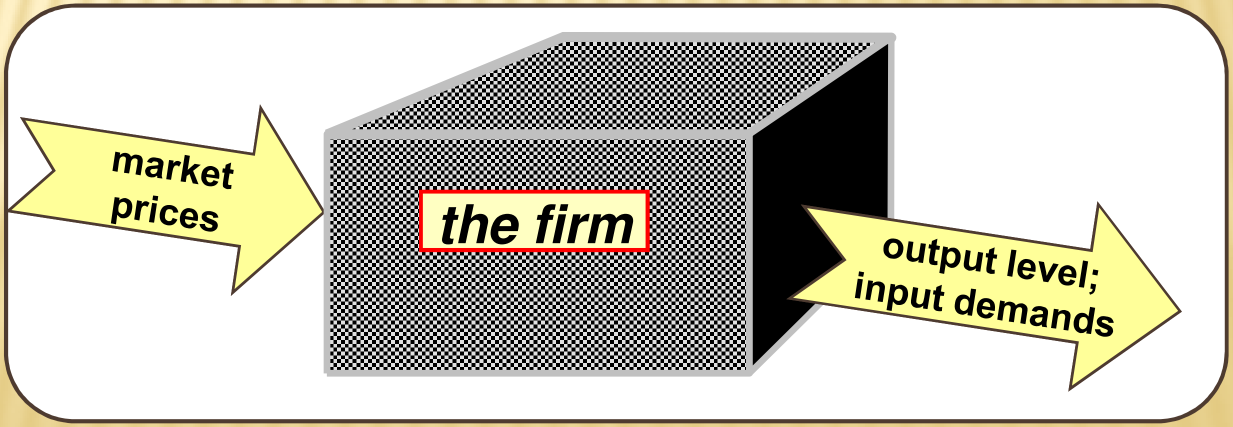
MICROECONOMICS

THE FIRM: DEMAND AND SUPPLY

MOVING ON FROM THE OPTIMUM…

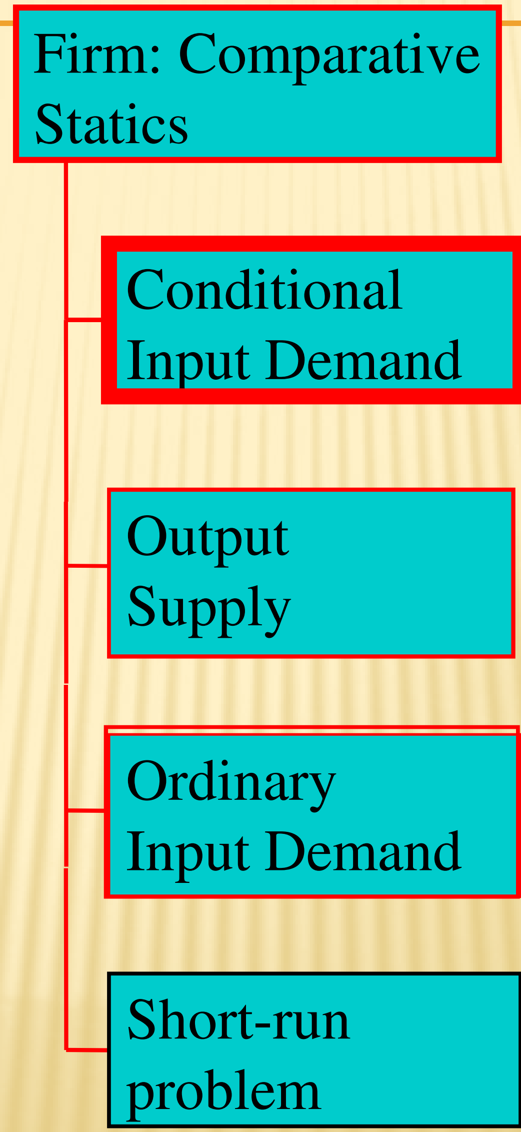
- We derive the firm's reactions to changes in its environment.
- These are the *response functions*.
  - We will examine three types of them
  - Responses to different types of market events.
- In effect we treat the firm as a Black Box.



THE FIRM AS A "BLACK BOX"

- Behaviour can be predicted by necessary and sufficient conditions for optimum.
- The FOC can be solved to yield behavioural response functions.
- Their properties derive from the solution function.
- We need the solution function's properties…
- …again and again.

OVERVIEW…



Response function for stage 1 optimisation

THE FIRST RESPONSE FUNCTION

- Review the cost-minimisation problem and its solution
- Choose $\mathbf{z}$ to minimise

$$\sum_{i=1}^{m} w_i z_i \text{ subject to } q \leq \phi(\mathbf{z}),\ \mathbf{z} \geq \mathbf{0}$$

- The firm's cost function:

$$C(\mathbf{w}, q) := \min_{\{\phi(\mathbf{z}) \geq q\}} \sum w_i z_i$$

- Cost-minimising value for each input:

$$z_i^* = H^i(\mathbf{w}, q),\ i=1,2,\ldots,m$$

may be a well-defined function or may be a correspondence

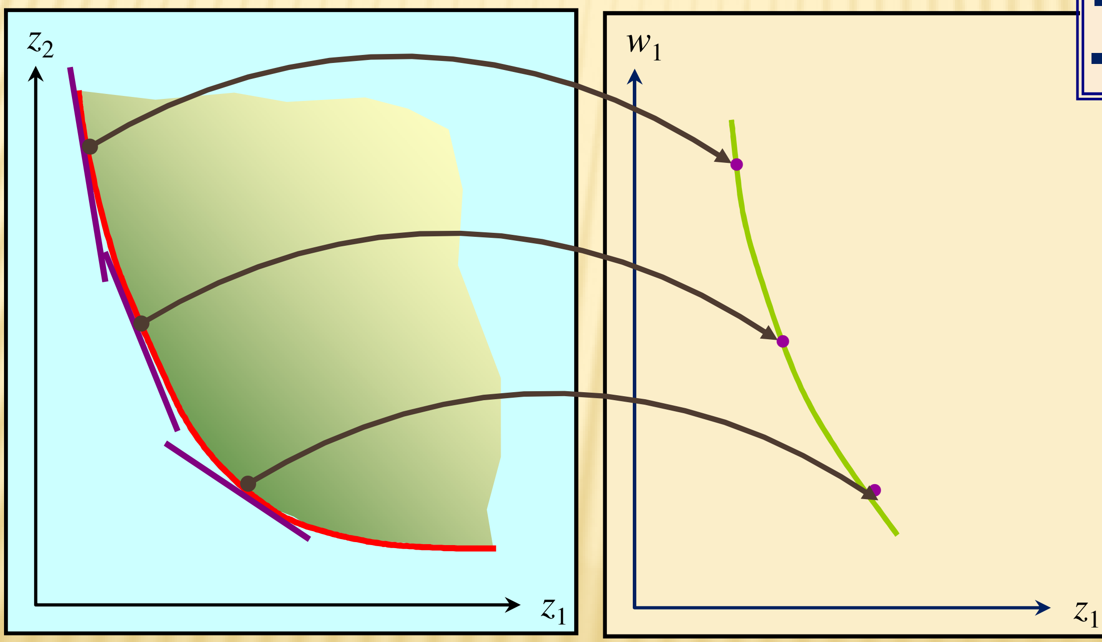
vector of input prices

Specified output level

- *The "stage 1" problem*
- *The solution function*
- *H^i is the <u>conditional input demand function</u>.*
- *Demand for input i, conditional on given output level q*

A graphical approach

MAPPING INTO (Z_1,W_1)-SPACE



- **Conventional case of Z.**
- **Start with any value of w_1 (the slope of the tangent to Z).**
- **Repeat for a lower value of w_1.**
- **...and again to get...**
- **...the conditional demand curve**

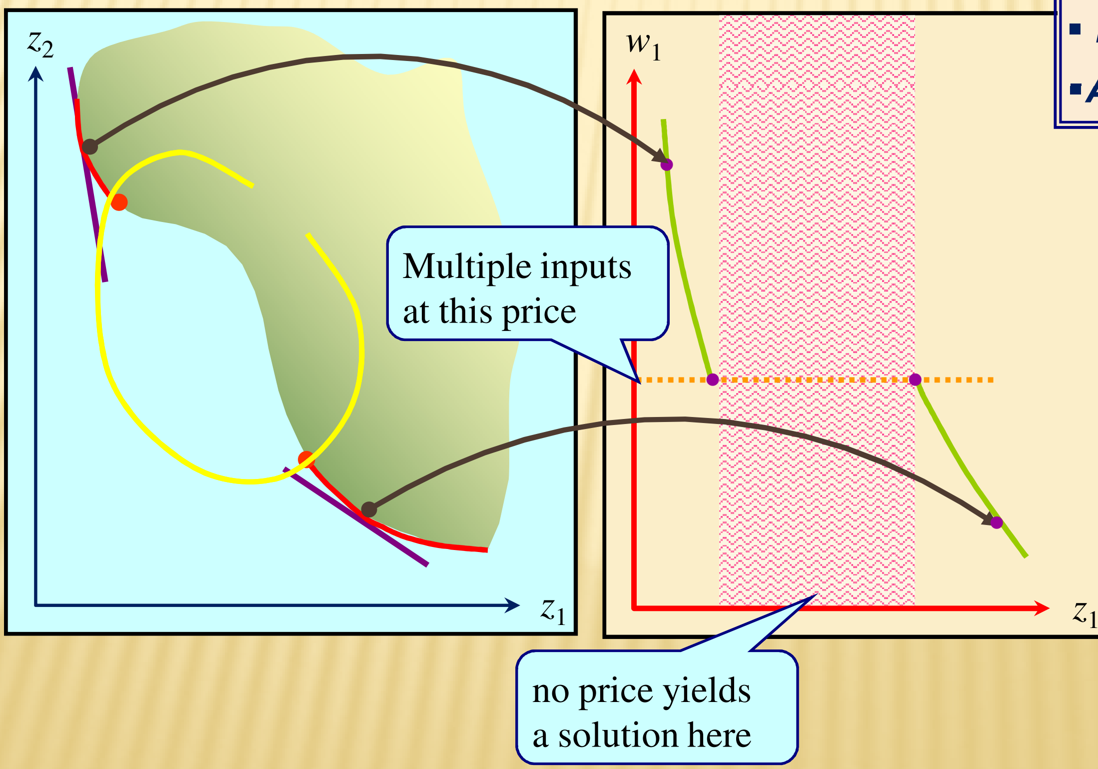
- *Constraint set is convex, with smooth boundary*
- *Response function is a continuous map:*

$$H^1(\mathbf{w},q)$$

Now try a different case

ANOTHER MAP INTO (Z_1,W_1)-SPACE

- *Now take case of nonconvex Z.*
- *Start with a high value of w_1.*
- *Repeat for a very low value of w_1.*
- *Points "nearby" work the same way.*
- *But what happens in between?*
- *A demand correspondence*



- *Constraint set is nonconvex.*
- *Response is a discontinuous map: jumps in z**
- *Map is multivalued at the discontinuity*

CONDITIONAL INPUT DEMAND FUNCTION

- Assume that single-valued input-demand functions exist.
- How are they related to the cost function?
- What are their properties?
- How are they related to properties of the cost function?

Do you remember these...?

USE THE COST FUNCTION

The slope:
$$\frac{\partial C(\mathbf{w}, q)}{\partial w_i}$$

Optimal demand for input i

- Recall this relationship?

$$C_i(\mathbf{w}, q) = z_i^*$$

- ...yes, it's Shephard's lemma

conditional input demand function

- So we have:

$$C_i(\mathbf{w}, q) = H^i(\mathbf{w}, q)$$

- Link between conditional input demand and cost functions

Second derivative

- Differentiate with respect to w_j

$$C_{ij}(\mathbf{w}, q) = H_j^i(\mathbf{w}, q)$$

- Slope of input demand function

Two simple results:

SIMPLE RESULT 1

- Use a standard property

$$\frac{\partial^2(\bullet)}{\partial w_i \partial w_j} = \frac{\partial^2(\bullet)}{\partial w_j \partial w_i}$$

- So in this case

$$C_{ij}(\mathbf{w}, q) = C_{ji}(\mathbf{w}, q)$$

- Therefore we have:

$$H_j^i(\mathbf{w}, q) = H_i^j(\mathbf{w}, q)$$

- *second derivatives of a function "commute"*
- *The order of differentiation is irrelevant*
- *The effect of the price of input i on conditional demand for input j equals the effect of the price of input j on conditional demand for input i.*

SIMPLE RESULT 2

- Use the standard relationship:

$$C_{ij}(\mathbf{w}, q) = H_j^i(\mathbf{w}, q)$$

- Slope of conditional input demand function derived from second derivative of cost function

- We can get the special case:

$$C_{ii}(\mathbf{w}, q) = H_i^i(\mathbf{w}, q)$$

- We've just put j=i

- Because cost function is concave:

$$C_{ii}(\mathbf{w}, q) \leq 0$$

- A general property

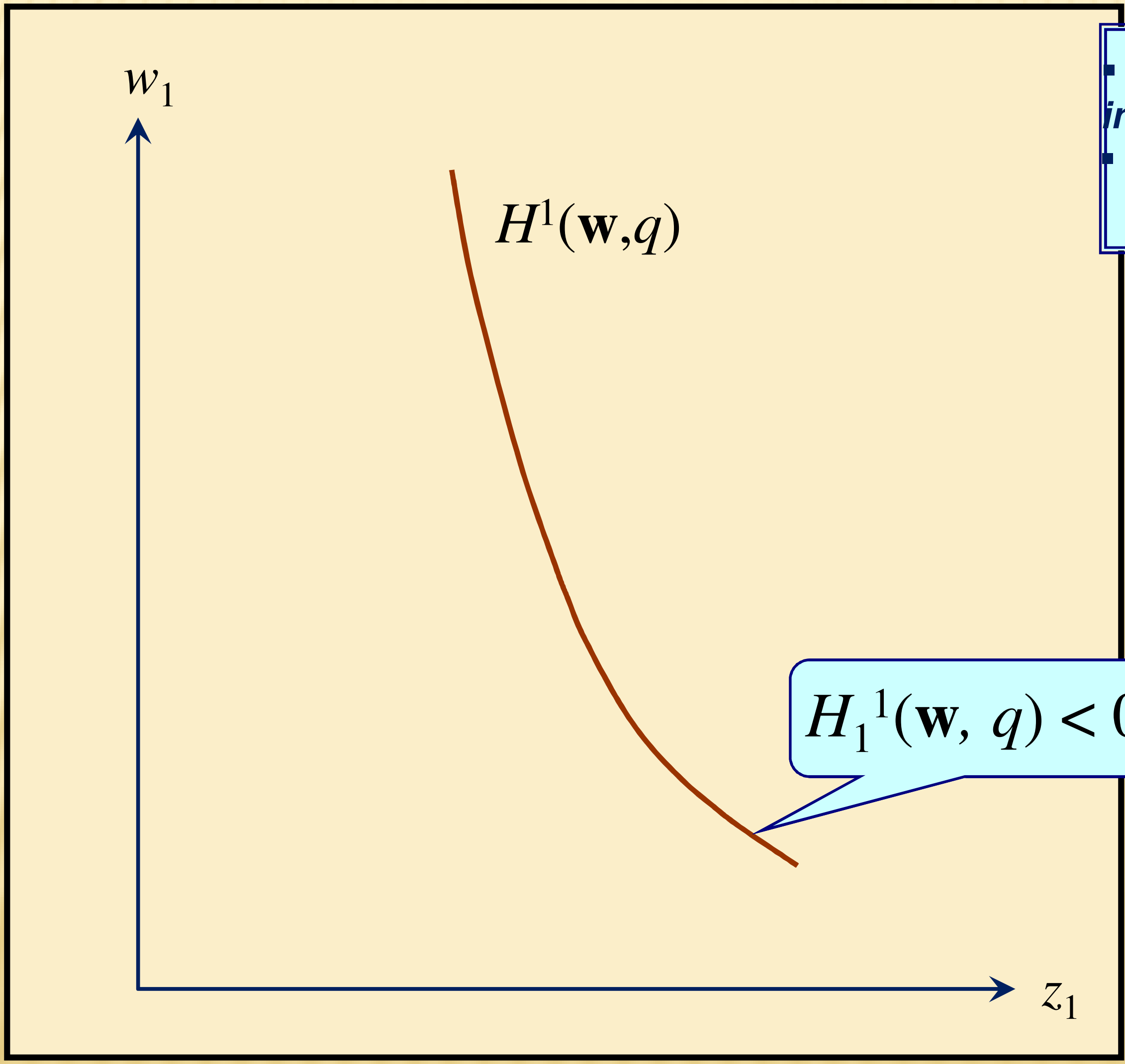
- Therefore:

$$H_i^i(\mathbf{w}, q) \leq 0$$

- The relationship of conditional demand for an input with its own price cannot be positive.

and so...

CONDITIONAL INPUT DEMAND CURVE



- Consider the demand for input 1
- Consequence of result 2?

- "Downward-sloping" conditional demand
- In some cases it is also possible that $H_i^i = 0$
- Corresponds to the case where isoquant is kinked: multiple **w** values consistent with same **z***.

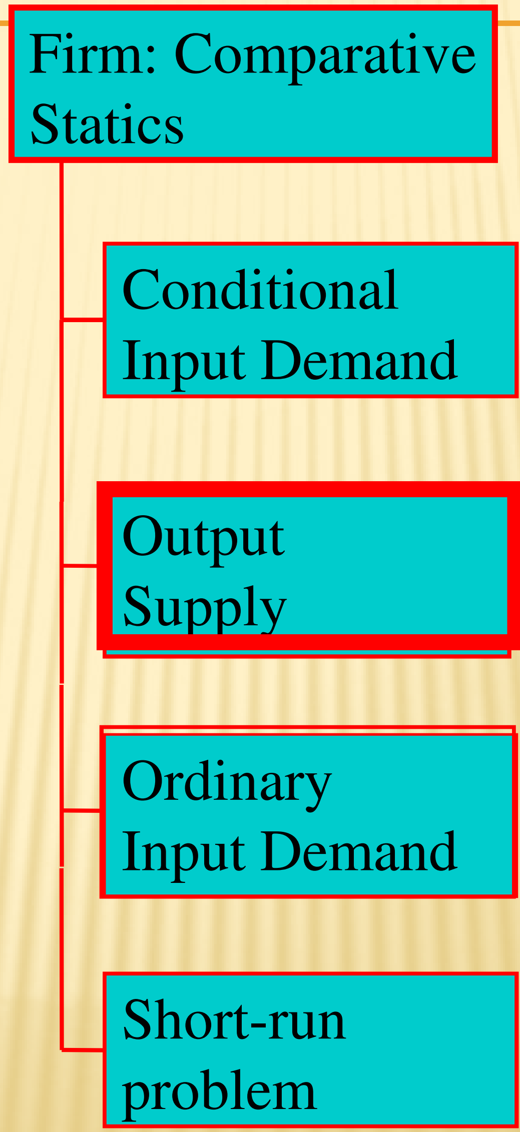
FOR THE CONDITIONAL DEMAND FUNCTION…

- Nonconvex Z yields discontinuous H
- Cross-price effects are symmetric
- Own-price demand slopes downward.

- (exceptional case: own-price demand could be constant)

OVERVIEW...

Response function for stage 2 optimisation



THE SECOND RESPONSE FUNCTION

- Review the profit-maximisation problem and its solution
- Choose q to maximise:

$$pq - C(\mathbf{w}, q)$$

- From the FOC:

$$p \leq C_q(\mathbf{w}, q^*)$$

$$pq^* \geq C(\mathbf{w}, q^*)$$

- profit-maximising value for output:

$$q^* = S(\mathbf{w}, p)$$

input prices

output price

- *The "stage 2" problem*
- *"Price equals marginal cost"*
- *"Price covers average cost"*
- *S is the <u>supply</u> function*
- *(again it may actually be a correspondence)*

SUPPLY OF OUTPUT AND OUTPUT PRICE

- Use the FOC:

$C_q(\mathbf{w}, q) = p$

- *"marginal cost equals price"*

- Use the supply function for q:

$C_q(\mathbf{w}, S(\mathbf{w}, p)) = p$

- *Gives an equation in w and p*

Differential of S with respect to p

- Differentiate with respect to p

$C_{qq}(\mathbf{w}, S(\mathbf{w}, p))\, S_p(\mathbf{w}, p) = 1$

- *Use the "function of a function" rule*

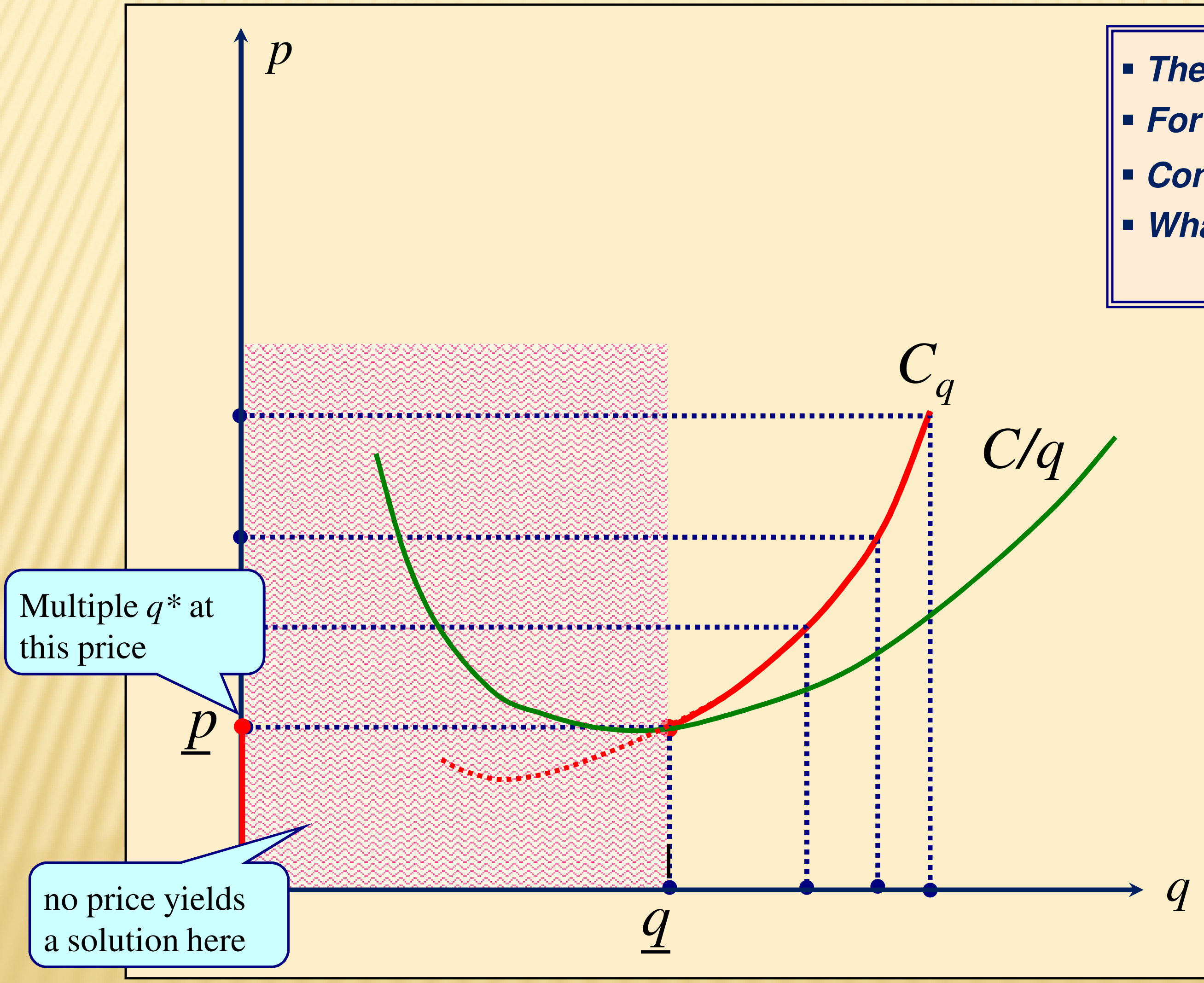
- Rearrange:

$$S_p(\mathbf{w}, p) = \frac{1}{C_{qq}(\mathbf{w}, q)}.$$

Positive if MC is increasing.

- *e of the supply*
nction.

THE FIRM'S SUPPLY CURVE



- *The firm's AC and MC curves.*
- *For given p read off optimal q**
- *Continue down to p*
- *What happens below p*

- *Supply response is given by* $q=S(\mathbf{w},p)$
- *Case illustrated is for* ϕ *with first IRTS, then DRTS. Response is a discontinuous map: jumps in q**
- *Map is multivalued at the discontinuity*

SUPPLY OF OUTPUT AND PRICE OF INPUT J

- Use the FOC:

$$C_q(\mathbf{w}, S(\mathbf{w}, p)) = p$$

- *Same as before: "price equals marginal cost"*

- Differentiate with respect to w_j

$$C_{qj}(\mathbf{w}, q^*) + C_{qq}(\mathbf{w}, q^*)\, S_j(\mathbf{w}, p) = 0$$

- *Use the "function of a function" rule again*

- Rearrange:

$$S_j(\mathbf{w}, p) = -\frac{C_{qj}(\mathbf{w}, q^*)}{C_{qq}(\mathbf{w}, q^*)}$$

Remember, this is positive

- *Supply of output must fall with w_j if marginal cost increases with w_j.*

FOR THE SUPPLY FUNCTION…

- Supply curve slopes upward.
- Supply decreases with the price of an input, if MC increases with the price of that input.
- Nonconcave ϕ yields discontinuous *S*.
- IRTS means ϕ is nonconcave and so *S* is discontinuous.

OVERVIEW…

Response function for combined optimisation problem

- Firm: Comparative Statics
  - Conditional Input Demand
  - Output Supply
  - Ordinary Input Demand
  - Short-run problem

THE THIRD RESPONSE FUNCTION

- Recall the first two response functions:

$$z_i^* = H^i(\mathbf{w},q)$$

$$q^* = S(\mathbf{w}, p)$$

- Now substitute for q^* :

$$z_i^* = H^i(\mathbf{w}, S(\mathbf{w}, p))$$

- Use this to define a new function:

$$D^i(\mathbf{w},p) := H^i(\mathbf{w}, S(\mathbf{w}, p))$$

input prices

output price

- *Demand for input i, conditional on output q*
- *Supply of output*
- *Stages 1 & 2 combined…*
- *Demand for input i (unconditional)*
- *Use this relationship to analyse further the firm's response to price changes*

DEMAND FOR *I* AND THE PRICE OF OUTPUT

- Take the relationship

$$D^i(\mathbf{w}, p) = H^i(\mathbf{w},$$

"function of a function" rule again

- Differentiate with respect p:

$$D_p^i(\mathbf{w}, p) = H_q^i(\mathbf{w}, q^*)\, S_p(\mathbf{w}, p)$$

- D^i *increases with p iff* H^i *increases with q. Reason? Supply increases with price (* $S_p > 0$*).*

- But we also have, for any q:

$$H^i(\mathbf{w}, q) = C_i(\mathbf{w}, q)$$
$$H_q^i(\mathbf{w}, q) = C_{iq}(\mathbf{w}, q)$$

- *Shephard's Lemma again*

- Substitute in the above:

$$D_p^i(\mathbf{w}, p) = C_{qi}(\mathbf{w}, q^*) S_p(\mathbf{w}, p)$$

- *Demand for input i (*D^i*) increases with p iff marginal cost (*C_q*) increases with* w_i *.*

DEMAND FOR *I* AND THE PRICE OF *J*

- Again take the relationship

$$D^i(\mathbf{w}, p) = H^i(\mathbf{w}, S(\mathbf{w}, p)).$$

- Differentiate with respect to w_j:

$$D_j^i(\mathbf{w}, p) = H_j^i(\mathbf{w}, q^*) + H_q^i(\mathbf{w}, q^*)S_j(\mathbf{w}, p)$$

"function of a function" rule yet again

- Use Shephard's Lemma again:

$$H_q^i(\mathbf{w}, q) = C_{iq}(\mathbf{w}, q) = C_{qi}(\mathbf{w}, q)$$

- Use this and the previous r… p) to give a decomposition into a "substitu… effect" and an "output effect":

"substitution effect"

"output effect"

$$D_j^i(\mathbf{w}, p) = H_j^i(\mathbf{w}, q^*) - \frac{C_{iq}(\mathbf{w}, q^*)C_{jq}(\mathbf{w}, q^*)}{C_{qq}(\mathbf{w}, q^*)}$$

RESULTS FROM DECOMPOSITION FORMULA

- Take the general relationship:

$$D_j^i(\mathbf{w}, p) = H_j^i(\mathbf{w}, q^*) - \frac{C_{iq}(\mathbf{w}, q^*)C_{jq}(\mathbf{w}, q^*)}{C_{qq}(\mathbf{w}, q^*)}.$$

We already know this is symmetric in *i* and *j*.

Obviously symmetric in *i* and *j*.

- *The effect w_i on demand for input j equals the effect of w_j on demand for input i.*

- Now take the special case where $j = i$:

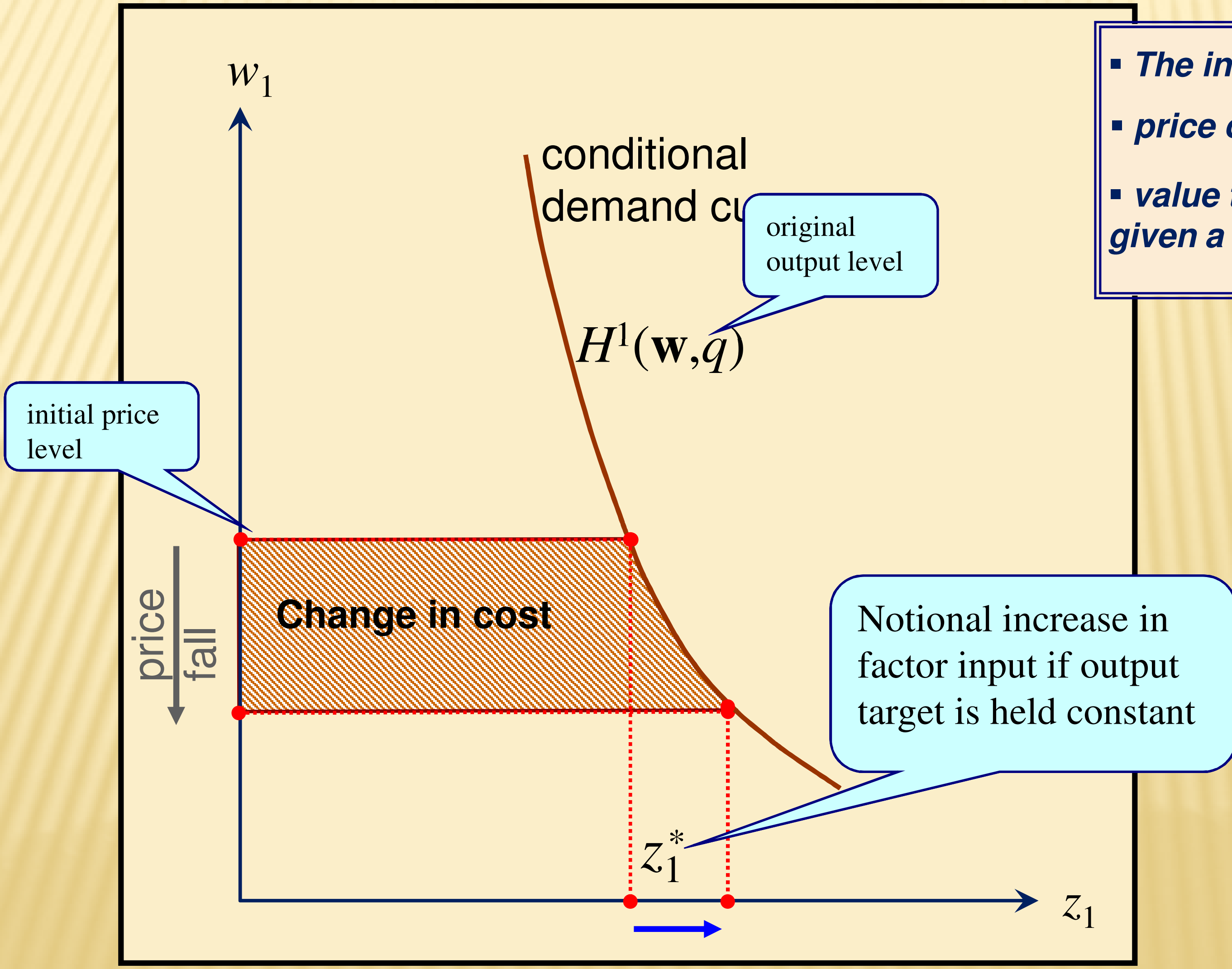
$$D_i^i(\mathbf{w}, p) = H_i^i(\mathbf{w}, q^*) - \frac{C_{iq}(\mathbf{w}, q^*)^2}{C_{qq}(\mathbf{w}, q^*)}$$

We already know this is negative or zero.

cannot be positive.

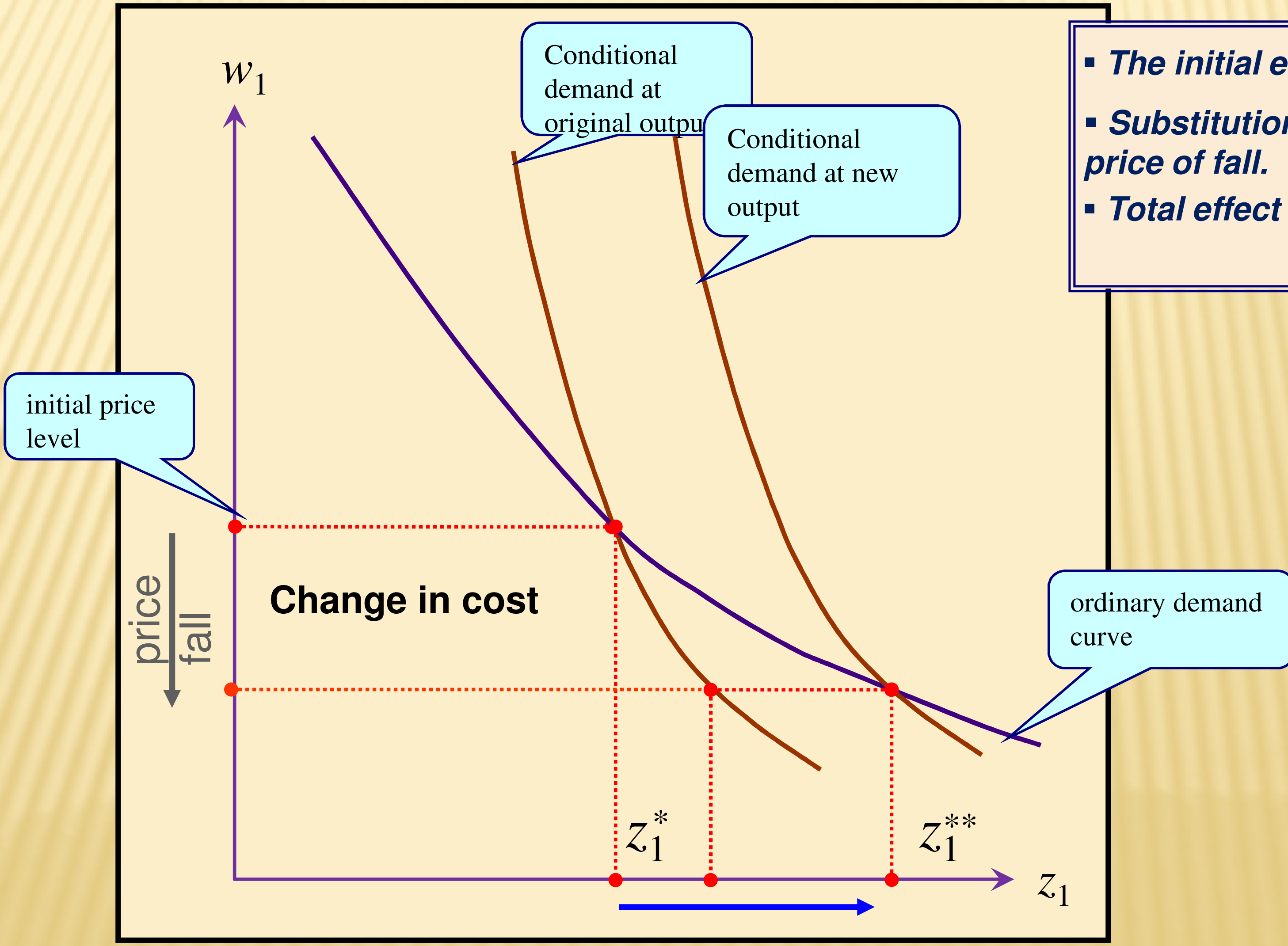
- *If w_i increases, the demand for input i cannot rise.*

INPUT-PRICE FALL: SUBSTITUTION EFFECT



- *The initial equilibrium*
- *price of input falls*
- *value to firm of price fall, given a fixed output level*

INPUT-PRICE FALL: TOTAL EFFECT



- ***The initial equilibrium***
- ***Substitution effect of input-price of fall.***
- ***Total effect of input-price fall***

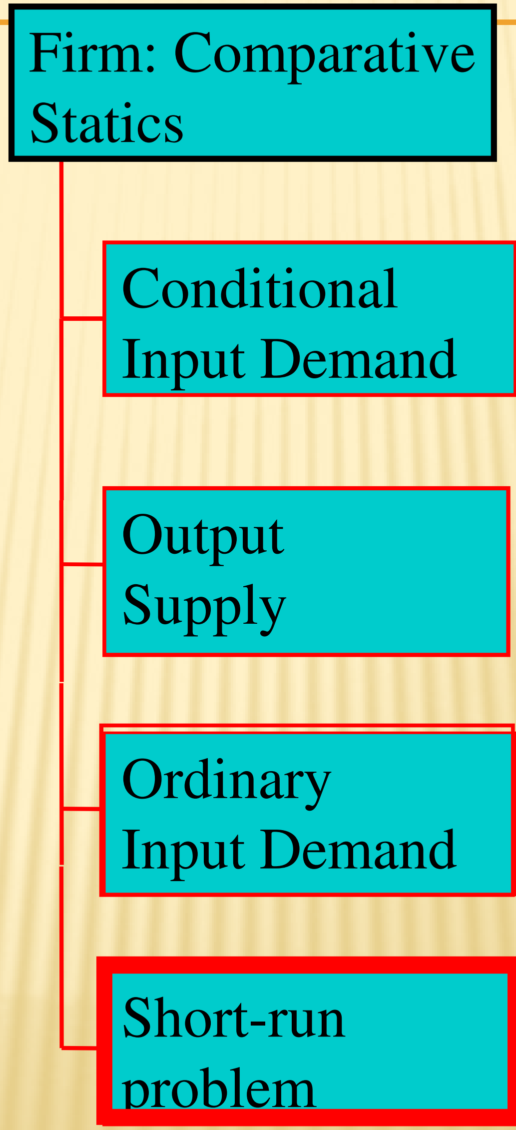
THE ORDINARY DEMAND FUNCTION…

- Nonconvex *Z* may yield a discontinuous *D*
- Cross-price effects are symmetric
- Own-price demand slopes downward

- Same basic properties as for *H* function

OVERVIEW…

Optimisation subject to side-constraint



THE SHORT RUN…

- This is not a moment in time but…
- … is defined by additional constraints within the model
- Counterparts in other economic applications where we sometimes need to introduce side constraints

THE SHORT-RUN PROBLEM

- We build on the firm's standard optimisation problem
- Choose q and $\mathbf{z}$ to maximise

$$\Pi := pq - \sum_{i=1}^{m} w_i z_i$$

- subject to the standard constraints:

$$q \leq \phi(\mathbf{z})$$

$$q \geq 0, \mathbf{z} \geq \mathbf{0}$$

- But we add a *side condition* to this problem:

$$z_m = \bar{z}_m$$

- Let $\bar{q}$ be the value of q for which $z_m = \bar{z}_m$ would have been freely chosen in the unrestricted cost-min problem…

THE SHORT-RUN COST FUNCTION

$$\tilde{C}(\mathbf{w}, q, \bar{z}_m) := \min_{\{z_m = \bar{z}_m\}} \Sigma\, w_i\, z_i$$

- *The solution function with the side constraint.*

- Short-run demand for input i:

$$H^i(\mathbf{w}, q, \bar{z}_m) = \tilde{C}_i(\mathbf{w}, q, \bar{z}_m)$$

- *Follows from Shephard's Lemma*

- Compare with the ordinary cost function

$$C(\mathbf{w}, q) \leq \tilde{C}(\mathbf{w}, q, \bar{z}_m)$$

- *By definition of the cost function. We have "=" if $q = \bar{q}$.*

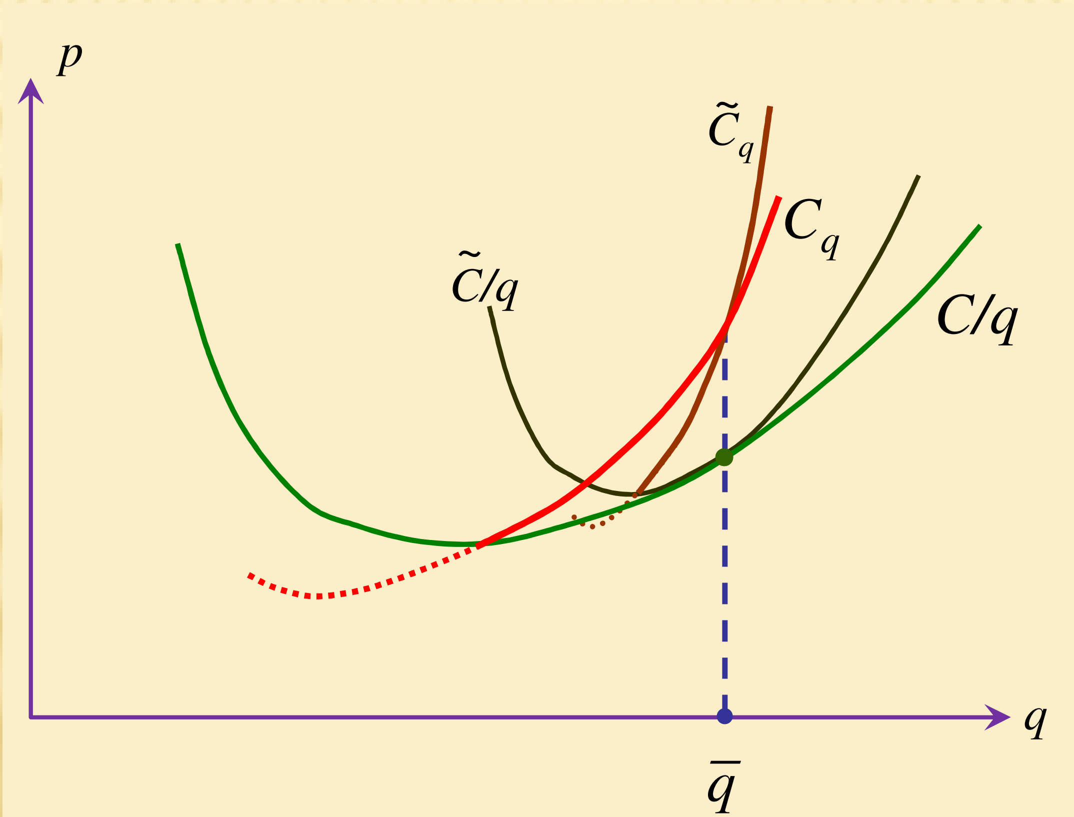
- So, dividing by q:

$$\frac{C(\mathbf{w}, q)}{q} \leq \frac{\tilde{C}(\mathbf{w}, q, \bar{z}_m)}{q}$$

- *Short-run AC ≥ long-run AC. SRAC = LRAC at $q = \bar{q}$*

Supply curves

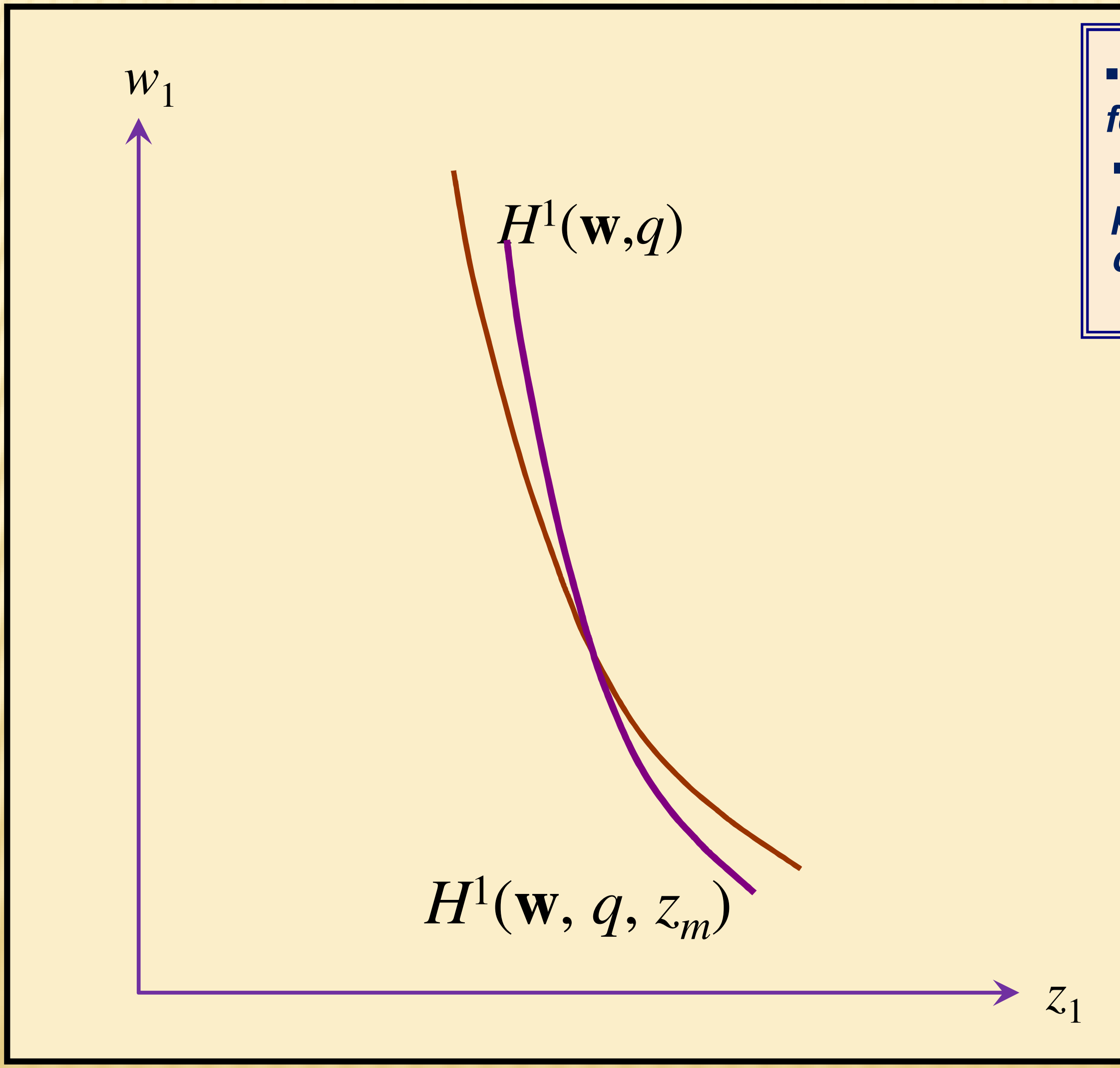
MC, AC AND SUPPLY IN THE SHORT AND LONG RUN



- AC if all inputs are variable
- MC if all inputs are variable
- Fix an output level.
- AC if input m is now kept fixed
- MC if input m is now kept fixed
- Supply curve in long run
- Supply curve in short run

- *SRAC touches LRAC at the given output*
- *SRMC cuts LRMC at the given output*
- *The supply curve is steeper in the short run*

CONDITIONAL INPUT DEMAND



- *The original demand curve for input 1*
- *The demand curve from the problem with the side constraint.*

- *"Downward-sloping" conditional demand*
- *Conditional demand curve is steeper in the short run.*

KEY CONCEPTS

- Basic functional relations
- price signals → firm → input/output responses
  - $H^i(\mathbf{w},q)$ — demand for input *i*, conditional on output
  - $S\,(\mathbf{w},p)$ — supply of output
  - $D^i(\mathbf{w},p)$ — demand for input *i* (unconditional)

And they all hook together like this:

- $H^i(\mathbf{w}, S(\mathbf{w},p)) = D^i(\mathbf{w},p)$

WHAT NEXT?

- Analyse the firm under a variety of market conditions.
- Apply the analysis to the consumer’s optimisation problem.