6 Public Goods

6.1 Introduction

When a government provides a level of national defense sufficient to make a country secure, all inhabitants are simultaneously protected. Equally, when a radio program is broadcast, it can be received simultaneously by all listeners in range of the transmitter. The possibility for many consumers to benefit from a single unit of provision violates the assumption of the private nature of goods underlying the efficiency analysis of chapter 2. The Two Theorems relied on all goods being private in nature, so they can only be consumed by a single consumer. If there are public goods such as national defense in the economy, then market failure occurs and the unregulated competitive equilibrium will fail to be efficient. This inefficiency implies that there is a potential role for government intervention.

The chapter begins by defining a public good, and distinguishing between public goods and private goods. Doing so provides considerable insight into why market failure arises when there are public goods. The inefficiency is demonstrated by analyzing the equilibrium that is achieved when it is left to the market to provide public goods. The Samuelson rule characterizing the efficient provision of a public good is then derived. This permits a comparison of the equilibrium and with an efficient allocation.

The focus of the chapter next turns to the consideration of methods through which efficiency can be achieved. The first of these, the Lindahl equilibrium, is based on the observation that the price each consumer pays for the public good should reflect their valuation of it. The Lindahl equilibrium achieves efficiency, but since the valuations are private information, it generates incentives for consumers to provide false information. Mechanisms designed to elicit the correct statement of these valuations are then considered. The theoretical results are contrasted with the outcomes of experiments designed to test the extent of false statement of valuations and the use of market data to calculate valuations. These results are primarily static in nature. To provide some insight into the dynamic aspects of public good provision, the chapter is completed by the analysis of two different forms of fund-raising campaign that permit sequential contributions.

6.2 Definitions

The *pure public good* has been the subject of most economic analyses of public goods. In many ways the pure public good is an abstraction that is adopted to provide a benchmark case against which other, more realistic, cases can be assessed. A pure public good has the following two properties:

- *Nonexcludability* If the public good is supplied, no consumer can be excluded from consuming it.
- *Nonrivalry* Consumption of the public good by one consumer does not reduce the quantity available for consumption by any other.

In contrast, a *private good* is excludable at no cost and is perfectly rivalrous: if it is consumed by one person, then none of it remains for any other. Although they were not made explicit, these properties of a private good have been implicit in how we have analyzed market behavior in earlier chapters. As we will see, the efficiency of the competitive economy is dependent on them.

The two properties that characterize a public good have important implications. Consider a firm that supplies a pure public good. Since the good is nonexcludable, if the firm supplies one consumer, then it has effectively supplied the public good to all. The firm can charge the initial purchaser but cannot charge any of the subsequent consumers. This prevents it from obtaining payment for the total consumption of the public good. The fact that there is no rivalry in consumption implies that the consumers should have no objection to multiple consumption. These features prevent the operation of the market equalizing marginal valuations as it does to achieve efficiency in the allocation of private goods.

In practice, it is difficult to find any good that perfectly satisfies both the conditions of nonexcludability and nonrivalry precisely. For example, the transmission of a television signal will satisfy nonrivalry, but exclusion is possible at finite cost by scrambling the signal. Similar comments apply, for example, to defense spending, which will eventually be rivalrous as a country of fixed size becomes crowded and from which exclusion is possible by deportation. Most public goods eventually suffer from congestion when too many consumers try to use them simultaneously. For example, parks and roads are public goods that can become congested. The effect of congestion is to reduce the benefit the public good yields to each user. Public goods that are excludable, but at a cost, or suffer from congestion beyond some level of use are called *impure*. The properties

	Rivalrous	Non-rivalrous
Excludable	Private good	Club good
Non-excludable	Common property resource	Public good

Figure 6.1
Typology of goods

of impure public goods place them between the two extremes of private goods and pure public goods.

A simple diagram summarizing the different types of good and the names given to them is shown in figure 6.1. These goods vary in the properties of excludability and rivalry. In fact, it is helpful to envisage a continuum of goods that gradually vary in nature as they become more rivalrous or more easily excludable.

The pure private good and the pure public good have already been identified. An example of a common property good is a lake that can be used for fishing by anyone who wishes, or a field that can be used for grazing by any farmer. This class of goods (usually called *the commons*) is studied in chapter 8. The problem with the commons is the tendency of overusing them, and the usual solution is to establish property rights to govern access. This is what happened in the sixteenth century in England where common land was enclosed and became property of the local landlords. The landlords then charged grazing fees, and so cut back the use. In some instances property rights are hard to define and enforce, as is the case of the control over the high seas or air quality. For this reason only voluntary cooperation can solve the international problems of overfishing, acid rain, and the greenhouse effect. Club goods are public goods for which exclusion is possible. The terminology is motivated by sport clubs whose facilities are a public good for members but from which nonmembers can be excluded. Clubs are studied in chapter 7.

6.3 Private Provision

Public goods do not conform to the assumptions required for a competitive economy to be efficient. Their characteristics of nonexcludability and nonrivalry lead to the wrong incentives for consumers. Since they can share in consumption, each consumer has an incentive to rely on others to make purchases of the public good. This reliance on others to purchase is call *free-riding*, and it is this that leads to inefficiency.

To provide a model that can reveal the motive for free-riding and its consequences, consider two consumers who have to allocate their incomes between purchases of a private good and of a public good. Assume that the consumers take the prices of the two goods as fixed when they make their decisions. If the goods were both private, we could move immediately to the conclusion that an efficient equilibrium would be attained. What makes the public good different is that each consumer derives a benefit from the purchases of the other. This link between the consumers, which is absent with private goods, introduces strategic interaction into the decision processes. With the strategic interaction the consumers are involved in a game, so equilibrium is found using the concept of a Nash equilibrium.

The consumers have income levels M^1 and M^2. Income must be divided between purchases of the private good and the public good. Both goods are assumed to have a price of 1. With x^h used to denote purchase of the private good by consumer h and g^h to denote purchase of the public good, the choices must satisfy the budget constraint $M^h = x^h + g^h$. The link between consumers comes from the fact that the consumption of the public good for each consumer is equal to the total quantity purchased, $g^1 + g^2$. Hence, when making the purchase decision, each consumer must take account of the decision of the other.

This interaction is captured in the preferences of consumer h by writing the utility function as

$$U^h\left(x^h,\ g^1 + g^2\right). \tag{6.1}$$

The standard Nash assumption is now imposed that each consumer takes the purchase of the other as given when they make their decision. By this assumption, consumer 1 chooses g^1 to maximize utility given g^2, while consumer 2 chooses g^2 given g^1. This can be expressed by saying that the choice of consumer 1 is the best reaction to g^2 and that of consumer 2 the best reaction to g^1. The Nash equilibrium occurs when these reactions are mutually compatible, so that the choice of each is the best reaction to the choice of the other.

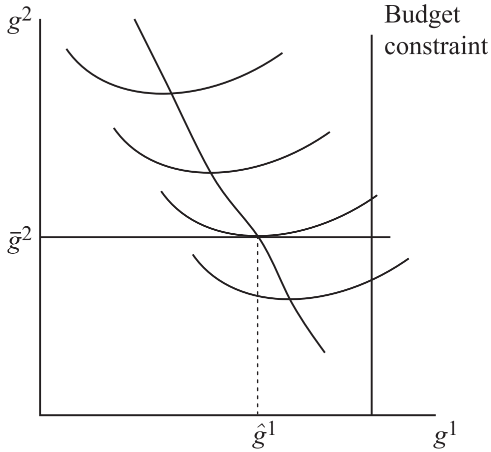

Figure 6.2
Preferences and choice

The Nash equilibrium can be displayed by analyzing the preferences of the two consumers over different combination of g^1 and g^2. Consider consumer 1. Using the budget constraint, we can write their utility as $U^1(M^1 - g^1, g^1 + g^2)$. The indifference curves of this utility function are shown in figure 6.2. These can be understood by noting that an increase in g^2 will always lead to a higher utility level for any value of g^1. For given g^2, an increase in g^1 will initially increase utility as more preferred combinations of private and public good are achieved. Eventually, further increases in g^1 will reduce utility as the level of private good consumption becomes too small relative to that of public good. The income level places an upper limit upon g^1.

Consumer 1 takes the provision of 2 as given when making their choice. Consider consumer 2 having chosen $\overline{g}^2$. The choices open to consumer 1 then lie along the horizontal line drawn at $\overline{g}^2$ in figure 6.2. The choice that maximizes the utility of consumer 1 occurs at the tangency of an indifference curve and the horizontal line—this is the highest indifference curve they can reach. This is shown as the choice $\hat{g}^1$. In the terminology we have chosen, $\hat{g}^1$ is the best reaction to $\overline{g}^2$. Varying the level of $\overline{g}^2$ will lead to another best reaction for consumer 1. Doing this for all possible $\overline{g}^2$ traces out the optimal choices of g^1 shown by the locus through the lowest point on each indifference curve. This locus is known as the *Nash reaction function* (or *best-response function*) and depicts the value of g^1 that will be chosen in response to a value of g^2. This construction can be repeated for consumer 2 and leads to figure 6.3. For consumer 2, utility increases with g^1, and thus indifference curves further to the right reflect

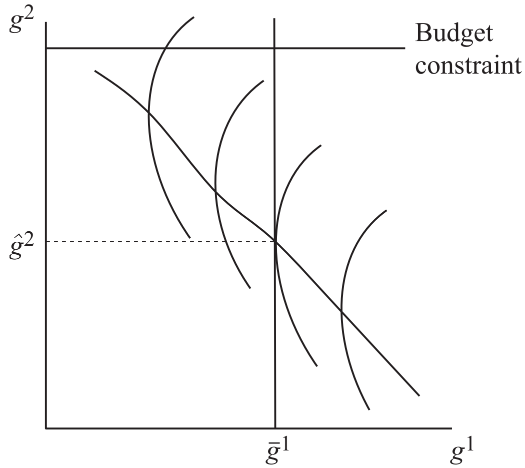

Figure 6.3
Best reaction for consumer 2

higher utility levels. The best reaction for consumer 2 is shown by $\hat{g}^2$, which occurs where the indifference curve is tangential to the vertical line at $\overline{g}^1$. The Nash reaction function links the points where the indifference curves are vertical.

The Nash equilibrium occurs where the choices of the two consumers are the best reactions to each other, so neither has an incentive to change their choice. This can only hold at a point where the Nash reaction functions cross. The equilibrium is illustrated in figure 6.4 in which the reaction functions are simultaneously satisfied at their intersection. By definition, $\hat{g}^1$ is the best reaction to $\hat{g}^2$ and $\hat{g}^2$ is the best reaction to $\hat{g}^1$. The equilibrium is privately optimal: if a consumer were to unilaterally raise or reduce his purchase, then he would move to a lower indifference curve.

Having determined the equilibrium, its welfare properties can now be addressed. From the construction of the reaction functions, it follows that at the equilibrium the indifference curve of consumer 1 is horizontal and that of consumer 2 is vertical. This is shown in figure 6.5. It can be seen that all the points in the shaded area are Pareto-preferred to the equilibrium—moving to one of these points will make both consumers better off. Starting at the equilibrium, these points can be achieved by both consumers simultaneously raising their purchase of the public good. The Nash equilibrium is therefore not Pareto-efficient, although it is privately efficient. No further Pareto improvements can be made when a point is reached where the indifference curves are tangential. The locus of these tangencies, which constitutes the set of Pareto-efficient allocations, is also shown in figure 6.5.

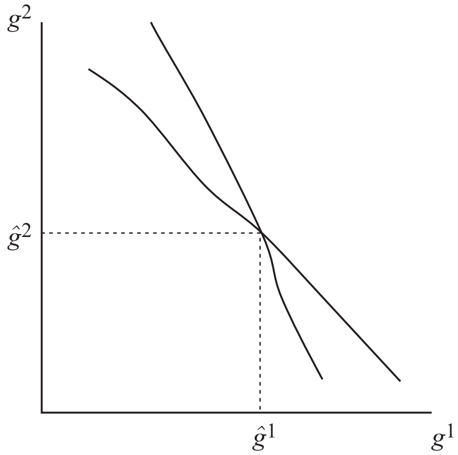

Figure 6.4
Nash equilibrium

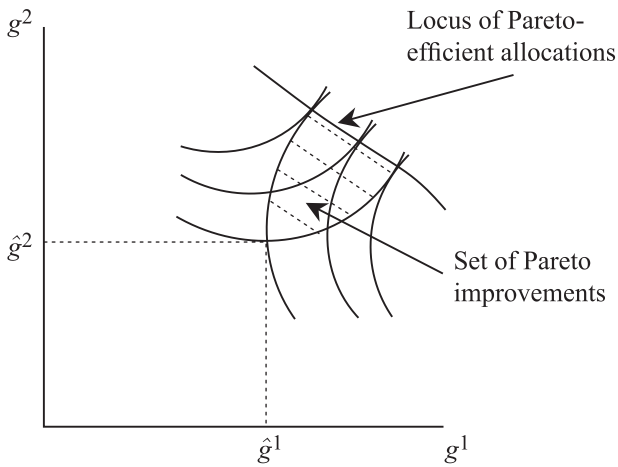

Figure 6.5
Inefficiency of equilibrium

The analysis has demonstrated that when individuals privately choose the quantity of the public goods they purchase, the outcome is Pareto-inefficient. A Pareto improvement can be achieved by all consumers increasing the purchases of public goods. Consequently, compared to Pareto-preferred allocations, the total level of the public good consumed is too low. Why is this so? The answer can be attributed to strategic interaction and the free-riding that results. The free-riding emerges from each consumer relying on the other to provide the public good and thus avoiding the need to provide for themselves. Since both consumers are attempting to free-ride in this way, too little of the public good is ultimately purchased. In the absence of government intervention or voluntary cooperation, inefficiency arises.

6.4 Efficient Provision

Efficiency in consumption for private goods is guaranteed by each consumer equating their marginal rate of substitution to the price ratio. The strategic interaction inherent with public goods does not ensure such equality. At a Pareto-efficient allocation with the public good, the indifference curves are tangential. However, this does not imply equality of the marginal rates of substitution because the indifference curves are defined over quantities of the public good purchased by the two consumers. As will soon be shown, the efficiency condition involves the sum of marginal rates of substitution and is termed the *Samuelson rule* in honor of its discoverer.

The basis for deriving the Samuelson rule can be observed in figure 6.5, where the locus of Pareto-efficient allocations has the property that the indifference curves of the two consumers are tangential. The gradient of these indifference curves is given by the rate at which g^2 can be traded for g^1 while keeping utility constant. The tangency conditions can then be expressed by requiring that the gradients are equal so that

$$\frac{dg^2}{dg^1}|_{U^1 const.} = \frac{dg^2}{dg^1}|_{U^2 const.}. \tag{6.2}$$

Calculating the derivatives using the utility functions (6.1), we write the efficiency condition (6.2) as

$$\frac{U_x^1 - U_G^1}{U_G^1} = \frac{U_G^2}{U_x^2 - U_G^2}. \tag{6.3}$$

The marginal rate of substitution between the private and the public good for consumer h is defined by $MRS^h_{G,x} \equiv \frac{U^h_G}{U^h_x}$. This can be used to rearrange (6.3) in the form

$$\left[\frac{1}{MRS^1_{G,x}} - 1\right]\left[\frac{1}{MRS^2_{G,x}} - 1\right] = 1. \tag{6.4}$$

Multiplying across by $MRS^1_{G,x} \times MRS^2_{G,x}$, we solve (6.4) and get the final expression

$$MRS^1_{G,x} + MRS^2_{G,x} = 1. \tag{6.5}$$

This is the two-consumer version of the Samuelson rule.

To interpret this rule, the marginal rate of substitution should be viewed as a measure of the marginal benefit of another unit of the public good. The marginal cost of a unit of public good is one unit of private good. Therefore the rule says that an efficient allocation is achieved when the total marginal benefit of another unit of the public good, which is the sum of the individual benefits, is equal to the marginal cost of another unit. The rule can easily be extended to incorporate additional consumers: the total benefit remains the sum of the individual benefits.

Further insight into the Samuelson rule can be obtained by contrasting it with the corresponding rule for efficient provision of two private goods. For two consumers, 1 and 2, and two private goods, i and j, this is

$$MRS^1_{i,j} = MRS^2_{i,j} = MRT_{i,j}, \tag{6.6}$$

where $MRT_{i,j}$ denotes the marginal rate of transformation, the number of units of one good the economy has to given up to obtain an extra unit of the other good. (The $MRT_{G,x}$ between public and private good was assumed to be equal to 1 in the derivation of the Samuelson rule in equation 6.5.) The difference between (6.5) and (6.6) arises because an extra unit of the public good increases the utility of all consumers so that the social benefit of this extra unit is found by summing the marginal benefits. This does not require equalization of the marginal benefit of all consumers. In contrast, an extra unit of private good can only be given to one consumer or another. Efficiency then occurs when it does not matter who the extra unit is given to so that the marginal benefits of all consumers are equalized.

The Samuelson rule provides a very simple description of the efficient outcomes, but this does not mean that efficiency is easily obtained. It was already shown that efficiency will not be achieved if there is no government intervention and agents act noncooperatively (i.e., adopt Nash behavior). But what form should government intervention take? The most direct solution would be for the government to take total responsibility for provision of the public good and to finance it through lump-sum taxation. Because lump-sum taxes do not cause any distortions, this would ensure satisfaction of the rule. However, there are numerous difficulties in using lump-sum taxation, which will be explored in detail in chapter 13. The same shortcomings apply here, thus ruling out the employment of lump-sum taxes. The use of other forms of taxation would introduce their own distortions, and these would prevent efficiency from being achieved. In addition, to apply the Samuelson rule, the government must know the individual benefits from public good provision. In practice, this information is not readily available, and the government must rely on what individuals choose to reveal.

The consequence of these observations is that efficiency will not be attained through direct public good provision if financed by distortionary taxes. Hence we have the motivation for considering alternative allocation mechanisms that can provide the correct level of public good by eliciting preferences from consumers.

6.5 Voting

The failure of private actions to provide a public good efficiently suggests that alternative allocation mechanisms need to be considered. There are a range of responses that can be adopted to counteract the market failure, ranging from intervention with taxation through to direct provision by the government. In practice, the level of provision for public goods is frequently determined by the political process, with competing parties in electoral systems differing in the level of public good provision they promise. The selection of one of the parties by voting then determines the level of public good provision.

We have already obtained a first insight into the provision of public goods by voting in chapter 5. That analysis focused on voting over the tax rate as a proxy for government size when people had different income levels. What we wish to do here is provide a contrast between the voting outcome and the efficient level of public good provision when people differ in tastes and (possibly) income levels. Consider a population of consumers who determine the quantity of public good to be provided by a majority vote. The cost of the public good is shared equally among the consumers, so, if G units of the public good are supplied, the cost to each consumer is $\frac{G}{H}$. With income M^h, a consumer can purchase private goods to the value of $M^h - \frac{G}{H}$ after paying for the public good. This provides an effective price of $\frac{1}{H}$ for each unit of the public good and a level of utility $U^h\left(M^h - \frac{G}{H}, G\right)$. The budget constraint, the highest attainable indifference curves and the most preferred quantity of public good are shown in the upper part of figure 6.6 (assuming, for convenience, the same income levels for all consumers).

So that the Median Voter Theorem can be applied (see chapter 11 for details), assume that there is an odd number, H, of consumers, where $H > 2$, and that each of the consumers has single-peaked preferences for the public good. This second assumption implies that when the level of utility is graphed against the quantity of public good, there will be a single value of G^h that maximizes utility for consumer h. Such preferences are illustrated in the lower panel of figure 6.6. The consumers are numbered so that their preferred levels of public good satisfy $G^1 < G^2 < \ldots < G^H$.

By these assumptions, the Median Voter Theorem ensures that the consumer with the median preference for the public good will be decisive in the majority vote. The

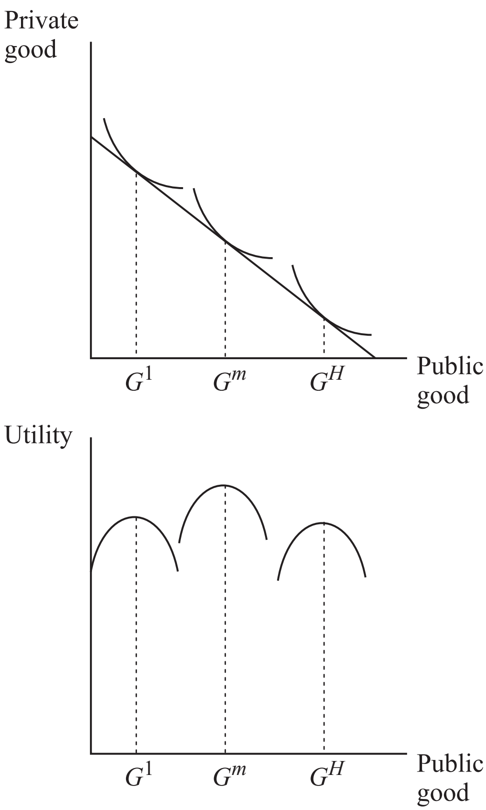

Figure 6.6
Allocation through voting

median preference belongs to the consumer at position $\frac{H+1}{2}$ in the ranking. We label the median consumer as m and denote their chosen quantity of the public good by G^m. A remarkable feature of the majority voting outcome is that nobody is able to manipulate the outcome to their advantage by misrepresenting their preference, so sincere voting is the best strategy. The reason is that anyone to the left of the median can only affect the final outcome by voting for a quantity to the right of the median that would move the outcome further away from their preferred position, and vice versa for anyone to the right of the median.

Having demonstrated that voting will reveal preferences and that the voting outcome will be the quantity G^m, it now remains to ask whether the voting outcome is efficient. The value G^m is the preferred choice of consumer m, so it solves

$$\max_{\{G\}} U^m\left(M^m - \frac{G}{H}, G\right), \tag{6.7}$$

where M^m denotes the income of the median voter that can differ from the median income with heterogeneous preferences. The first-order condition for the maximization can be expressed in terms of the marginal rate of substitution to show that the voting outcome is described by

$$MRS^m = \frac{1}{H}. \tag{6.8}$$

In contrast, because the marginal rate of transformation is equal to 1, the efficient outcome satisfies the Samuelson rule

$$\sum_{h=1}^{H} MRS^h = 1. \tag{6.9}$$

Contrasting these, the voting outcome is efficient only if

$$MRS^m = \sum_{h=1}^{H} \frac{MRS^h}{H}. \tag{6.10}$$

Therefore majority voting leads to efficient provision of the public good only if the median voter's *MRS* is equal to the mean *MRS* of the population of voters. There is no reason to expect that it will, so it must be concluded that majority voting will not generally achieve an efficient outcome. This is because the voting outcome does not take account of preferences other than those of the median voter: changing all the preferences except those of the median voter does not affect the voting outcome (although it would affect the efficient level of public good provision).

Can any comments be offered on whether majority voting typically leads to too much or too little public good? In general, the answer has to be no, since no natural restrictions can be appealed to and the median voter's *MRS* may be lower or higher than the mean. If it is lower, then too little public good will provided. The converse holds if it is higher. The only approach that might give an insight is to note that the distribution of income has a very long right tail. If the *MRS* is higher for lower income voters, then the nature of the income distribution suggests that the median *MRS* is higher than the mean. Thus voting will lead to an excess quantity of public good being provided. Alternatively, if the *MRS* is increasing with income, then voting would lead to underprovision.

6.6 Personalized Prices

We have now studied two allocation mechanisms that lead to inefficient outcomes. The private market fails because of free-riding, and voting fails because the choice of the decisive median voter need not match the efficient choice. What these have in common is that the consumers face incorrect incentives. In both cases the decision makers take account only of the private benefit of the public good rather than the broader social benefit (i.e., that public good contribution also benefits others). As a rule, efficiency will only be attained by modifying the incentives to align private and social benefits.

The first method for achieving efficiency involves using an extended pricing mechanism for the public good. This mechanism uses prices that are "personalized," with each consumer paying a price that is designed to fit their situation. These personalized prices modify the actual price in two ways. First, they adjust the price of the public good in order to align social and private benefits. Second, they further adjust the price to capture each consumer's individual valuation of the public good.

This latter aspect can be understood by considering the differences between public and private goods. With a private good, consumers face a common price but choose to purchase different quantities according to their preferences. In contrast, with a pure public good, all consumers consume the same quantity. This can only be efficient if the consumers wish to purchase the same given quantity of the public good. They can be induced to do so by correctly choosing the price they face. For instance, a consumer who places a low value on the public good should face a low price, while a consumer with a high valuation should face a high price. This reasoning is illustrated in table 6.1.

Table 6.1
Prices and quantities

	Private good	Public good
Price	Same	Different
Quantity	Different	Same

The idea of personalized pricing can be captured by assuming that the government announces the share of the cost of the public good that each consumer must bear. For example, it may say that each of two consumers must pay half the cost of the public good. Having heard the announcement of these shares, the consumers then state how much of the public good they wish to have supplied. If they both wish to have the same level, then that level is supplied. If their wishes differ, the shares are adjusted and the process repeated. The adjustment continues until shares are reached at which both wish to have the same quantity. This final point is called a *Lindahl equilibrium*. It can easily be seen how this mechanism overcomes the two sources of inefficiency. The fact that the consumers only pay a share of the cost reduces the perceived unit price of the public good. Hence the private cost appears lower, and the consumers increase their demands for the public good. Additionally the shares can be tailored to match the individual valuations.

To make this reasoning concrete, let the share of the public good that has to paid by consumer h be denoted τ^h. The scheme must be self-financing, so, with two consumers, $\tau^1 + \tau^2 = 1$. Now let G^h denote the quantity of the public good that household h would choose to have provided when faced with the budget constraint

$$x^h + \tau^h G^h = M^h. \tag{6.11}$$

The Lindahl equilibrium shares $\{\tau^1, \tau^2\}$ are found when $G^1 = G^2$. The reason why efficiency is attained can be seen in the illustration of the Lindahl equilibrium in figure 6.7. The indifference curves reflect preferences over levels of the public good and shares in the cost. The shape of these captures the fact that each consumer prefers more of the public good but dislikes an increased share. The highest indifference curve for consumer 1 is to the northwest and the highest for consumer 2 to the northeast. Maximizing utility for a given share (which gives a vertical line in the figure) achieves the highest level of utility where the indifference curve is vertical. Below this point the consumer is willing to pay a higher share for more public good, and above it is just the other way around. Hence the indifference curves are backward-bending. The *Lindahl*

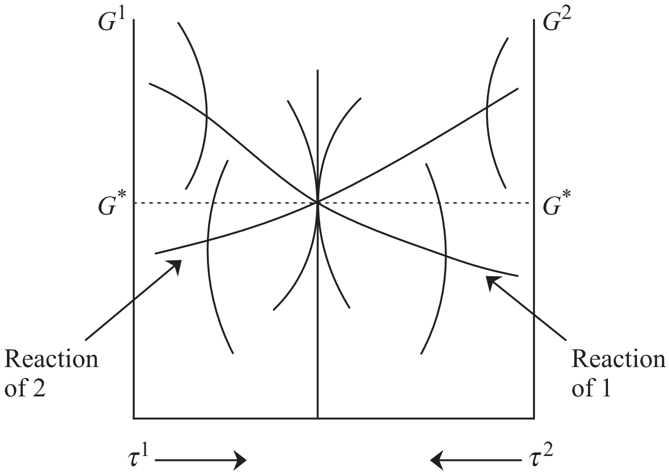

Figure 6.7
Lindahl equilibrium

reaction functions are then formed as the loci of the vertical points of the indifference curve. The equilibrium requires that both consumers demand the same level of the public good; this occurs at the intersection of the reactions functions. At this point the indifference curves of the two consumers are tangential and the equilibrium is Pareto-efficient.

To derive the efficiency result formally, note that utility is given by the function $U^h\left(M^h - \tau^h G^h, G^h\right)$. The first-order condition for the choice of the quantity of public good is

$$\frac{U_G^h}{U_x^h} = \tau^h, \qquad h = 1, 2. \tag{6.12}$$

Summing these conditions for the two consumers yields

$$\frac{U_G^1}{U_x^1} + \frac{U_G^2}{U_x^2} \equiv MRS_{G,x}^1 + MRS_{G,x}^2 = \tau^1 + \tau^2 = 1. \tag{6.13}$$

This is the Samuelson rule for the economy, and it establishes that the equilibrium is efficient. The personalized prices equate the individual valuations of the supply of public goods to the cost of production in a way that uniform pricing cannot. They also correct for the divergence between private and social benefits.

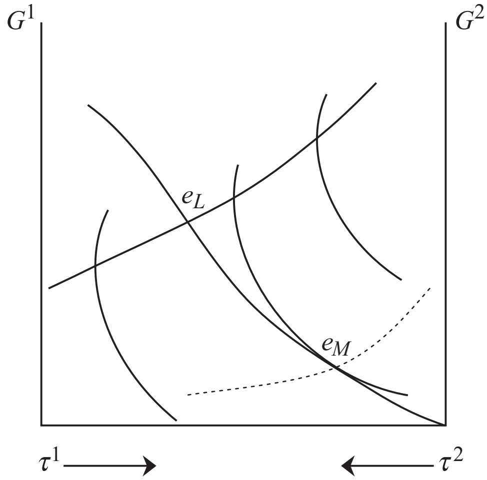

Figure 6.8
Gaining by false announcement

Although personalized prices seem a very simple way of resolving the public good problem, when considered more closely a number of difficulties arise in actually applying them. First, there is the very practical problem of determining the prices in an economy with many consumers. The practical difficulties involved in announcing and adjusting the individual shares are essentially insurmountable. Second, there are issues raised concerning the incentives for consumers to reveal their true demands.

The analysis assumed that the consumers were honest in revealing their reactions to the announcement of cost shares, meaning they simply maximize utility by taking the share of cost as given. However, there will be a gain to any consumer who attempts to cheat, or *manipulate*, the allocation mechanism. By announcing preferences that do not coincide with their true preferences, it is possible for a consumer to shift the outcome in their favor, provided that the other does not do likewise. To see this, assume that consumer 1 acts honestly and that consumer 2 knows this and knows the reaction function of 1. In figure 6.8 an honest announcement on the part of consumer 2 would lead to the equilibrium e_L where the two Lindahl reaction functions cross. However, by claiming their preferences to be given by the dashed Lindahl reaction function rather than the true function, the equilibrium can be driven to point e_M, which represents the maximization of 2's utility given the Lindahl reaction function of 1. This improvement for consumer 2 reveals the incentive for dishonest behavior.

The use of personalized prices can achieve efficiency only if the consumers act honestly. A consumer who acts strategically is able to manipulate the outcome to her advantage. Indeed it is clear that the search for a means of attaining an allocation that satisfies the Samuelson rule should be restricted to allocation mechanisms that cannot be manipulated in this way. This is the focus of the next section.

6.7 Mechanism Design

The previous section showed how consumers have an incentive to reveal false information on demand when personalized prices are being determined. From the consistent application of the assumption of utility maximization we observed that a consumer will behave dishonestly if it is in their interests to do so. This fact has led to the search for allocation mechanisms that are immune from attempted manipulation. As will be shown, the design of some of these mechanisms leads households to reveal their true preferences. Because of this property these mechanisms are called *preference revelation mechanisms*.

6.7.1 Examples of Preference Revelation

The general problem of preference revelation is now illustrated by considering two simple examples. In both examples people are shown to gain by making false statements of their preferences. If they act rationally, then they will choose to make false statements. Since these situations have the nature of strategic games, we call the participants *players*.

Example 1: False Understatement

The decision that has to be made is whether to produce or not produce a fixed quantity of a public good. If the public good is not produced, then $G = 0$. If it is produced, $G = 1$. The cost of the public good is given by $C = 1$. The gross benefit of the public good for players 1 and 2 is given by $v^1 = v^2 = 1$. Since the social benefit of providing the good is $v^1 + v^2 = 2$, which is greater than the cost, it is socially beneficial to provide the public good.

Each player makes a report, r^h, of the benefit they receive from the public good. This report can either be false, in which case $r^h = 0$, or truthful so that $r^h = v^h = 1$. Based on the reports, the public good is provided if the sum of announced valuations is at least as high as the cost. This gives the collective decision rule to choose $G = 1$ if $r^1 + r^2 \geq C = 1$, and to choose $G = 0$ otherwise. The cost of the public good is shared between the two players, with the shares proportional to the announced valuations. In detail,

$$c^h = 1 \quad \text{if } r^h = 1 \text{ and } r^{h'} = 0, \tag{6.14}$$

$$c^h = \frac{1}{2} \quad \text{if } r^h = 1 \text{ and } r^{h'} = 1, \tag{6.15}$$

$$c^h = 0 \quad \text{if } r^h = 0 \text{ and } r^{h'} = 0 \text{ or } 1. \tag{6.16}$$

The net benefit, the difference between true benefit and cost, which is termed the *payoff* from the mechanism, is then given by

$$U^h = v^h - c^h \text{ if } r^1 + r^2 \geq 1, \tag{6.17}$$

$$= 0 \quad \text{otherwise.} \tag{6.18}$$

This information is summarized in the payoff matrix in figure 6.9.

From the payoff matrix it can be seen that the announcement $r^h = 0$ is a weakly dominant strategy for both players. For instance, if player 2 chooses $r^2 = 1$, then player 1 will choose $r^1 = 0$. Alternatively, if player 2 chooses $r^2 = 0$, then player 1 is indifferent between the two strategies of $r^1 = 0$ and $r^1 = 1$. The Nash equilibrium of the game is therefore $\hat{r}^1 = 0$, $\hat{r}^2 = 0$.

In equilibrium both players will understate their valuation of the public good. As a result the public good is not provided, despite provision being socially beneficial. The reason is that the proportional cost-sharing rule gives an incentive to underreport preferences for public good. With both players underreporting, the public good is not provided. To circumvent this problem, we can make contributions independent of the reports. This is our next example.

		Announcement of player 2	
		0	1
Announcement of player 1	0	0 0	0 1
	1	1 0	½ ½

Figure 6.9
Announcements and payoffs

Example 2: False Overstatement

The second example is distinguished from the first by considering a public good that is socially nondesirable with a cost greater than the social benefit. The possible announcements and the charging scheme are also changed.

It is assumed that the gross payoffs when the public good is provided are

$$v_1 = 0 < v_2 = \frac{3}{4}. \tag{6.19}$$

With the cost of the public good remaining at 1, these payoffs imply that

$$v_1 + v_2 = \frac{3}{4} < C = 1, \tag{6.20}$$

so the social benefit from the public good is less than its cost.

The possible announcements of the two players are given by $r^1 = 0$ or 1 and $r^2 = \frac{3}{4}$ or 1. These announcements permit the players to either tell the truth or overstate the benefit so as to induce public good provision. Assume that there is also a uniform charge for the public good if it is provided, so $c^h = \frac{1}{2}$ if $r^1 + r^2 \geq c = 1$, and $c^h = 0$ otherwise. These valuations and charges imply the net benefits

$$U^h = v^h - c^h \quad \text{if } r^1 + r^2 \geq 1, \tag{6.21}$$

$$U^h = 0 \quad \text{otherwise.} \tag{6.22}$$

These can be used to construct the payoff matrix in figure 6.10.

The weakly dominant strategy for player 1 is to play $r^1 = 0$ and the best response of player 2 is to select $r^2 = 1$ (which is also a dominant strategy). Therefore the Nash equilibrium is $\hat{r}^1 = 0$, $\hat{r}^2 = 1$, which results in the provision of a socially

Announcement of player 1 \ Announcement of player 2	$\frac{3}{4}$	1
0	0, 0	$-\frac{1}{2}$, $\frac{1}{4}$
1	$-\frac{1}{2}$, $\frac{1}{4}$	$-\frac{1}{2}$, $\frac{1}{4}$

Figure 6.10
Payoffs and overstatement

nondesirable public good. The combination of payoffs and charging scheme has resulted in overstatement and unnecessary provision. The explanation for this is that the player 2 is able to guarantee the good is provided by announcing $r^2 = 1$. Their private gain is $\frac{1}{4}$, but this is more than offset by the loss of $-\frac{1}{2}$ for player 1.

6.7.2 Clarke–Groves Mechanism

The preceding examples showed that true valuations may not be revealed for some mechanisms linking announcement to contribution. Even worse, it is possible for the wrong social decision to be made. The question then arises as to whether there is a mechanism that will always ensure that true values are revealed (as for voting), and at the same time that the level of public good provided is efficient (which voting cannot do).

The potential for constructing such a mechanism, and the difficulties in doing so, can be understood by retaining the simple allocation problem of the examples that involves the decision on whether to provide a single public good of fixed size. The construction of a length of road or the erection of a public monument both fit this scenario. It is assumed that the cost of the project is known, and it is also known how the cost is allocated among the consumers that make up the population. What needs to be found from the consumers is how much their valuation of the public good exceeds, or falls short of, their contribution to the cost. Each consumer knows the benefit they will gain if the public good is provided, and they know the cost they will have to pay. The difference between the benefit and the cost is called the *net benefit*. This can be positive or negative. The decision rule is that the public good is provided if the sum of reported net benefits is (weakly) positive.

Consider two consumers with true net benefits v^1 and v^2. The mechanism we consider is the following. Each consumer makes an announcement of his or her net benefit. Denote by r^h the report of h, $h = 1, 2$. The public good is provided if the sum of announced net benefits satisfies $r^1 + r^2 \geq 0$. If the public good is not provided, each consumer receives a payoff of 0. If the good is provided, then each consumer receives a *side payment* equal to the reported net benefit of the other consumer; hence, if the public good is provided, consumer 1 receives a total payoff of $v^1 + r^2$ and consumer 2 receives $v^2 + r^1$. It is these additional side payments that will lead to the truth being told by inducing each consumer to "internalize" the net benefit of the public good for the other. If the public good is not provided, no side payments are made.

To see how this mechanism works, assume that the true net benefits and the reports can take the values of either -1 or $+1$. The public good will not be provided if both report a value of -1, but if at least one reports $+1$, it will be provided. The payoffs to the mechanism are summarized in the payoff matrix in figure 6.11. The claim we now wish to demonstrate is that this mechanism provides no incentive to make a false announcement of the net benefit. To do this, it is enough to focus on player 1 and show that the player will report truthfully when $v^1 = -1$ and when $v^1 = +1$. The payoffs relating to the true values are in the two payoff matrices in figure 6.12.

Take the case of $v^1 = -1$. Then consumer 1 finds the true announcement to be weakly dominant—the payoff from being truthful (the top row) is greater if $r^2 = -1$ and equal if $r^2 = +1$. Next take the case of $v^1 = +1$. Consumer 1 is indifferent between truth and nontruth. But the point is that there is now no incentive to provide a false announcement. Hence truth should be expected.

The problem with this mechanism is the side payments that have to be made. If the public good is provided and $v^1 = v^2 = +1$, then the total side payments are equal to

Announcement of player 1 \ Announcement of player 2	-1	+1
-1	0, 0	$v^1 + 1$, $v^2 - 1$
+1	$v^1 - 1$, $v^2 + 1$	$v^1 + 1$, $v^2 + 1$

Figure 6.11
Clarke–Groves mechanism

$v^1 = -1$:

Announcement of player 1 \ Announcement of player 2	-1	+1
-1	0	0
+1	–2	0

$v^1 = +1$:

Announcement of player 1 \ Announcement of player 2	-1	+1
-1	0	2
+1	0	2

Figure 6.12
Payoffs for player 1

2—which is equal to the total net benefit of the public good. These side payments are money that has to be put into the system to support the telling of truth. Obtaining the truth is possible, but it is costly.

6.7.3 Clarke Tax

The problem caused by the existence of the side payments can be reduced but can never be eliminated. The reason it cannot be eliminated entirely is simply that the mechanism is extracting information, and this can never be done for free. The way in which the side payments can be reduced is to modify the structure of the mechanism.

One way to do this is for side payments to be made only if the announcement of a player *changes* the social decision. To see what this implies, consider calculating the sum of the announced benefits of all players but one. Whether this is positive or negative will determine a social decision for those players. Now add the announcement of the final player. Does this change the social decision? If it does, then the final player is said to be *pivotal*, and a set of side payments is implemented that requires taxing the pivotal agent for the cost inflicted on the other agent through the changed social decision. This process is repeated for each player in turn. These side payments are the *Clarke taxes* that ensure that the correct decision is made so that the public good is produced if it is socially desirable and not otherwise. The use of Clarke taxes reduces the number of circumstances in which the side payments are made.

In a game with only two players, the payoffs for player 1 when the Clarke taxes are used are given by

$$v^1 \quad \text{if} \quad r^1 + r^2 \geq 0 \quad \text{and} \quad r^2 \geq 0, \tag{6.23}$$

$$v^1 - t^1 \quad \text{if} \quad r^1 + r^2 \geq 0 \quad \text{and} \quad r^2 < 0, \text{ with } t^1 = -r^2 > 0, \tag{6.24}$$

$$-t^1 \quad \text{if} \quad r^1 + r^2 < 0 \quad \text{and} \quad r^2 \geq 0, \text{ with } t^1 = r^2 \geq 0, \tag{6.25}$$

$$0 \quad \text{if} \quad r^1 + r^2 < 0 \quad \text{and} \quad r^2 < 0. \tag{6.26}$$

Only in the second and third cases is player 1 pivotal (respectively, by causing provision and stopping provision of the public good), and for these cases a tax is levied on player 1 reflecting the cost to the other agent of changing public good provision ($t^1 = -r^2 > 0$ for the cost of imposing provision, and $t^1 = r^2 \geq 0$ for the cost of stopping provision).

The Clarke taxes induce truth-telling and guarantee that the public good is provided if and only if it is socially desirable. The explanation is that any misreport that changes the decision about the public good would induce the payment of a tax in excess of the benefit from the change in decision. Indeed, suppose that the public good is socially desirable, so $v^1 + v^2 \geq 0$, but that player 1 dislikes it, so $v^1 < 0$. Then, given an honest announcement from player 2 with $r^2 = v^2$, by underreporting sufficiently to prevent provision of the public good (so $r^1 < -r^2$), player 1 becomes pivotal. Player 1 will have to pay a tax of $t^1 = r^2 = v^2$, which is in excess of the gain from nonprovision, $-v^1$ (since $v^1 + v^2 \geq 0 \Rightarrow v^2 \geq -v^1$). Hence player 1 is better off telling the truth, and given this truth-telling, player 2 is also better off telling the truth (although in this case he is the pivotal agent, inducing provision and paying a tax equal to the damage of public good provision for player 1, $t^2 = -r^1 = -v^1$).

The conclusion is that the Clarke tax induces preference revelation, and by restricting side payments to pivotal agents only, it lowers the cost of information revelation.

6.7.4 Further Comments

The theory of mechanism design shows that it is possible to construct schemes that ensure that the truth will be revealed and correct the social decision made. These mechanisms may work, but they are undoubtedly complex to implement. Putting this objection aside, it can still be argued that such revelation mechanisms are not actually needed in practice. Two major reasons can be provided to support this contention.

First, the mechanisms are built on the basis that the players will be rational and precise in their strategic calculations. In practice, many people may not act as strategically as the theory suggests. As in the theory of tax evasion, which we discuss in chapter 17, nonmonetary benefits may be derived simply from acting honestly. These benefits may provide a sufficient incentive that the true valuation is reported. In such circumstances the revelation mechanism will not be needed.

Second, the market activities of consumers often indirectly reveal the valuation of public goods. To give an example of what is meant by this, consider the case of housing. A house is a collection of characteristics, such as the number of rooms, size of garden, and access to amenities. The price that a house purchaser is willing to pay is determined by their assessment of the total value of these characteristics. Equally the cost of supplying a house is also dependent on the characteristics supplied. By observing the equilibrium prices of houses with different characteristics, it is possible to determine the value assigned to each characteristic separately. If one of the characteristics relates to a public good, for example, the closeness to a public park, the value of this public good can then be inferred. Such implicit valuation methods can be applied to a broad range of public goods by carefully choosing the related private good. Since consumers have no incentive to act strategically in purchasing private goods, the true valuations should be revealed.

The fact that consumers have an incentive to falsely reveal their valuations can also be exploited to obtain an approximation of the true value. This can be done by running two preference revelation mechanisms simultaneously. If one is designed to lead to an underreporting of the true valuation and the other one to overreporting, then the true value of the public good can be taken as lying somewhere between the over- and underreports. The Swedish economist Peter Bohm has conducted an experimental implementation of this procedure. In the experiment 200 people from Stockholm had to evaluate the benefit of seeing a previously unshown television program. The participants were divided into four groups each of which faced the following payment mechanisms: (1) pay stated valuation, (2) pay a fraction of stated valuation such that costs are covered from all payments, (3) pay a low flat fee, and (4) no payment. Although the first two provide an incentive to underreport and the latter two to overreport, the experiment found that there was no significant difference in the stated valuations, suggesting that misrevelation may not be as important as suggested by the theory.

6.8 More on Private Provision

The analysis of the private purchase of a public good in section 6.3 focused on the issue of efficiency. The analysis showed that a Pareto improvement can be made from the equilibrium point if both consumers simultaneously raise their contributions, so the equilibrium cannot be efficient. This finding was sufficient to develop the contrast with efficient provision and to act as a basis for investigating mechanism design.

Although useful, these are not the only results that emerge from the private purchase model. The model also generates several remarkably precise predictions about the effect of income transfers and increases in the number of purchasers. These results are now described and then contrasted with empirical and experimental evidence.

6.8.1 Neutrality and Population Size

The first result concerns the effect of redistributing income. Consider transferring an amount of income Δ from consumer 1 to consumer 2 so that the income of consumer 1 falls to $M^1 - \Delta$ and that of consumer 2 rises to $M^2 + \Delta$. The objective is to calculate the effect that this transfer has on the equilibrium level of public good purchases. To do this, notice that the equilibrium in figure 6.5 is identified by the fact that it occurs where an indifference curve for consumer 1 crosses an indifference curve for consumer 2 at right angles. Hence the effect of the transfer on the equilibrium can be found by determining how it affects the indifference curves.

Consider consumer 1 who has their income reduced by Δ. If we reduce their public good purchase by Δ and raise that of consumer 2 by Δ, the utility of consumer 1 is unchanged because

$$U^1\left(M^1 - g^1, g^1 + g^2\right) = U^1\left(\left[M^1 - \Delta\right] - \left[g^1 - \Delta\right], \left[g^1 - \Delta\right] + \left[g^2 + \Delta\right]\right). \tag{6.27}$$

This transfer of income causes the indifference curves and the best-reaction function of consumer 1 to move as illustrated in figure 6.13. The indifference curve through any point g^1, g^2 before the income transfer shifts to pass through the point $g^1 - \Delta, g^2 + \Delta$ after the income transfer.

The transfer of income has the same effect on the indifference curves and best-reaction function of consumer 2. From the reduction in purchase of consumer 1 and the increase by consumer 2, it follows that

$$U^2\left(M^2 - g^2, g^1 + g^2\right) = U^2\left(\left[M^2 + \Delta\right] - \left[g^2 + \Delta\right], \left[g^1 - \Delta\right] + \left[g^2 + \Delta\right]\right).$$

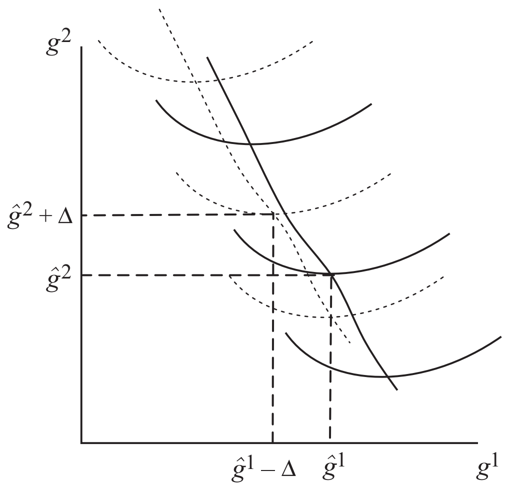

Figure 6.13
Effect of a transfer

For consumer 2 the indifference curve through g^1, g^2 before the income transfer becomes that through $g^1 - \Delta, g^2 + \Delta$ after the transfer.

These shifts in the indifference curves result in the equilibrium moving as in figure 6.14. The point where the indifference curves cross at right angles shifts in the same way as the individual indifference curves. If the equilibrium was initially at $\hat{g}^1, \hat{g}^2$ before the income transfer, it is located at $\hat{g}^1 - \Delta, \hat{g}^2 + \Delta$ after the transfer.

The important result now comes from noticing that in the move from the original to the new equilibrium, consumer 1 reduces his purchase of the public good by Δ, but consumer 2 increases her purchase by the same amount Δ. These changes in the value of purchases exactly match the change in income levels. The net outcome is that the levels of private consumption remain unchanged for the two consumers, and the total supply of the public good is also unchanged. As a consequence the income transfer does not affect the levels of consumption in equilibrium—all it does is to redistribute the burden of purchase. Income redistribution is entirely offset by an opposite redistribution of the responsibility for purchases of the public good. This result, known as *income distribution invariance*, is a consequence of the fact that the utility levels of the consumers are linked via the quantity of a public good.

The second interesting result is that the transfer of income leaves the utility levels of the two consumers unchanged. This has to be so because, as we have just seen,

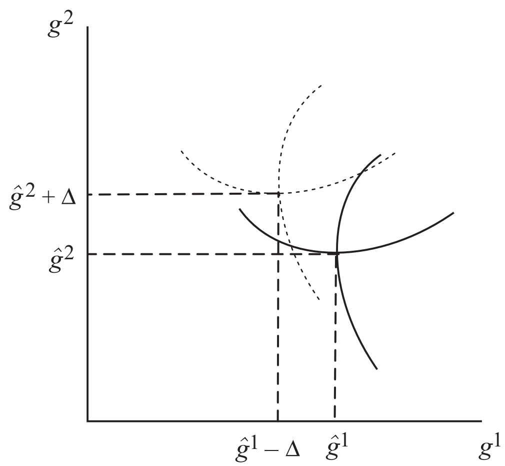

Figure 6.14
New equilibrium

the consumption levels do not change. Therefore the redistribution of income has not affected the distribution of welfare; the transfer is simply offset by the change in public good purchases. If the income redistribution was due to government policy, this becomes an example of *policy neutrality*: by changing their behavior, the individuals in the economy are able to undo what the government is trying to do. Income redistribution will always be neutral until the point is reached at which one of the consumers no longer purchases the public good. Only then will further income transfers affect the distribution of utility.

A third result follows easily from income invariance. Let both consumers have the same utility function but possibly different income levels. Since the quantity of public good consumed by both must be the same, the first-order conditions require that both must also consume the same quantity of private good; hence $x^1 = x^2$. Further these common levels of consumption imply that the consumers must have the same utility levels even if there is an initial income disparity. The private-purchase model therefore implies that when the consumers have identical utilities, the choices made by the consumers will equalize utilities even in the face of income differentials. The poor set their purchases sufficiently lower than the purchases of the rich to make them equally well off.

This model can also be used to consider the consequence of variations in the number of households. Maintaining the assumption that all the consumers are identical in terms of both preferences and income, for an economy with H consumers the total provision of the public good is $G = \sum_{h=1}^{H} g^h$ and the utility of h is

$$U^h = U\left(M - g^h, G\right) = U\left(M - g^h,\ \bar{G}^h + g^h\right). \tag{6.28}$$

Here $\bar{G}^h$ is the total contributions of all consumers other than h . Since all consumers are identical, it makes sense to focus on symmetric equilibria where all consumers make the same contribution. Hence let $g^h = g$ for all h. It follows that at at symmetric equilibrium

$$g = \frac{\bar{G}}{H - 1}. \tag{6.29}$$

In a graph of g against $\bar{G}$ an allocation satisfying (6.29) must lie on a ray through the origin with gradient $H - 1$. For each level of H, the equilibrium is given by the intersection of the appropriate ray with the best-reaction function. This is shown in figure 6.15.

The important point is what happens to the equilibrium level of provision as the number of consumer tends to infinity (the idealization of a "large" population). What

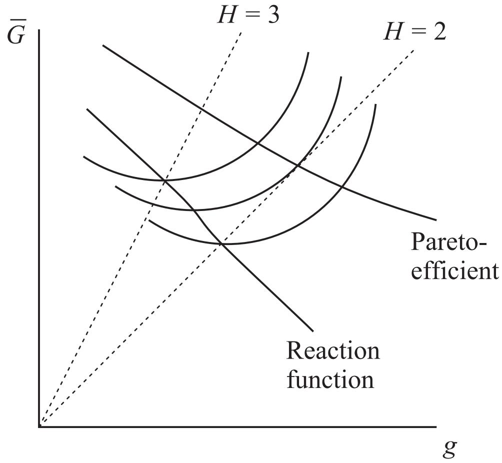

Figure 6.15
Additional consumers

happens can be seen by considering the consequence of the ray in figure 6.15 becoming vertical: the equilibrium will be at the point where the reaction function crosses the vertical axis. As this point is reached, the provision of each consumer will tend to zero, but aggregate provision will not, since it is the sum of infinitely many zeros. This result can be summarized by saying that in a large population each consumer will effectively contribute nothing.

6.8.2 Experimental Evidence

The analysis of private purchase demonstrated that the equilibrium will not be Pareto-efficient and that, compared to Pareto-preferred allocations, too little of the public good will be purchased. A simple explanation of this result can be given in terms of each consumer relying on others to purchase and hence deciding to purchase too little. Each consumer is free-riding on others' purchases, and since all attempt to free-ride, the total value of purchases fails to reach the efficient level. This conclusion has been subjected to close experimental scrutiny.

The basic form of the experiment is to give participants a number of tokens that can be invested in either a private good or a public good. Each participant makes a single purchase decision. The private good provides a benefit only to its purchaser while purchase of public good provides a benefit to all participants. The values are set so that the private benefit is less than the social benefit. The benefits are known to the participants and the total benefit from purchases is the payoff to the participant at the end of the experiment. It is therefore in the interests of each participant to maximize his or her payoff.

To see how this works in detail, assume that there are 10 participants in the game. Allow each participant to have 10 tokens to spend. A unit of the private good costs 1 token and provides a benefit of 5 units (private benefit = social benefit = 5). A unit of the public good also costs 1 token but provides a benefit of 1 unit to *all* the participants in the game (private benefit = 1 < social benefit = 10). The returns are summarized in figure 6.16.

If the game is played once (a "one-shot" game), the Nash equilibrium strategy is to purchase only the private good, since each token spent on the private good yields a return five times higher than for the public good. In equilibrium, the total return to each player is 50. In contrast, the socially efficient outcome is for all players to purchase only the public good and to generate a payoff of 100 to each player. The fact that the Nash equilibrium differs from the efficient outcome is because the private benefits diverge from the social benefits. Thus, in the one-shot game, all tokens should be spent on the private good.

In experimental implementations of this game the average value of purchases of the public good has been approximately 30 to 90 percent of tokens, with most observations falling in the 40 to 50 percent range. Interestingly, among student participants contributions have been lowest for those studying economics, and fall with the number of years of economics taken. Since the purchase of the public good is significantly different from 0, these results clearly do not support the predictions of the private-purchase model.

Some experiments have repeated the purchase decision over several rounds, with the view that this should allow time for the participants to learn about free-riding and develop the optimal strategy. The results from such experiments are not as clear and a wider range of purchases occurs. Free-riding is not completely supported, but instances

	Private good	Public good
Private benefit	5	1
Social benefit	5	10

Figure 6.16
Public good experiment

have been reported in which it does occur. However, this finding should be treated with caution, since having several rounds of the game introduces aspects of repeated game theory. While it remains true that the only credible equilibrium of the repeated game is the private-purchase equilibrium of the corresponding single-period game, it is possible that in the experiments some participants were attempting to establish cooperative equilibria by playing in a fashion that invited cooperation. Additionally those not trained in game theory would have been unable to derive the optimal strategy even though they could solve the single-period game.

Other results show that increasing group size leads to increased divergence from the efficient outcome when accompanied by a decrease in marginal return from the public good, but the results do not support a pure numbers-in-group effect. This finding is compatible with the theoretical finding that the effect of group size on the divergence from optimality is, in general, indeterminate.

These results indicate that there is little evidence of free-riding in single-period, or one-shot, games but in the repeated games the purchases fall toward the private-purchase level as the game is repeated. In total, these experiments do not provide great support for the equilibrium based on the private-purchase model with Nash behavior. In the single-period games free-riding is unambiguously rejected. Although it appears after several rounds in repeated games, the explanation for the strategies involved is not entirely apparent. Neither a strategic nor a learning hypothesis is confirmed. What seems to be occurring is that the participants are initially guided more by a sense of fairness than by Nash behavior. When this fairness is not rewarded, the tendency is then to move toward the Nash equilibrium. The failure of experimentation to support free-riding lends some encouragement to the views that although such behavior may be individually optimal, it is not actually observed in practice.

6.8.3 Modifications

The experimental evidence has produced a number of conflicts with the predictions of the theoretical model. The analysis of private purchase was based on two fundamental assumptions. The utility of consumers was assumed to depend only on the consumption of the private good and the total supply of the public good. This ensures that consumers do not care directly about the size of their own contribution nor do they care about the behavior of other consumers, except for how it affects the total level of the public good. The second assumption was that the consumers acted noncooperatively and played according to the assumptions of Nash equilibrium.

The simplest modification that can be made to the model is to consider the game being played in a different way. The foundation of the Nash equilibrium is that each player takes the behavior of the others as given when optimizing. One way to change this is to consider "conjectural variations" so that each player forms an opinion as to how their choice will affect that of others. If the conjectural variation is positive, each player predicts that the others will respond to an increase in purchase by also making additional purchases. Such a positive conjecture can be interpreted as being more cooperative than the zero conjecture that arises in the Nash equilibrium and leads to the equilibrium having greater total public good supply than the Nash equilibrium.

Moving to non-Nash conjectures may alter the equilibrium level of the public good, but it does not eliminate the neutrality properties. Furthermore the major objection to this approach is that it is entirely arbitrary. There are sensible reasons founded in game theory for focusing on the Nash equilibrium, and no other set of conjectures can appeal to similar justification. If the Nash equilibrium of the private-purchase model does not agree with observations, it would seem that the objectives of the consumers and the social rules they observe should be reconsidered, not the conjectures they hold when maximizing.

One approach to modified preferences is to assume that consumers derive utility directly from the contributions they make. For instance, making a donation to charity can make a consumer feel good about herself; she is acting as a "good citizen." This is often referred to as the *warm glow* effect. With a warm glow, a purchase of the public good provides a return from direct consumption of the public good and a further return from the warm glow. The private warm glow effect increases the value of the purchase and so raises the equilibrium level of total purchases. The equilibrium also no longer has the same invariance properties. This would seem a significant advance were it not that the specification of the warm glow is entirely arbitrary.

A final modification is to remove the individualism and allow for social interaction by modifying the rules of social behavior. In the same way that social effects can arise with tax evasion, they can also occur with public goods. One way to do this is to introduce reciprocity, by which each consumer considers the contributions of others and contrasts them to what she feels she should have made. If the contributions of others match, or exceed, what is expected, then the consumer is assumed to feel under an obligation to make a similar contribution. This again raises the equilibrium level of contribution.

6.9 Fund-Raising Campaigns

The model of voluntary purchase that we have considered so far has involved a single one-off contribution decision. It is easy to appreciate that once these contributions have been made, the consumers may look again at the situation and realize that it is inefficient. This could give them an incentive to conduct a second round of contribution that will move the equilibrium closer to efficiency. Repeatedly applying this argument suggests that it may be possible to eventually reach efficiency. We now assess this claim by addressing it within a simple fund-raising game.

The basis of the fund-raising game is that a target level of funds must be achieved before a public good can be provided. For example, consider the target as the minimum cost of construction for a public library. Subscribers to the campaign take it in turn to make either a contribution or a pledge to contribute. Only when the target is met does the process cease. The basic question is whether such a fund-raising campaign can be successful given the possibility of free-riding.

We model a campaign as a game with an infinite horizon, meaning that solicitation for donations can continue until the goal is met. There is one public good (or joint project) whose production cost is C and two identical players X and Y. These players derive the same benefit, B, from the public good, so the total benefit is $2B$. Both also have the same discount rate δ, $0 < \delta < 1$, for delaying completion of the project by one period.

The players alternate in making contributions. The sequential (marginal) contributions are denoted $(\ldots, x_{t-1}, y_t, x_{t+1}, \ldots)$, where x_{t-1} denotes the contribution of player X at time $t-1$ and y_t denotes the contribution of player Y at time t. The game ends, and the public good is provided, only when the total contributions cover the cost of the public good. Individuals derive no benefits from the public good before completion of the fund-raising, so the marginal contributions yield no return until the cost is met. It follows that the incentive of each player to wait for the other one to contribute (free-riding) must be balanced against the cost of delaying completion of the project. We suppose that the public good is "socially desirable" ($C < 2B$) but that no single player values the public good enough to bear the full cost ($B < C$). We now contrast two different forms of fund-raising campaigns. In the first, the *contribution campaign*, the contributions are paid at the time they are made. In the second, the *subscription campaign*, players are asked in sequence to make donation pledges that are not paid until the cost is met.

6.9.1 The Contribution Campaign

In the contribution campaign, contributions are sunk at the time they are made because a credible commitment cannot be made to make contributions later. The lack of commitment leads each player to back his contribution to ensure that the other players contribute their share. This is because past contributions are sunk and cannot influence the division of the remaining cost between the players. As a result we show that it is never possible to raise the money, even though the project is worthwhile.

The two players are asked in sequence to make a contribution. While there is no natural end period, there is a total contribution level that is close enough to the cost C that the contributor whose turn it is should complete the fund-raising rather than waiting for the other one to make up the difference. Suppose that it is player X's turn to make a contribution offer at that final round T. There exists a deficit sufficiently small that player X is indifferent between making up the difference and getting a payoff of $B - x_T$ or between waiting in the expectation (at best) that player Y will make up the difference in the next round and producing a payoff with delayed completion of δB. Hence the maximal contribution of player 1 in the final round T is

$$x_T = (1 - \delta)B, \tag{6.30}$$

so the contribution is equal to the benefit of speeding up completion of the project. We suppose that $[1 - \delta]B < C$ so that such a contribution cannot cover the full cost and a donation from player Y must be solicited. Working backward, it is now player Y's turn to make a contribution at time $T - 1$. Player Y anticipates that in bringing (total) contributions up to $C - x_T$ at date $T - 1$, player X will complete the project the next period. So there exists a sufficiently small deficit such that player Y is indifferent between bringing total contributions up to that level, giving a payoff $\delta B - y_{T-1}$, and waiting for the other player to make such contribution while himself making the final contribution x_T, which produces a payoff $\delta^2[B - x_T]$ (i.e., two periods later you get the completed project benefit B and pay the last contribution x_T). Hence, substituted for x_T, the contribution at time $T - 1$ that makes player Y indifferent is

$$y_{T-1} = \delta\left(1 - \delta^2\right)B. \tag{6.31}$$

Proceeding backward to date $T - 2$, it is now the turn of player X to make a contribution. Using the same line of argument, there exists a total contribution level at date $T - 2$ such that player X is indifferent between bringing total contribution up to that level to get a payoff $\delta^2[B - x_T] - x_{T-2}$ from completion in two periods or waiting and delaying completion to get a payoff $\delta^3 B - \delta^2 y_{T-1}$ (in which from the switching position it becomes worthwhile to contribute y_{T-1}). Substituting for x_T and y_{T-1} gives

$$x_{T-2} = \delta^3\left[1 - \delta^2\right]B. \tag{6.32}$$

Moving back to round $T-3$ and following the same reasoning, the potential contribution at time $T-3$ from player Y is

$$y_{T-3} = \delta^5\left[1 - \delta^2\right]B, \tag{6.33}$$

and the potential contribution at time $T-4$ is

$$x_{T-4} = \delta^7\left[1 - \delta^2\right]B. \tag{6.34}$$

Going back further, it is possible to calculate how much each player is willing to contribute at each stage. This is illustrated in figure 6.17.

Summing these contributions by starting from the end of the campaign, we have the total potential for contributions as

$$[1-\delta]B + \delta\left[1 - \delta^2\right]B + \delta^3\left[1 - \delta^2\right]B + \delta^5\left[1 - \delta^2\right]B + \delta^7\left[1 - \delta^2\right]B + \ldots = B. \tag{6.35}$$

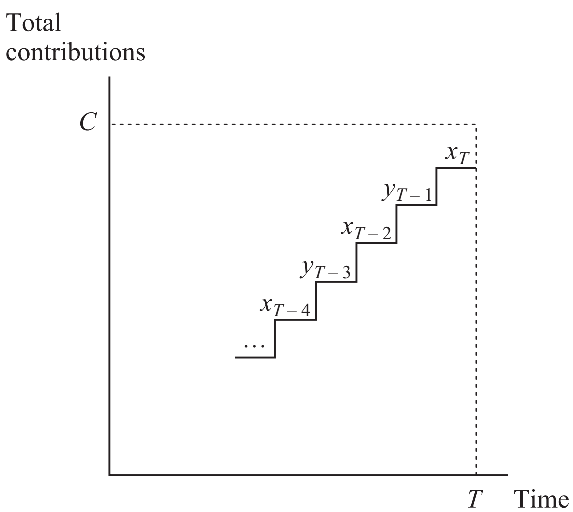

Figure 6.17
Two-player contribution campaign

In (6.35) we used the geometric progression fact that $1 + \delta^2 + \delta^4 + \delta^6 + \ldots = \frac{1}{[1-\delta^2]}$. The remarkable feature is that the total potential for contributions never exceeds the individual benefit from the project, and because $B < C$, it is not possible to raise sufficient contributions for a successful campaign.

6.9.2 The Subscription Campaign

In the subscription game, agents alternate in making donation pledges and bear the cost of their contribution only *when* and *if* enough contributions are pledged to complete the project. In a sense, agents are able to make certain conditional commitments to contribute in the future. This possibility to commit modifies the strategic structure of the game and alters the total amount that can be raised. As we now show, in this case it becomes possible to raise an amount equal to the total valuation of all the contributors.

Once again, we start when the fund-raising operation is over and work backward. Fix an arbitrary end point T with player X's turn to make a donation pledge at date T. There must exists a contribution deficit sufficiently small to make player X indifferent between financing the deficit himself to obtain a payoff $B - x_T$ and waiting for player Y to make up the difference in the next period, with a delayed completion payoff of δB. So the potential pledge of player X at date T is

$$x_T = [1 - \delta]B. \tag{6.36}$$

We continue to assume that $[1 - \delta]B < C$ so that we can solicit player Y's donation. Working backward, it is then up to player Y to pledge at date $T - 1$. Player Y anticipates that in bringing the total amount pledged up to $C - x_T$ at date $T - 1$, player X will complete the project in the next period. So there exists a sufficiently small deficit such that player Y is indifferent between making up the difference to get a payoff $\delta\,[B - y_{T-1}]$ and leaving player X to make up the difference, thereby delaying completion to get a payoff of $\delta^2\,[B - x_T]$ (in which case it becomes worthwhile for Y to pledge himself x_T at date T). Hence, substituting for x_T, we obtain

$$y_{T-1} = \left[1 - \delta^2\right]B. \tag{6.37}$$

Going back to date $T - 2$, it is up to player X to pledge. Again, there exists a total amount pledged close enough to $C - x_T - y_{T-1}$ such that player X is indifferent between bringing the total contribution up to that level, anticipating completion in two rounds with a payoff $\delta^2[B - x_T - x_{T-2}]$, and waiting for Y to pledge instead with a payoff from switching position of $\delta^3\,[B - y_{T-1}]$. Substituting for x_T and y_{T-1} gives

$$x_{T-2} = \delta\left[1 - \delta^2\right]B. \tag{6.38}$$

Proceeding likewise, we can go back further and calculate how much player Y will pledge at date $T - 3$ as

$$y_{T-3} = \delta^2\left[1 - \delta^2\right]B, \tag{6.39}$$

and player X will pledge at date $T - 4$ the amount

$$x_{T-4} = \delta^3\left[1 - \delta^2\right]B. \tag{6.40}$$

Going back to calculate how much each player is willing to pledge at each stage and summing up potential pledges, we get

$$[1-\delta]B + \left[1 - \delta^2\right]B + \delta\left[1 - \delta^2\right]B + \delta^2\left[1 - \delta^2\right]B + \delta^3\left[1 - \delta^2\right]B + \ldots = 2B. \tag{6.41}$$

This is the maximum amount that can be raised and is equal to the total valuations of all the contributors. Hence it is always possible to raise enough money for any worthwhile project because $C < 2B$.

These results have shown how allowing contributions to be repeated may lead to efficient private provision of the public good. But this conclusion is sensitive to the assumptions made on the ability of contributors to make binding commitments.

6.10 Conclusions

This chapter has reviewed the standard analysis of the efficient level of provision of a public good leading to the Samuelson rule. The analysis of private purchase emphasized the fact that this outcome will not be achieved without government intervention. The efficiency rule describes an allocation that can only be achieved if the government is unrestricted in its policy tools or, as the Lindahl equilibrium demonstrates, using prices that are personalized for each consumer.

One aspect of public goods that prevents the government from making efficient decisions is the government's lack of knowledge of consumers' preferences and their willingness to pay for public goods. Mechanisms were constructed that provide the right incentives for consumers to correctly reveal their true valuation of the public good. Experimental evidence suggests that consumer behavior when confronted with decision problems involving public goods does not fully conform with the theoretical prediction and that the private-purchase equilibrium may not be as inefficient as theory suggests. Furthermore misrevelation has not been confirmed as the inevitable outcome.

Further Reading

The classic paper on the efficient provision of public goods is:

Samuelson, P. A. 1954. The pure theory of public expenditure. *Review of Economics and Statistics* 36: 387–89.

The private provision model is developed fully in:

Cornes, R. C., and Sandler, T. 1996. *The Theory of Externalities, Public Goods and Club Goods*. Cambridge: Cambridge University Press.

The independence between income distribution and public good allocation is in:

Warr, P. 1983. The private provision of public goods is independent of the distribution of income. *Economic Letters* 13: 207–11.

Further developments of the model are in:

Bergstrom, T. C., Blume, L., and Varian, H. 1986. On the private provision of public goods. *Journal of Public Economics* 29: 25–49.

Bergstrom, T. C., and Cornes, R. 1983. Independence of allocative efficiency from distribution in the theory of public goods. *Econometrica* 51: 1753–65.

Itaya, J.-I., de Meza, D., and Myles, G. D. 2002. Income distribution, taxation and the private provision of public goods. *Journal of Public Economic Theory* 4: 273–97.

The effect of group size on private provision is in:

Andreoni, J. 1988. Privately provided public goods in a large economy: The limits of altruism. *Journal of Public Economics* 35: 57–73.

Chamberlin, J. 1974. Provision of collective goods as a function of group size. *American Political Science Review* 68: 707–16.

The effect of altruism on private provision is in:

Hindriks, J., and Pancs, R. 2002. Free riding on altruism and group size. *Journal of Public Economic Theory* 4: 335–46.

Preference revelation for public goods was first described as a dominant strategy mechanism in:

Groves, T., and Ledyard, J. 1977. Optimal allocation of public goods: A solution to the “free rider” problem. *Econometrica* 45: 783–809.

A simple mechanism for preference revelation as a Nash equilibrium is the “round table” scheme in:

Walker, M. 1981. A simple incentive-compatible scheme for attaining Lindahl allocations. *Econometrica* 49: 65–71.

There is also a mechanism that induces truth-telling as a Bayesian–Nash equilibrium in:

d'Aspremont, C., and Gerard-Varet, L. A. 1979. Incentives and incomplete information. *Journal of Public Economics* 11: 25–45.

A very good survey of the preference revelation mechanisms is in:

Laffont, J.-J. 1987. Incentives and the allocation of public goods. In A. Auerbach and M. Feldstein, eds., *Handbook of Public Economics*. Amsterdam: North Holland.

The fund-raising campaign is based on private provision of discrete public good in:

Admati, A. R., and Perry, M. 1991. Joint projects without commitment. *Review of Economic Studies* 58: 259–76.

More on private provision of discrete public goods (such as the volunteer dilemma) is in:

Palfrey, T., and Rosenthal, H. 1984. Participation and the provision of discrete public goods: A strategic analysis. *Journal of Public Economics* 24: 171–93.

Experimental results are surveyed in:

Bohm, P. 1972. Estimating demand for public goods: An experiment. *European Economic Review* 3: 55–66.

Isaac, R. M., McCue, K. F., and Plott, C. R. 1985. Public goods in an experimental environment. *Journal of Public Economics* 26: 51–74.

Ledyard, J. O. 1993. Public goods: A survey of experimental research. In J. Kagel and R. Roth, eds., *Handbook of Experimental Economics*. Princeton: Princeton University Press.

Exercises

6.1 Which of the following are public goods? Explain why.

a. Snowplowing services during the winter.
b. A bicycle race around France during the summer.
c. Foreign aid to Africa to feed its famine-stricken people.
d. Cable television programs.
e. Radio programs.
f. Back roads in the country.
g. Waste collection services.
h. Public schools.

6.2 What are their features with respect to the properties of rivalry and excludability?

6.3 How does a nonrival good differ from a nonexcludable good?

6.4 In the United Kingdom the lifeboat service is funded by charitable donations. How can this work? How are the rescue services funded in other countries?

6.5 Discuss how television technology can turn a public good into a private good.

6.6 What is a public good? How can one determine the efficient level of provision of a public good?

6.7 Let each dollar spent on a private good give you 10 units of utility but each dollar spent on a public good give you and your two neighbors 5 units each. If you have a fixed income of $10, how much would you spend on the public good? What is the value of the total purchases at the Nash equilibrium if your neighbors also have $10 each? What level of expenditure on the public good maximizes the total level of utility?

6.8 How many allocations satisfy the Samuelson rule?

6.9 How do prices ensure that the efficiency condition is satisfied for private goods? Why is the same not true when there is a public good?

6.10 Consider two consumers with the following demand functions for a public good:

$$p_1 = 10 - \frac{1}{10}G,$$

$$p_2 = 20 - \frac{1}{10}G,$$

where p_i is the price that i is willing to pay for quantity G.

a. What is the optimal level of the public good if the marginal cost of the public good is $25?

b. Suppose that the marginal cost of the public good is $5. What is the optimal level?

c. Suppose that the marginal cost of the public good is $40. What is the optimal level? Should the consumers make an honest statement of their demand functions?

6.11 There are three consumers of a public good. The demands for consumers are as follows:

$$p_1 = 50 - G,$$

$$p_2 = 110 - G,$$

$$p_3 = 150 - G,$$

where G measures the number of units of the good and p_i the price in dollars. The marginal cost of the public good is $190.

a. What is the optimal level of provision of the public good? Illustrate your answer with a graph.

b. Explain why the public good may not be supplied at all because of the free-rider problem.

c. If the public good is not supplied at all, what is the size of the deadweight loss arising from this market failure?

6.12 Take an economy with 2 consumers, 1 private good, and 1 public good. Let each consumer have an income of M. The prices of public and private good are both 1. Let the consumers have utility functions

$$U^A = \log\left(x^A\right) + \log(G), \quad U^B = \log\left(x^B\right) + \log(G).$$

a. Assume that the public good is privately provided so that $G = g^A + g^B$. Eliminating x^A from the utility function using the budget constraint, show that along an indifference curve

$$dg^A\left[\frac{1}{g^A + g^B} - \frac{1}{M - g^A}\right] + dg^B\left[\frac{1}{g^A + g^B}\right] = 0,$$

and hence thatpt

$$\frac{dg^B}{dg^A} = \frac{g^A + g^B}{M - g^A} - 1.$$

Solve the last equation to find the locus of points along which the indifference curve of A is horizontal, and use this to sketch the indifference curves of A.

b. Consider A choosing g_A to maximize utility. Show that the optimal choice satisfiespt

$$g^A = \frac{M}{2} - \frac{g^B}{2}.$$

c. Repeat part b for B, and calculate the level of private provision of the public good.

d. Calculate the optimal level of provision for the welfare function

$$W = U^A + U^B.$$

Contrast this with the private provision level.

6.13 Let there be H consumers all with the utility function $U^h = \log\left(x^h\right) + \log(G)$ and an income of 1. Noting that the utility with private purchase can be written

$$U^h = \log\left(x^h\right) + \log\left(g^h + \sum_{h' \neq h} g^{h'}\right),$$

and that the equilibrium must be symmetric, calculate the private purchase equilibrium and the social optimum for the welfare function

$$W = \sum_{h=1}^{H} U^h.$$

Comment on the effect of changing H on the contrast between the equilibrium and the optimum.

6.14 Assume there are two consumers with incomes M^1 and M^2. The preferences of consumer h, $h = 1, 2$, are described by the utility function $U = \log\left(x^h\right) + \log\left(g^1 + g^2\right)$, where x^h is consumption of the private good and g^h contribution to the public good. The consumers must determine how to allocate income between the private good and contribution to the public good. Each consumer takes the other consumer's decision as given.

a. Assuming that the solution to the contribution game is interior ($x^h > 0$, $g^h > 0$, $h = 1, 2$), find the individual contributions.

b. Show that the total contribution $g^1 + g^2$ depends only on the sum, $M^1 + M^2$, of income.

c. Contrast the outcome of the contribution game to outcome that maximizes a utilitarian social welfare function.

Now let $M^2 = \bar{M} - M^1$, for given $\bar{M}$.

d. Find the limits on M^1 as function of $\bar{M}$ for which the solution is interior.

e. Find the solution inside and outside of these limits. Plot the amount of public good as a function of M^2.

f. Derive the level of welfare as a function of M^2. Show that the function is kinked, and increases (locally) on one side of each kink point.

6.15 (Pareto-improving transfers; Cornes and Sandler 2000). Let there be one consumer with income M and H consumers with income m, where $m < M$. All consumers have preferences described by $U = \log(x^h) + \log(G)$, where x^h is consumption of a private good and $G = \sum_{h=1}^{H+1} g^h$ is total contribution to a public good.

a. Find the condition for the consumers with income m not to contribute.

b. Derive the condition for a transfer of income dm from each consumer with income m to the consumer with income M to raise welfare.

c, Explain how a Pareto improvement can arise when H consumers have their incomes reduced.

6.16 Consider two consumers (1, 2), each with income M to allocate between two goods. Good 1 provides 1 unit of consumption to its purchaser and α, $0 \leq \alpha \leq 1$, units of consumption to the other consumer. Each consumer i, $i = 1, 2$, has the utility function $U^i = \log\left(x_1^i\right) + x_2^i$, where x_1^i is consumption of good 1 and x_2^i is consumption of good 2.

a. Provide an interpretation of α.

b. Suppose that good 2 is a private good. Find the Nash equilibrium levels of consumption when both goods have a price of 1.

c. By maximizing the sum of utilities, show that the equilibrium is Pareto-efficient if $\alpha = 0$ but inefficient for all other values of α.

d. Now suppose that good 2 also provides 1 unit of consumption to its purchaser and α, $0 \leq \alpha \leq 1$, units of consumption to the other consumer. For the same preferences, find the Nash equilibrium and show that it is efficient for all values of α.

e. Explain the conclusion in part d.

6.17 Consider four students deciding to jointly share a textbook. Describe a practical method for using the Lindahl equilibrium to determine how much each should pay.

6.18 Let there be two identical consumers. What would be the share of the cost each should pay for a public good at the Lindahl equilibrium? Use this result to argue that there must be a subsidy to the price of the public good that makes the private purchase equilibrium efficient.

6.19 Assume there are two consumers with preferences described by $U = \log(x) + \log(G)$. Both consumers have income M. The government asks each consumer to announce their demand for the public good as a function of the share of the cost they pay. The cost shares are chosen so that both consumers demand the quantity of public good in equilibrium. Consumer 1 pays share τ_1 and consumer 2 pays share τ_2, with $\tau_1 + \tau_2 = 1$. The government insists that the demand function is linear, and consumers announce the intercept they think is best. Hence consumer i announces the value of a_i in the demand function $G = a_i - b\tau_i$.

a. Determine the efficient level of public good provision.

b. Given a pair of announcements $G = a_1 - b\tau_1$ and $G = a_2 - b\tau_2$, find the resulting values of G and of τ_1 and τ_2 as functions of a_1, a_2, and b.

c. Express the utility of each consumer as a function of a_1 and a_2. Solve for the Nash equilibrium in announcements and derive the equilibrium value of G.

d. How is the equilibrium value of G affected by changes in b?

6.20 What would be the equilibrium outcome if both consumers tried to manipulate the Lindahl equilibrium?

6.21 Discuss the effect that an increase in the number of consumers involved in a mechanism has on the consequences of manipulation.

6.22 Consider a two-good economy (one private good and one public good) and a large number H of individuals with single-peaked preferences for the public good. Suppose that the provision of the public good is decided by majority voting, and that it costs one unit of private good to produce one unit of public good. The cost is equally divided among the H individuals. Show that the majority voting outcome is Pareto-efficient if the median marginal rate of substitution is equal to the average marginal rate of substitution.

6.23 Consider a collective decision by three individuals to produce, or not, one public good that costs \$150. Suppose that if the public good is produced, the cost is equally shared among the three individuals, namely \$50 each. Assume that the gross benefits from the public good differ among individuals and are respectively \$20, \$40, and \$100 for individuals 1, 2, and 3. Each individual is asked to announce his own benefit for the public good, and the public good is produced only if the sum of reported benefits exceeds the total cost.

a. Show that the Groves–Clarke tax induces truth-telling as a dominant strategy if each individual reports independently his own benefit.

b. Show that the resulting provision of public good is optimal.

c. Show that the Groves–Clarke tax is not robust to collusion in the sense that two individuals could be better off by jointly misreporting their benefit from the public good.

d. What would be the provision of public good if the decision were taken by a majority vote, assuming that the cost is equally shared in the event of public good provision? Compare your answer with part b, and interpret the difference.

6.24 Consider three consumers ($i = 1, 2, 3$) who care about their consumption of a private good and their consumption of a public good. Their utility functions are respectively $u_1 = x_1 G$, $u_2 = x_2 G$, and $u_3 = x_3 G$, where x_i is consumer i's consumption of private good and G is the amount of public good jointly consumed by all of them. The unit cost of the private good is \$1 and the unit cost of the public good is \$10. Individual wealth levels in are $w_1 = 30$, $w_2 = 50$, and $w_3 = 20$. What is the efficient amount of public good for them to consume?

6.25 Albert and Beth are thinking of buying a sofa. Albert's utility function is $u_a(s, m_a) = [1 + s]\,m_a$ and Beth's utility function is $u_b(s, m_b) = [2 + s]\,m_b$, where $s = 0$ if they don't get the sofa and $s = 1$ if they do, and m_a and m_b are the amounts of money they have respectively to spend on private consumption. Albert and Beth each have a total of $w = 100$ (in \$) to spend. What is the maximum amount that they could pay for the sofa and still both be better off than without it?

6.26 Are the following statements true or false? Explain why.

a. If the supply of public good is determined by majority vote, then the outcome must be Pareto-efficient.

b. If preferences are single-peaked, then everyone will agree about the right amount of public goods to be supplied.

c. Public goods are those goods that are supplied by the government.

d. The source of the free-rider problem is the absence of rivalry in the consumption of public goods.

e. The source of the preference revelation problem is the nonexcludability of public goods.

f. If a public good is provided by voluntary contributions, too little will be supplied relative to the efficient level.

6.27 Why does the free-rider problem make it difficult for markets to provide the efficient level of public goods?

6.28 Four people are considering whether to hire a boat for a day out. Describe questions that will elicit over- and undervaluations of the boat hire.

6.29 People are observed traveling a long distance to visit a scenic countryside. How can this fact be used to place a lower bound on their valuation of the countryside?