MICROECONOMICS

Principles and Analysis

CONVEXITY

CONVEX SETS

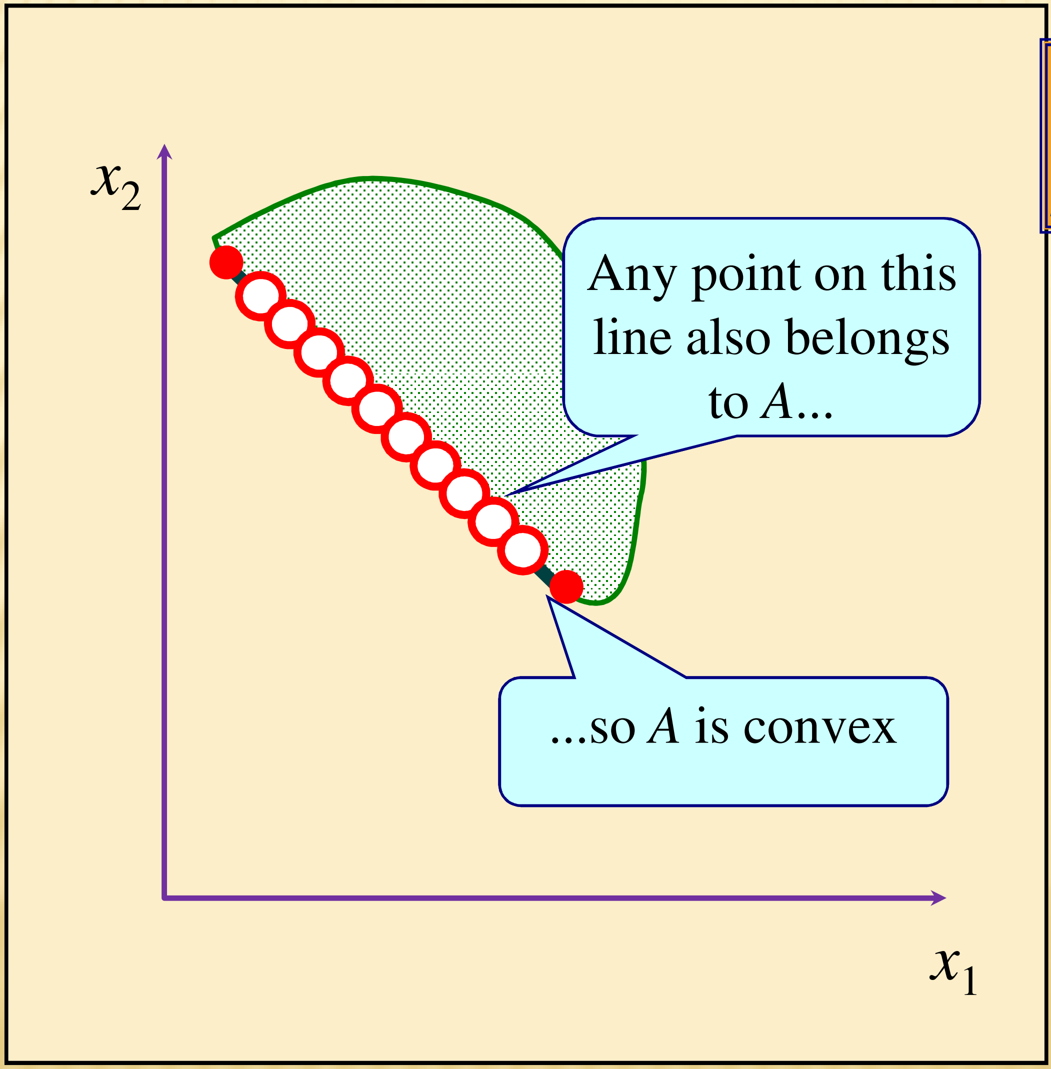
- Ideas of convexity used throughout microeconomics
- Restrict attention to real space R^n
- I.e. sets of vectors $(x_1, x_2, \ldots, x_n)$
- Use the concept of convexity to define
  - Convex functions
  - Concave functions
  - Quasiconcave functions

OVERVIEW…

Basic definitions

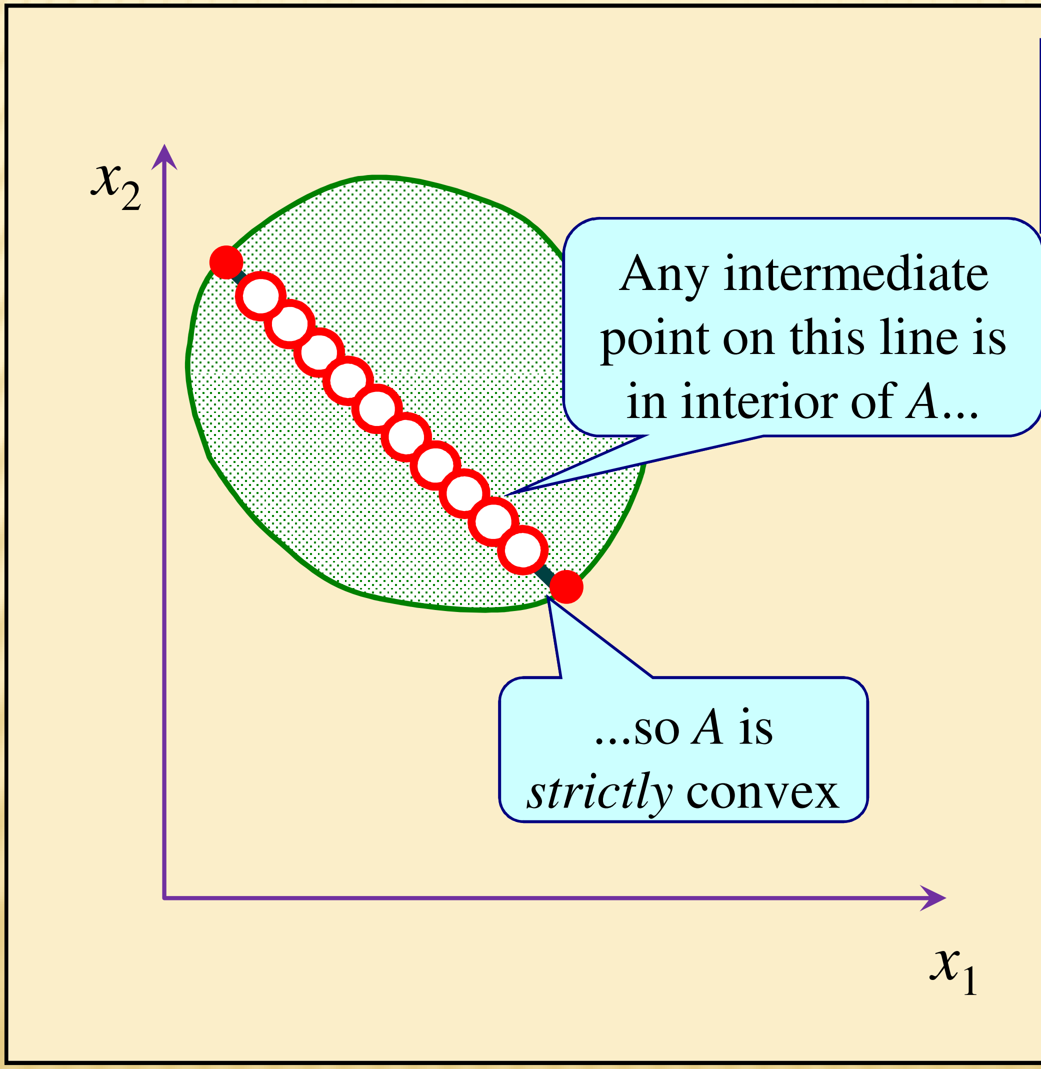
- Convexity
  - Sets
  - Functions
  - Separation

CONVEXITY IN R^2



- *A set A in* R^2
- *Draw a line between any two points in A*

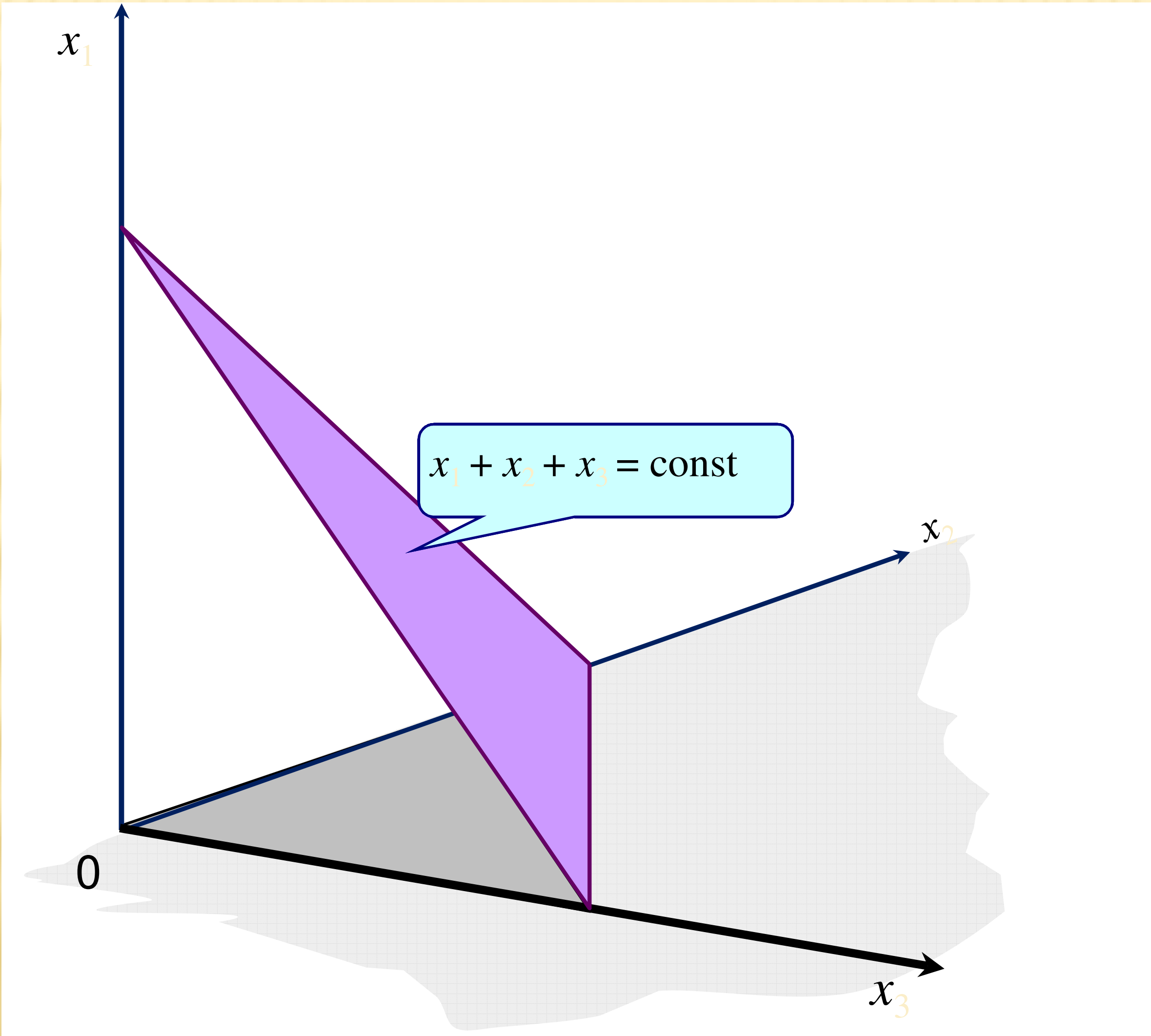
STRICT CONVEXITY IN R²



- ***A set A in*** R²
- ***Draw a line between any two boundary points of A***

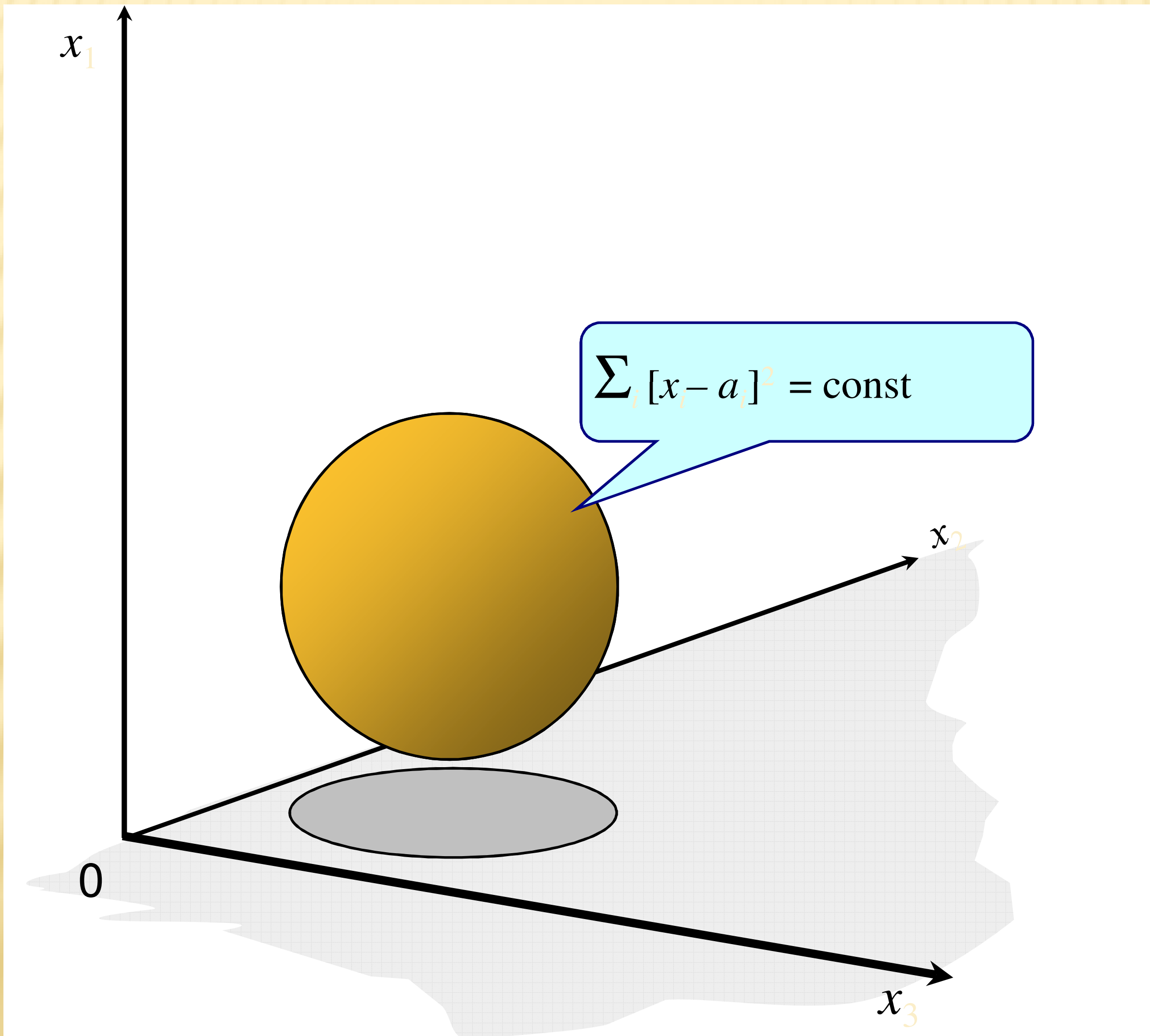
Examples of convex sets in R³

THE SIMPLEX



- *The simplex is convex, but not strictly convex*

THE BALL



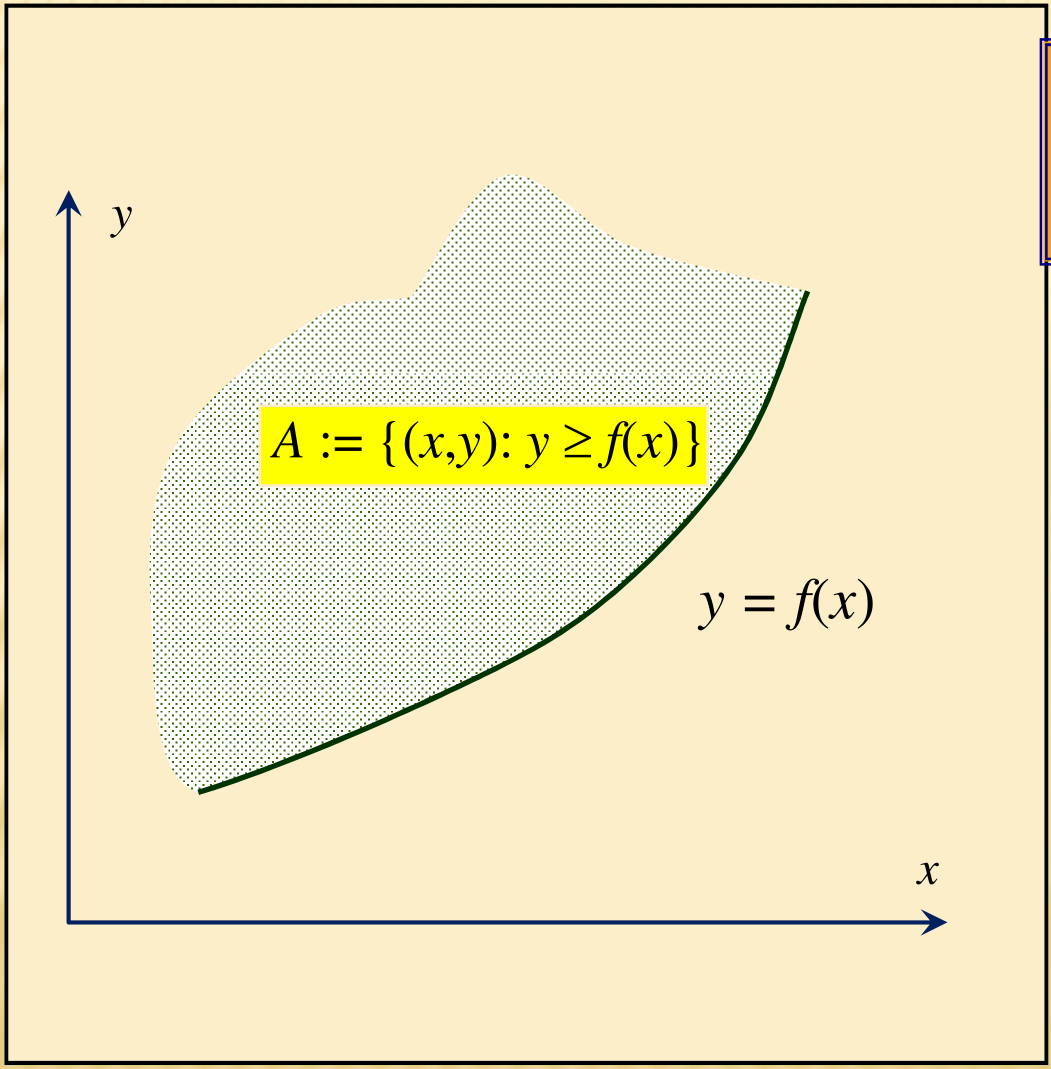
- *A ball centred on the point* $(a_1,a_2,a_3) > 0$
- *It is strictly convex*

OVERVIEW…

For scalars and vectors

- Convexity
  - Sets
  - Functions
  - Separation

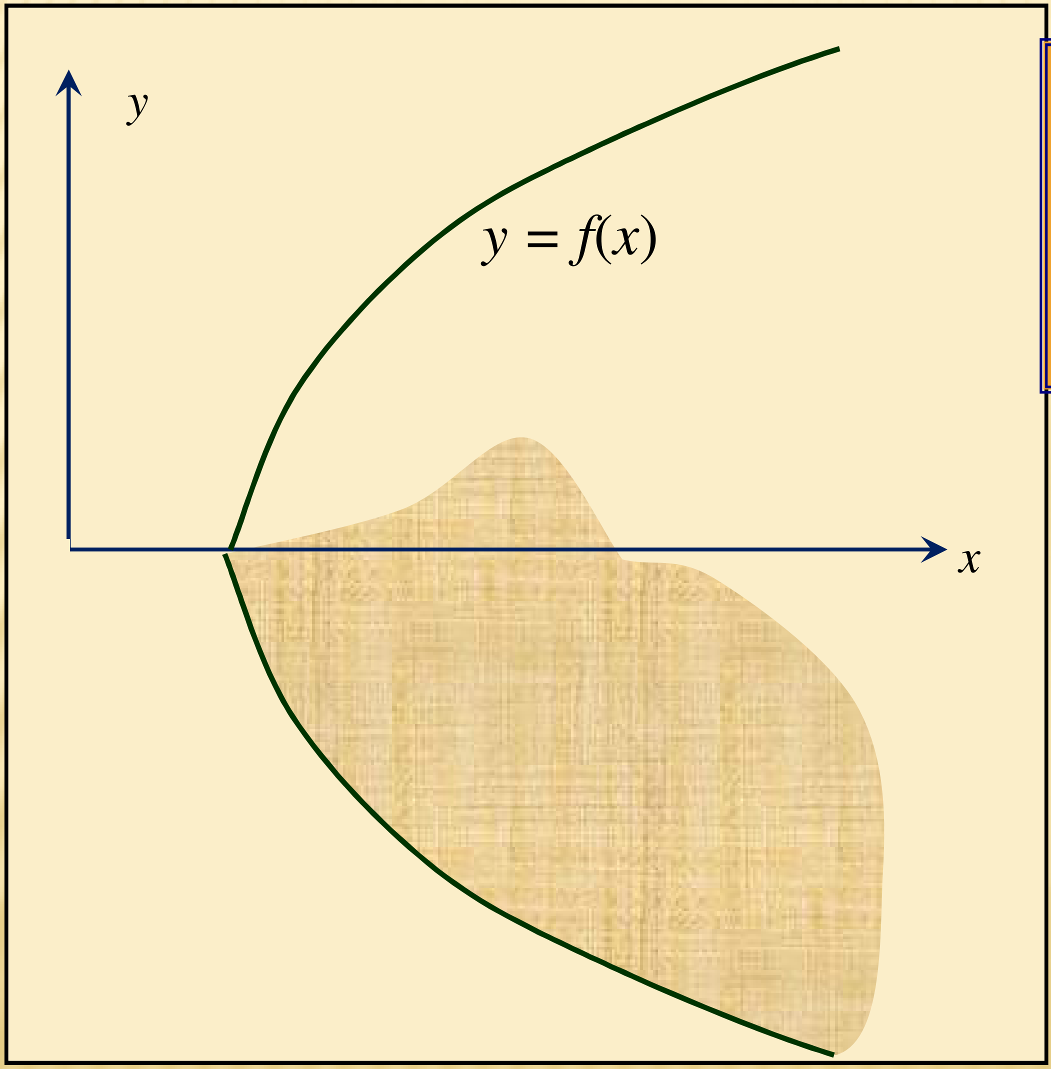
CONVEX FUNCTIONS



- ***A function f:*** R→R
- ***Draw A, the set "above" the function f***

- *If A is convex, f is a convex function*
- *If A is strictly convex, f is a strictly convex function*

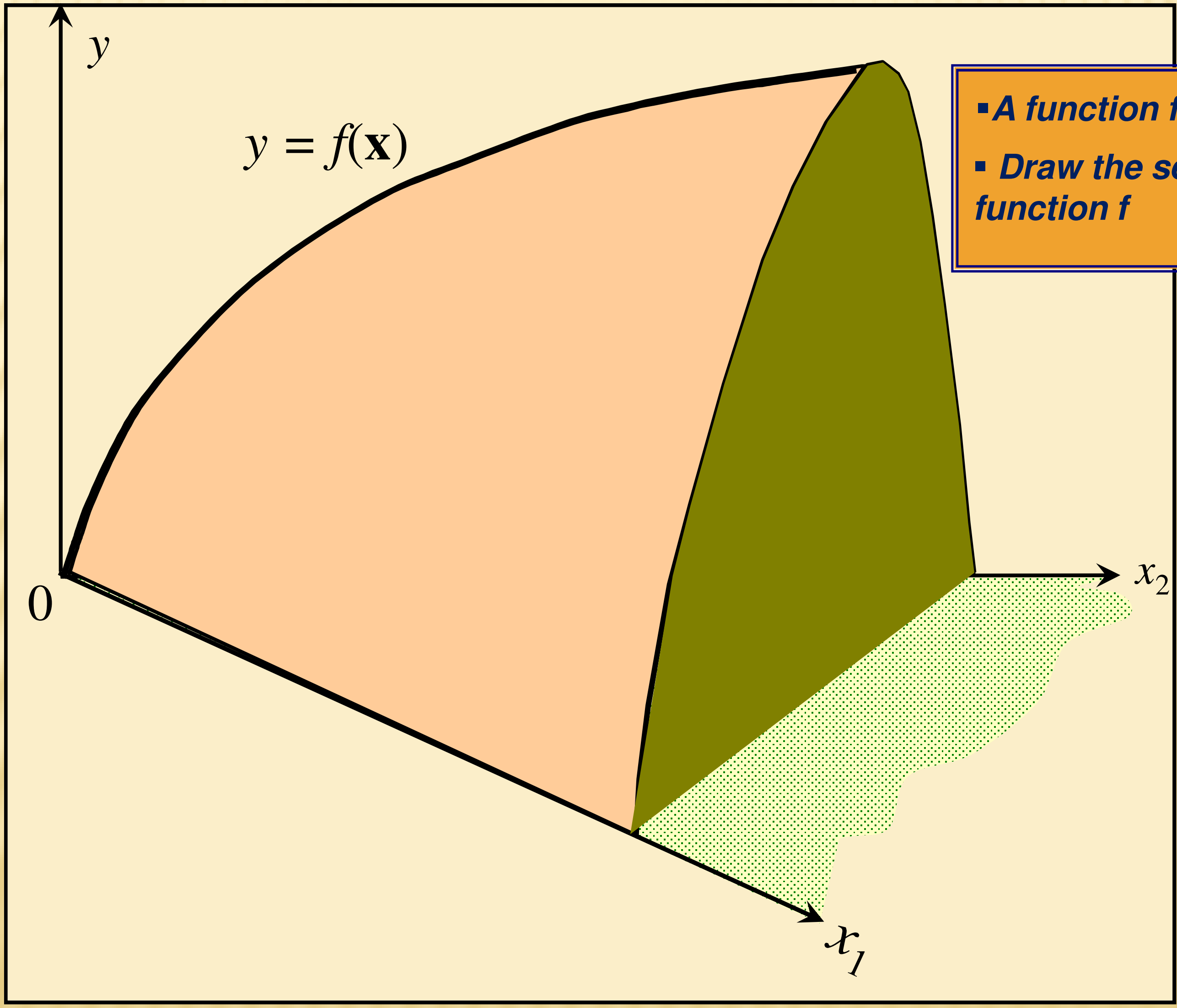
CONCAVE FUNCTIONS (1)



- ***A function f:*** R→R
- ***Draw the function –f***
- ***Draw A, the set "above" the function –f***

- *If –f is a convex function, f is a concave function*
- *Equivalently, if the set "below" f is convex, f is a concave function*
- *If –f is a strictly convex function, f is a strictly concave function*

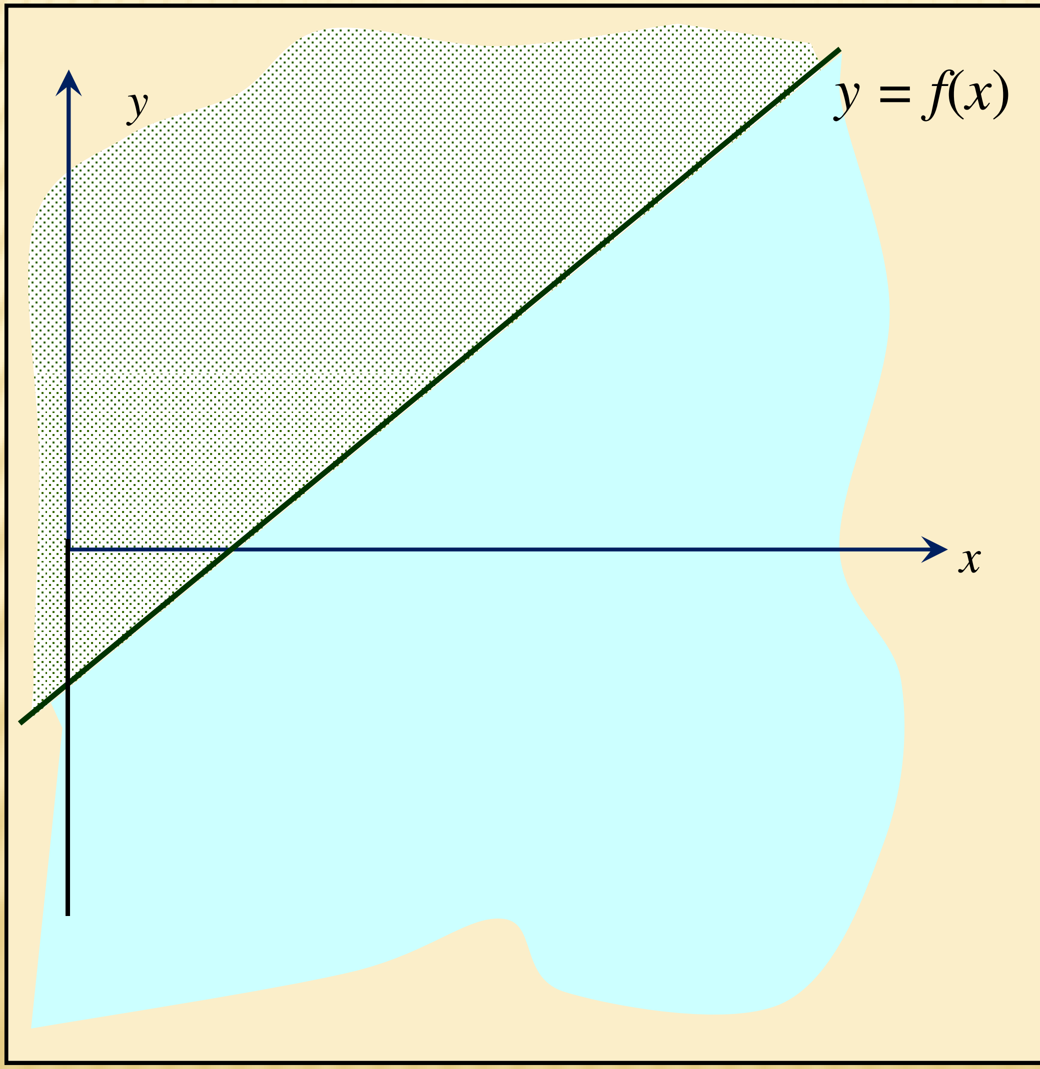
CONCAVE FUNCTIONS (2)



- ***A function f:*** $R^2 \to R$
- ***Draw the set "below" the function f***

- *Set "below" f is strictly convex, so f is a strictly concave function*

CONVEX AND CONCAVE FUNCTION



- ***An affine function f:*** R→R
- ***Draw the set "above" the function f***
- ***Draw the set "below" the function f***

- *The graph in* R^2 *is a straight line.*
- *Both "above" and "below" sets are convex.*
- *So f is both concave and convex.*
- *Corresponding graph in* R^3 *would be a plane.*
- *The graph in* R^n *would be a* *hyperplane*.

QUASICONCAVITY

- In mathematics, a *quasiconvex* function is a real-valued function defined on an interval or on a convex subset of a real vector space such that the inverse image of any set of the form is a convex set.
- Equivalently, a function defined on a convex subset S of a real vector space is quasiconvex if whenever $x, y \in S$ and $\lambda \in [0, 1]$ then

QUASICONCAVITY

$$f(\lambda x + (1-\lambda)y) \leq \max\left(f(x), f(y)\right).$$

If instead

$$f(\lambda x + (1-\lambda)y) < \max\left(f(x), f(y)\right)$$

for any $x \neq y$ and $\lambda \in (0,1)$

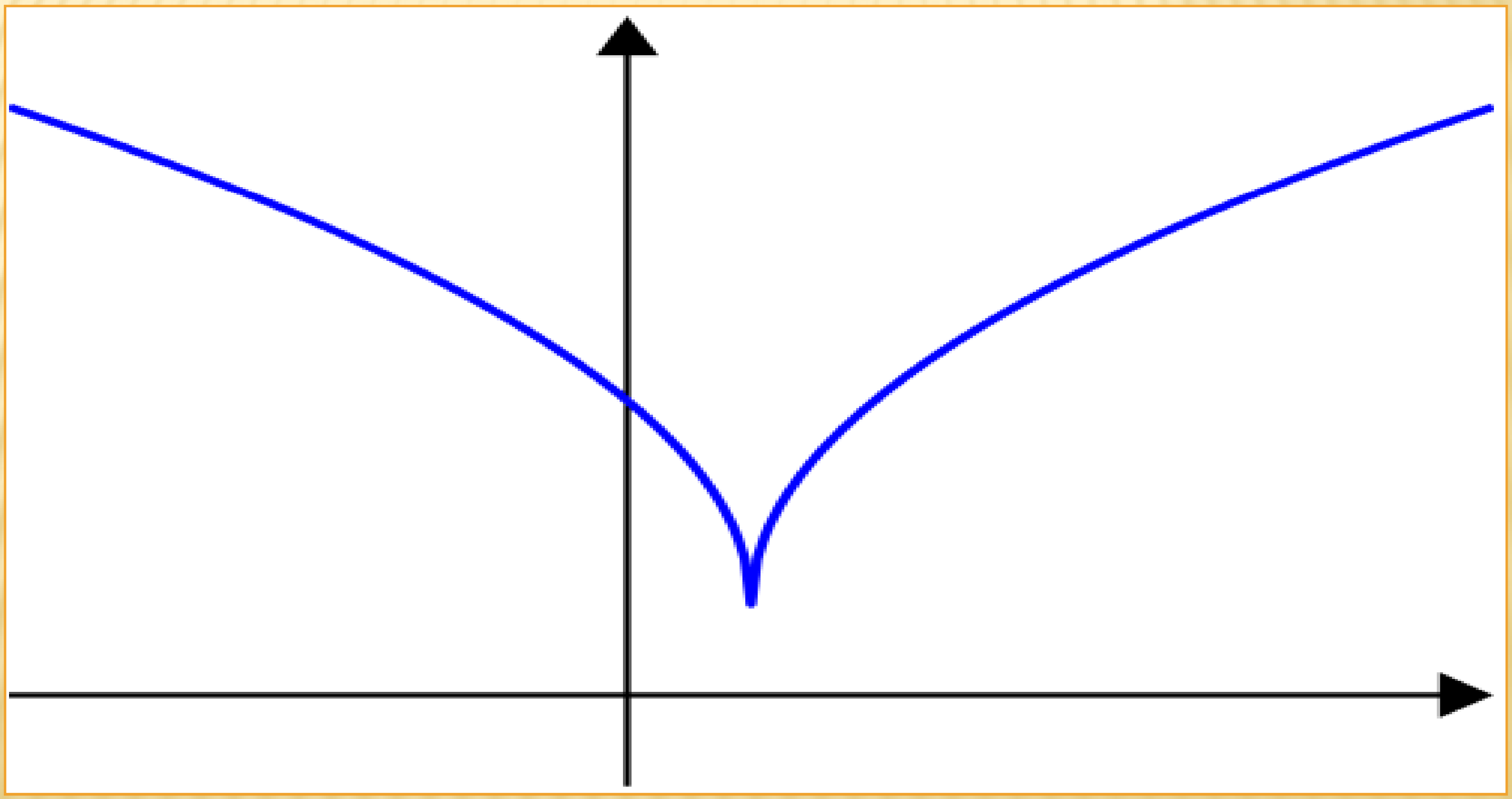
, then *f* is **strictly quasiconvex**.

QUASICONCAVITY

- A quasiconcave function is a function whose negative is quasiconvex, and a strictly quasiconcave function is a function whose negative is strictly quasiconvex.
- A (strictly) quasiconvex function has (strictly) convex lower contour sets, while a (strictly) quasiconcave function has (strictly) convex upper contour sets.

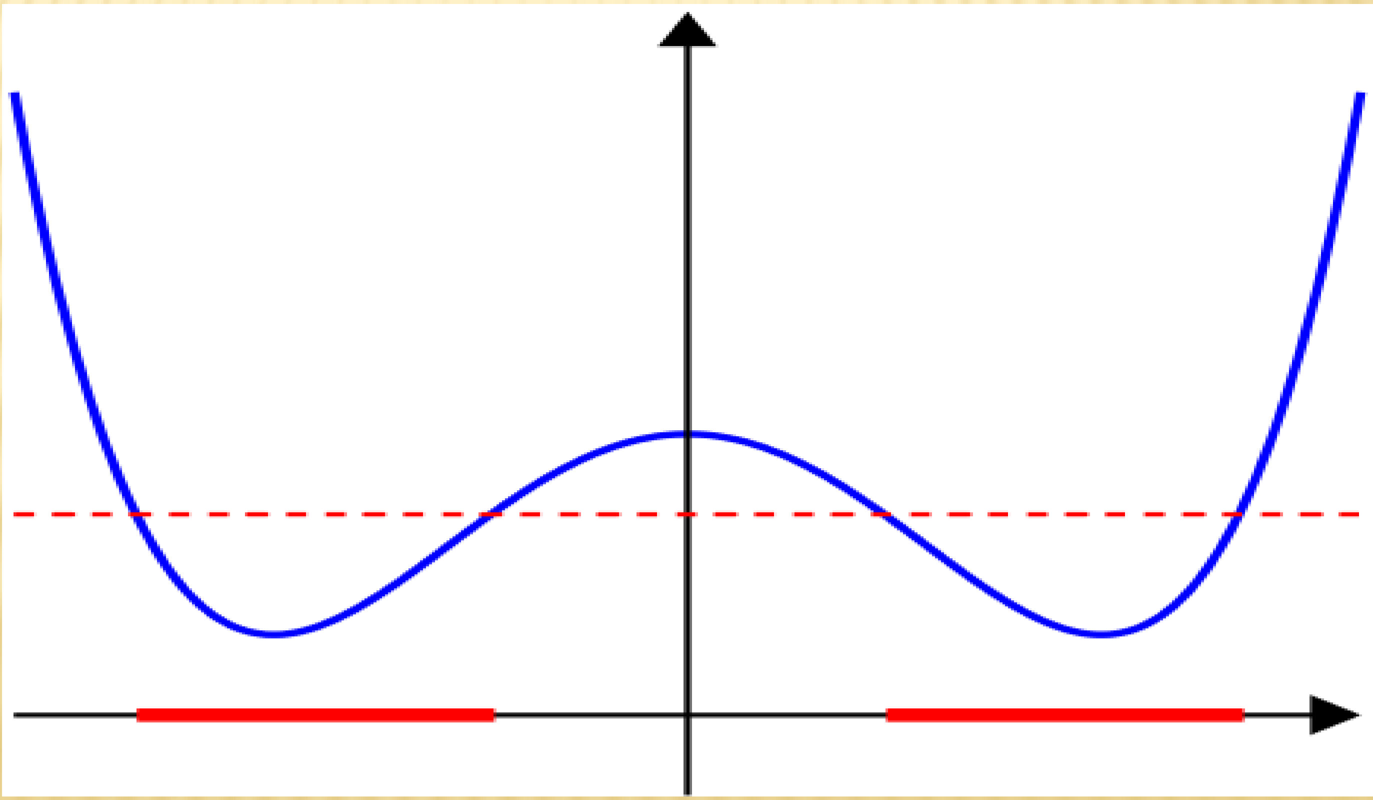
QUASICONCAVITY, EXAMPLE

- A quasiconvex function which is not convex.

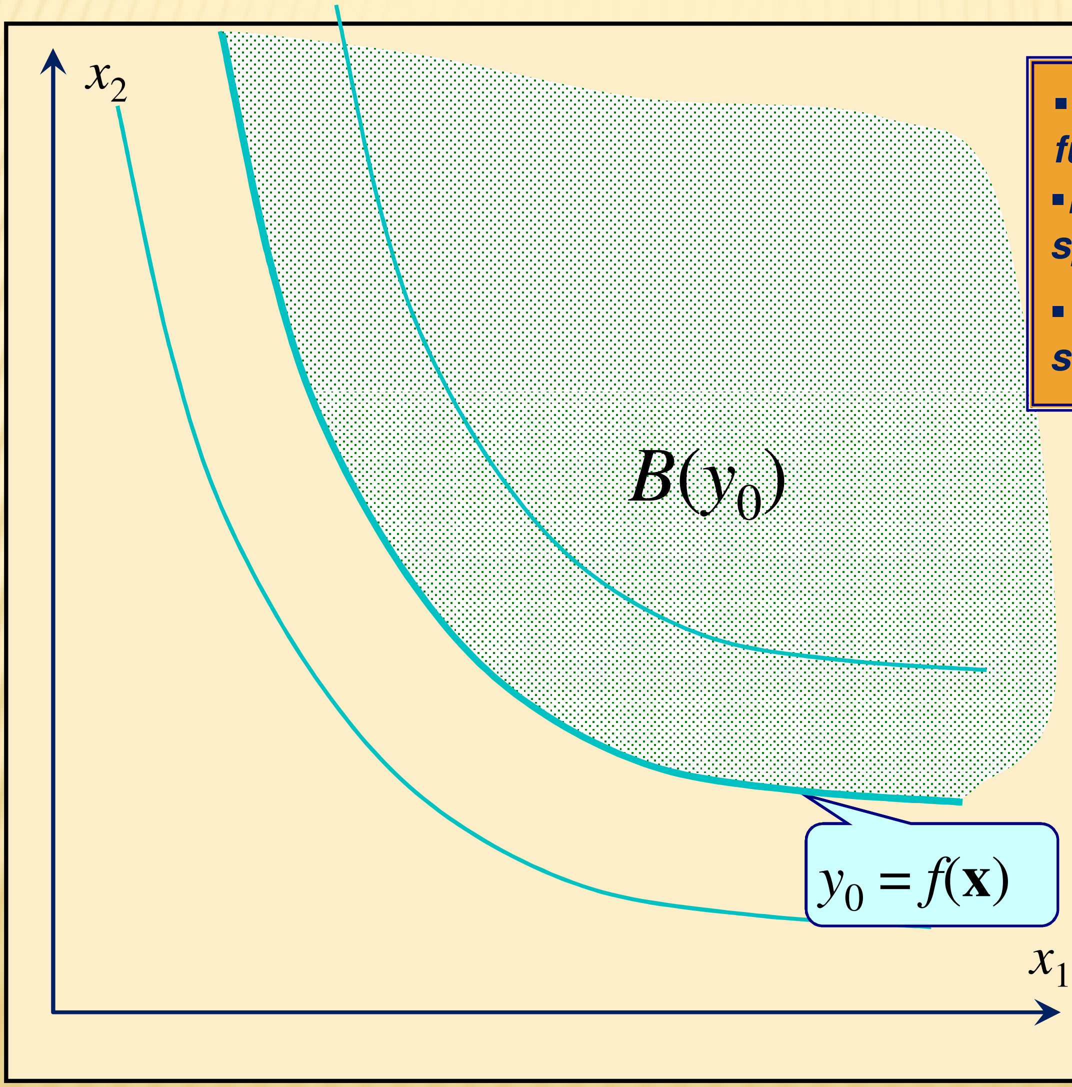


QUASICONCAVITY, EXAMPLE

- A function which is not quasiconvex: the set of points in the domain of the function for which the function values are below the dashed red line is the union of the two red intervals, which is not a convex set.



QUASICONCAVITY



- ***Draw contours of a function f:*** $R^2 \to R$
- ***Pick the contour for some specific y-value y_0 .***
- ***Draw the "better-than" set for y_0 .***

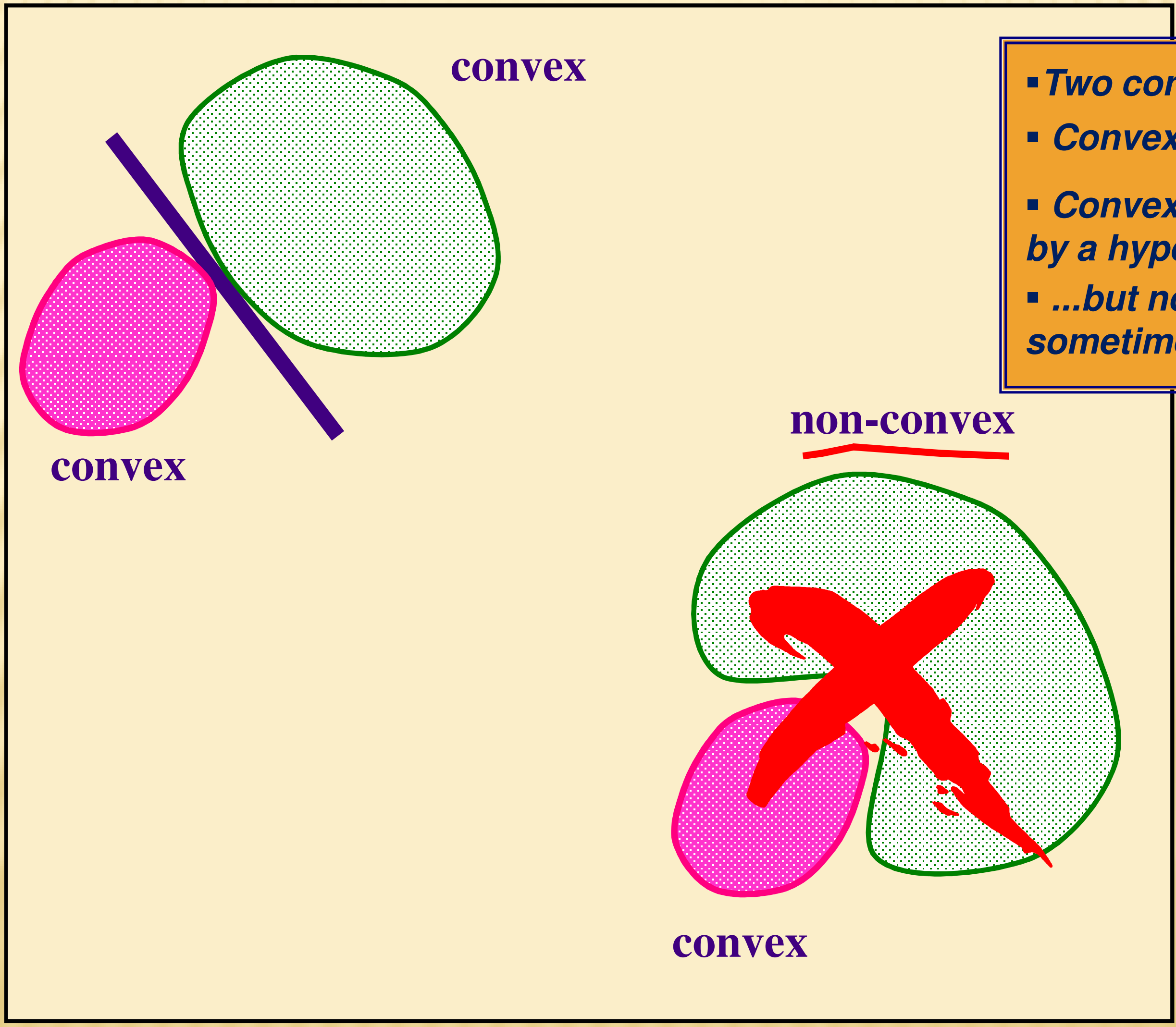
- *If the "better-than" set $B(y_0)$ is convex, f is a <u>concave-contoured</u> function*
- *An equivalent term is a "<u>quasiconcave</u>" function*
- *If $B(y_0)$ is strictly convex, f is a <u>strictly quasiconcave</u>" function*

OVERVIEW…

- Convexity
  - Sets
  - Functions
  - Separation

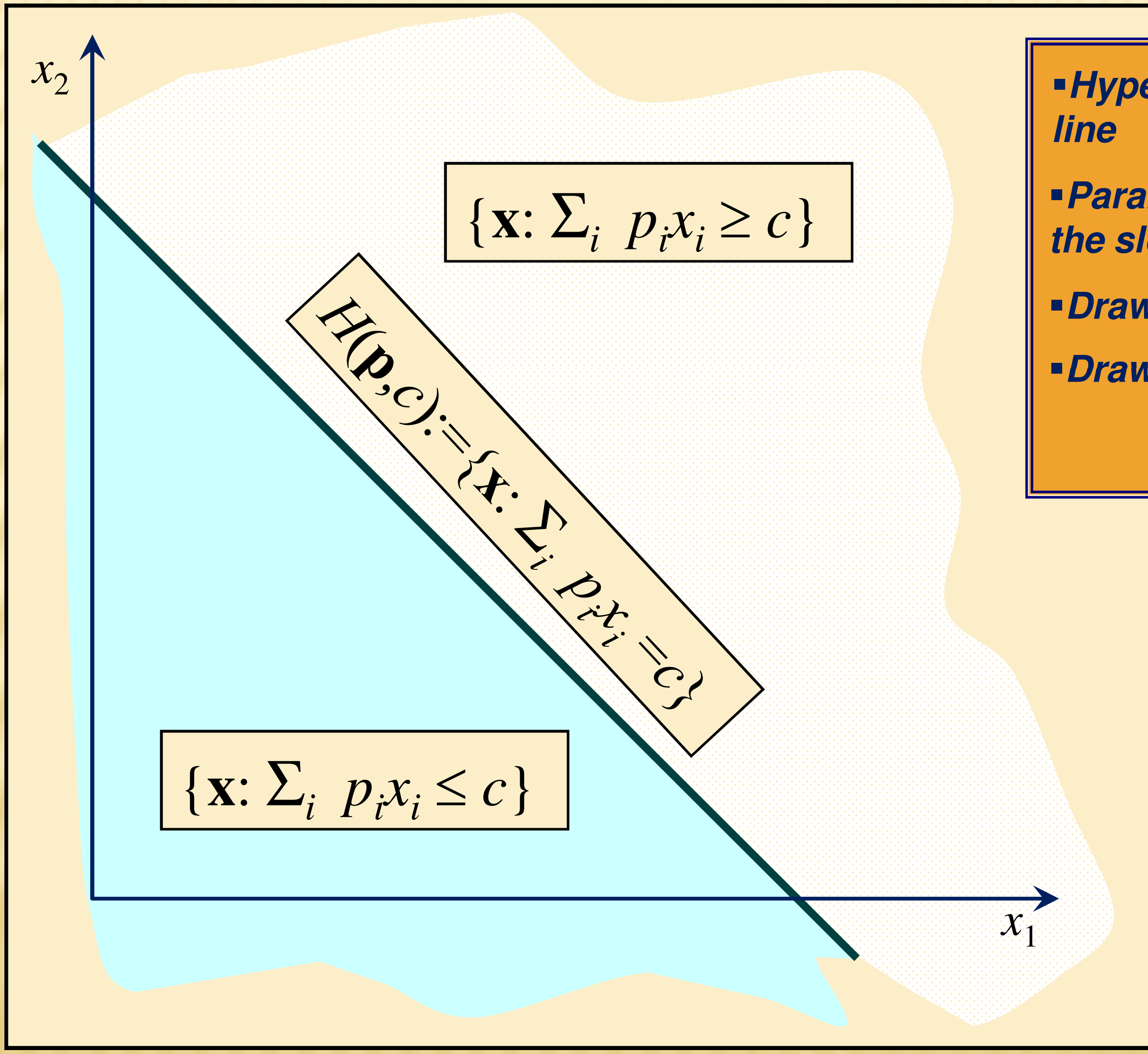
Fundamental relations

CONVEXITY AND SEPARATION



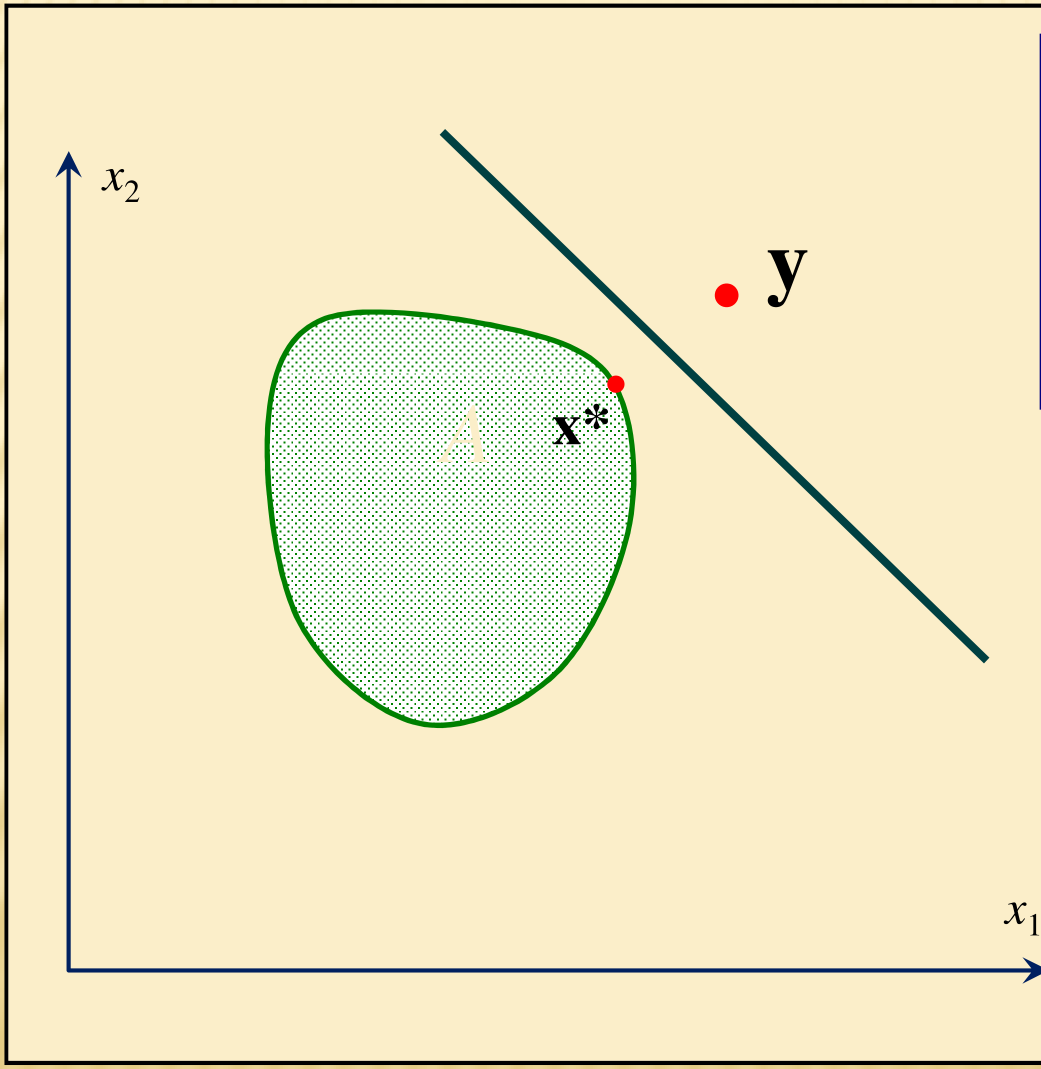
- *Two convex sets in* R^2
- *Convex and nonconvex sets*
- *Convex sets can be separated by a hyperplane...*
- *...but nonconvex sets sometimes can't be separated*

A HYPERPLANE IN $\mathbb{R}^2$



- ***Hyperplane in*** $\mathbb{R}^2$ ***is a straight line***
- ***Parameters*** $\mathbf{p}$ ***and*** c ***determine the slope and position***
- ***Draw in points "above" H***
- ***Draw in points "below" H***

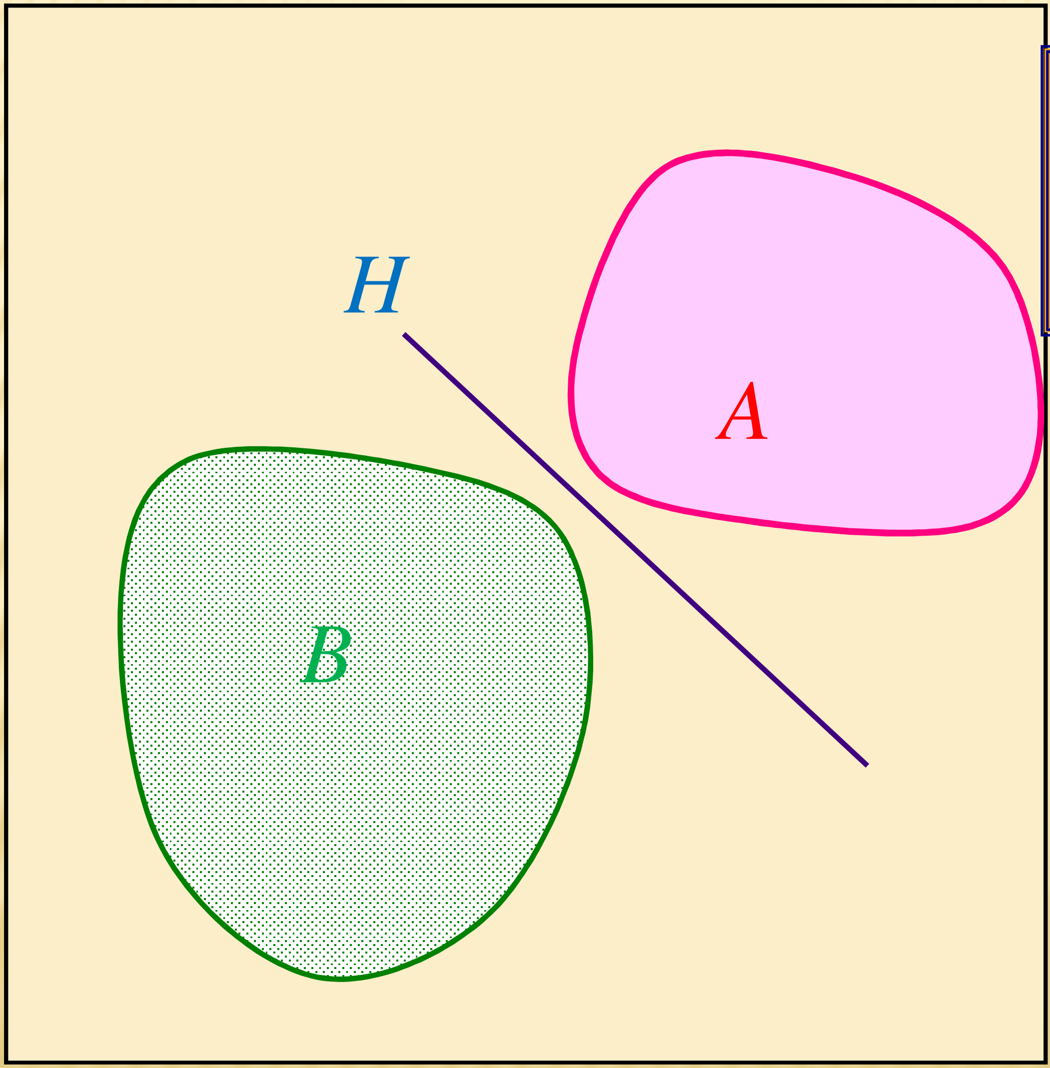
A HYPERPLANE SEPARATING *A* AND Y



- *A convex set A*
- *A point y "outside" A*
- *The point x* in A that is closest to y*
- *The separating hyperplane*

- $y \notin A$.
- *y lies in the "above-H" set*
- *x* lies in the "below-H" set*

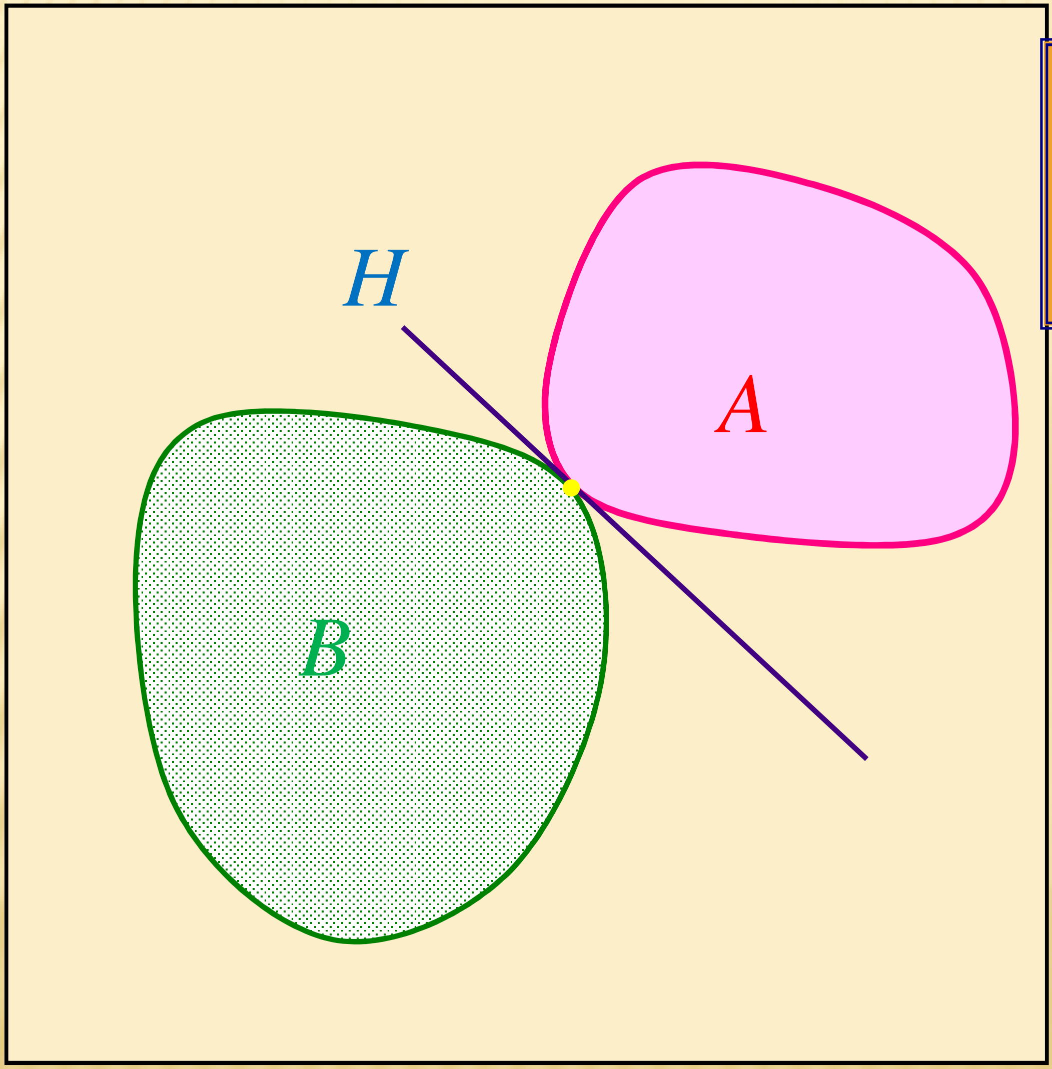
A HYPERPLANE SEPARATING TWO SETS



- *Convex sets A and B.*
- *A and B only have no points in common.*
- *The separating hyperplane.*

- *All points of A lie strictly above H*
- *All points of B lie strictly below H*

SUPPORTING HYPERPLANE



- Convex sets A and B.
- A and B only have boundary points in common.
- The supporting hyperplane.

- Interior points of A lie strictly above H
- Interior points of B lie strictly below H