MICROECONOMICS

Principles and Analysis

THE FIRM: BASICS

OVERVIEW…

The environment for the basic model of the firm.

- The Firm: Basics
  - The setting
  - Input require-ment sets
  - Isoquants
  - Returns to scale
  - Marginal products

THE BASICS OF PRODUCTION...

- We set out some of the elements needed for an analysis of the firm.
  - Technical efficiency
  - Returns to scale
  - Convexity
  - Substitutability
  - Marginal products
- This is in the context of a single-output firm...
- ...and assuming a competitive environment.
- First we need the building blocks of a model...

NOTATION

- **Quantities**

z_i — •amount of input i

$\mathbf{z} = (z_1, z_2, \ldots, z_m)$ — •input vector

q — •amount of output

- **Prices**

w_i — •price of input i

$\mathbf{w} = (w_1, w_2, \ldots, w_m)$ — •Input-price vector

p — •price of output

MOTIVATION OF THE FIRM

- Almost without exception we shall assume that the objective of the firm is to maximise profits: this assumes either that the firm is run by owner-managers or that the firm correctly interprets shareholders' interests.
- More formally, we define the expression for profits as

$$\Pi = pq - \sum_{i=1}^{n} w_i z_i$$

FEASIBLE PRODUCTION

- The basic relationship between output and inputs:

The production function

$$q \le \phi(z_1, z_2, \ldots, z_m)$$

•single-output, multiple-input production relation

- This can be written more compactly as:

Vector of inputs

$$q \le \phi(\mathbf{z})$$

•Note that we use "≤" and not "=" in the relation. Why?
•Consider the meaning of ϕ

- ϕ gives the *maximum* amount of output that can be produced from a given list of inputs

distinguish two important cases...

TECHNICAL EFFICIENCY

- Case 1:

$$q = \phi(\mathbf{z})$$

- The case where production is *technically efficient*

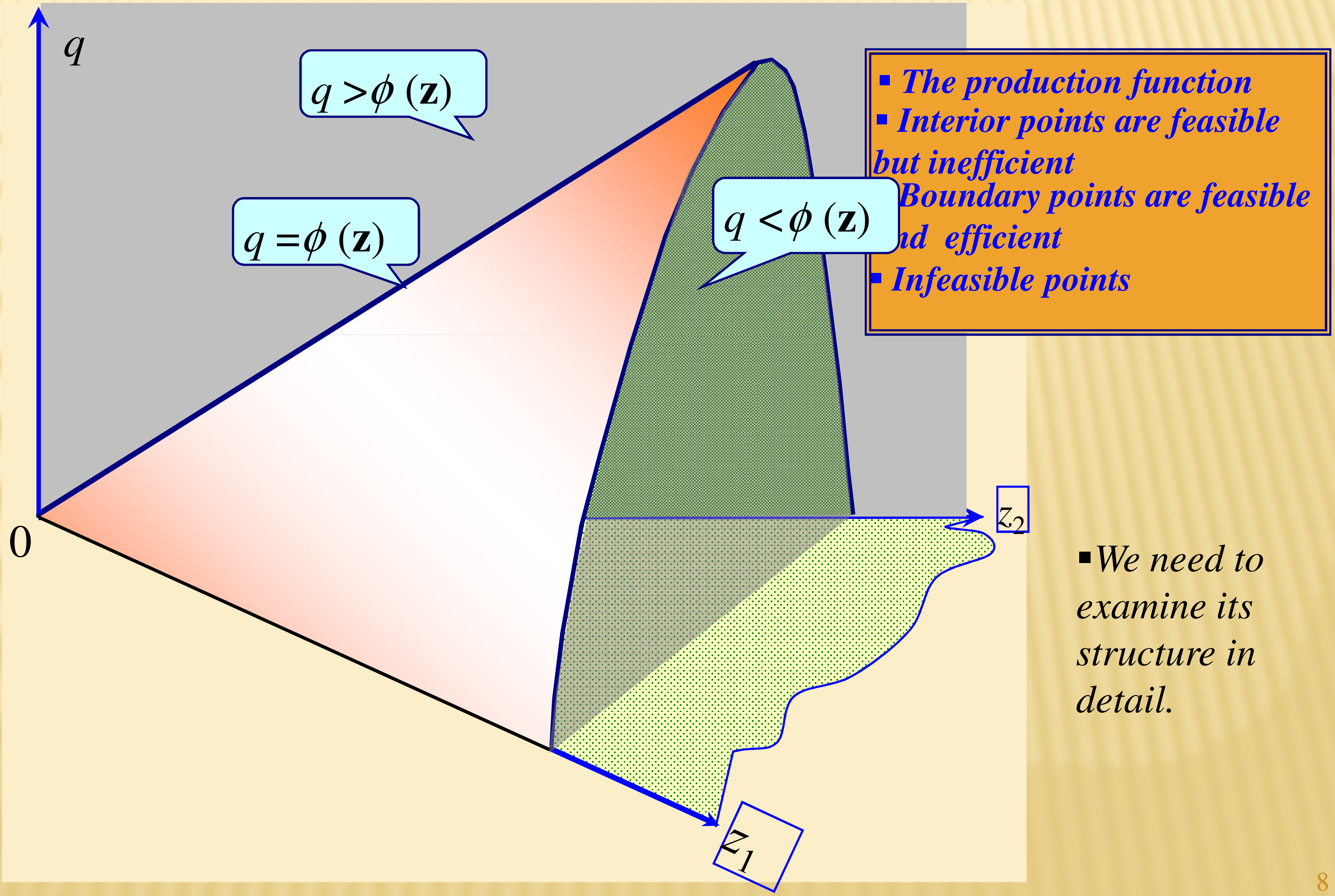
- Case 2:

$$q < \phi(\mathbf{z})$$

- The case where production is (technically) inefficient

Intuition: if the combination ($\mathbf{z}$,q) *is inefficient you can throw away some inputs and still produce the same output*

THE FUNCTION ϕ



- *We need to examine its structure in detail.*

OVERVIEW…

The structure of the production function.

- The Firm: Basics
  - The setting
  - Input requirement sets
  - Isoquants
  - Returns to scale
  - Marginal products

PROPERTIES OF THE PRODUCTION FUNCTION

- Let us examine more closely the production function given in $q \leq \phi(\mathbf{z})$.
- We will call a particular vector of inputs a technique.
- It is useful to introduce two concepts relating to the techniques available for a particular output level q:

THE INPUT REQUIREMENT SET

Pick a particular output level q

- Find a feasible input vector $\mathbf{z}$

Repeat to find all such vectors

- Yields the input-requirement set

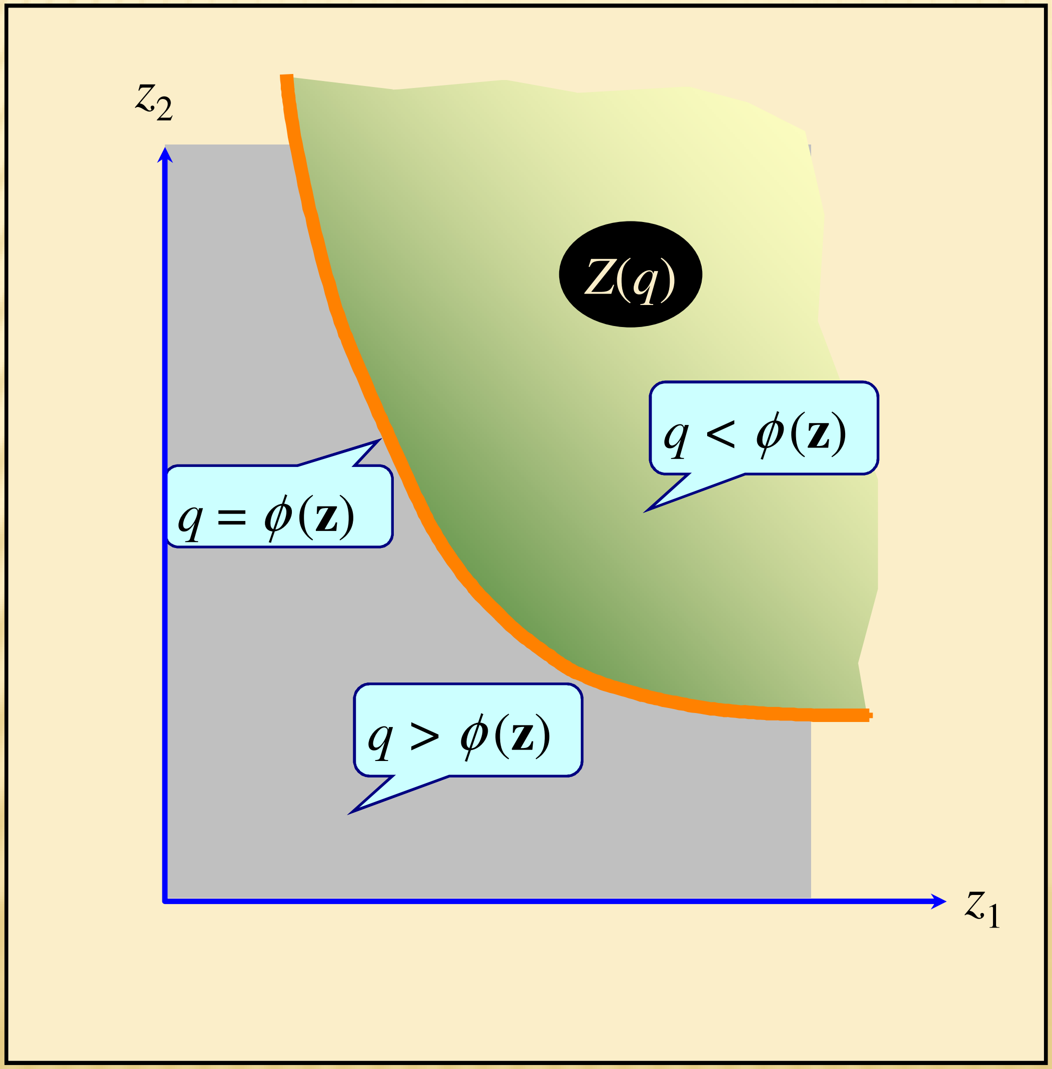
$$Z(q) := \{\mathbf{z}: \phi(\mathbf{z}) \geq q\}$$

- The shape of Z depends on the assumptions made about production...
- We will look at four cases.

- remember, we must have $q \leq \phi(\mathbf{z})$
- The set of input vectors that meet the technical feasibility condition for output q...

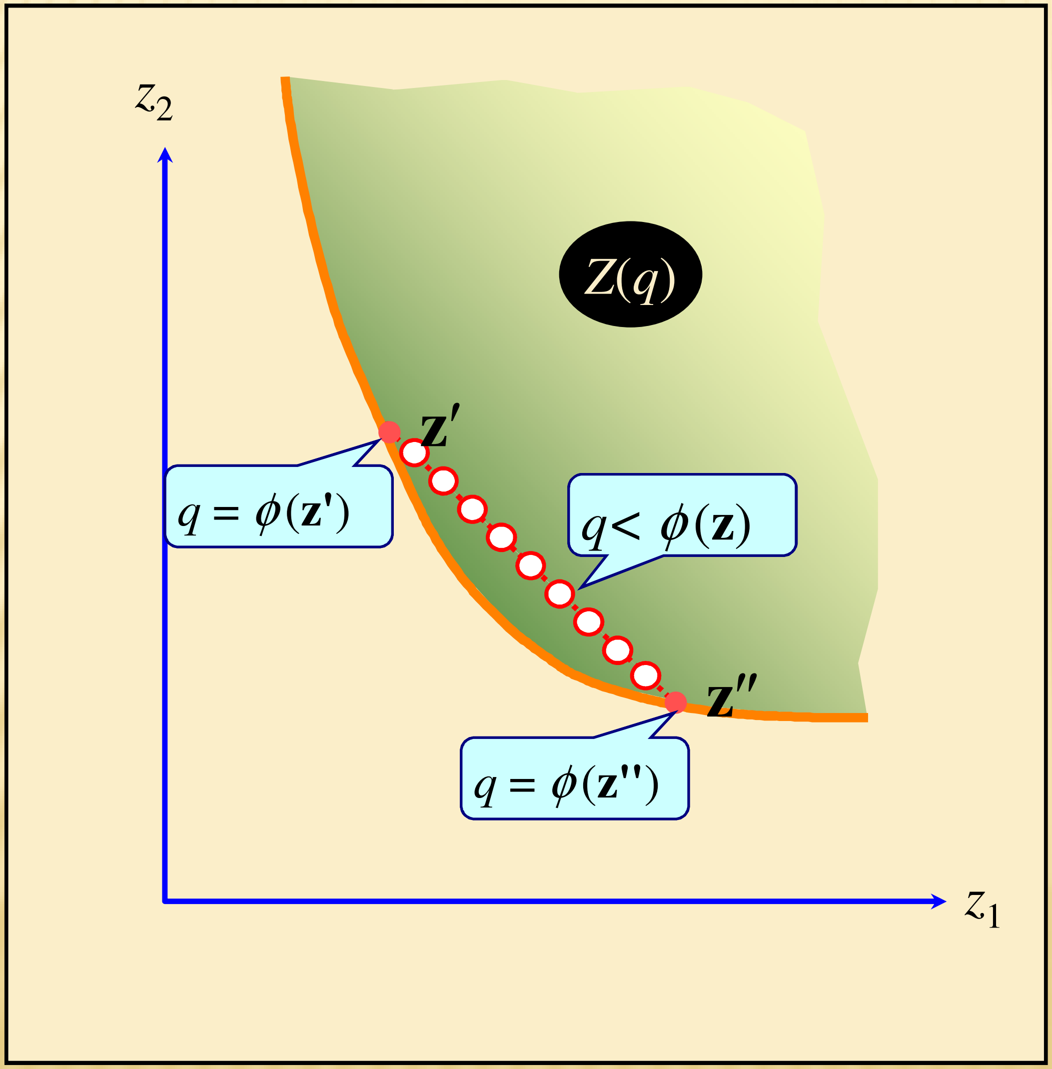
First, the "standard" case.

THE INPUT REQUIREMENT SET



- *Infeasible points.*
- *Feasible but inefficient*
- *Feasible and technically efficient*

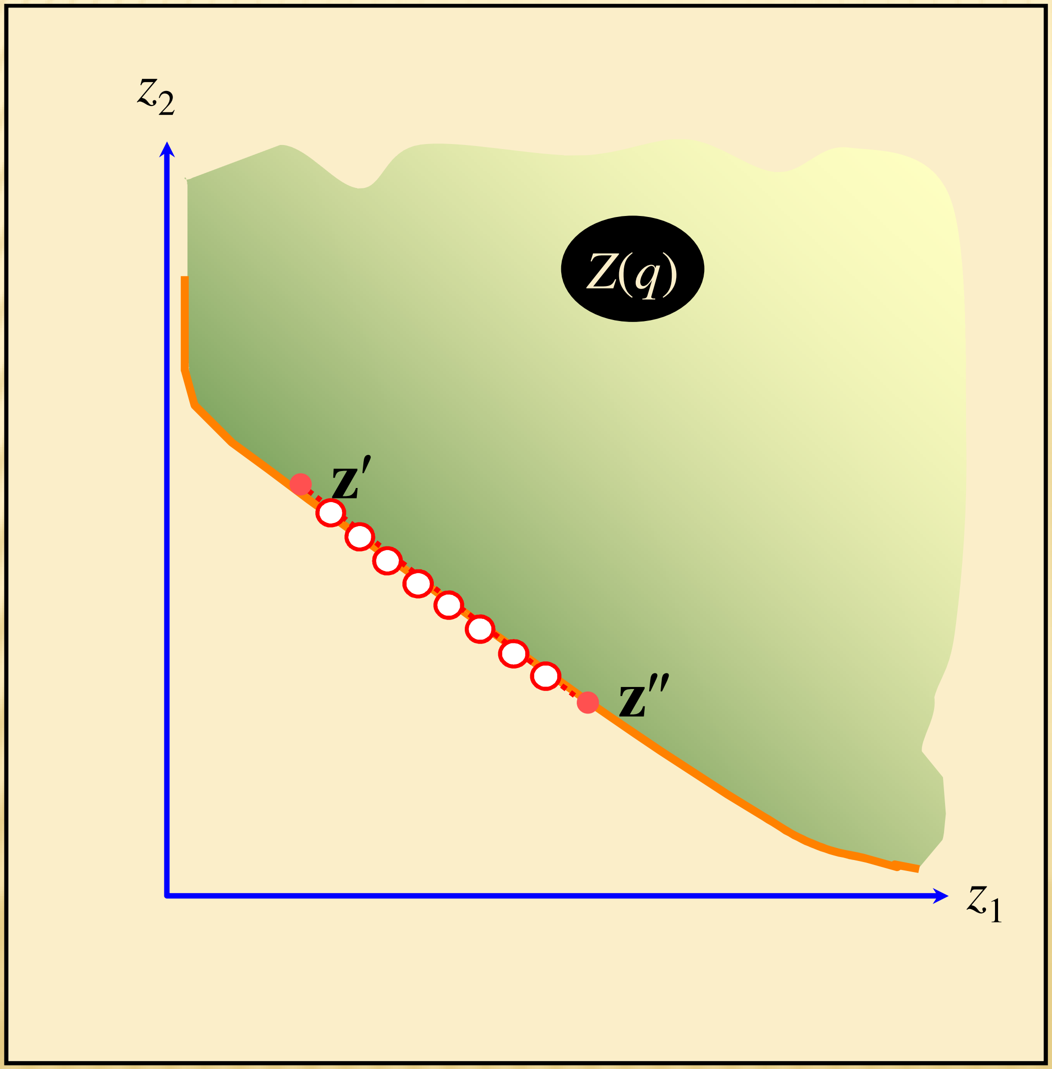
CASE 1: Z SMOOTH, STRICTLY CONVEX



- ***Pick two boundary points***
- ***Draw the line between them***
- ***Intermediate points lie in the interior of Z.***

- *Note important role of convexity.*
- *A combination of two techniques may produce more output.*
- *What if we changed some of the assumptions?*

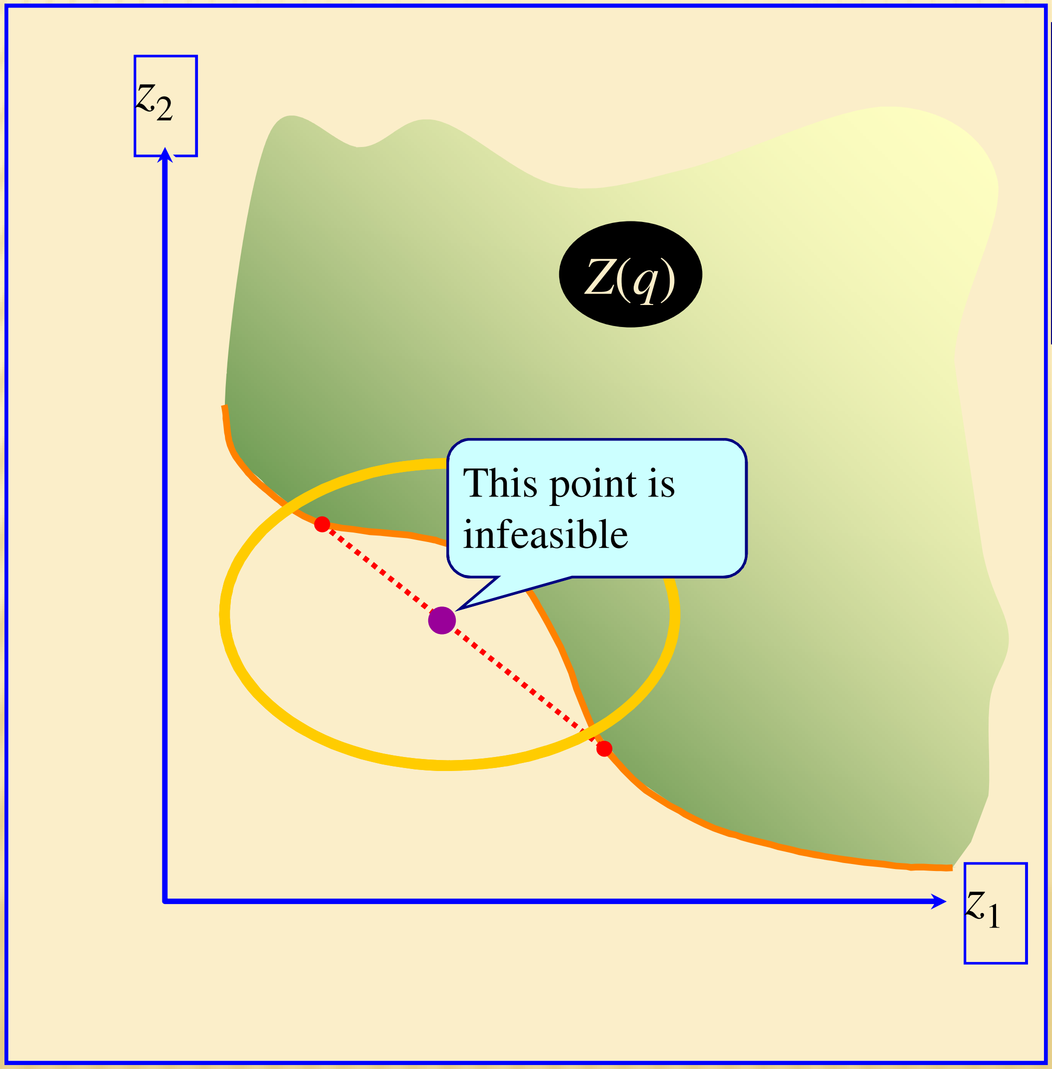
CASE 2: Z CONVEX (BUT NOT STRICTLY)



- ***Pick two boundary points***
- ***Draw the line between them***
- ***Intermediate points lie in Z (perhaps on the boundary).***

- *A combination of feasible techniques is also feasible*

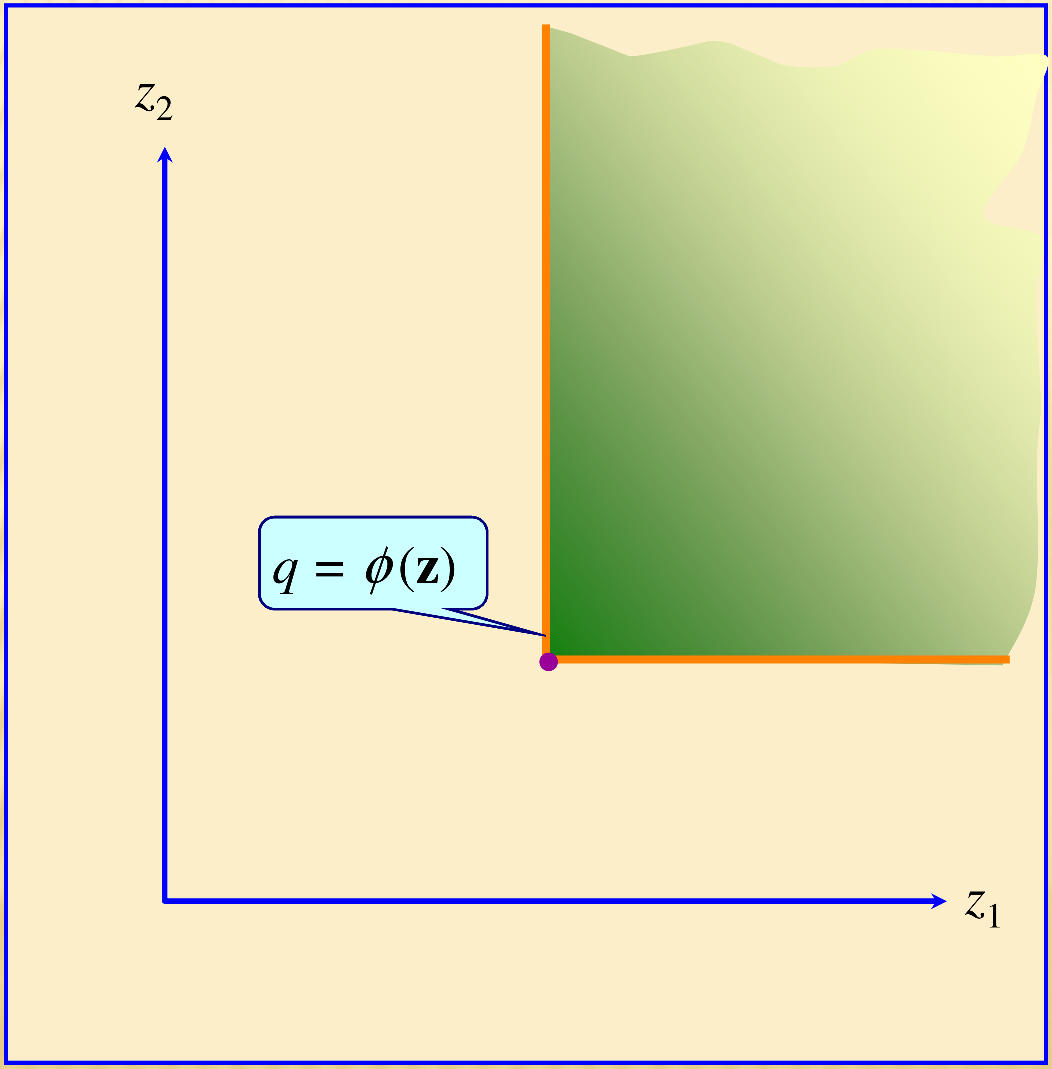
CASE 3: Z SMOOTH BUT *NOT* CONVEX



- ***Join two points across the "dent"***
- ***Take an intermediate point***
- ***Highlight zone where this can occur.***

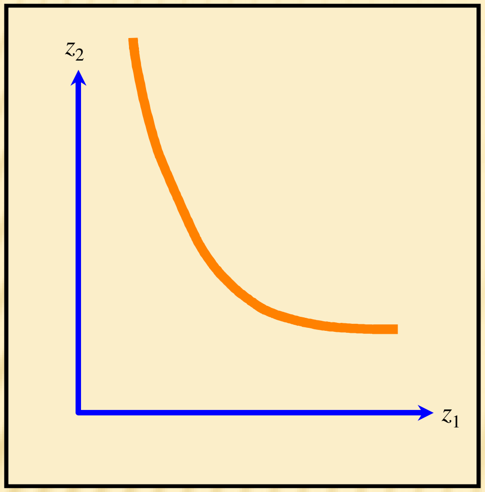
- *in this region there is an indivisibility*

CASE 4: Z CONVEX BUT NOT SMOOTH

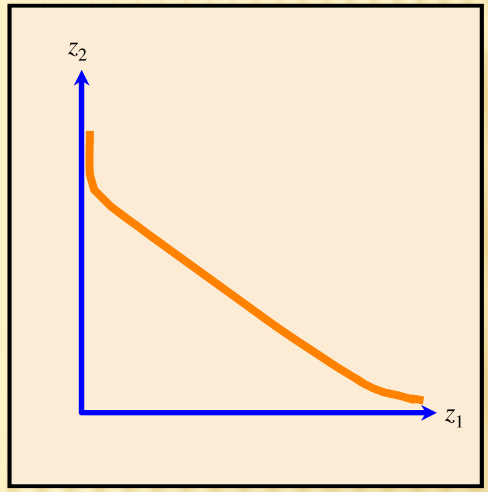


- *Slope of the boundary is undefined at this point.*

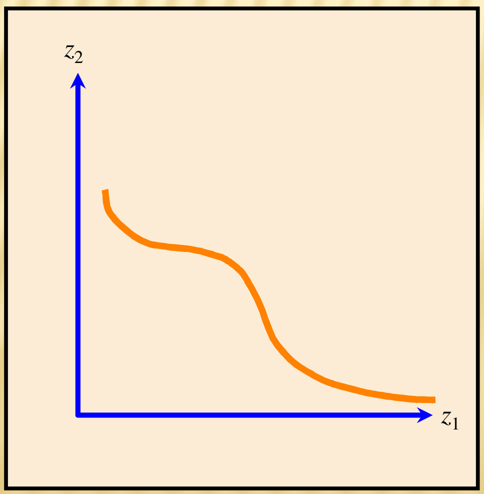
SUMMARY: 4 POSSIBILITIES FOR Z



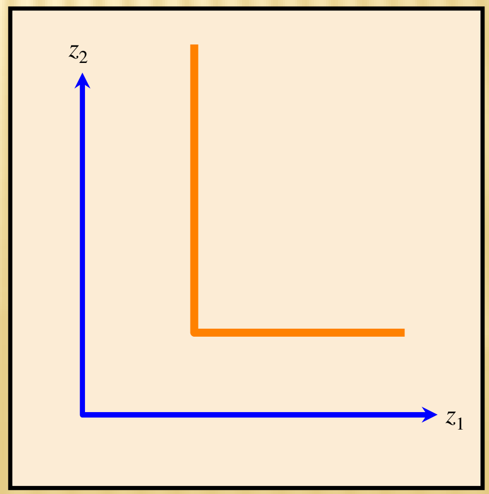
Standard case, but strong assumptions about divisibility and smoothness



Almost conventional: mixtures may be just as good as single techniques



Problems: the "dent" represents an indivisibility



Only one efficient point and not smooth. But not perverse.

OVERVIEW...

Contours of the production function.

- The Firm: Basics
  - The setting
  - Input require-ment sets
  - Isoquants
  - Returns to scale
  - Marginal products

ISOQUANTS

- Pick a particular output level q
- Find the input requirement set $Z(q)$
- The *isoquant* is the boundary of Z:

$$\{\ \mathbf{z} : \phi(\mathbf{z}) = q\ \}$$

- Think of the isoquant as an integral part of the set $Z(q)$...

- If the function ϕ is differentiable at $\mathbf{z}$ then the *marginal rate of technical substitution* is the slope at $\mathbf{z}$: $\frac{\phi_j(\mathbf{z})}{\phi_i(\mathbf{z})}$

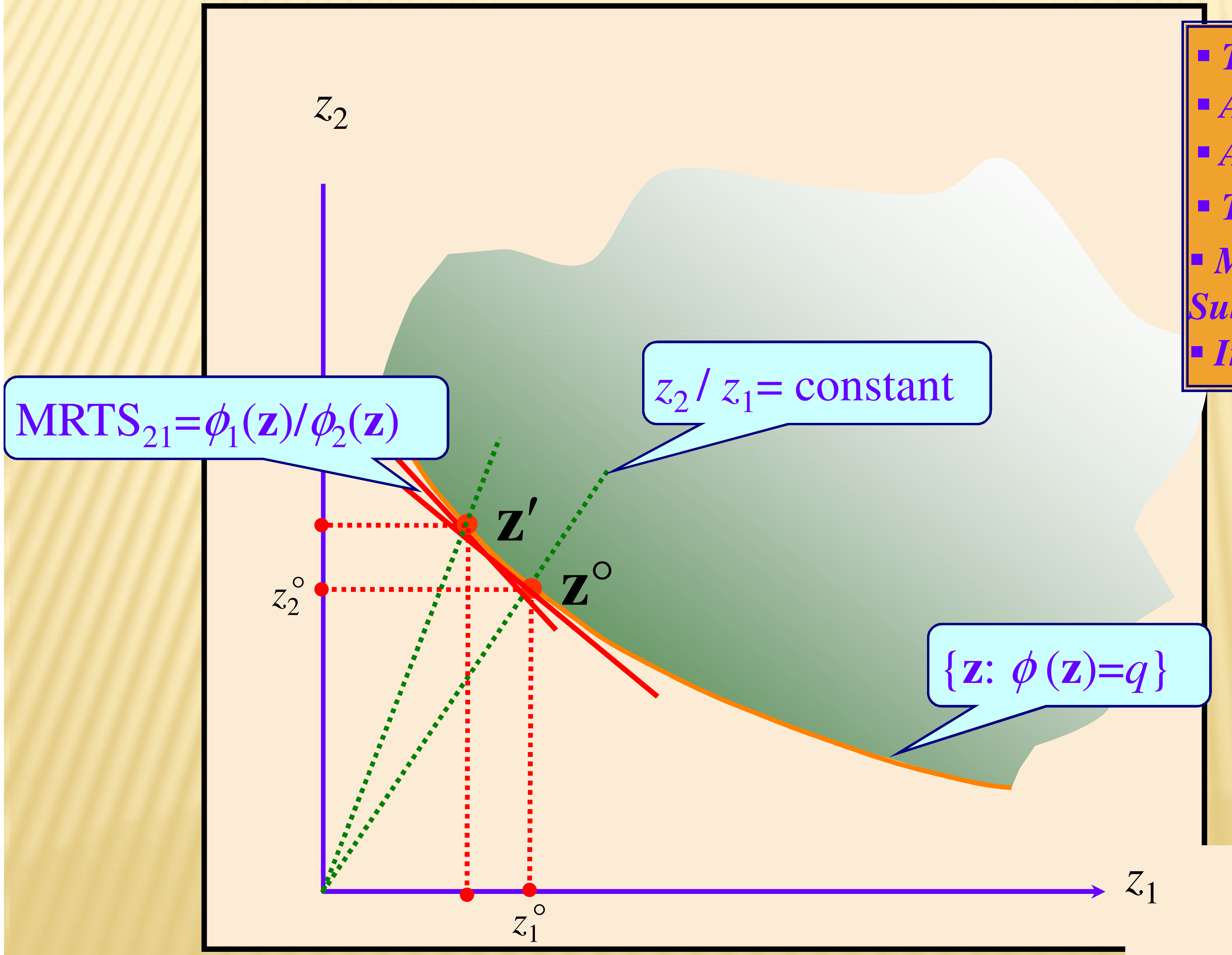
- Where appropriate, use subscript to denote partial derivatives. So

$$\phi_i(\mathbf{z}) := \frac{\partial \phi(\mathbf{z})}{\partial z_i}$$

- Gives the rate at which you can trade off one output against another along the isoquant – to maintain a constant q.

Let's look at its shape

ISOQUANT, INPUT RATIO, MRTS



- ***The set Z(q).***
- ***A contour of the function ϕ.***
- ***An efficient point.***
- ***The input ratio***
- ***Marginal Rate of Technical Substitution***
- ***Increase the MRTS***

- *The isoquant is the boundary of Z.*
- *Input ratio describes one particular production technique.*
- *Higher input ratio associated with higher MRTS..*

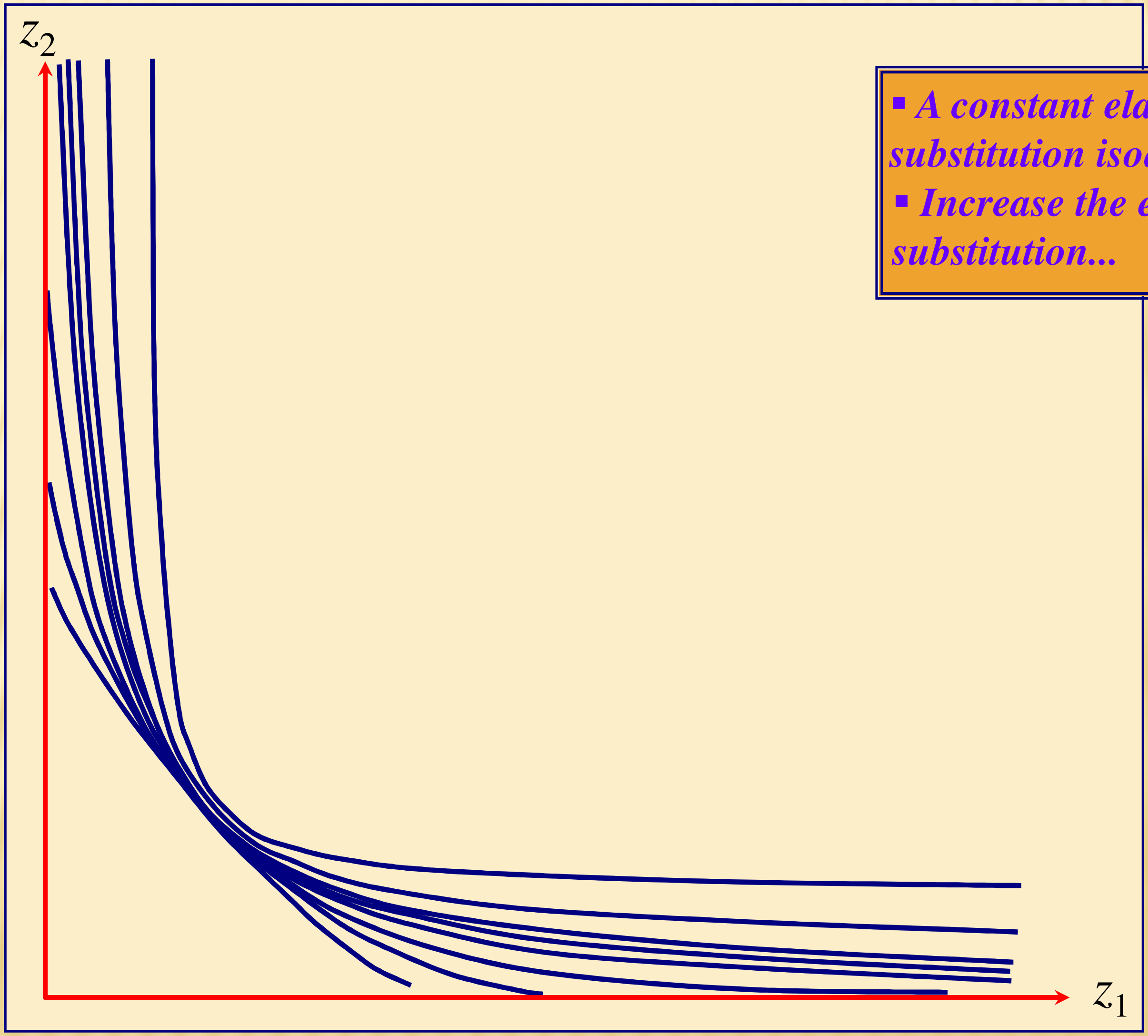
INPUT RATIO AND MRTS

- $MRTS_{21}$ is the implicit "price" of input 1 in terms of input 2.
- The higher is this "price", the smaller is the relative usage of input 1.
- Responsiveness of input ratio to the MRTS is a key property of ϕ.
- Given by the *elasticity of substitution*

$$\sigma_{ij} = - \frac{\partial \log(z_1/z_2)}{\partial \log(\phi_1/\phi_2)}$$

- Can think of it as measuring the isoquant's "curvature" or "bendiness"

ELASTICITY OF SUBSTITUTION



- *A constant elasticity of substitution isoquant*
- *Increase the elasticity of substitution...*

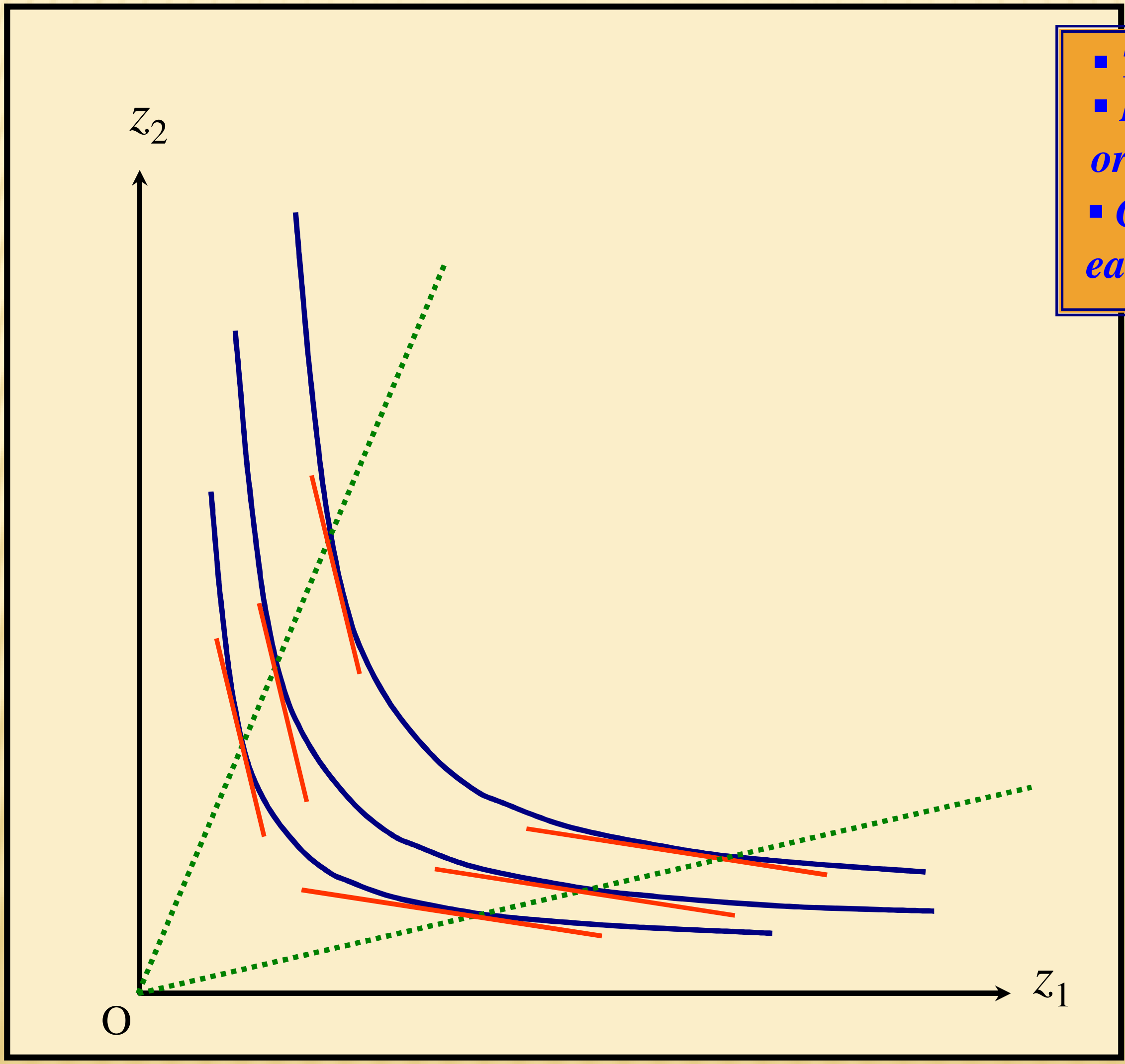
ELASTICITY OF SUBSTITUTION

- Higher values of σ mean that the production function is more "flexible" in that there is a proportionately larger change in the production technique in response to a given proportionate change in the implicit relative valuation of the factors:

HOMOTHETIC CONTOURS

- With homothetic contours, each isoquant appears like a photocopied enlargement; along any ray through the origin all the tangents have the same slope so that the MRTS depends only on the relative proportions of the inputs used in the production process.

HOMOTHETIC CONTOURS



- *The isoquants*
- *Draw any ray through the origin…*
- *Get same MRTS as it cuts each isoquant.*

CONTOURS OF A HOMOGENEOUS FUNCTION

- An important subcase of the family of homothetic functions is the *homogeneous* production functions, for which the map looks the same but where the labelling of the contours has to satisfy the following rule: for any scalar $t > 0$ and any input vector $z \geq 0$:

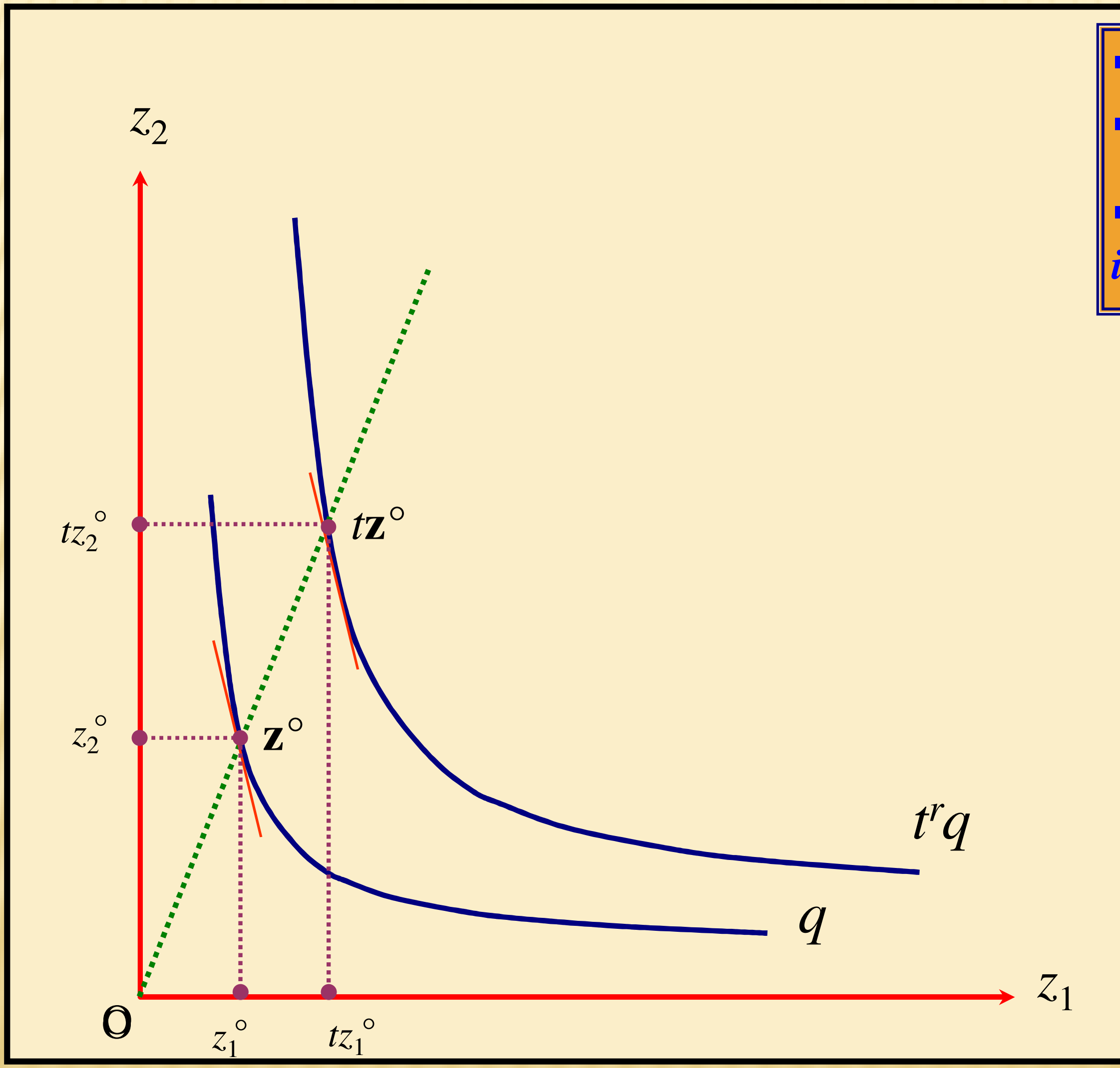
$$\phi(tz) = t^r \phi(z)$$

where r is a positive scalar.

CONTOURS OF A HOMOGENEOUS FUNCTION

- If $\varphi(.)$ satisfies the property in the above equation then it issaid to be homogeneous of degree r. Clearly the parameter r carries important information about the way output responds to a proportionate change in all inputs together:
- If $r > 1$, for example then doubling more inputs will more than double output.

CONTOURS OF A HOMOGENEOUS FUNCTION



- *The isoquants*
- *Coordinates of input z°*
- *Coordinates of "scaled up" input tz°*

$$\phi(t\mathbf{z}) = t^r \phi(\mathbf{z})$$

OVERVIEW…

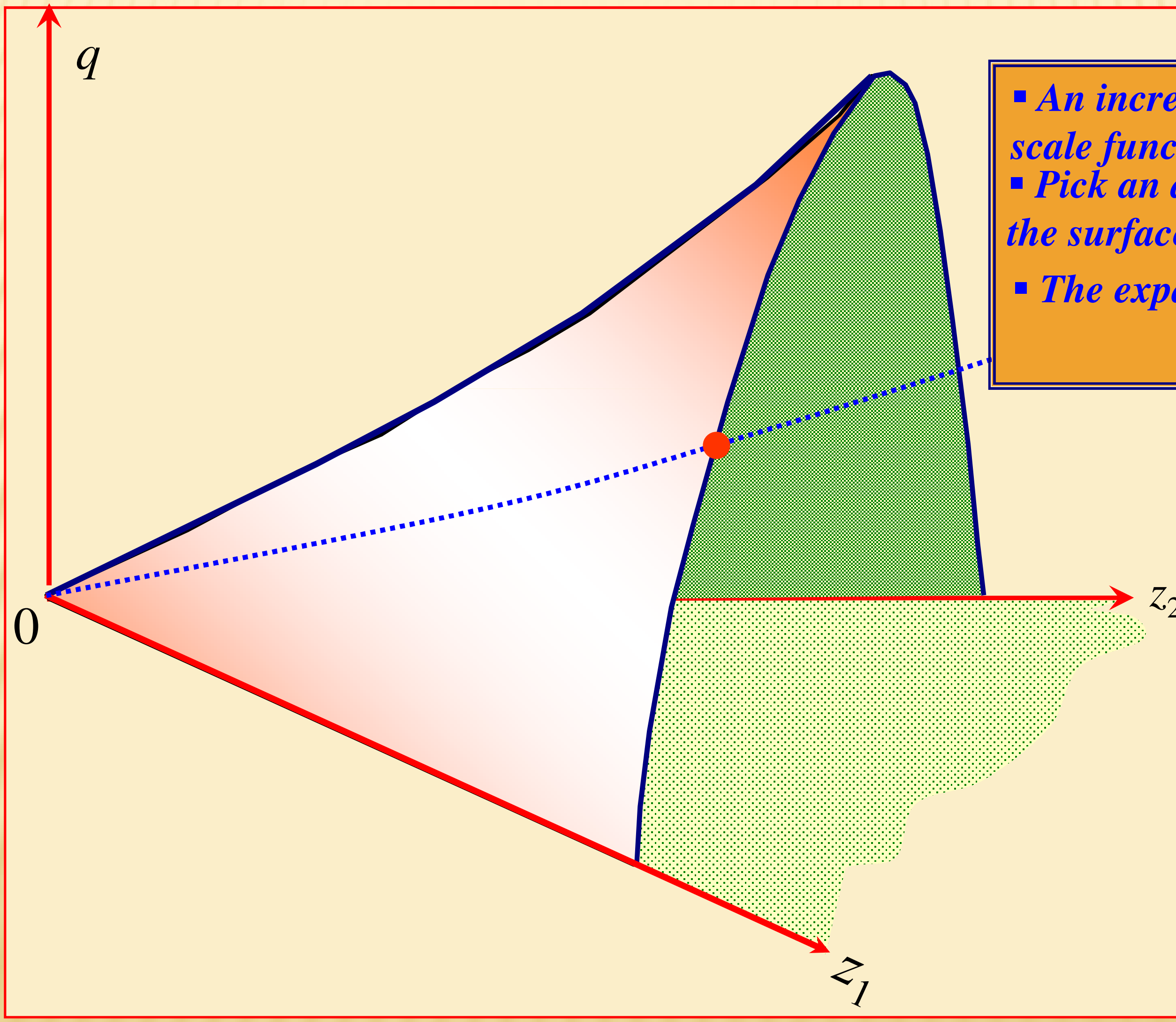
Changing all inputs together.

- The Firm: Basics
  - The setting
  - Input require-ment sets
  - Isoquants
  - **Returns to scale**
  - Marginal products

LET'S REBUILD FROM THE ISOQUANTS

- The isoquants form a contour map.
- If we looked at the “parent” diagram, what would we see?
- Consider ***returns to scale*** of the production function.
- Examine effect of varying all inputs together:
  - Focus on the expansion path.
  - q plotted against proportionate increases in **z**.
- Take three standard cases:
  - Increasing Returns to Scale
  - Decreasing Returns to Scale
  - Constant Returns to Scale
- Let's do this for 2 inputs, one output…

CASE 1: IRTS

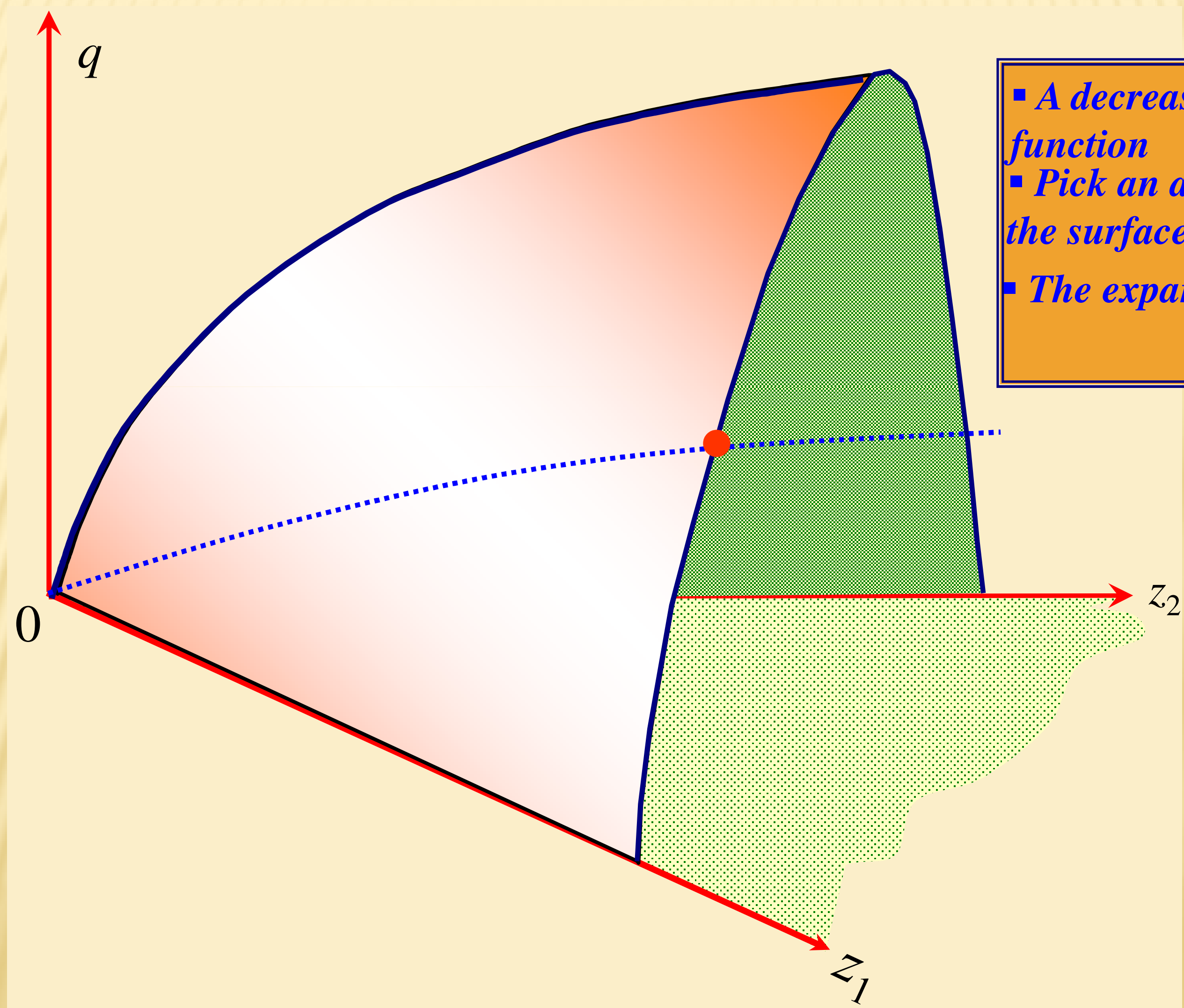


- *An increasing returns to scale function*
- *Pick an arbitrary point on the surface*
- *The expansion path…*

- *t*>1 *implies* $\phi(t\mathbf{z}) > t\phi(\mathbf{z})$

- *Double inputs and you more than double output*

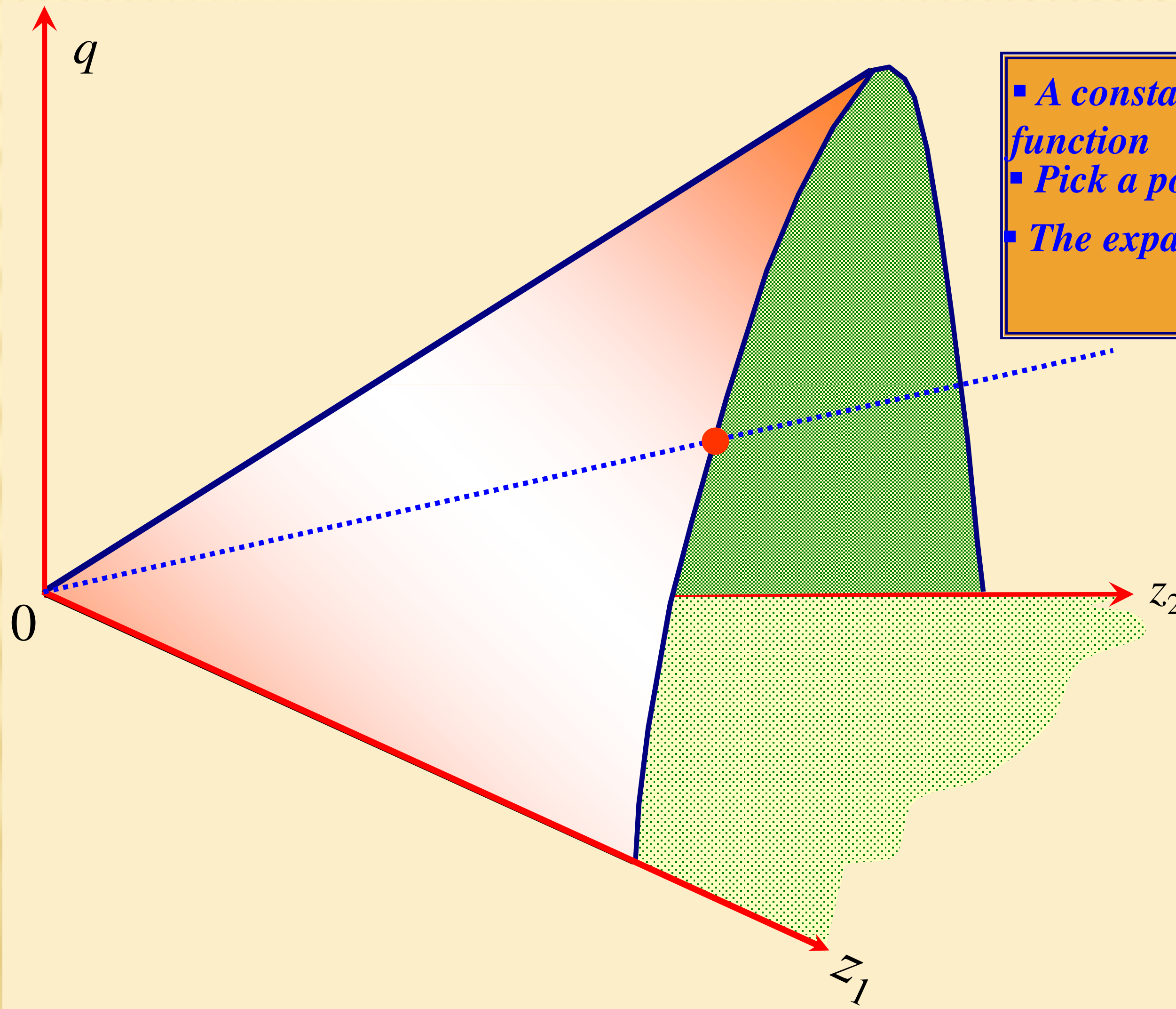
CASE 2: DRTS



- *A decreasing returns to scale function*
- *Pick an arbitrary point on the surface*
- *The expansion path…*

- t>1 *implies* $\phi(t\mathbf{z}) < t\phi(\mathbf{z})$
- *Double inputs and output increases by less than double*

CASE 3: CRTS

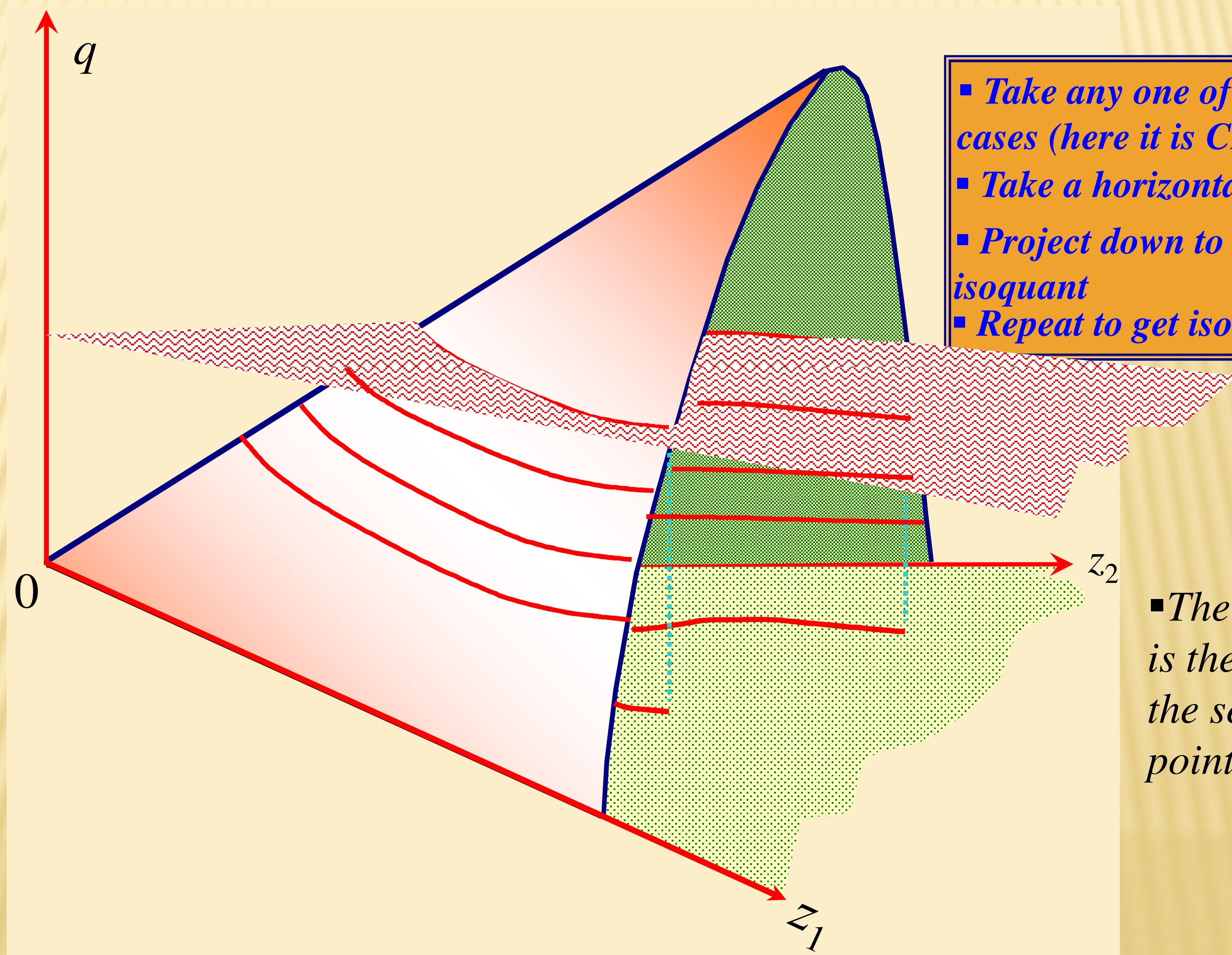


- *A constant returns to scale function*
- *Pick a point on the surface*
- *The expansion path is a ray*

- $\phi(t\mathbf{z}) = t\phi(\mathbf{z})$

- *Double inputs and output exactly doubles*

RELATIONSHIP TO ISOQUANTS



- ***Take any one of the three cases (here it is CRTS)***
- ***Take a horizontal "slice"***
- ***Project down to get the isoquant***
- ***Repeat to get isoquant map***

- *The isoquant map is the projection of the set of feasible points*

OVERVIEW…

Changing one input at time.

- The Firm: Basics
  - The setting
  - Input requirement sets
  - Isoquants
  - Returns to scale
  - Marginal products

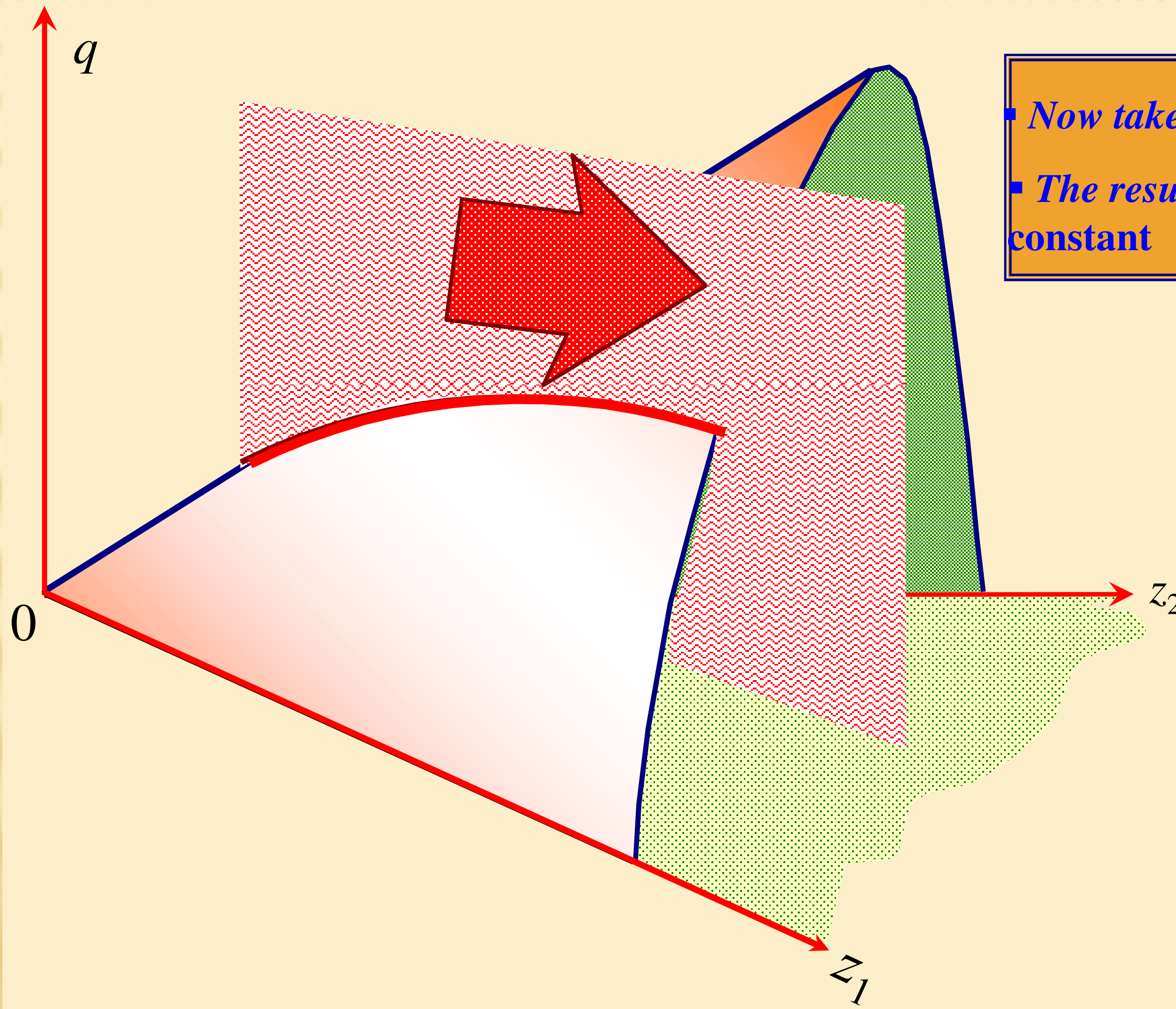
MARGINAL PRODUCTS

- Pick a technically efficient input vector
- Keep all but one input constant
- Measure the marginal change in output w.r.t. this input

- Remember, this means a **z** such that $q = \phi(\mathbf{z})$
- The marginal product

$$\mathrm{MP}_i = \phi_i(\mathbf{z}) = \frac{\partial \phi(\mathbf{z})}{\partial z_i}$$

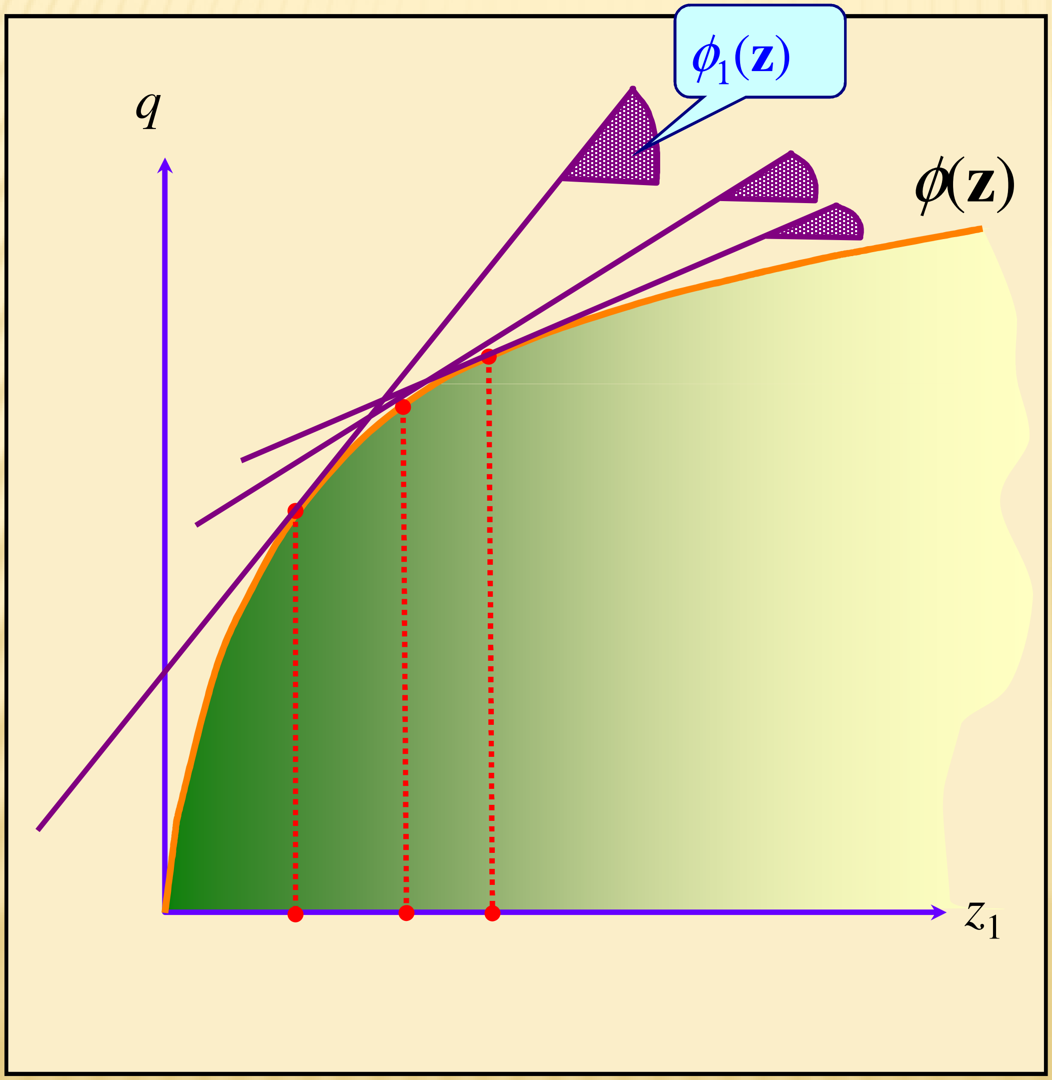
CRTS PRODUCTION FUNCTION AGAIN



- *Now take a vertical "slice"*
- *The resulting path for z_2 = constant*

Let's look at its shape

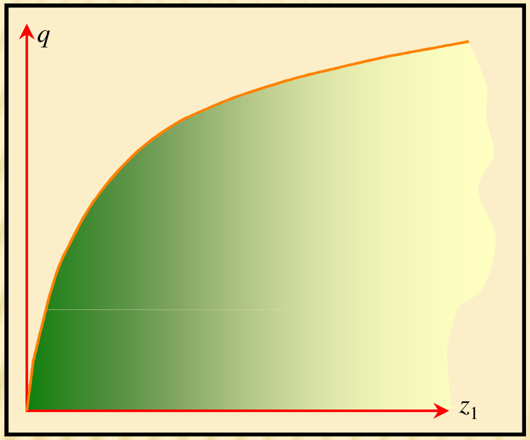
MP FOR THE CRTS FUNCTION



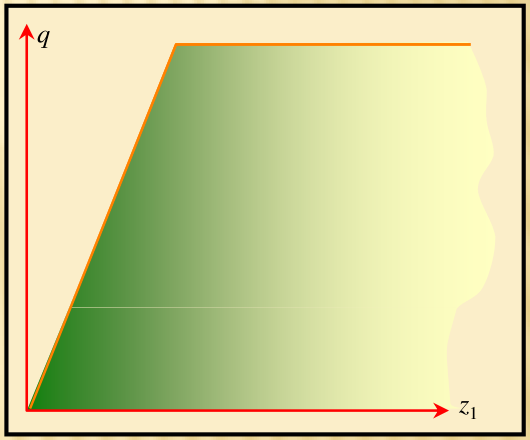
- *The feasible set*
- *Technically efficient points*
- *Slope of tangent is the marginal product of input 1*
- *Increase z_1…*

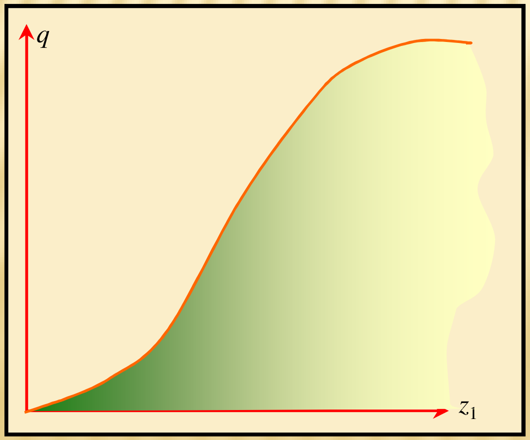
- *A section of the production function*
- *Input 1 is essential: If $z_1 = 0$ then $q = 0$*
- *$\phi_1(\mathbf{z})$ falls with z_1 (or stays constant) if ϕ is concave*

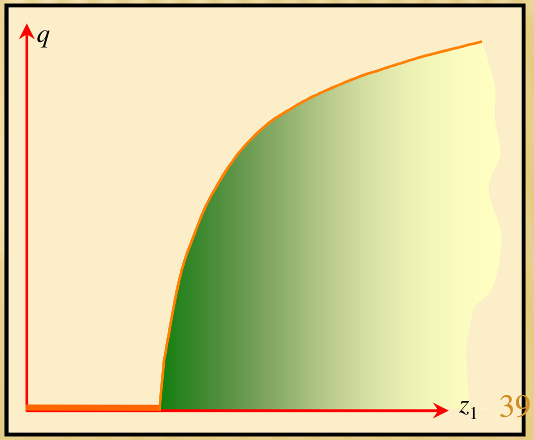
RELATIONSHIP BETWEEN Q AND Z_1



- *We've just taken the conventional case*
- *But in general this curve depends on the shape of* ϕ.
- *Some other possibilities for the relation between output and one input…*







KEY CONCEPTS

- Technical efficiency
- Returns to scale
- Convexity
- MRTS
- Marginal product

WHAT NEXT?

- Introduce the market
- Optimisation problem of the firm
- Method of solution
- Solution concepts.