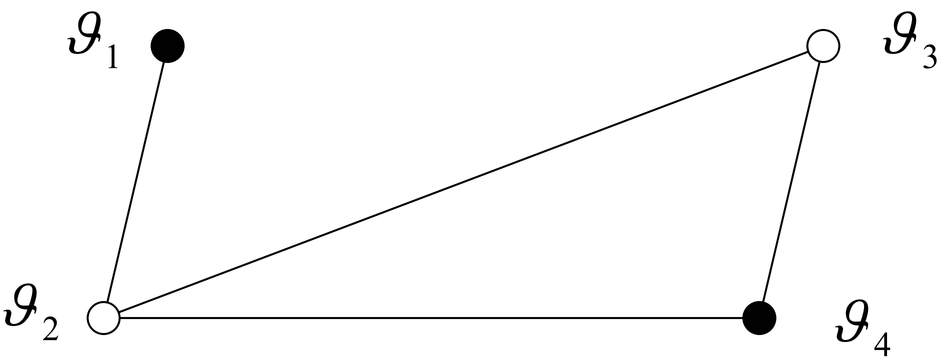
The Local Search: The MIS

example : The maximum Independent Set problem in a graph G(V,E)

$$"\text{Find } V' \subseteq V \text{ s.t. } \forall\, u, \vartheta \in V' \Rightarrow [u, \vartheta] \notin E"$$



$\{\vartheta_1, \vartheta_3\}$ a solution

The MIS by the Local Search

Solution coding : $x_1, x_2, \ldots, x_i, \ldots, x_n$

$$x_i = 1 \Rightarrow \vartheta_i \in V'$$

$$x_i = 0 \Rightarrow \vartheta_i \notin V'$$

Function :

$$\mathrm{Max} \sum_{i=1}^{n} x_i - \lambda |E'|, \ \lambda \in \Re$$

$$E' = \{[u, \vartheta] \in E \text{ with } u, \vartheta \in V'\}$$

Neighborhood : FLIP

$$x_1, x_2, x_3, \ldots, x_i, \ldots, x_n \quad \text{current sol.}$$

$$x_1, x_2, x_3, \ldots, \overline{x_i}, \ldots, x_n \quad \text{neighboor sol.}$$

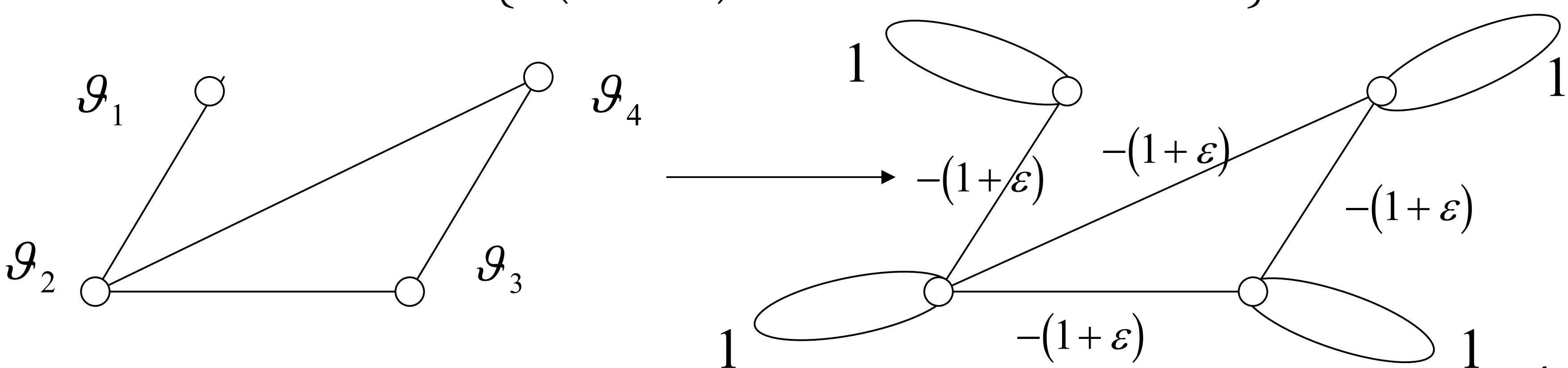
The maximum Independent Set problem

Another solution :

$$G(V,E) \xrightarrow{\text{tranf.}} G'(V,E',W)$$

$$E' = E \cup \{(\vartheta,\vartheta) \mid \vartheta \in V\}$$

$$W = (W_{ij}) = \begin{cases} 1 \text{ si} & i = j \\ -(1+\varepsilon) \text{si} & i \neq j,\ \varepsilon \in \Re \end{cases}$$



Solution coding : n-binary vector of 0,1

$$\vartheta \rightarrow \underset{\vartheta_1}{\square} \quad \underset{\vartheta_2}{\square} \quad \cdots \quad \underset{\vartheta_n}{\square}$$

initial configuration : randomly (not feasible)

Rule :

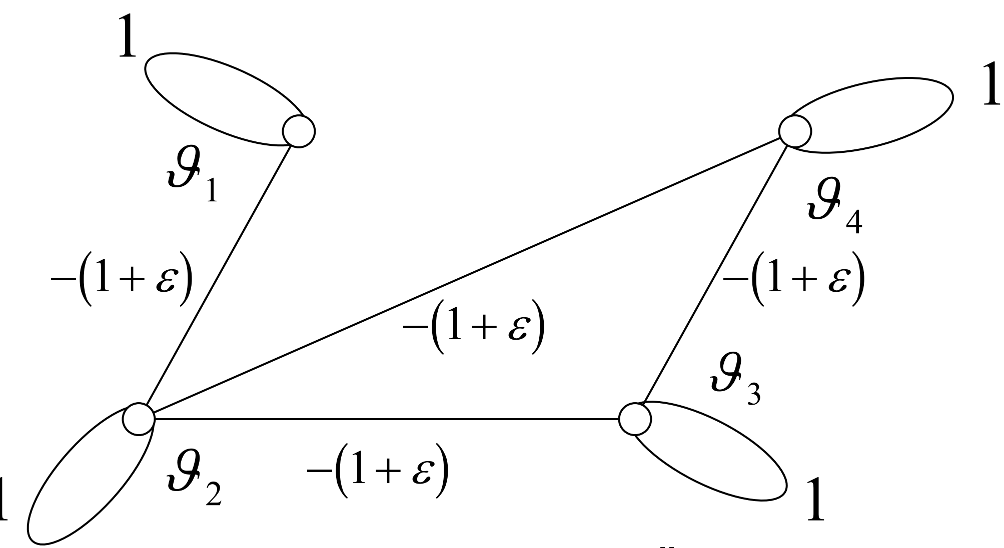
$$\text{change bit } \vartheta_i \begin{cases} \text{yes} & \text{if } \Delta f(\vartheta_i) \rangle 0 \\ \text{non} & \text{if } \Delta f(\vartheta_i) \le 0 \end{cases}$$

$$\Delta f(\vartheta_i) = (1 - 2\vartheta_i)\left(\sum_{j \in N(\vartheta)} w_{ij}\vartheta_j + w_{ii} \right)$$

Example (LS) with the neighborhood FLIP

$\bullet \leftrightarrow$ "1"

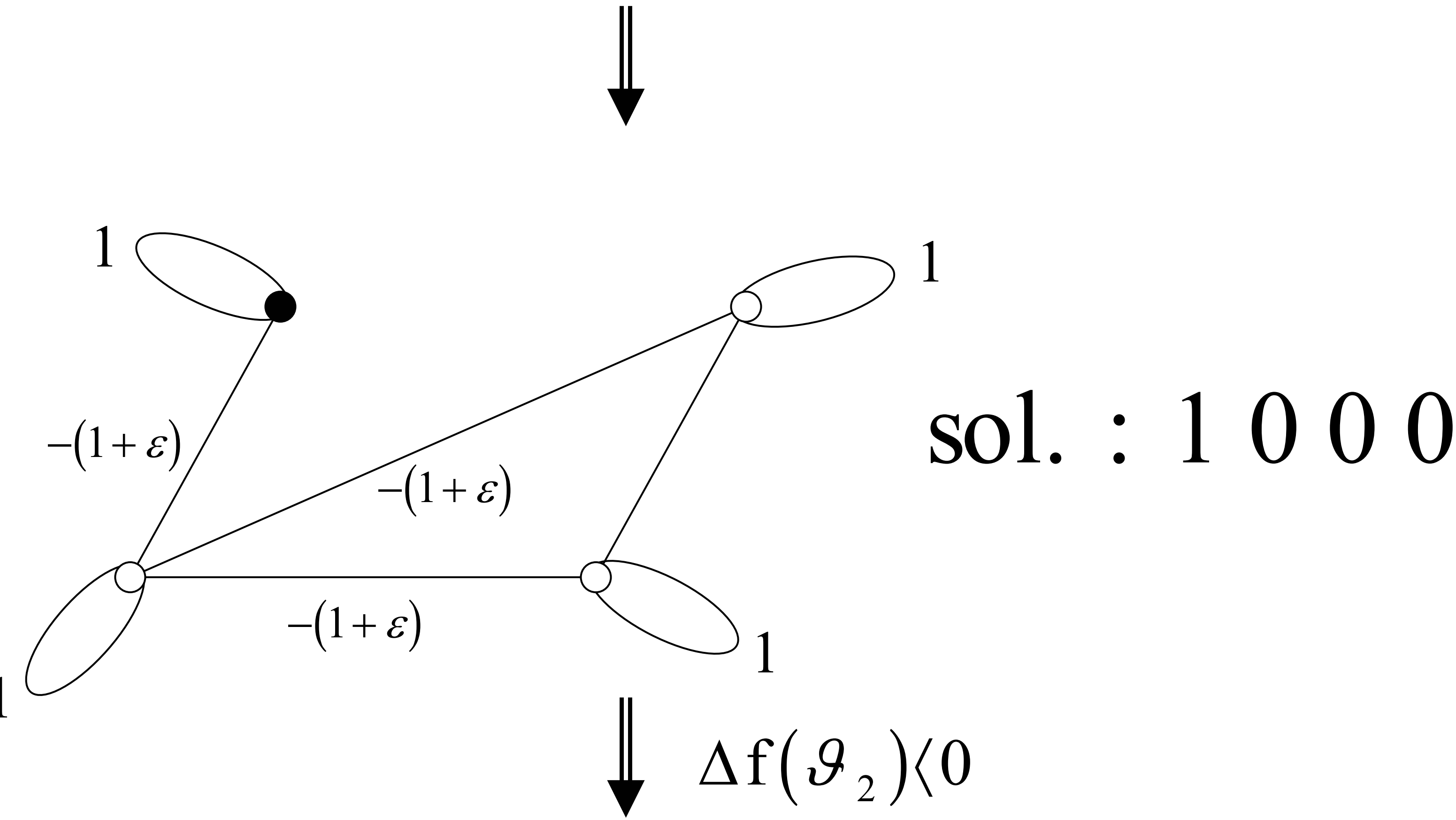
$\circ \leftrightarrow$ "$\varnothing$"



sol. : ϑ_1 $\circ$, ϑ_2 $\circ$, ϑ_3 $\circ$, ϑ_4 $\circ$

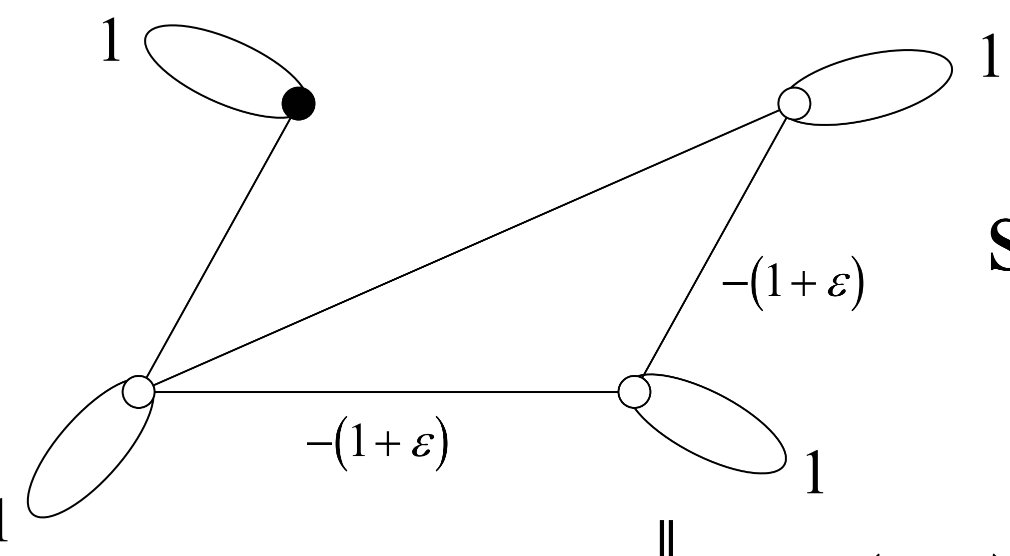
$$\Downarrow \quad \Delta f(\vartheta_1) \rangle 0$$

MIS: Another Solution (the second vertex is happy)



The third vertex becomes Happy

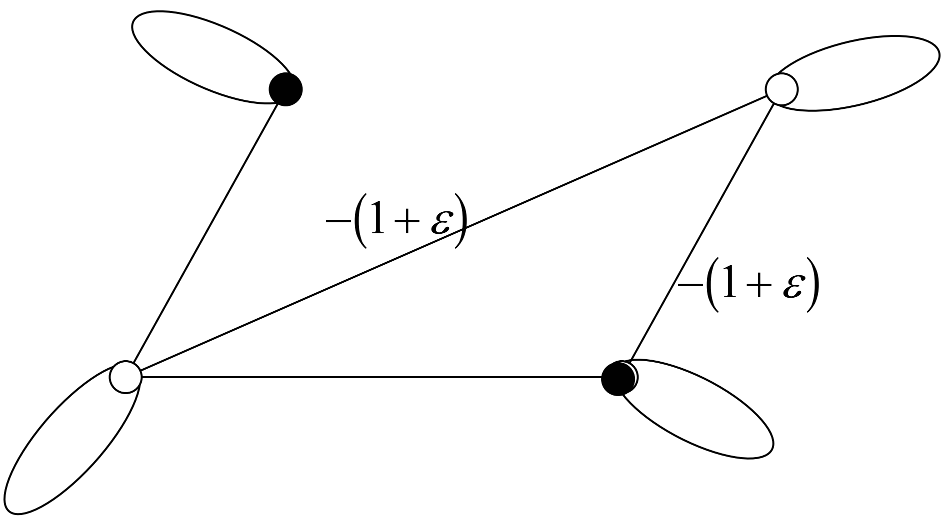
$\Downarrow$



sol. : 1 0 0 0

$\Downarrow \quad \Delta f(\vartheta_3) \rangle 0$

$\Downarrow$



sol. : 1 0 1 0

system

" HAPPY "

$\Downarrow$

$\Delta f(\vartheta_4) \langle 0$

$\Delta f(\vartheta_1) \langle 0$

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Rermarks

- each vertex ⟶ ¨ happy ¨ locally
- vertices ⟶ ¨ happy ¨ asynchronously
- each vertex ⟶ simple operations (addition , comp.)

¨ Massively Parallel algorithm ¨ (Hopfield model)

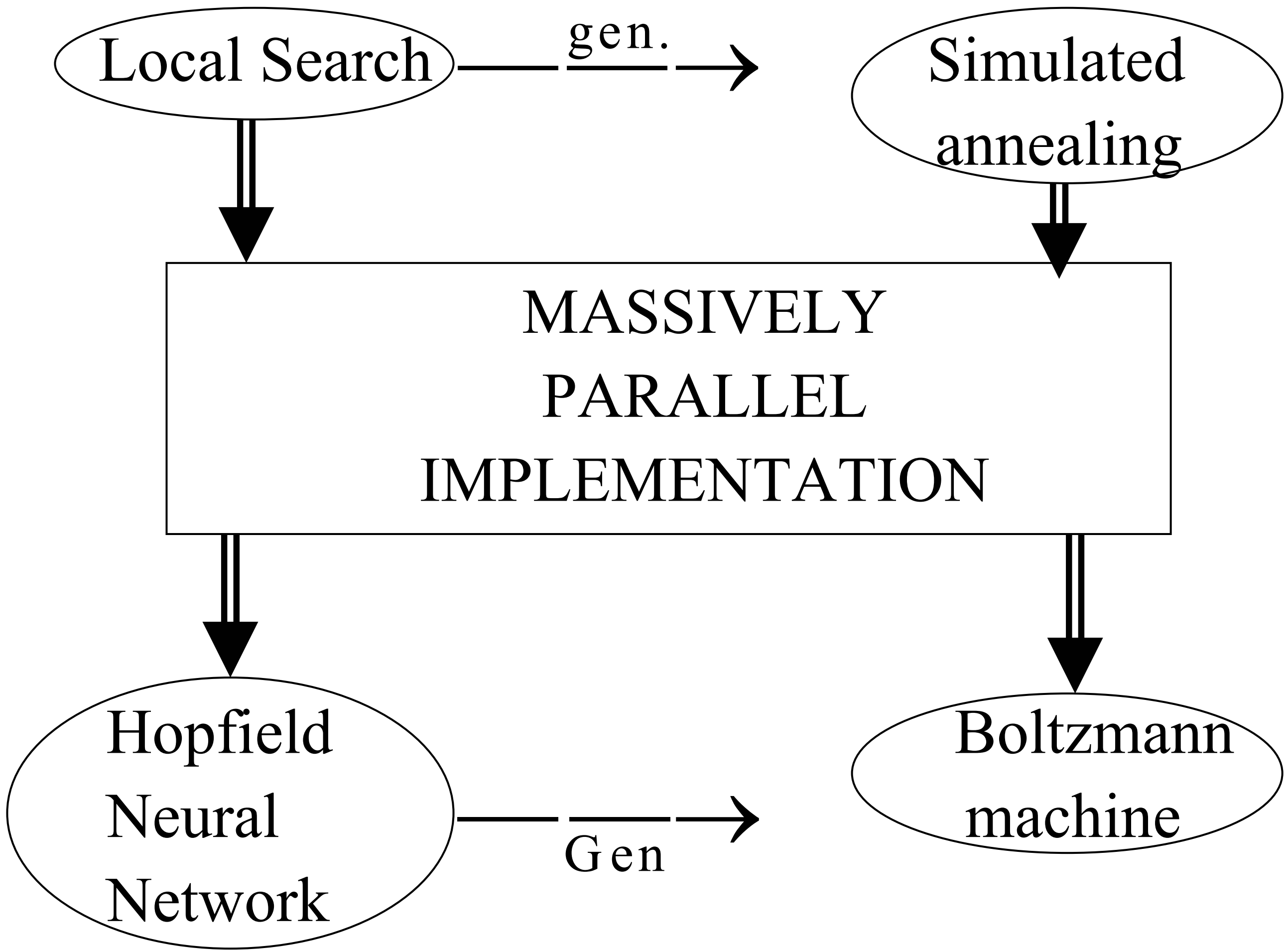
- + Simulated annealing

⇓

Asymptotically optimal algorithm

(Bolzmann machine)

CONCLUSION


Local Search
gen.
Simulated annealing
MASSIVELY PARALLEL IMPLEMENTATION
Hopfield Neural Network
Gen
Boltzmann machine

Future Work

- A problems classification relatively to HARDNESS with meta-heuristics

  - Neighborhoods for using in practice !

  - Landscapes RUGGEDNESS

    … A THEORY … → ?