by collective agreements that leave little flexibility for variation.

The labor-supply elasticity of single mothers is, on average, the largest of the three sets. This result is probably a consequence of the participation effect. For single mothers part-time work is an unattractive option, since this usually implies the loss of state benefits. Consequently labor supply becomes an all or nothing decision. Married women represent the intermediate case. For them part-time work is quite common, and this often opens the way to some degree of flexibility in hours of work. As expected, these factors lead to a labor-supply elasticity greater than that of married men but lower than that of single mothers.

Although the estimates vary widely within the groups, indicating some imprecision in the estimates, some general conclusions can still be drawn. First, the elasticity of labor supply is not uniform across the population of workers. It clearly varies among the three groups identified in this discussion and probably varies within these groups. Despite this variation, it is still apparent that the uncompensated labor-supply elasticity for married men is small with estimates grouped around zero. In contrast, the elasticity of women is higher and reflects the participation effect and the greater flexibility they have in the choice of hours.

Table 16.1
Labor-supply elasticities

	Married women		Married men		Single mothers	
	United States	United Kingdom	United States	United Kingdom	United States	United Kingdom
Uncompensated wage	0.45	0.43	−0.03	−0.23	0.53	0.76
Compensated wage	0.90	0.65	0.95	0.13	0.65	1.28
Income	−0.45	−0.22	−0.98	−0.36	−0.18	−0.52

Source: Blundell (1992).

16.5 Optimal Income Taxation

The analysis to this point has considered the positive question of how income taxation affects labor supply. Having understood this, it is now possible to turn to the normative question of how the income tax structure should be determined. This is by nature a complex issue. As has already been noted, in practice, income tax systems generally have a number of thresholds at which the marginal tax rate rises. An investigation of the optimal system must at least be flexible enough to consider such tax systems without limiting the number of thresholds or the rates of tax at each. In fact it must do more much than this. The model of income taxation introduced by Mirrlees (1971) has several important attributes. First, there is an unequal distribution of income, so there are equity motivations for taxation. Second, the income tax affects the labor supply decisions of the consumers so that it has efficiency consequences. Third, in view of the comments above, the structure is sufficiently flexible that no prior restrictions are placed on the optimal tax functions that may arise.

In the model all consumers have identical preferences but differ in their level of skill in employment. The hourly wage received by each consumer is determined by their level of skill. This combines with the labor-supply decision to determine income. The economy is competitive, so the wage rate is also equal to the marginal product of labor and firms price their output at marginal cost. A tax levied on skill would be the first-best policy as it would be a lump-sum tax on the unalterable characteristic that differentiates consumers. But this first-best is not feasible, since the level of skill is assumed to be private information and so unobservable by the government. As the discussion of chapter 13 showed, this makes it impossible to tax skill directly. Since the government cannot observe a consumer's skill level (which is essentially the initial endowment of the consumer), it employs an income tax as a second-best policy. The income tax function is chosen to maximize social welfare subject to it raising enough revenue to meets the government's requirements.

There are two commodities: a consumption good and labor. A consumer's labor supply is denoted by ℓ and consumption by x. Each consumer is characterized by a skill level s. The value of s measures the hourly output of the consumer, and since the economy is competitive, it is equal to the wage rate. If a consumer of ability s supplies ℓ hours of labor, that consumer earns income of $s\ell$ before tax. Denote the income of the consumer with skill s by $z(s) = s\ell(s)$. The amount of tax paid on an income z is given by $T(z)$. This is the tax function that the analysis aims to determine. Equivalently, denote the consumption function by $c(z)$ so that a consumer who earns income z can consume

$$x = c(z) = z - T(z). \tag{16.3}$$

The relationship among income, the tax function, and consumption is depicted in figure 16.7. In the absence of taxation, income would be equal to consumption and this is depicted by the 45 degree line. Where the consumption function lies above the 45 degree line, the tax payment is negative. It is positive when the consumption function is below the line. For example, the consumer earning $\hat{z}$ in the figure pays an amount of tax $T(\hat{z})$ and can consume $\hat{x}$. The gradient of the consumption function is equal to one minus the marginal rate of tax.

All consumers have the same utility function (so that the possibility of workers displaying different aversion to work is ruled out):

$$U = U\left(x, \frac{z}{s}\right). \tag{16.4}$$

The indifference curves are dependent on the skill level of the consumer, since a high-skill consumer takes less labor time to achieve a given level of income than a low-skill consumer. This is reflected in the fact that at any income and consumption pair $\{\hat{x}, \hat{z}\}$ the indifference curve of a high-skill consumer passing through that point is flatter than the indifference curve of a low-skill consumer. This single-crossing property is termed *agent monotonicity* and is illustrated in figure 16.8.

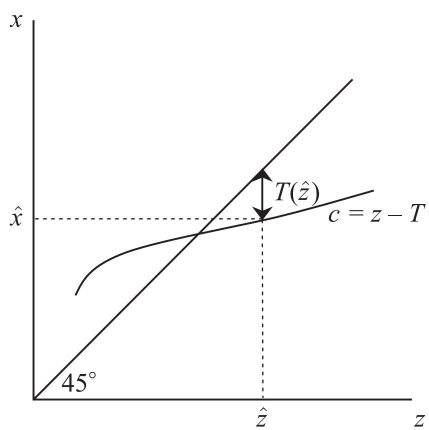


Figure 16.7
Taxation and the consumption function

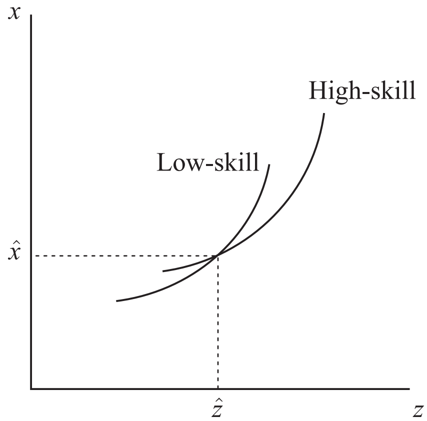


Figure 16.8
Agent monotonicity

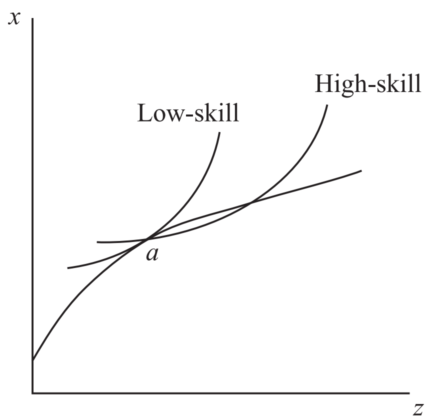


Figure 16.9
Income and skill

An immediate consequence of agent monotonicity is that high-skill consumers will never earn less income than low-skill. Generally, they will earn strictly more. This result is shown in figure 16.9. It arises because at the point where the indifference curve of the low-skill consumer is tangential to the consumption function (and so determines the optimal choice for that consumer), the indifference curve of the high-skill consumer is flatter and so cannot be at a tangency. Recall that all consumers face the same tax function and thus the same consumption function no matter what their skills are. The optimal choice for the high-skill consumer therefore has to be to the right of a, which implies a higher level of income.

The first property of the optimal tax function relates to the maximum tax rate that will be charged. If the consumption function slopes downward, as shown in figure 16.10, then the shape of the indifference curves ensures that no consumer will choose to locate on the downward-sloping section. This part of the consumption function is therefore redundant and can be replaced by the flat-dashed section without altering any of the consumers' choices. Economically, along the downward-sloping section, increased work effort is met with lower consumption. Hence there is no incentive to work harder, and such points will not be chosen. Since $c(z) = z - T(z)$, it follows that $c'(z) = 1 - T'(z)$. The argument has shown that $c'(z) \geq 0$, which implies $T'(z) \leq 1$, so the marginal tax rate is less than 100 percent.

It is also possible to put a lower limit on the marginal tax rate. If the gradient of the consumption function is greater than one, meaning $c'(z) > 1$, then $T'(z) < 0$. A

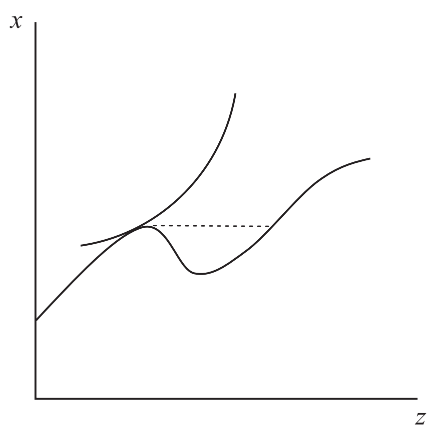


Figure 16.10
Upper limit on tax rate

negative tax rate represents a marginal subsidy to the tax payer from the tax system. That is, the after-tax wage for additional work will be greater than the before-tax wage. To demonstrate the logic behind this claim most clearly it is assumed that the social welfare function is utilitarian and that there are only two types of consumers (low-skill and high-skill), and equal numbers of the two types. Begin with the tax function denoted $c^1(z)$ in figure 16.11, which has gradient greater than one. Along this are located a high-skill consumer at h_1 and a low-skill consumer at l_1.

The effect of moving to the new tax function $c^2(z)$ where the gradient is equal to one is now analyzed in two stages. In the first stage the income levels of the consumers are kept fixed, but the consumption levels change to $\tilde{x}_l$ and $\tilde{x}_h$. This is shown in figure 16.12. The position of $c^2(z)$ is chosen so that the increase in consumption for the low-skill is exactly matched by the decrease in consumption of the high-skill. Because aggregate income and aggregate consumption have not changed, the budget position of the government is unaffected. The change in consumption levels must raise welfare because the marginal utility of consumption for the low-skill is higher than that for the high-skill.

The second stage of the process is to allow the consumers to respond to the tax function $c^2(z)$ and relocate to utility-maximizing positions. The high-skill consumer will choose $\{z_h^2, x_h^2\}$ and the low-skill will choose $\{z_h^1, x_h^1\}$ as shown in figure 16.13.

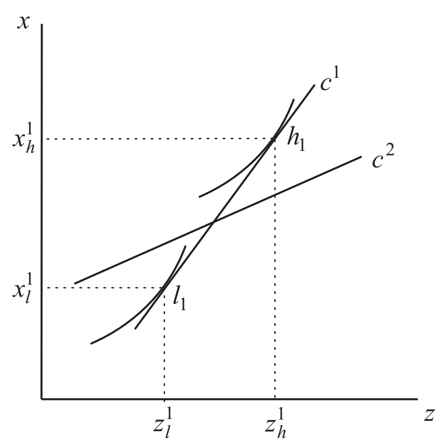


Figure 16.11
Initial position and new tax function

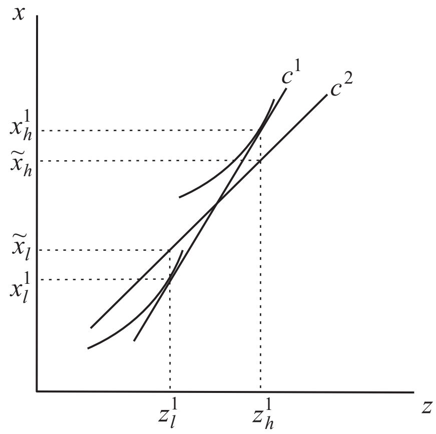


Figure 16.12
Transfer of consumption

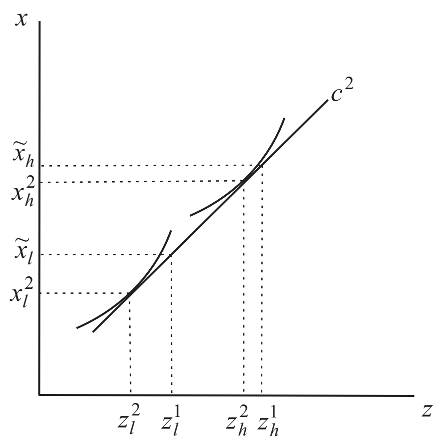


Figure 16.13
Allowing relocation

Being voluntary, the relocation must raise the utilities of both types of consumer. Furthermore the relocations do not affect the budget position of the government, since they are moves along a consumption function with a zero marginal rate of tax.

The fact that social welfare increases in stage 1 and then increases further in stage 2 ensures that consumption function $c^2(z)$ leads to a higher level of social welfare than consumption function $c^1(z)$. The process has also ensured that government revenue is unchanged. Consumption function $c^1(z)$ with a negative marginal rate of tax cannot therefore be optimal. From this it follows that the marginal tax rate must be nonnegative so that $T'(z) \geq 0$. It should be noted that this result does not restrict the average rate of tax, $\frac{T(z)}{z}$, to be nonnegative. The average rate of tax is negative whenever the consumption function is above the 45 degree line, and if the system is redistributive, this will be the case at low incomes.

The final result determines the marginal tax rate faced by the highest skill consumer. Let the consumption function ABC in figure 16.14 be a candidate for optimality. It is now shown that ABC cannot be optimal unless its gradient is one (so the marginal rate of tax is zero) at point B where the highest skill consumer locates. To prove this result, assume that the gradient is less than one (so the marginal tax rate is positive) at point B. A better consumption function than ABC will now be constructed. To do this, define ABD by following the old consumption function up to point B, and then let the new section BD have gradient of one. The highest ability consumer will now relocate to point b. Consequently the highest ability consumer is better off, but their actual tax payment (the vertical distance from the consumption point to the 45 degree line) is

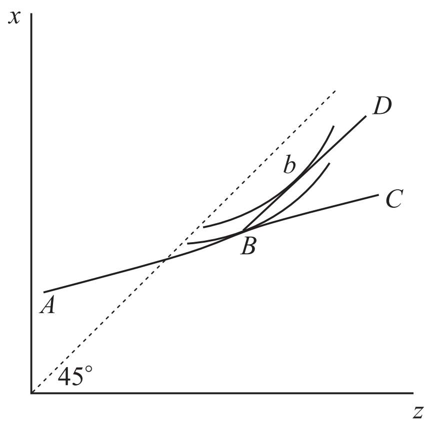


Figure 16.14
Zero marginal rate of tax

unchanged. So replacing ABC with ABD leaves aggregate tax revenue unchanged, makes one person better off, and makes no one worse off. This must be an improvement for society, so no consumption function, like ABC, that has the highest ability person facing a positive marginal rate of tax can be optimal. In other words, the optimal tax function must have a zero marginal rate of tax for the highest skill consumer.

This result is important for assessing the optimality of actual tax schedules. Those observed, in practice, invariably have a marginal rate of tax that rises with income. This leaves the highest income consumers facing the highest marginal tax rate rather than a zero rate. Accordingly such tax systems cannot be optimal. The result also carries implications for discussions about how progressive the income tax system should be. A tax system displays marginal rate progressivity if the marginal rate of tax increases with income. Since it has been shown that the marginal rate of tax should be zero at the top of the income distribution, the optimal tax system cannot be a fully (marginal rate) progressive one.

There has been considerable debate about this result due partly to its contrast with what is observed in practice. There are several points that can be made in this respect. The result is valid only for the highest skill consumer, and it makes no prediction about the tax rate that will be faced by consumer with the second-highest skill. Therefore it does not demonstrate that those close to the top of the skill range should face a tax rate of zero or even close to zero. For them the tax rate may have to be significantly different from zero. If this is the case, observed tax systems may only be "wrong" at the very top, which will not result in too great a divergence from optimality. The result also relies on the fact that the highest skill person can be identified and the tax system adjusted around her needs. Putting this into practice is clearly an impossible task. In summary, the result is important in that it questions preconceptions about the structure of taxes, but it has limited immediate policy relevance.

The results described in this section capture the general properties of an optimal income tax system that can be derived within this framework. They have shown that the marginal tax rate should be between zero and one and that the highest skill consumer should face a zero marginal rate. Moreover they have established that the tax system should not involve marginal rate progressivity everywhere. It is possible to derive further results only by adding further specification. The next section looks at two special cases that give alternative routes for proceeding in this direction. However, even these do not provide entirely transparent insights into the level of optimal tax rates. This can only be done through the use of numerical results, and these are the subject matter of section 16.7.

16.6 Two Specializations

To provide some more insight into the optimal income tax, two specializations of the model are worth considering. These are noteworthy for the very clear view they give into the trade-offs involved in setting the tax rate.

16.6.1 Quasi-Linearity

The first specialization is to consider a special form of the utility function. It is assumed in this section that utility is quasi-linear with respect to labor income,

$$U\left(x, \frac{z}{s}\right) = u(x) - \frac{z}{s}, \tag{16.5}$$

so that the marginal disutility of labor $\ell = \frac{z}{s}$ is constant. The utility of consumption, $u(x)$, is increasing and concave (so $u' > 0$ and $u'' < 0$). For this utility function the marginal rate of substitution between consumption and income is $MRS_{x,z} = \frac{1}{su'(x)}$. Since the marginal rate of substitution is decreasing in s, the gradient of the indifference curve through any value of x falls as s rises. This makes the utility function consistent with agent monotonicity.

We simplify further by assuming that there are just two consumers, one with a high level of skill, s_h, and the other with a low level, s_l. It is assumed that $s_l < s_h < 3s_l$ (the reason for this is explained later). With only two consumers the problem of choosing the optimal tax (or consumption) function can be given the following formulation: whatever consumption function is selected by the government, the fact that there are only two consumers ensures that at most two locations on it will ever be chosen. For example, in figure 16.15, a_l is the allocation chosen by the low-skill consumer and a_h the chosen allocation of the high-skill. Having observed this, it is apparent that selecting the consumption function is equivalent to specifying the two allocations. The rest of the consumption function can then be chosen to ensure that no point on it is better for the consumers than the two chosen allocations. Essentially the consumption function just needs to link the two points, while elsewhere remaining below the indifference curves through the points. Following this reasoning reduces the choice of tax function to a simple maximization problem involving the two locations.

A consumer will only choose the allocation intended for him if he prefers his own location to that of the other consumer. In other words, the allocations must be *incentive compatible*. Since the high-skill consumer can mimic the low-skill, but not vice versa,

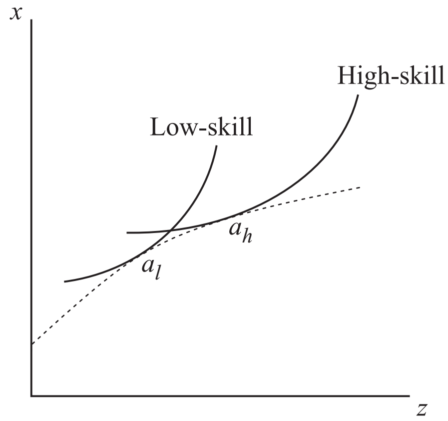


Figure 16.15
Allocations and the consumption function

the incentive compatibility constraint must be binding on the high-skill consumer. Denoting the location intended for low-skill consumer by $\{x_l, z_l\}$ and that for high-skill by $\{x_h, z_h\}$, the incentive compatibility constraint is

$$u(x_h) - \frac{z_h}{s_h} = u(x_l) - \frac{z_l}{s_h}. \tag{16.6}$$

A pair of allocations that satisfy the incentive compatibility constraint (16.6) are shown in figure 16.16. The high-skill consumer is indifferent between the two allocations (a_l and a_h are on the same indifference curve for the high-skill) while the low-skill consumer strictly prefers allocation a_l.

The optimization facing a government that maximizes a utilitarian social welfare function is

$$\max_{\{x_l, x_h, z_l, z_h\}} u(x_l) - \frac{z_l}{s_l} + u(x_h) - \frac{z_h}{s_h} \tag{16.7}$$

subject to the incentive compatibility constraint (16.6) and the resource constraint

$$x_l + x_h = z_l + z_h. \tag{16.8}$$

The resource constraint makes the simplifying assumption that no revenue is to be raised so that the tax system is purely redistributive. What is now shown is that the quasi-linearity of utility allows this maximization problem to be considerably simplified. The simplification then permits an explicit solution to be given.

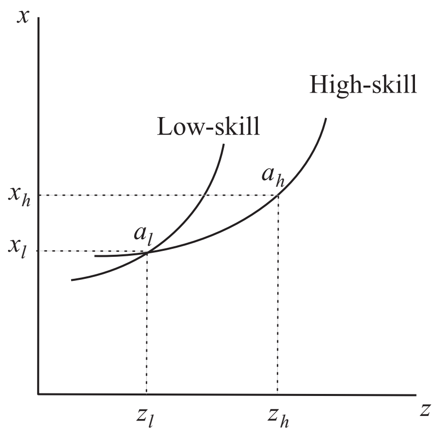


Figure 16.16
Binding incentive compatibility

Given that (16.6) is an equality, it can be solved to write

$$z_h = s_h[u(x_h) - u(x_l)] + z_l. \tag{16.9}$$

Combining this equation with the resource constraint and eliminating z_h by using (16.9), the income of the low-skill consumer can be written

$$z_l = \frac{1}{2}\left[x_l + x_h - s_h[u(x_h) - u(x_l)]\right]. \tag{16.10}$$

Using the resource constraint again gives the income of the high-skill consumer as

$$z_h = \frac{1}{2}\left[x_l + x_h + s_h[u(x_h) - u(x_l)]\right]. \tag{16.11}$$

These solutions for the income levels can then be substituted into the objective function (16.7). Collecting terms shows that the original constrained optimization is equivalent to

$$\max_{\{x_l, x_h\}} \beta_l u(x_l) + \beta_h u(x_h) - \left[\frac{s_l + s_h}{2 s_l s_h}\right][x_l + x_h], \tag{16.12}$$

where $\beta_l = \frac{3s_l - s_h}{2s_l}$ and $\beta_h = \frac{s_l + s_h}{2s_l}$. (The assumption $s_h < 3s_l$ ensures that β_l is greater than zero so that the low-skill consumer has a positive social weight. Without this assumption the analysis becomes more complex.)

Comparing (16.7) and (16.12) allows a new interpretation of the optimal tax problem. The construction undertaken has turned the maximization of the utilitarian social welfare function subject to constraint into the maximization of a weighted welfare function without constraint. The incentive compatibility and resource constraints have been incorporated by placing a greater weight on the welfare of the high-skill consumer (since $\beta_h > \beta_l$), which in turn ensures that their consumption level must be higher at the optimum. From (16.10) and (16.11) this feeds back into a higher level of income for the high-skill consumer. It can also be seen that as the skill difference between the two consumers increases, so does the relative weight given to the high-skill.

Carrying out the optimization in (16.12), the consumption levels of the consumers satisfy the first-order conditions

$$\beta_i u'(x_i) - \frac{s_l + s_h}{2 s_l s_h} = 0, \qquad i = l, h, \tag{16.13}$$

so the consumption levels are proportional to the welfare weights. For the high-skill consumer, substituting in the value of β_h gives

$$u'(x_h) = \frac{1}{s_h}. \tag{16.14}$$

Consequently the marginal utility of the high-skill consumer is inversely proportional to their skill level. With $u'' < 0$ (decreasing marginal utility) this implies that consumption is proportional to skill. Combining this result with the fact that $MRS^h_{x,z} = \frac{1}{su'(x)}$, it follows that at the optimum allocation $MRS^h_{x,z} = 1$. The finding that the marginal rate of substitution is unity shows that the high-skill consumer faces a zero marginal tax rate. This is the no-distortion-at-the-top result we have already seen. For the low-skill consumer

$$u'(x_l) = \frac{s_l + s_h}{s_h\,[3s_l - s_h]}, \tag{16.15}$$

and $MRS^l_{x,z} = \frac{s_h[3s_l - s_h]}{s_l[s_l + s_h]} < 1$. These show that consumer l faces a positive marginal rate of tax.

The use of quasi-linear utility allows the construction of an explicit solution to the optimal income tax problem, which shows how the general findings of the previous section translate into this special case. It is interesting to note the simple dependence of consumption levels on the relative skills and the manner in which the constraints become translated into a higher effective welfare weight for the high-skill consumer. This shows that this consumer needs to be encouraged to supply more labor through the reward of additional consumption.

16.6.2 Rawlsian Taxation

The second specialization restricts the form of the social welfare function. In chapter 13 we introduced the Rawlsian social welfare function, which represents a society that is concerned only with the utility of the worst-off individual. The worst-off person is typically at the bottom of the income distribution and his welfare depends on the extent of redistribution. We now assume that tax revenue is entirely redistributed in the form of lump-sum grants. Consequently, for a Rawlsian government, the optimal income tax is simply that which maximizes the lump-sum grant or, equally, that which maximizes the revenue extracted from taxpayers.

Given a tax schedule $T(z)$, a consumer of skill level s makes the choice of income, z (which is equivalent to choosing labor supply $\ell = \frac{z}{s}$) and consumption x to maximize his utility subject to satisfying the budget constraint $x = z - T(z)$. Let $z = z(s)$ denote the optimal income choice of type s (conditional on the tax function T). It has been seen that agent monotonicity implies that high-skill consumers never earn less income than the low-skill. So $z(s)$ is increasing in s and can be inverted to give the increasing inverse function $s = z^{-1}(z)$ that represents the skill level s associated to each income choice. Different tax schemes will induce different relationships between skill and income from the same underlying distribution of skills.

Assume that skill levels are continuously distributed in the population according to a cumulative distribution function $F(s)$ (indicating the proportion of the population below any skill level s) with associated density function $f(s) > 0$ (representing the probability associated with a small interval of the continuous skill). The tax scheme $T(z)$ induces the income distribution $G(z) = F(z^{-1}(z))$ with density $g(z) = f(z^{-1}(z))$.

Now we are in a position to derive the optimal income tax associated with a Rawlsian social welfare function following a simple method originally proposed by Piketty (1997). The Rawlsian optimal tax structure maximizes tax revenue, so no alternative tax structure must exist that can raise more revenue from the taxpayers given their optimal labor supply response to that new tax structure. From the first-order condition of the revenue maximization problem, a small deviation from the optimal tax scheme must have no effect on total tax revenue (and larger deviations must lower tax revenue). It follows that a small change of the tax rate at any given income level z must not change total revenue. Using this simple argument, we can derive the optimal tax structure.

Take income level z, and consider a small increase in the marginal tax rate at that point of amount $\Delta T'$. This change has two effects on tax revenue. First, holding labor supply constant, it will increase the tax payment by amount $z\Delta T'$ for all those taxpayers with an income above or equal to the level, z, at which the higher marginal tax rate applies. These taxpayers represent a proportion $1 - G(z)$ of the population. Therefore the revenue gain from this marginal tax change is

$$[1 - G(z)]\, z\Delta T'. \tag{16.16}$$

Obviously the labor supply is not fixed, and it is expected to vary in response to a change in the tax rate. Let ε_s denote the elasticity of labor supply with respect to the net price of labor (the percentage change in labor supply in response to a 1 percent reduction in the net price of labor). With perfect competition on the labor market, the price of labor decreases by the amount of the tax rate (i.e., there is no shifting of the tax burden to employers in the form of a higher gross wage). Now the marginal tax rate increase $\Delta T'$ at income level z induces a proportional reduction $\frac{\Delta T'}{1-T'}$ in the price of labor. Those facing this marginal tax rate change will reduce their labor supply by $\varepsilon_s \frac{\Delta T'}{1-T'}$, inducing a reduction of their taxable income equal to $z\varepsilon_s \frac{\Delta T'}{1-T'}$. They represent a proportion $g(z)$ of the population (since those with an income level above the level at which higher marginal tax rate applies continue to face the same marginal tax rate). Therefore the revenue loss associated with the incentive effect of the tax change is

$$[g(z)]\, T' z\varepsilon_s \frac{\Delta T'}{1 - T'}. \tag{16.17}$$

The revenue-maximizing tax scheme (Rawlsian tax) is found by equating the revenue loss to the revenue gain from a marginal tax change for every income level. This yields

$$[1 - G(z)]\, z\Delta T' = [g(z)]\, T' z\varepsilon_s \frac{\Delta T'}{1 - T'}, \tag{16.18}$$

and the Rawlsian tax structure is easily seen to be such that for all income level z,

$$\frac{T'(z)}{1 - T'(z)} = \frac{1 - G(z)}{\varepsilon_s g(z)}. \tag{16.19}$$

This expression has the following interpretation. High marginal tax rates over some middle-income interval $[z, z + dz]$ mean that for those middle-income individuals, but also for the upper-income individuals, the government is collecting more taxes. Altogether, they represent a proportion $1 - G(z)$ that is decreasing with z and converging to zero for the highest income level (hence the zero marginal tax rate at the top). The cost of the high marginal tax rate over this interval is a greater distortion for those with income in the range $[z, z + dz]$. The total distortion (and revenue loss) will be low, however, if there are relatively few taxpayers in this interval (low$g(z)$), or if those in it have a relatively low labor supply elasticity (ε_s).

Interestingly, even though the redistributive motive is maximal under the Rawlsian social welfare function, the optimal tax structure does not require marginal rate progressivity. Indeed we do not really know how the labor supply elasticity changes with income. Suppose that it is constant. Next take the Pareto distribution of income, which is supposed to be a good fit to the empirical distribution of income. For the Pareto distribution, the *hazard rate* $\frac{g(z)}{1-G(z)}$ is increasing almost everywhere. Therefore, from the optimal tax structure given above, it follows that the marginal tax rate must decrease everywhere. Maximal redistribution is better achieved when the tax schedule is regressive (concave) instead of progressive (convex).

16.7 Numerical Results

The standard analysis of optimal income taxation was introduced above, and a number of results were derived that provide some characterization of the shape of the tax schedule. The marginal rate was seen to be between zero and one, but as yet no idea was developed, except for the upper end point, of how close it should be to either. Similarly, although equity considerations are expected to raise the marginal rate, this was not demonstrated formally nor was consideration given to how efficiency criteria, particularly the effect of taxation on labor supply, affects the choice of tax schedule. Because of the analytical complexity of the model, these questions are best addressed via numerical analysis.

The numerical results (from Mirrlees 1971) are based on a social welfare function that takes the form

$$W = \begin{cases} \int_0^S -\frac{1}{\varepsilon} e^{-\varepsilon U} f(s)\, ds, & \varepsilon > 0, \\ \int_0^S U f(s)\, ds, & \varepsilon = 0, \end{cases} \tag{16.20}$$

where S is the maximum level of skill in the population and 0 the lowest, and $f(s)$ is the density function for the skill distribution. The form of this social welfare function permits variations in the degree of concern for equity by changes in ε. Higher values of ε represent greater concern for equity, with $\varepsilon = 0$ representing the utilitarian case. (This is an alternative specification to that of equation 14.24). The individual utility function is the Cobb–Douglas form

$$U = \log(x) + \log(1 - \ell), \tag{16.21}$$

The skill distribution is lognormal with a standard deviation of 0.39. This value of the standard deviation corresponds approximately to a typical value for the income distribution. If the skill distribution matches the income distribution, then this is a value of particular interest.

A selection of the numerical results obtained from this model are given in tables 16.2 and 16.3. In table 16.2, $\varepsilon = 0$, so this is the case of a utilitarian social welfare function. Table 16.3 introduces equity considerations by using $\varepsilon = 1$. In both cases the government revenue requirement is set at 10 percent of national income.

The first fact to be noticed from these results is that the average rate of tax for low-skill consumers is negative. These consumers are receiving an income supplement from the government. This is in the nature of a negative income tax where incomes below a chosen cutoff are supplemented by the government through the tax system. The average rate of tax then increases with skill. The maximum average rate of tax is actually quite small. The value of 34 percent in table 16.3 is not far out of line with the actual rate in many countries.

Table 16.2
Utilitarian case ($\varepsilon = 0$)

Income	Consumption	Average tax (%)	Marginal tax (%)
0	0.03	—	23
0.055	0.07	−34	26
0.10	0.10	−5	24
0.20	0.18	9	21
0.30	0.26	13	19
0.40	0.34	14	18
0.50	0.43	15	16

Table 16.3
Some equity considerations ($\varepsilon = 1$)

Income	Consumption	Average tax (%)	Marginal tax (%)
0	0.05	—	30
0.05	0.08	−66	34
0.10	0.12	−34	32
0.20	0.19	7	28
0.30	0.26	13	25
0.40	0.34	16	22
0.50	0.41	17	20

The behavior of the marginal rate of tax is rather different from that of the average rate. It first rises and then falls. The maximum rate is reached around the median of the skill distribution. Except at the extremes of the skill distribution, there is not much variation in the marginal tax rate. To a first approximation, the optimal tax systems reported in these tables have a basically constant marginal rate of tax so that the consumption function is close to being a straight line. This is one of the most surprising conclusions of the analysis of income taxation: the model allows nonconstancy in the marginal tax rate, but this does not feature to a great degree in the optimal solution. Finally the zero tax rate for the highest skill consumer is reflected in the fall of the marginal rate at high skills, but this is not really significant until close to the top of the skill distribution.

These results provide an interesting picture of the optimal income tax function. They suggest that the tax function should subsidize low-skill consumers through a negative income tax but should still face them with a high marginal rate of tax. The maximum marginal rate of tax should not be at the top of the skill distribution but should occur much lower. Generally, the marginal rate should be fairly constant.

16.8 Voting over a Flat Tax

Having identified the properties of the optimal tax structure, we now consider the tax system that emerges from the political process. To do this, we consider people voting over tax schedules that have some degree of redistribution. Because it is difficult to model voting on nonlinear tax schemes given the high dimensionality of the problem, we will restrict attention to a linear tax structure as originally proposed by Romer (1975). We specify the model further with quasi-linear preferences to avoid unnecessary complications and to simplify the analysis of the voting equilibrium.

Assume, as before, that individuals differ only in their level of skill. We assume that skills are distributed in the population according to a cumulative distribution function $F(s)$ that is known to everyone, with mean skill $\overline{s}$ and median s_m. Individuals work and consume. They also vote on a linear tax scheme that pays a lump-sum benefit b to each individual financed by a proportional income tax at rate t. The individual utility function has the quasi-linear form

$$u\left(x, \frac{z}{s}\right) = x - \frac{1}{2}\left[\frac{z}{s}\right]^2, \tag{16.22}$$

and the individual budget constraint is

$$x = (1 - t)z + b. \tag{16.23}$$

It is easy to verify that in this simple model the optimal income choice of a consumer with skill level s is

$$z(s) = [1 - t]\, s^2. \tag{16.24}$$

The quasi-linear preferences imply that there is no income effect on the labor supply (i.e., $z(s)$ is independent of the lump-sum benefit b). This simplifies the expression of the tax distortion and makes the analysis of the voting equilibrium easier. Less surprisingly, a higher tax rate induces taxpayers to work less and earn less income.

The lump-sum transfer b is constrained by the government budget balance condition

$$b = t\,E(z(s)) = t\,[1 - t]\,E(s^2), \tag{16.25}$$

where $E(\cdot)$ is the mathematical expectation, and we used the optimal income choice to derive the second equality. This constraint says that the lump-sum benefit paid to each individual must be equal to the expected tax payment $t\,E(z(s))$. This expression is termed the *Dupuit–Laffer curve* and describes tax revenue as a function of the tax rate. In this simple model the Dupuit–Laffer curve is bell-shaped with a peak at $t = \frac{1}{2}$ and no tax collected at the ends $t = 0$ and $t = 1$. We can now derive individual preferences over tax schedules by substituting (16.23) and (16.24) into (16.22). After re-arrangement, (indirect) utility can be written as

$$v(t, b, s) = b + \frac{1}{2}\,[1 - t]^2\, s^2. \tag{16.26}$$

Taking the total differential of (16.26) gives

$$dv = db - [1 - t]\, s^2 dt, \tag{16.27}$$

so that along an indifference curve where $dv = 0$,

$$\frac{db}{dt} = [1 - t]\, s^2. \tag{16.28}$$

It can be seen from this that for given t, the indifference curve becomes steeper in (t, b) space as s increases. This monotonicity is a consequence of the single-crossing property of the indifference curves. As we saw in chapter 11, the single-crossing property is a sufficient condition for the Median Voter Theorem to apply. It follows that there is only one tax policy that can result from majority voting: it is the policy preferred by the median voter (half of the voters are poorer than the median and prefer higher tax rates, and the other half are richer and prefer lower tax rates). Letting t_m be the tax rate preferred by the median voter, we have t_m implicitly defined by the solution to the first-order condition for maximizing the median voter's utility. We differentiate (16.26) with respect to t, taking into account the government budget constraint (16.25) to obtain

$$\frac{\partial v}{\partial t} = [1 - 2t]\, E(s^2) - [1 - t]\, s^2. \tag{16.29}$$

Setting this expression equal to zero for the median skill level s_m yields the tax rate preferred by the median voter

$$t_m = \frac{E(s^2) - s_m^2}{2E(s^2) - s_m^2}, \tag{16.30}$$

or, using the optimal income choice (16.24),

$$t_m = \frac{E(z) - z_m}{2E(z) - z_m}. \tag{16.31}$$

This simple model predicts that the political equilibrium tax rate is determined by the position of the median voter in the income distribution. The greater the income inequality, as measured by the distance between median and mean income, the higher is the tax rate. If the median voter is relatively badly off, with income well below the mean income, then equilibrium redistribution is large. In practice, the income distribution has a median income below the mean income, so a majority of voters would favor redistribution through proportional income taxation. More general utility functions would also predict that the extent of this redistribution decreases with the elasticity of labor supply.

16.9 Conclusions

This chapter introduced the issues surrounding the design of the income tax structure. It was first shown how income and substitution effects left the theoretical impact of a tax increase on labor supply indeterminate. If the income effect is sufficiently strong, it is possible for a tax increase to lead to more labor being supplied. The participation decision was also discussed, and it was argued that taxation could be significant in affecting this choice.

The lack of theoretical predictions places great emphasis on empirical research for determining the actual effects of taxation. The labor-supply responses of different groups in the population to tax changes were discussed. The observations made were borne out by the empirical results that showed a very small or negative elasticity of supply for married men but a much large positive elasticity for single mothers. The latter can be interpreted as a reflection of the participation decision.

A model that was able to incorporate the important issues of efficiency and equity in income taxation was then introduced. A number of results were derived that capture the general features of an optimal tax system. Notably, the marginal rate of tax facing the highest skill person should be zero and the optimal tax rate is bounded between zero and one. This model was specialized to quasi-linear utility and to a Rawlsian social welfare function, and some further insights were obtained. The numerical simulation results showed the marginal rate of tax to remain fairly constant while the average rate of tax was negative for low-skill consumers. Finally the political economy of taxation was presented by means of a simple model of voting over linear income tax schedules.

Further Reading

For a comprehensive survey of recent income tax policy in the United States see:

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Mirrlees, J. A. 1971. An exploration in the theory of optimum income tax. *Review of Economic Studies* 38: 175–208.

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The case of Rawlsian taxation is analyzed in:

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Snyder, J., and Kramer, G. 1988. Fairness, self-interest and the politics of the progressive ;income tax. *Journal of Public Economics* 36: 197–230.

A comprehensive and more advanced presentation of the optimal taxation theory is:

Salanie, B. 2003. *Economics of Taxation*. Cambridge: MIT Press.

A bargaining approach to the income tax problem is in:

Aumann, R. J., and Kurz, M. 1977. Power and taxes. *Econometrica* 45: 1137–61.

For the relationship between existing income tax systems and the equal sacrifice principle, see:

Young, H. P. 1990. Progressive taxation and equal sacrifice. *American Economic Review* 80: 2531–66.

Exercises

16.1 Consider the budget constraint $x = b + [1 - t]w\ell$. Provide an interpretation of b. How does the average rate of tax change with income? Let utility be given by $U = x - \ell^2$. How is the choice of ℓ affected by increases in b and t? Explain these effects.

16.2 Assume that a consumer has preferences over consumption and leisure described by $U = x[1 - \ell]$, where x is consumption and ℓ is labor. For a given wage rate w, which leads to a higher labor supply: an income tax at constant rate t or a lump-sum tax T that raises the same revenue as the income tax?

16.3 Let the utility function be $U = \log(x) - \ell$. Find the level of labor supply if the wage rate, w, is equal to \$10. What is the effect of the introduction of an overtime premium that raises w to \$12 for hours in excess of that worked at the wage of \$10?

16.4 An individual has to choose her division of time, L, between labor, ℓ, and leisure, $L - \ell$. Her hourly wage is w.

a. Use a diagram to show how optimal choice without taxation.

b. Show how the choice changes when an income tax at rate t is introduced. Identify the income and substitution effects caused by the income tax. What is the total effect on labor supply?

c. Express the revenue raised by the income tax in terms of units of consumption.

d. What happens to consumption and the labor supply if a lump-sum tax is introduced instead of the income tax? Assume that the lump sum tax raises the same revenue (in units of consumption) as the income tax.

e. Use parts c and d to show the excess of burden of the income tax.

16.5 Assume that utility is $U = \log(x) + \log(1 - \ell)$. Calculate the labor-supply function. Explain the form of this function by calculating the income and substitution effects of a wage increase.

16.6 (Stern 1976) The utility function of a consumer has the constant elasticity of substitution form

$$U = \left[\alpha\,(T - \ell)^{-\mu} + (1 - \alpha)\,x^{-\mu}\right]^{-1/\mu}.$$

Let the budget constraint be $x = b + w\ell$, where $b \geq 0$ is a lump-sum grant received from the government.

a. Show that the first-order condition for utility maximization can be written as

$$\left[\frac{(b - \mu w\ell)}{b + w\ell}\right]^{\mu+1} = \frac{\alpha}{(1 - \alpha)\,\ell}.$$

b. Totally differentiate the first-order condition to find $\frac{d\ell}{dw}$. Under what conditions is this negative?

16.7 Show that a tax function is average-rate progressive (the average rate of tax rises with income) if $MRT > ART$.

16.8 Which is better: a uniform tax on consumption or a uniform tax on income?

16.9 Consider the utility function $U = x - \ell^2$.

a. For $U = 10$, plot the indifference curve with ℓ on the horizontal axis and x on the vertical axis.

b. Now define $z = s\ell$. For $s = 0.5$, 1, and 2 plot the indifference curves for $U = 10$ with z on the horizontal axis and x on the vertical.

c. Plot the indifference curves for $s = 0.5$, 1, and 2 through the point $x = 20$, $z = 2$.

d. Prove that at any point (x, z) the indifference curve of a high-skill consumer is flatter than that of a low-skill.

16.10 Consider an economy with two consumers who have skill levels $s^1 = 1$ and $s^2 = 2$ and utility function $U = 10x - \ell^2$. Let the government employ an income tax function that leads to the allocation $x = 4$, $z = 5$ for the consumer of skill $s = 1$ and $x = 9$, $z = 8$ for the consumer of skill $s = 2$.

a. Show that this allocation satisfies the incentive compatibility constraint that each consumer must prefer his allocation to that of the other.

b. Keeping incomes fixed, consider a transfer of 0.01 units of consumption from the high-skill to the low-skill consumer.

i. Calculate the effect on each consumer's utility.

ii. Show that the sum of utilities increases.

iii. Show that the incentive compatibility constraint is still satisfied.

iv. Use parts i through iii to prove that the initial allocation is not optimal for a utilitarian social welfare function.

16.11 For the utility function $U = x - \ell^2$ and two consumers of skill levels s_1 and s_2, $s_2 > s_1$, show that the incentive compatibility constraints imply that the income and consumption levels of the high-skill consumer cannot be lower than those of the low-skill consumer.

16.12 Assume that skill is uniformly distributed between 0 and 1 and that total population size is normalized at 1. If utility is given by $U = \log(x) + \log(1 - \ell)$ and the budget constraint is $x = b + (1 - t)s\ell$, find the optimal values of b and t when zero revenue is to be raised. Is the optimal tax system progressive?

16.13 Consider the tax function $T(z) = az+bz^2-c$, with consumption given by $x = z-az+bz^2+c$.

a. For a consumer of skill s, find the level of income that maximizes $U = x - \ell^2$. (Hint: Substitute for x using the above and for ℓ using $z = s\ell$.)

b. Hence calculate labor supply and describe its relation to s.

c. Show that at income level z, the marginal rate of tax is $a + 2bx$.

d. Substitute the answer from part b into the expression for the marginal rate of tax and calculate the limiting marginal tax rate as $z \to \infty$.

e. Use the answer to part d to show that the tax function is not optimal.

16.14 Consider an economy with two consumers of skill levels s^1 and s^2, $s^2 > s^1$. Denote the allocation to the low-skill consumer by $\{x^1, z^1\}$ and that to the high-skill consumer by $\{x^2, z^2\}$.

a. For the utility function $U = u(x) - \frac{z}{s}$ show that incentive compatibility requires that $z^2 = z^1 + \left[u\left(x^2\right) - u\left(x^1\right)\right]$.

b. For the utilitarian social welfare function

$$W = u\left(x^1\right) - \frac{z^1}{s^1} + u\left(x^2\right) - \frac{z^2}{s^2},$$

express W as a function of x^1 and x^2 alone.

c. Assuming $u\left(x^h\right) = \log x^h$, derive the optimal values of x^1 and x^2 and hence of z^1 and z^2.

d. Calculate the marginal rate of substitution for the two consumers at the optimal ;allocation. Comment on your results.

16.15 How is the analysis of section 16.6.1 modified if $s^2 > 3s^1$? (*Hint:* Think about what must happen to the before-tax income of consumer 1.)

16.16 Suppose two types of consumers with skill levels 10 and 20. There is an equal number of consumers of both types. If the social welfare function is utilitarian and no revenue is to be raised, find the optimal allocation under a nonlinear income tax for the utility function $U = \log(x) - \ell$. Contrast this to the optimal allocation if skill is publicly observable.

16.17 Tax revenue is given by $R(t) = tB(t)$, where $t \in [0, 1]$ is the tax rate and $t \in [0, 1]$ is the tax base. Suppose that the tax elasticity of the tax base is $\varepsilon = -\frac{\gamma t}{1-\gamma t}$ with $\gamma \in [\frac{1}{2}, 1]$.

a. What is the revenue-maximizing tax rate?

b. Graph tax revenue as a function of the tax rate both for $\gamma = \frac{1}{2}$ and $\gamma = 1$. Discuss the implications of this Dupuit–Laffer curve.

16.18 Consider an economy populated by a large number of workers with utility function $U = x^{\alpha}[1 - \ell]^{1-\alpha}$, where x is disposable income, ℓ is the fraction of time worked ($0 \leq \ell \leq 1$), and α is a preference parameter (with $0 < \alpha < 1$). Each worker's disposable income depends on his fixed "skill" as represented by wage w and a tax-transfer scheme (t, B) so that $x = B + [1 - t]w\ell$, where $t \in (0, 1)$ is the marginal tax rate and $B > 0$ is the unconditional benefit payment.

a. Find the optimal labor supply for someone with ability w. Will the high-skill person work more than the low-skill person? Will the high-skill person have higher disposable income than the low-skill person? Show that the condition for job market participation is $w > \frac{1-\alpha}{\alpha}\frac{B}{1-t}$.

b. If tax proceeds are only used to finance the benefit B, what is the government's budget constraint?

c. Suppose that the mean skill in the population is $\overline{w}$ and that the lowest skill is a fraction $\gamma < 1$ of the mean skill. If the government wants to redistribute all tax proceeds to finance the cash benefit B, what condition should the tax-transfer scheme (t, B) satisfy?

d. Find the optimal tax rate if the government seeks to maximize the disposable income of the lowest skill worker subject to everyone working.

16.19 Consider an economy where each consumer has one of two skill levels. The low skill level is s_1 and the high skill level is s_2. There are N consumers in total and n_i have skill level i. Each consumer has preferences given by $U = x - \ell^2$.

a. State the incentive compatibility constraints for the economy. Which will bind at the optimum?

b. Assuming that no revenue is to be collected, write down the optimization describing the optimal allocation for a utilitarian social welfare function.

c. Show that the incentive compatibility constraint and the production constraint can be solved to express x_1 and x_2 in terms of incomes and skills.

d. Use these solutions to obtained a reduced form for the optimization problem and solve.

e. Discuss the effect of changing the relative population proportions.

16.20 Under the negative income tax all individuals are guaranteed a minimum standard of living by being awarded a grant, and the grant is reduced as their earnings rise (though by less than one for one). Alternatively, under wage subsidies, for each dollar of earnings up to some level, the government pays each person a refundable tax credit for each dollar earned up to that level. This tax credit is then phased out after reaching a maximum, so the credit goes to zero for middle-income taxpayers.

a. Compare the work incentives of the wage subsidy and the negative income tax for the entire income distribution. Use a diagram and explain.

b. Assume that the poverty line is fixed at \$20,000. Design a negative income tax to combat poverty by choosing a basic grant level and an implicit tax rate at which this grant is reduced as incomes rise. What are the trade-offs involved in setting the grant level and tax rate? What are the efficiency and equity effects of choosing different grant levels and tax rates? How will the program affect people with different incomes?

c. Now consider the possibility of using categorical welfare grants. Under categorical welfare grants all individuals possessing certain characteristics are guaranteed a minimum standard of living, and the grant is taken away one for one as income rises. How should the government choose the right categories for targeting grants to some welfare groups?

d. What are the advantages and disadvantages of categorical grants relative to a negative income tax?

16.21 Consider a single mother with the utility function $U = \frac{2}{3}\log(x) + \frac{1}{3}\log(l)$, where x is consumption and l is leisure. The mother can work up to 100 hours per month. Any of the 100 hours that are not worked are leisure hours. She earns a wage of \$10 per hour and pays no taxes. The consumption price is normalized to \$1. To be able to work, she has to incur a child care cost of \$5 for every hour worked.

a. Suppose that there are no tax and welfare benefits. How many hours will she work and what will be her consumption level? Draw the graph depicting her budget set, with consumption on the vertical axis and leisure on the horizontal axis.

b. Suppose that the government introduces a negative income tax (NIT) that guarantees an income of \$200 per month. The benefit is taken away one for one as earnings increase. Draw the new budget set. Compute the new number of hours worked and consumption level. Has consumption increased and is the mother better off? Why or why not?

c. Now suppose the income guarantee is reduced by one-half to the amount of \$100 per month. What is the new number of hours worked and the consumption level? Compare with your result in part b.

d. Consider again the income guarantee in part b of \$200 per month, and suppose that the government complements this benefit by offering free child care. Draw the new budget set, and calculate the number of hours worked and consumption level. Calculate the total cost of the program for the government. How does it compare with the program in part b? Define program efficiency as the ratio of the mother's consumption to government expenditure. Which program dominates on efficiency grounds?

16.22 (Stern 1976) The numerical analysis of income taxation has often assumed that the skill distribution is identical to the income distribution. This need not be the case. Assume that the utility function is defined as

$$U(x, \ell) = \begin{cases} 1 - \ell, & \text{if } x \geq \bar{x}, \\ -\infty & \text{otherwise.} \end{cases}$$

a. Provide an interpretation of this utility function.

b. Assume that the skill level of all consumers is strictly positive. What will be the observed distribution of income?

c. Will this distribution be identical to the skill distribution?

d. Comment on the practice of assuming that the income and skill distributions are identical.

16.23 Suppose that income is fixed and distributed according to $F(y)$ on the interval $[0, Y]$, with the median income below the average income, $y_m \leq \overline{y}$. Consider the quadratic tax function $t(y) = -c + by + ay^2$ with $c > 0$ the uniform grant, b the proportionality parameter, and $a > 0$ ($a < 0$) the progressivity (regressivity) parameter. Feasible taxation requires $t(y) \leq y$ for all $y \in [0, Y]$. Suppose that taxation is purely redistributive so that budget balance requires $\int t(y)dF(y) = 0$. With the quadratic tax schedule $t(y) = -c + by + ay^2$, the budget balance requirement implies that $c = b\overline{y} + a\overline{y}^2$, where $\overline{y}_2$ is the average of square income levels $\int y^2 dF(y)$. Show that for any feasible tax scheme $t_2 = -c_2 + b_2 y + a_2 y^2$ there exists a feasible more progressive (less regressive) tax scheme $t_1 = -c_1 + b_1 y + a_1 y^2$ with $a_1 > a_2$, $b_1 < (>) b_2$ and $c_1 > c_2$ such that more than half the voters would prefer t_1 to t_2.

16.24 (Joint taxation) In some countries the tax unit is the individual. In other countries, such as France, the tax unit is the household. Consider a two-person household where the first earner earns w_f and the second earner earns w_s. In an individual system, the household pays tax of amount $T_I(w_f, w_s) = t(w_f) + t(w_s)$. In a joint tax system, the household pays $T_j(w_f, w_s) = 2t\left(\frac{w_f + w_s}{2}\right)$.

a. Bill and Jody live together. Bill earns 30 and Jody earns 10. The tax system $t(w)$ has two brackets with a tax rate of 10 percent for income between 0 and 20 and a tax rate of 40 percent for income above 20. What would be the average tax rate of the household if the tax system is individual? What if it is a joint system?

b. Show that for all (w_f, w_s), $T_J(w_f, w_s) \leq T_I(w_f, w_s)$. Explain why this is true for any convex and increasing tax function.

c. Taxation schemes are often based on the idea of *horizontal equity*: households having the same ability to pay taxes (represented by taxable income) should pay a similar amount of tax. John is single and earns 40. What is John's average tax rate? Do you think that the individual taxation system respect the horizontal equity principe? And what about the joint taxation system?

16.25 (Kleven, Kreiner, and Saez 2009) Consider the previous exercise and assume that the first earner works full time and earns $w_f = \overline{w}$. The second earner chooses her labor supply $l \in [0, 1]$ and earns $w_s = \underline{w}l$, with $\underline{w} \leq \overline{w}$.

a. Show that for all increasing and weakly convex tax function $t(w)$, the marginal tax rate of the second earner is greater with a joint tax system than with an individual tax system.

b. What is the expected impact of the joint tax system on the labor supply of the second earner compared to the individual tax system? Is it possible to deduce the impact of joint taxation on the well-being of the household?