Πρόβλημα Μέγιστου Ανεξάρτητου Υποσυνόλου (MIS)

Συνδυαστική Βελτιστοποίηση

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Θεωριτική Πληροφορική
Εθνικό και Καποδιστριακό Πανεπιστήμιο Αθηνών

$\Delta/2$-προσεγγίσιμο

- Αλγόριθμος:
  - $A_1(G)$: Λύση του smallest degree
  - $A_2(G)$: Λύση του exposed vertices
  - $A(G) : \max\{A_1(G), A_2(G)\}$
- $\tau(G)$: ελάχιστη διαμέριση σε κλίκες του G

Matching-Exposed vertices

- $A(G) \leq \tau(G)$
- M: maximum matching
- $M \leq n/2$
- $A(G) \leq \tau(G) \leq M + |A_2| \leq n/2$
- $M = n/2(1 - \gamma, \ \ \gamma in[0,1])$
- $|E| = n - 2M = \gamma n$ (Independent Set)
- $A_1(G) = \gamma n$

Upper Bound for MIS

- $\max\{A_1(G), A_2(G)\}$
- $\tau(G)$: ελάχιστη διαμέριση σε κλίκες του G
- $A(G) \leq M + \gamma n = (n/2)(1+\gamma)$
- $A(G) \leq (n/2)(1+\gamma)$

Approximation ratio (Exposed vertices)

$$
\begin{aligned}
\frac{A(G)}{A_1(G)} &\leq \frac{(n/2)(1+\gamma)}{n\gamma} \\
&= \frac{(1+\gamma)}{2\gamma} \\
&= \rho_2(\gamma)
\end{aligned}
$$

Approximation ratio (smallest degree)

$$A_2(G) \geq \frac{n - (\delta + 1)}{\Delta} + 1$$

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$$\frac{A(G)}{A_2(G)} \leq \frac{(n/2)(1 + \gamma)}{n - (\delta + 1)/\Delta + 1}$$

Approximation ratio (smallest degree)

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\frac{A(G)}{A_2(G)} &\leq \frac{(n/2)(1+\gamma)}{n-(\delta+1)/\Delta+1} \\
&\leq \frac{(\Delta n)/2(\gamma+1)}{n-(\delta+1)+\Delta}
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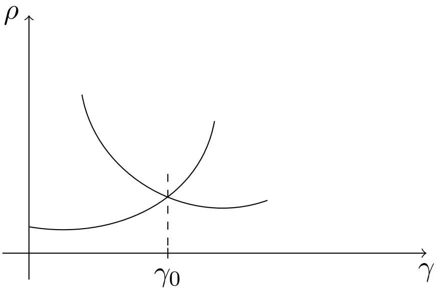
&\leq \frac{(\Delta n)/2(\gamma+1)}{n-(\delta+1)+\Delta} \\
&\leq \frac{\Delta n(\gamma+1)}{2(n-1)} \\
&\leq \rho_1(\gamma)
\end{aligned}
$$

Break point-approximation ratio



$$\rho_1(\gamma_0) = \rho_2(\gamma_0) \Rightarrow \gamma_0 = \frac{n-1}{\Delta n}$$

$$\rho_1(\gamma_0) = \frac{\Delta}{2}(1 + \frac{1}{n-1}) + \frac{1}{2}$$

$$\lim_{n \to \infty} \rho = \frac{\Delta}{2}$$

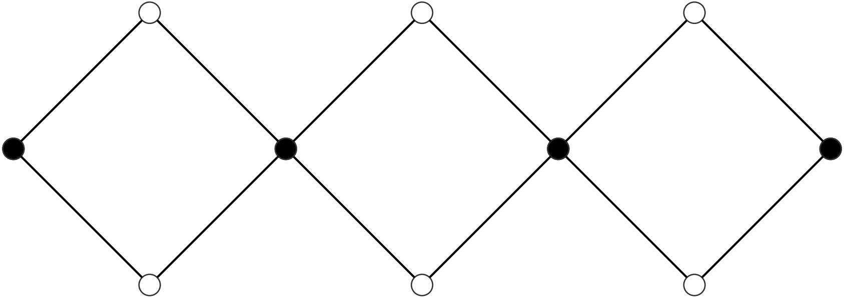
MIS Non Approximated

- Minimum coloring
- $\Delta_i = 2, \Delta = 4$

MIS: Polynomial cases

- lChains
- Cycles
- $\Delta = 2$
- Interval graphs
- Bipartite graphs
- Chordal graphs

Greedy MIS

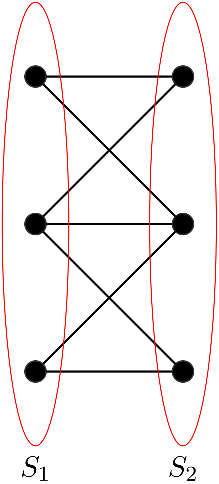


Σχήμα: $\rho = \frac{6}{4} = \frac{3}{2}$

MIS (efficient algorithms)

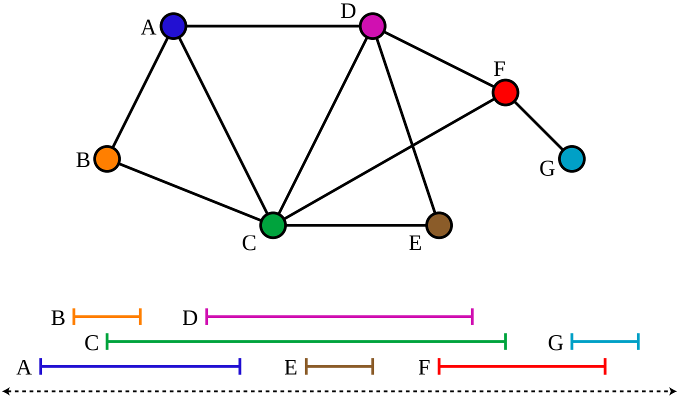
- $G = (V, E), d(v) = 2, \forall v \in V$
  - Find polynomial algorithm
- G bipartite
  - Pb polynomial ($\exists$ polynomial algorithm)

MIS (efficient algorithms)



Σχήμα: S_1, S_2 feasible solutions

MIS (efficient algorithms)

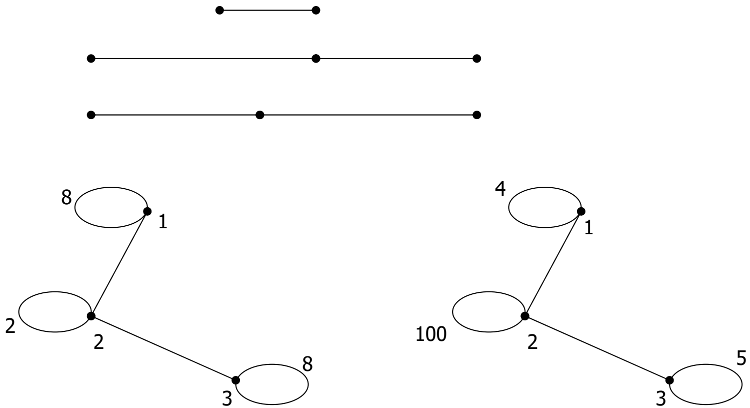


Σχήμα: Interval Graphs

Weighted IS Pb

- $G = (V, E, w)$
- WMIS: find IS maximum weight where $V' \subseteq V | V' IS$ and $\max \sum_{v \in V'} w(v)$

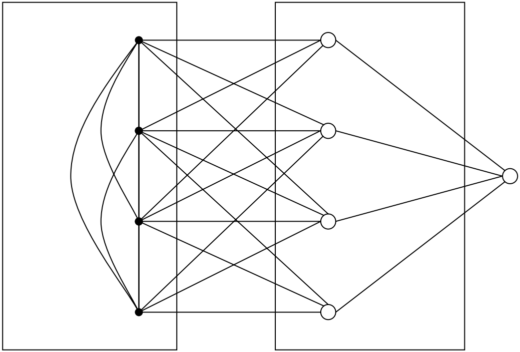
Παράδειγμα (max duration)



MIS non approximate

- MIS not so good as Max Knapsack! not in the method but difficulty of MIS
- Unless $P = NP$ No Approximation Algorithm can be exist for MIS

Non approximability of MIS (example for smallest degree algorithm)



Klique with k vertices

K independent vertices

$$\frac{A}{OPT} \geq \frac{2}{4}$$
$$\frac{A}{OPT} \geq \frac{2}{k} \to 0$$