Bayesian networks:

In naive Bayes:

$$p(x_1, x_2, \ldots, x_N) = p(x_1)\, p(x_2) \ldots p(x_N)$$

More generally

$$p(x_1, x_2) = p(x_2 | x_1)\; p(x_1)$$

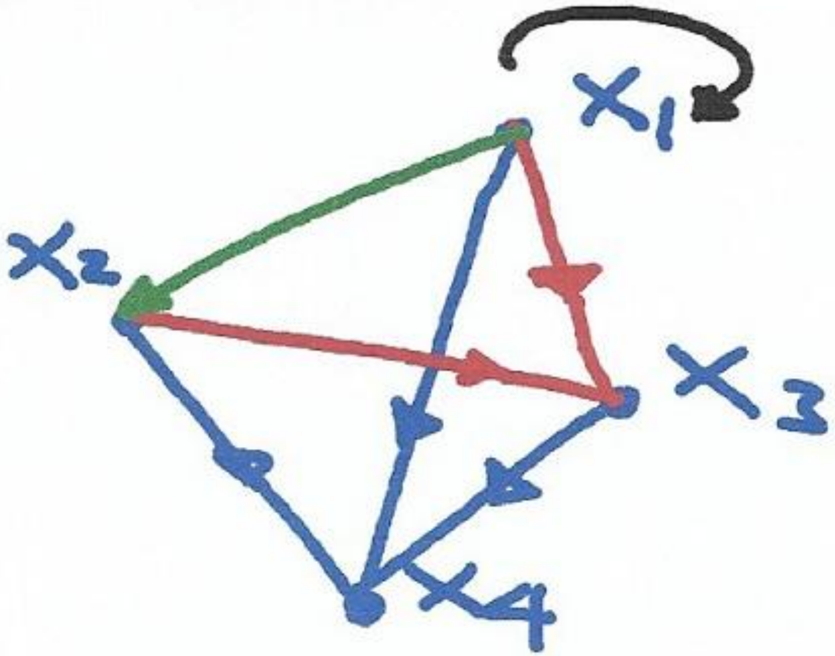
$$p(x_1, x_2, x_3) = p(x_3 | x_2, x_1) \cdot p(x_1, x_2)$$

$$= p(x_3 | x_2, x_1)\, p(x_2 | x_1)\, p(x_1)$$

$$p(x_1, x_2, x_3, x_4) = p(x_4 | x_3, x_2, x_1)\, p(x_1, x_2, x_3)$$

$$= p(x_4 | x_3, x_2, x_1)\, p(x_3 | x_2, x_1)\, p(x_2 | x_1)\, p(x_1)$$

$$\boxed{\begin{aligned} p(x_1, x_2, \dots, x_N) &= p(x_N \mid x_{N-1}, \dots, x_1) \cdot \\ &\quad \cdot p(x_{N-1} \mid x_{N-2}, \dots, x_1) \cdot \ldots \cdot p(x_2 \mid x_1)\, p(x_1) \end{aligned}}$$

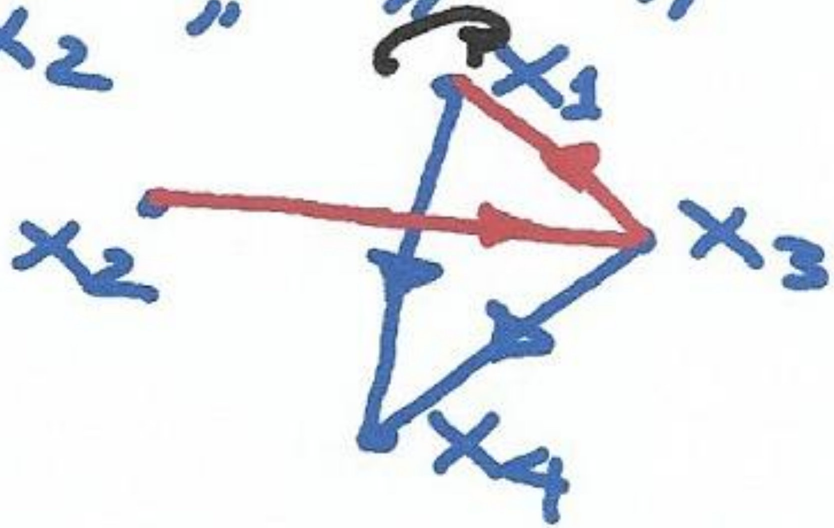


x_1, x_2, x_3, x_4

Knowledge (particular to the problem):

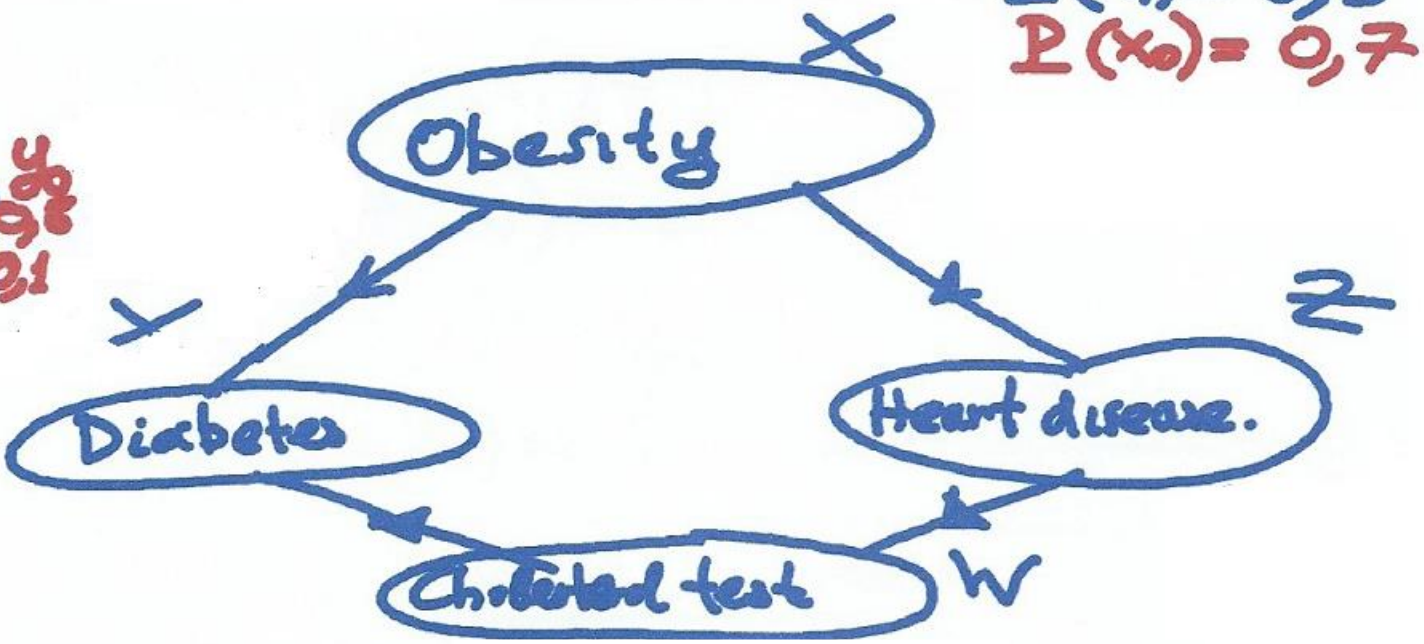
x_4 does not depend on x_2

x_2 " " " x_1



Bayesian network example:

$P(x_1) = 0{,}3$
$P(x_0) = 0{,}7$



		y_0
$P(y_1 \mid x_1) = 0{,}2$		$0{,}8$
$P(y_1 \mid x_0) = 0{,}9$		$0{,}1$

	z_0
$P(z_1 \mid x_1) = 0{,}3$	$0{,}7$
$P(z_1 \mid x_0) = 0{,}1$	$0{,}9$

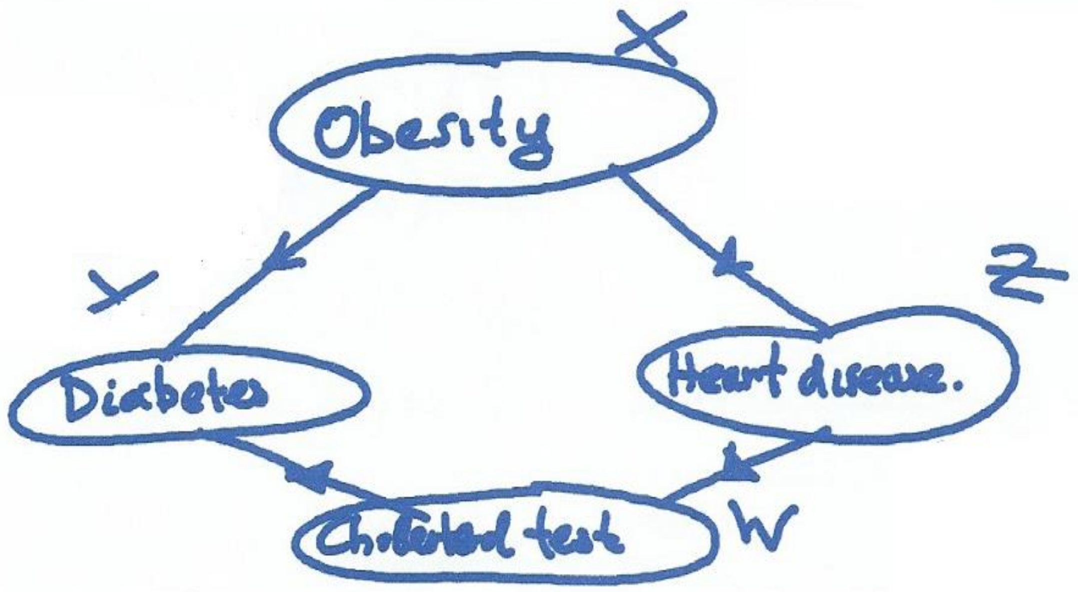
	w_0
$P(w_1 \mid y_1, z_1) = 0{,}9$	$0{,}1$
$P(w_1 \mid y_1, z_0) = 0{,}6$	$0{,}4$
$P(w_1 \mid y_0, z_1) = 0{,}3$	$0{,}7$
$P(w_1 \mid y_0, z_0) = 0{,}1$	$0{,}9$

$P(\text{Positive test} \mid \text{Obese})$

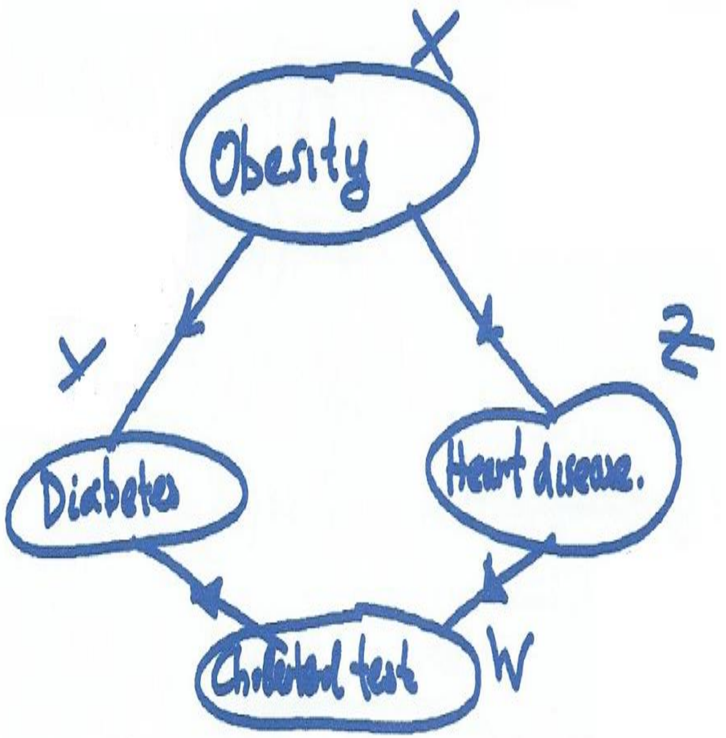
Evidence.

$= P(w_1 \mid x_1)$

Pearl's algorithm.



(1) $$P(w_1|x_1) = \frac{P(x_1, w_1)}{P(x_1)}$$



(2) $$P(w_1, x_1) = \sum_{y,z} P(w_1, y, z, x_1) =$$

$$= P(w_1, y_1, z_1, x_1) + P(w_1, y_1, z_0, x_1) + P(w_1, y_0, z_1, x_1) + P(w_1, y_0, z_0, x_1)$$

(3) $$P(w_1, y_1, z, x_1) = P(w_1|y_1, z_1)\, P(y_1|x_1)\, P(z_1|x_1) \cdot P(x_1)$$

③ $P(w_1, y_1, z_1, x_1) = P(w_1 | y_1, z_1) P(y_1 | x_1) P(z_1 | x_1) \cdot P(x_1)$

0,0162

$P(w_1, y_1, z_0, x_1) = P(w_1 | y_1, z_0) P(y_1 | x_1) P(z_0 | x_1) P(x_1)$

0,0168

$P(w_1, y_0, z_1, x_1) = 0{,}0216$

$P(w_1, y_0, z_0, x_1) = 0{,}0168$

$P(w_1, x_1) = 0{,}0714$

$P(w_1 | x_1) = \frac{P(x_1, w_1)}{P(x_1)} = \frac{0{,}0714}{0{,}3} = 0{,}238.$

Backward inference:

$$P(x_1|w_1) = \frac{P(x_1, w_1)}{P(w_1)}$$

$$= \frac{P(x_1, w_1)}{\sum_{x,y,z} P(w_1, \underbrace{x, y, z})}$$ 8 situations

Eg. $P(w_1, x_1, y_1, z_1) = P(w_1 | y_1, z_1) P(y_1 | x_1)$

$\cdot P(z_1 | x_1) P(x_1)$.

Hopfield network example.

5 neurons
3 patterns:

$$z_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad z_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \quad z_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$$w_{13} = \frac{1 \cdot 1 + 1 \cdot (-1) + 1 \cdot 1}{5} = \frac{1}{5}.$$

$$w_{25} = \frac{1 \cdot 1 + (-1) \cdot 1 + 1 \cdot (-1)}{5} = -\frac{1}{5}.$$

$$W = \frac{1}{5}\begin{bmatrix} 3 & 1 & 1 & -1 & 1 \\ 1 & 3 & 3 & 1 & -1 \\ 1 & 3 & 3 & 1 & -1 \\ -1 & 1 & 1 & 3 & 1 \\ 1 & -1 & -1 & 1 & 3 \end{bmatrix}$$

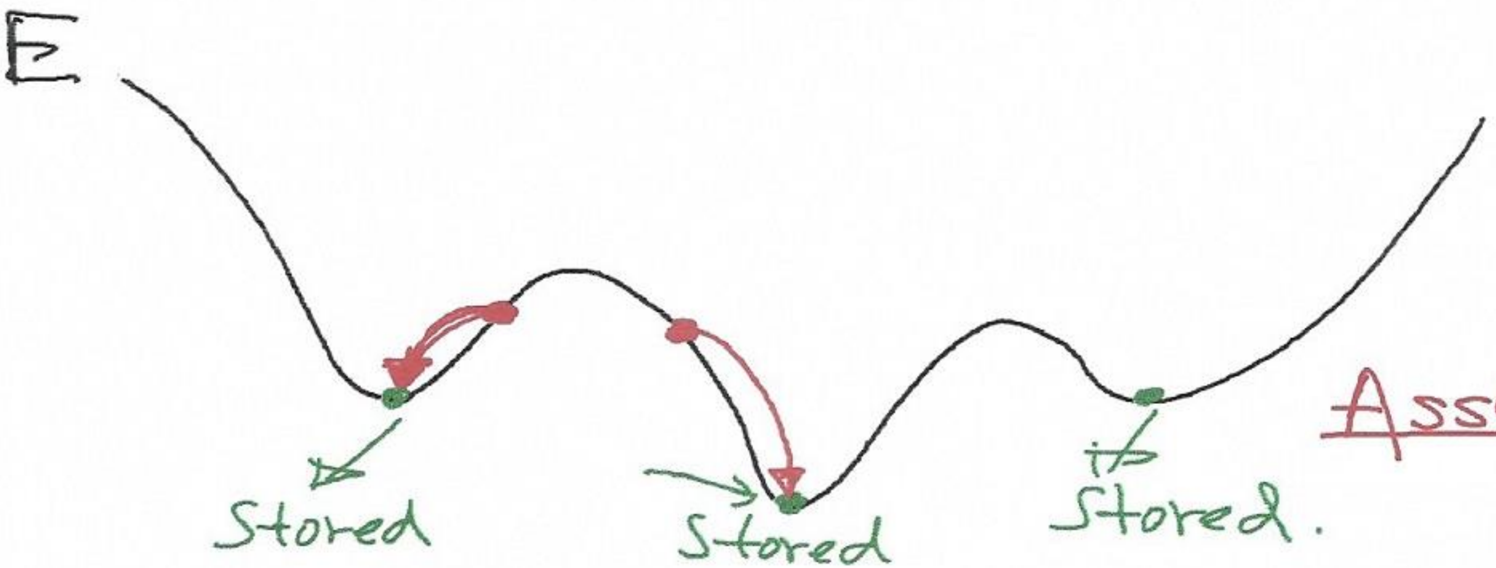
Input $z_1 \to$ output $\text{sign}[W z_1]$

$$W z_1 = W \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 5 \\ 7 \\ 7 \\ 5 \\ 3 \end{bmatrix} \xrightarrow{\text{sign}} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = z_1.$$

$$z_3 \rightarrow z_3' = \begin{bmatrix} 1 \\ 1 \\ \boxed{-1} \\ -1 \\ -1 \end{bmatrix} \rightarrow \underline{\text{flipped.}}$$

Output: $\text{sign}\left[W \cdot z_3'\right]$

$$W \cdot z_3' = W \cdot \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \\ -1 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 3 \\ 1 \\ 1 \\ -5 \\ -3 \end{bmatrix} \xrightarrow{\text{sign}} \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} = z_3.$$



Associative memory.

Over my capacity limit: