

Introduction to

GeoGebra⁴

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www.geogebra.org



Introduction to GeoGebra

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This book covers the basic introduction to the dynamic mathematics software GeoGebra. It can be used both for workshops and for self-learning.

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together will help form many other GeoGebra Team members

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How to Use this Book

“Introduction to GeoGebra” covers all basics of the dynamic mathematics software GeoGebra. On the one hand, this book can serve as a basis for introductory workshops guided by an experienced GeoGebra presenter. On the other hand, you can use this document to learn the use of the software yourself.

By working through this book you will learn about the use of GeoGebra for teaching and learning mathematics from middle school (age 10) up to college level. The provided sequence of activities introduces you to geometry tools, algebraic input, commands, and a selection of different GeoGebra features. Thereby, a variety of different mathematical topics is covered in order to familiarize you with the versatility of the software and to introduce you to some methods of integrating GeoGebra into your everyday teaching.

Additionally, practice blocks are included in the structure of this introductory book. They allow you to practice new skills learned before and to explore the software on your own.

All construction files covered in this book, as well as accompanying files (additional GeoGebra files, dynamic worksheets, picture files) are available online at <http://www.geogebra.org/book/intro-en.zip>

We wish you a lot of fun and success working with GeoGebra!
Judith, Markus, and the GeoGebra Team



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1. Installation and Introduction of GeoGebra

Activity 1: Installing GeoGebra

Preparations

Create a new folder called GeoGebra_Introduction on your desktop.

Hint: During the workshop, save all files into this folder so they are easy to find later on.

Installation WITH Internet Access

Install GeoGebra WebStart

- Open your Internet browser and go to www.geogebra.org/webstart.
- Click on the button called *WebStart 4.0*.
Note: The software is automatically installed on your computer. You only need to confirm all messages that might appear with *OK* or *YES*.

Hint: Using GeoGebra WebStart has several advantages for you provided that you have an Internet connection available for the initial installation:

- You don't have to deal with different files because GeoGebra is installed automatically on your computer.
- You don't need to have special user permissions in order to use GeoGebra WebStart, which is especially useful for computer labs and laptop computers in schools.
- Once GeoGebra WebStart was installed you can use the software off-line as well.
- Provided you have Internet connection after the initial installation, GeoGebra WebStart frequently checks for available updates and installs them automatically. Thus, you are always working with the newest version of GeoGebra.

Installation WITHOUT Internet Access

- Your workshop presenter will provide the GeoGebra installer files via USB drives or CDs.
- Copy the installer file from the storage device into the created GeoGebra_Introduction folder on your computer.
Hint: Make sure you have the correct version for your operating system
Examples: MS Windows: GeoGebra_4_0_0_0.exe
MacOS: GeoGebra_4_0_0_0.zip
- Double-click the GeoGebra installer file and follow the instructions of the installer assistant.



Activity 2: Save the Accompanying Files

Download the Accompanying Files and Save them on your Computer

- Your workshop presenter will provide the accompanying files via USB drives or CDs. Copy the folder *GeoGebra_Introduction_Files* from the storage device into the created *GeoGebra_Introduction* folder on your computer.

OR

- Download the zipped file from www.geogebra.org/book/intro-en.zip
 - Save the zipped file in your *GeoGebra_Introduction* folder.
 - Extract the files on your computer. Depending on your operating system there are different ways to do this:
Examples:
MS Windows XP: Right-click the zipped file and follow the instructions of the Extraction Wizard.
MacOS: Double click on the zipped file.

Introduction: What is GeoGebra and How Does it Work?

Background Information About GeoGebra

GeoGebra is dynamic mathematics software for schools that joins geometry, algebra and calculus.

On the one hand, GeoGebra is an interactive geometry system. You can do constructions with points, vectors, segments, lines, and conic sections as well as functions while changing them dynamically afterwards.

On the other hand, equations and coordinates can be entered directly. Thus, GeoGebra has the ability to deal with variables for numbers, vectors, and points. It finds derivatives and integrals of functions and offers commands like Root or Vertex.

These two views are characteristic of GeoGebra: an expression in the algebra view corresponds to an object in the graphics view and vice versa.

GeoGebra's User Interface

GeoGebra's user interface consists of a graphics view, an algebra view and a spreadsheet view. On the one hand you can operate the provided geometry tools with the mouse in order to create geometric constructions in the graphics view. On the other hand, you can directly enter algebraic input, commands, and functions into the input bar by using the keyboard. While the graphical representation of all objects is displayed in the graphics view, their algebraic numeric representation is shown in the algebra view.



The user interface of GeoGebra is flexible and can be adapted to the needs of your students. If you want to use GeoGebra in early middle school, you might want to hide the algebra view, input bar, and coordinate axes and just work with the graphics view and geometry tools. Later on, you might want to introduce the coordinate system using a grid to facilitate working with integer coordinates. In high school, you might want to use algebraic input in order to guide your students through algebra on into calculus.

Basic Use of Tools

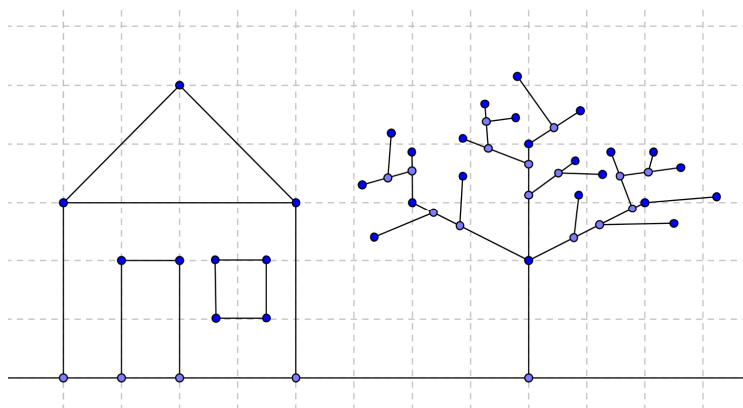
- Activate a tool by clicking on the button showing the corresponding icon.
- Open a toolbox by clicking on the lower part of a button and select another tool from this toolbox.
Hint: You don't have to open the toolbox every time you want to select a tool. If the icon of the desired tool is already shown on the button it can be activated directly.
Hint: Toolboxes contain similar tools or tools that generate the same type of new object.
- Check the toolbar help in order to find out which tool is currently activated and how to operate it.

2. Drawing versus Geometric Construction

Activity 3: Drawing Geometric Figures and Other Objects

Preparations

- Go to *Perspectives – Geometry*.



Drawing Pictures with GeoGebra

Use the mouse and the following selection of tools in order to draw figures in the graphics view (e.g. square, rectangle, house, tree,...).



	New Point	New!
	Move	New!
	Line Through Two Points	New!
	Segment Between Two Points	New!
	Delete Object	New!
	Undo / Redo Buttons	New!
	Move Graphics View	New!
	Zoom In / Zoom Out	New!

What to practice

- How to select an already existing object.
Hint: When the pointer hovers above an object, it highlights and the pointer changes its shape from a cross to an arrow. Clicking selects the corresponding object.
- How to create a point that lies on an object.
Hint: The point is displayed in a light blue color. Always check if the point really lies on the object by dragging it with the mouse.
- How to correct mistakes step-by-step using the *Undo* and *Redo* buttons.

Tip: Several tools allow the creation of points “on the fly”. Therefore, no existing objects are required in order to use the tool.

Example: The tool *Segment Between Two Points* can be applied to two already existing points or to the empty graphics view. By clicking on the graphics view the corresponding points are created and a segment is drawn in between them.

Activity 4: Saving GeoGebra Files

Save your drawing

- Open the *File* menu and select *Save*.
- Select the folder *GeoGebra_Introduction* in the appearing dialog window.
- Type in a *name* for your GeoGebra file.
- Click *Save* in order to finish this process.

Hint: A file with the extension ‘*.ggb*’ is created. This extension identifies GeoGebra files and indicates that they can only be opened with GeoGebra.

Hint: Name your files properly: Avoid using spaces or special symbols in a file name since they can cause unnecessary problems when transferred to other



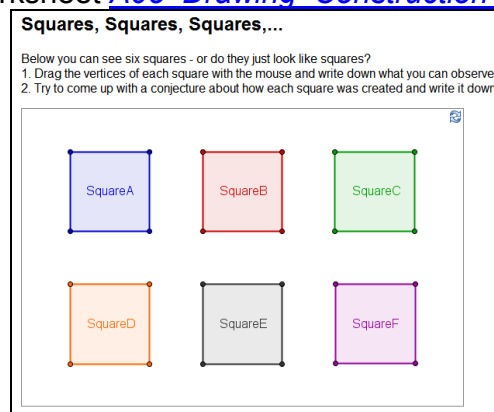
computers. Instead you can use underscores or upper case letters within the file name (e.g. First_Drawing.ggb).

What to practice

- How to open a new GeoGebra window using *New Window* from the *File* menu.
- How to open a blank GeoGebra interface within the same window (menu *File – New*)
Hint: If you didn't save the existing construction yet GeoGebra will ask you to do so before the blank screen is opened.
- How to open an already existing GeoGebra file (menu *File – Open*).
 - Navigate through the folder structure in the appearing window.
 - Select a GeoGebra file (extension '.ggb').
 - Click *Open*.

Activity 5: Drawings, Constructions and Drag Test

Open the dynamic worksheet [A05 Drawing Construction Squares.html](#).



The dynamic figure shows several squares constructed in different ways.

- Examine the squares by dragging ALL their vertices with the mouse.
- Find out which of the quadrilaterals real squares are and which ones just happen to look like squares.
- Try to come up with a conjecture about how each square was created.
- Write down your conjectures on paper.

Discussion

- What is the difference between a drawing and a construction?
- What is the “drag test” and why is it important?
- Why is it important to construct figures instead of just drawing them in interactive geometry software?
- What do we have to know about the geometric figure before we are able to construct it using dynamic mathematics software?



Activity 6: Rectangle Construction

Preparations

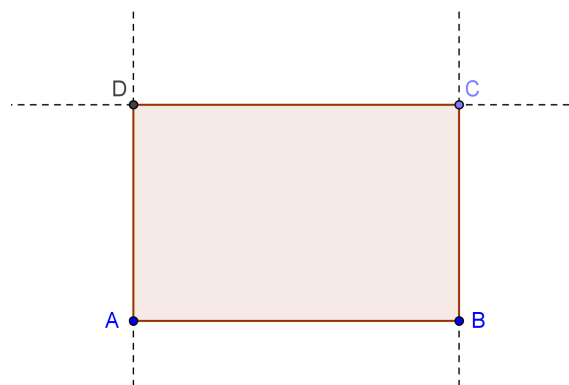
- Summarize the properties of a rectangle before you start the construction.
Hint: If you don't know the construction steps necessary for a rectangle you might want to open the file [A06 Rectangle Construction.ggb](#). Use the buttons of the navigation bar in order to replay the construction steps.
- Open new GeoGebra file.
- Switch to *Perspectives - Basic Geometry*.
- Change the labeling setting to *New Points Only* (menu *Options – Labeling*).

Introduction of New Tools

- *Perpendicular Line* and *Parallel Line Tools*
Hint: Click on an already existing line and a point in order to create a perpendicular line / parallel line through this point.
- *Intersect Two Objects Tool*
Hint: Click on the intersection point of two objects to get this one intersection point. Successively click on both objects to get all intersection points.
- *Polygon Tool*
Hints: Click on the graphics view or already existing points in order to create the vertices of a polygon. Connect the last and first vertex to close the polygon! Always connect vertices counterclockwise!

Hint: Don't forget to read the toolbar help if you don't know how to use a tool.

Hint: Try out all new tools before you start the construction.





Construction Steps

1		Segment AB
2		Perpendicular line to segment AB through point B
3		New point C on perpendicular line
4		Parallel line to segment AB through point C
5		Perpendicular line to segment AB through point A
6		Intersection point D
7		Polygon $ABCD$ <u>Hint</u> : To close the polygon click on the first vertex again.
8		Save the construction

Check your Construction

1. Apply the drag test to check if the construction is correct.
2. Show the *Navigation bar for construction steps* (View menu) to review your construction step-by-step using the buttons.
3. Show the *Construction protocol* (View menu) and use it to review your rectangle construction step-by-step.
 - Try to change the order of some construction steps by dragging a line with the mouse. Why does this NOT always work?
 - Group several constructions steps by setting breakpoints:
 - Show the column *Breakpoint* (Right click on the construction protocol window *Columns - Breakpoint*)
 - Group construction steps by checking the *Breakpoint* box of the last one of the group
 - Change setting to *Show Only Breakpoints* (Right click on the construction protocol window *Options - Show Only Breakpoints*)
 - Use the navigation bar to review the construction step-by-step. Did you set the breakpoints correctly?



Activity 7: Equilateral Triangle Construction

Preparations

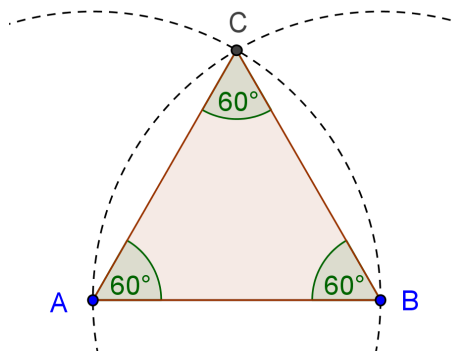
- Summarize the properties of an equilateral triangle before you start the construction.
Hint: If you don't know the construction steps necessary for an equilateral triangle you might want to have a look at the following file: [A07 Equilateral Triangle Construction.ggb](#). Use the buttons of the navigation bar in order to replay the construction steps.
- Open a new GeoGebra window.
- Hide algebra view, input bar and coordinate axes (*View* menu).
- Change the labeling setting to *New points only* (menu *Options – Labeling*).

Introduction of New Tools

- *Circle With Center Through Point* tool
Hint: First click creates center, second click determines radius of the circle.
- *Show / Hide Object* tool
Hints: Highlight all objects that should be hidden, then switch to another tool in order to apply the visibility changes!
- *Angle* tool
Hint: Click on the points in counterclockwise direction! GeoGebra always creates angles with mathematically positive orientation.









Hint: Don't forget to read the toolbar help if you don't know how to use a tool.

Hint: Try out all new tools before you start the construction.





Construction Steps

1		Segment AB
2		Circle with center A through B <u>Hint</u> : Drag points A and B to check if circle is connected to them.
3		Circle with center B through A <u>Hint</u> : Drag points to check if circle is connected to them.
4		Intersect both circles to get point C
5		Polygon ABC in counterclockwise direction
6		Hide circles
7		Show interior angles of triangle <u>Hint</u> : Click in the middle of the polygon to create all interior angles!
8		Save the construction

Check your Construction

1. Apply the drag test to check if the construction is correct.
2. Show the navigation bar or use the construction protocol to review your construction step-by-step.

Use the Properties Dialog to Enhance your Construction

There are different ways to access the Properties dialog:

- Right-click (MacOS: Ctrl-click) an object
- Select *Object Properties...* from the Edit menu
- Double click an object in *Move* mode
- Toggle *Styling Bar* on the right hand side of the graphics view

What to practice...

- Select different objects from the list on the left hand side and explore the available properties tabs for different types of objects
- Select several objects in order to change a certain property for all of them at the same time
Hint: Hold the *Ctrl*-key (MacOS: Open Apple-click) pressed and select all desired objects.
- Select all objects of one type by clicking on the corresponding heading.
- Show the value of different objects and try out different label styles.



3. Practice Block I

This practice block consists of a pool of geometry activities of two different difficulty levels: *Basic tasks* and *Advanced tasks*. You can pick tasks of your interest and work on them either on your own or together with a colleague.

Tips and Tricks

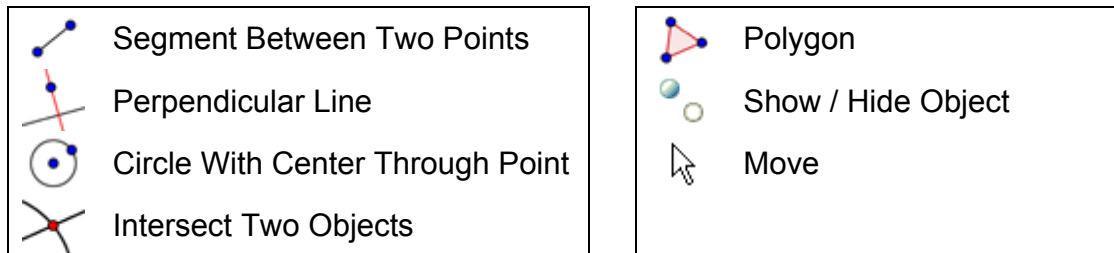
- Summarize the properties of the geometric figure you want to create.
- Try to find out which GeoGebra tools can be used in order to construct the figure using some of these properties (e.g. right angle – tool *Perpendicular line*).
- Make sure, you know how to use each tool before you begin the construction. If you don't know how to operate a certain tool, activate it and read the toolbar help.
- For each of these activities, open a new GeoGebra file, hide the algebra view, input bar and the coordinate axes.
- You might want to save your files before you start a new activity.
- Don't forget about the *Undo* and *Redo* buttons in case you make a mistake.
- Frequently use the *Move* tool in order to check your construction (e.g. are objects really connected, did you create any unnecessary objects).
- If you have questions, please ask a colleague before you address the presenter or assistant(s).



Activity I.a: Square Construction

Classification: Basic task

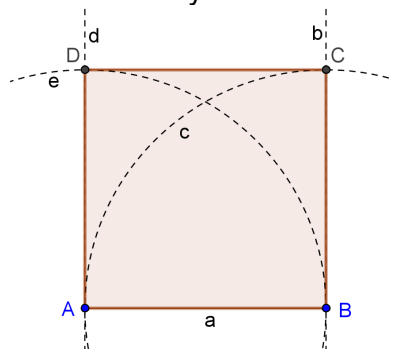
In this activity you are going to use the following tools. Make sure you know how to use each tool before you begin with the actual construction of the square:



Hint: You might want to have a look at the file [A 1a Square Construction.html](#) if you are not sure about the construction steps.

Construction Steps

1. Draw segment $a = AB$ between points A and B
2. Construct perpendicular line b to segment AB through point B
3. Construct circle c with center B through point A
4. Intersect circle c with perpendicular line b to get intersection point C
5. Construct perpendicular line d to segment AB through point A
6. Construct circle e with center A through point B
7. Intersect perpendicular line d with circle e to get intersection point D
8. Create polygon ABCD.
Hint: Don't forget to close the polygon by clicking on point A after selecting point D.
9. Hide circles and perpendicular lines
10. Perform the drag test to check if your construction is correct









Challenge: Can you come up with a different way of constructing a square?



Activity I.b: Regular Hexagon Construction

Classification: Basic task

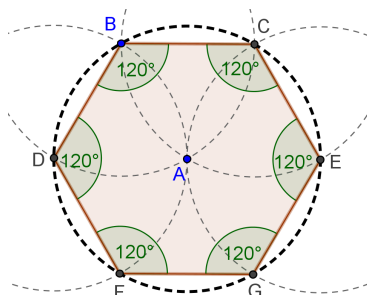
In this activity you are going to use the following tools. Make sure you know how to use each tool before you begin with the actual construction of the hexagon:

	Circle With Center Through Point		Angle
	Intersect Two Objects		Show / Hide Object
	Polygon		Move

Hint: You might want to have a look at the file [A 1b Hexagon Construction.html](#) if you are not sure about the construction steps.

Construction Steps

1. Draw a circle with center A through point B
2. Construct another circle with center B through point A
3. Intersect the two circles in order to get the vertices C and D.
4. Construct a new circle with center C through point A.
5. Intersect the new circle with the first one in order to get vertex E.
6. Construct a new circle with center D through point A.
7. Intersect the new circle with the first one in order to get vertex F.
8. Construct a new circle with center E through point A.
9. Intersect the new circle with the first one in order to get vertex G.
10. Draw hexagon FGECBD.
11. Create the angles of the hexagon.
12. Perform the drag test to check if your construction is correct.



Challenge: Try to find an explanation for this construction steps.






Hint: Which radius do the circles have and why?



Activity I.c: Circumscribed Circle of a Triangle

Classification: Advanced task

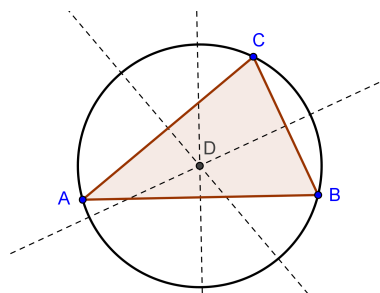
In this activity you are going to use the following tools. Make sure you know how to use each tool before you begin with the actual construction of the circumscribed circle:

	Polygon		Circle With Center Through Point
	Perpendicular Bisector New!		Move
	Intersect Two Objects		

Hint: If you are not sure about the construction steps, you might want to have a look at the file [A 1c Circumcircle Triangle Construction.html](#).

Construction Steps

1. Create an arbitrary triangle ABC
2. Construct the line bisector for each side of the triangle.
Hint: The tool, Line bisector, can be applied to an existing segment.
3. Create intersection point D of two of the line bisectors
Hint: The tool, Intersect two objects, can't be applied to the intersection of three lines. Either select two of the three line bisectors successively, or click on the intersection point and select one line at a time from the appearing list of objects in this position.
4. Construct a circle with center D through one of the vertices of triangle ABC
5. Perform the drag test to check if your construction is correct.



Challenge: Modify your construction to answer the following questions:

1. Can the circumcenter of a triangle lie outside the triangle? If yes, for which types of triangles is this true?
2. Try to find an explanation for using line bisectors in order to create the circumcenter of a triangle.

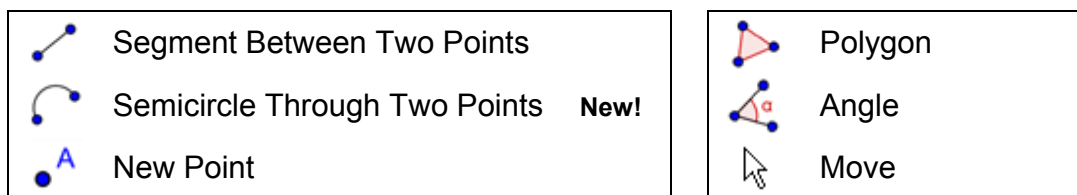


Activity 1.d: Visualize the Theorem of Thales

Classification: Advanced task

Before you begin this construction, check out the dynamic worksheet called [04 Theorem Thales.html](#) in order to see how students could rediscover what Thales found out about 2600 years ago.

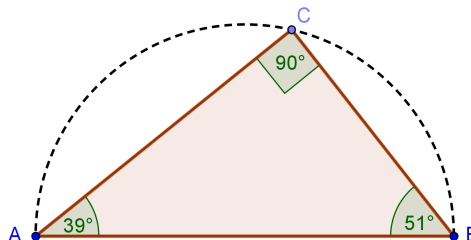
In this activity you are going to use the following tools. Make sure you know how to use each tool before you begin with the actual construction:



Hint: If you are not sure about the construction steps, you might want to have a look at the file [A 1d Theorem Thales Construction.html](#).

Construction Steps

1. Draw segment AB
2. Construct a semicircle through points A and B
Hint: The order of clicking points A and B determines the direction of the semicircle.
3. Create a new point C on the semicircle
Hint: Check if point C really lies on the arc by dragging it with the mouse.
4. Create triangle ABC
5. Create the interior angles of triangle ABC




Challenge: Try to come up with a graphical proof for this theorem.

Hint: Create midpoint O of segment AB and display the radius OC as a segment.



4. Basic Algebraic Input, Commands and Functions

Tips and Tricks

- **Name a new object** by typing `name =` into the input bar in front of its algebraic representation. Example: `P = (3, 2)` creates point *P*.
- **Multiplication** needs to be entered using an asterisk or space between the factors. Example: `a*x` or `a x`
- **GeoGebra is case sensitive!** Thus, upper and lower case letters must not be mixed up. Note:
 - Points are always named with upper case letters
Example: `A = (1, 2)`
 - Vectors are named with lower case letters
Example: `v = (1, 3)`
 - Segments, lines, circles, functions... are always named with lower case letters.
Example: circle `c: (x - 2)^2 + (y - 1)^2 = 16`
 - The variable *x* within a function and the variables *x* and *y* in the equation of a conic section always need to be lower case.
Example: `f(x) = 3*x + 2`
- If you want to use an **object within an algebraic expression** or command you need to create the object prior to using its name in the input bar. Examples:
 - `y = m x + b` creates a line whose parameters are already existing values *m* and *b* (e.g. numbers / sliders).
 - `Line[A, B]` creates a line through existing points *A* and *B*.
- **Confirm an expression** you entered into the input bar by pressing the *Enter* key.
- **Open the help window** for using the input bar and commands by selecting *Help* from the Menu *Help* (or shortcut *F1*).
- **Error messages**: Always read the messages – they could possibly help to fix the problem!
- **Commands** can be typed in or selected from the list  next to the input bar.
Hint: If you don't know which parameters are required within the brackets of a certain command, type in the full command name and press key *F1* to open the GeoGebra Wiki.
- **Automatic completion of commands**: After typing in the first two letters of a command into the input bar, GeoGebra tries to complete the command.
 - If GeoGebra suggests the desired command, hit the *Enter* key in order to place the cursor within the brackets.
 - If the suggested command is not the one you wanted to enter, just keep typing until the suggestion matches.



Activity 8a: Constructing Tangents to a Circle (Part 1)

Back to School...

Open the dynamic worksheet [A08 Tangents Circle.html](#). Follow the directions on the worksheet in order to find out how to construct tangents to a circle.

Constructing Tangents to a Circle

1. Use the **arrow buttons** in the figure below to review the construction process of tangents to a circle.
2. Try to do this **construction on your own** using the figure to the right.
3. Write down a construction protocol and **explain** every construction step.

Move: Drag or select objects (Esc)

Discussion

- Which tools did you use in order to recreate the construction?
- Were there any new tools involved in the suggested construction steps? If yes, how did you find out how to operate the new tool?
- Did you notice anything about the toolbar displayed in the right applet?
- Do you think your students could work with such a dynamic worksheet and find out about construction steps on their own?

Activity 8b: Constructing Tangents to a Circle (Part 2)

What if my Mouse and Touchpad wouldn't work?

Imagine your mouse and / or touchpad stop working while you are preparing GeoGebra files for tomorrow's lesson. How can you finish the construction file?

GeoGebra offers algebraic input and commands in addition to the geometry tools. Every tool has a matching command and therefore could be applied without even using the mouse.

Note: GeoGebra offers more commands than geometry tools. Therefore, not every command has a corresponding geometry tool!



Task 1: Check out the list of commands next to the input bar and look for commands whose corresponding tools were already introduced in this workshop.

As you saw in the last activity, the construction of tangents to a circle can be done by using geometric construction tools only. You will now recreate this construction by just using keyboard input.

Preparations

- Open a new GeoGebra window.
- Show the algebra view and input bar, as well as coordinate axes (*View* menu)

Construction Steps

1	$A = (0, 0)$	Point A
2	$(3, 0)$	Point B <u>Hint:</u> If you don't specify a name objects are named in alphabetical order.
3	$c = \text{Circle}[A, B]$	Circle with center A through point B <u>Hint:</u> Circle is a dependent object

Note: GeoGebra distinguishes between free and dependent objects. While free objects can be directly modified either using the mouse or the keyboard, dependent objects adapt to changes of their parent objects. Thereby, it is irrelevant in which way (mouse or keyboard) an object was initially created!

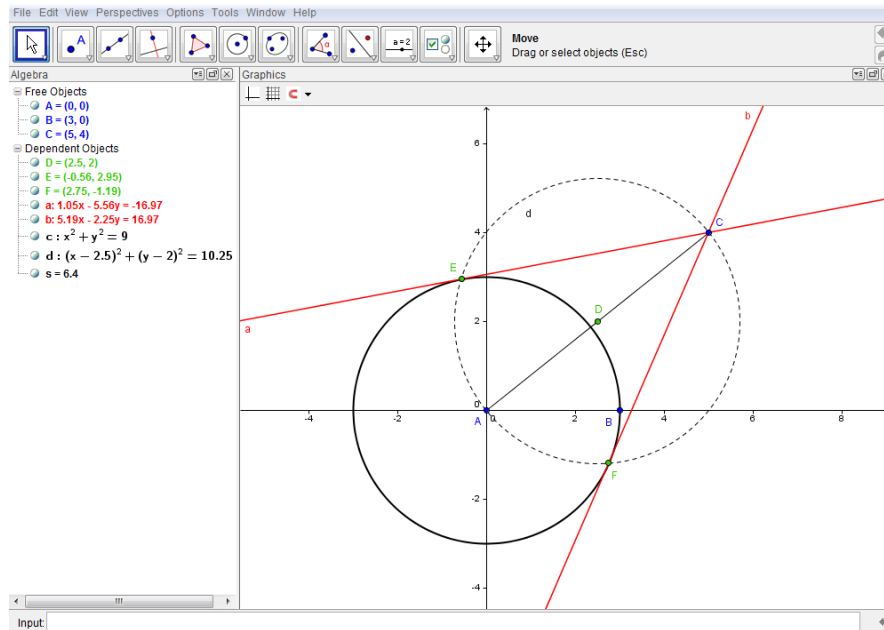
Hint 1: Activate *Move* mode and double click an object in the algebra view in order to change its algebraic representation using the keyboard. Hit the *Enter* key once you are done.

Hint 2: You can use the arrow keys in order to move free objects in a more controlled way. Activate *Move* mode and select the object (e.g. a free point) in either window. Press the up / down or left / right arrow keys in order to move the object into the desired direction.

4	$C = (5, 4)$	Point C
5	$s = \text{Segment}[A, C]$	Segment AC
6	$D = \text{Midpoint}[s]$	Midpoint D of segment AC



7	$d = \text{Circle}[D, C]$	Circle with center D through point C
8	$\text{Intersect}[c, d]$	Intersection points E and F of the two circles
9	$\text{Line}[C, E]$	Tangent through points C and E
10	$\text{Line}[C, F]$	Tangent through points C and F



Checking and Enhancing the Construction

- Perform the drag-test in order to check if the construction is correct.
- Change properties of objects in order to improve the construction's appearance (e.g. colors, line thickness, auxiliary objects dashed,...)
- Save the construction.

Discussion

- Did any problems or difficulties occur during the construction steps?
- Which version of the construction (mouse or keyboard) do you prefer and why?
- Why should we use keyboard input if we could also do it using tools?
Hint: There are commands available that have no equivalent geometric tool.
- Does it matter in which way an object was created? Can it be changed in the algebra view (using the keyboard) as well as in the graphics view (using the mouse)?




Activity 9: Exploring Parameters of a Quadratic Polynomial

Back to school...

In this activity you will explore the impact of parameters on a quadratic polynomial. You will experience how GeoGebra could be integrated into a 'traditional' teaching environment and used for active, student-centered learning.

Follow the instructions on the paper worksheet and write down your results and observations while working with GeoGebra. Your notes will help you during the following discussion of this activity.

Exploring Parameters of a Quadratic Polynomial

1. Open a **new GeoGebra window**
2. **Type** in $f(x) = x^2$ and hit the *Enter* key. Which **shape** does the function graph have? Write down your answer on paper.
3. In  *Move* mode, highlight the polynomial in the algebra view and use the **↑ up and ↓ down arrow keys**.
 - a. How does this **impact the graph** of the polynomial? Write down your observations.
 - b. How does this **impact the equation** of the polynomial? Write down your observations.
4. Again, in *Move* mode, highlight the function in the algebra view and use the **← left and → right arrow keys**.
 - a. How does this **impact the graph** of the polynomial? Write down your observations.
 - b. How does this **impact the equation** of the polynomial? Write down your observations.
5. In *Move* mode, double click the equation of the polynomial. Use the keyboard to **change the equation to $f(x) = 3x^2$** .

Hint: Use an asterisk * or space in order to enter a multiplication.

 - a. **Describe** how the function graph changes.
 - b. **Repeat changing the equation** by typing in different values for the parameter (e.g. 0.5, -2, -0.8, 3). **Write down** your observations.



Discussion

- Did any problems or difficulties concerning the use of GeoGebra occur?
- How can a setting like this (GeoGebra in combination with instructions on paper) be integrated into a 'traditional' teaching environment?
- Do you think it is possible to give such an activity as a homework problem to your students?
- In which way could the dynamic exploration of parameters of a polynomial possibly affect your students' learning?
- Do you have ideas for other mathematical topics that could be taught in similar learning environment (paper worksheets in combination with computers)?

Activity 10: Using Sliders to Modify Parameters

Let's try a more dynamic way of exploring the impact of a parameter on a polynomial $f(x) = a x^2$ by using a slider to modify the parameter value.

Preparation

- Open a new GeoGebra window
- Switch to *Perspectives – Algebra & Graphics*

Construction Steps


1	$a = 1$	Create the variable a
2	$f(x) = a * x^2$	Enter the quadratic polynomial f <u>Hint</u> : Don't forget to enter an asterisk $*$ or space between a and x^2 .

Representing a Number as a Slider

To display number as a slider in the graphics view you need to right click (MacOS: *Ctrl*-click) the variable in the algebra view and select *Show object*.

Enhancing the Construction

Let's create another slider b that controls the constant in the polynomial's equation $f(x) = a x^2 + b$.

3		Create a slider b using the <i>Slider</i> tool <u>Hint</u> : Activate the tool and click on the graphics view. Use the default settings and click <i>Apply</i> .
---	---	---



4	$f(x) = a * x^2 + b$	Enter the polynomial f <u>Hint:</u> GeoGebra will overwrite the old function f with the new definition.
---	----------------------	--

Tasks

- Change the parameter value a by moving the point on the slider with the mouse. How does this influence the graph of the polynomial?
- What happens to the graph when the parameter value is (a) greater than 1, (b) between 0 and 1, or (c) negative? Write down your observations.
- Change the parameter value b . How does this influence the graph of the polynomial?


Activity 11: Library of Functions

Apart from polynomials there are different types of functions available in GeoGebra (e.g. trigonometric functions, absolute value function, exponential function). Functions are treated as objects and can be used in combination with geometric constructions.

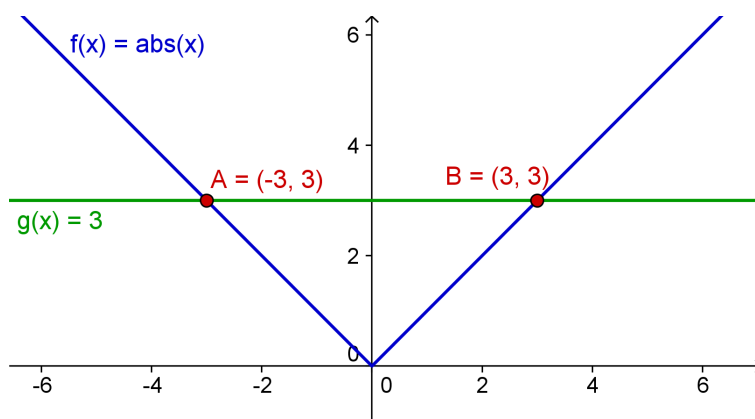
Note: Some of the functions available can be selected from the menu next to the input bar. Please find a complete list of functions supported by GeoGebra in the GeoGebra Wiki (<http://wiki.geogebra.org/en/>).

Task 1: Visualizing absolute values

Open a new GeoGebra window. Make sure the algebra view, input bar and coordinate axes are shown.

1	$f(x) = \text{abs}(x)$	Enter the absolute value function f
2	$g(x) = 3$	Enter the constant function g
3		Intersect both functions <u>Hint:</u> You need to intersect the functions twice in order to get both intersection points.

Hint: You might want to close the algebra view and show the names and values as labels of the objects.



(a) Move the constant function with the mouse or using the arrow keys. The y -coordinate of each intersection point represents the absolute value of the x -coordinate.

(b) Move the absolute value function up and down either using the mouse or the arrow keys. In which way does the function's equation change?

(c) How could this construction be used in order to familiarize students with the concept of absolute value?

Hint: The symmetry of the function graph indicates that there are usually two solutions for an absolute value problem.

Task 2: Superposition of Sine Waves


Sound waves can be mathematically represented as a combination of sine waves. Every musical tone is composed of several sine waves of the form $y(t) = a \sin(\omega t + \varphi)$. The amplitude a influences the volume of the tone while the angular frequency ω determines the pitch of the tone. The parameter φ is called phase and indicates if the sound wave is shifted in time.

If two sine waves interfere, superposition occurs. This means that the sine waves amplify or diminish each other. We can simulate this phenomenon with GeoGebra in order to examine special cases that also occur in nature.

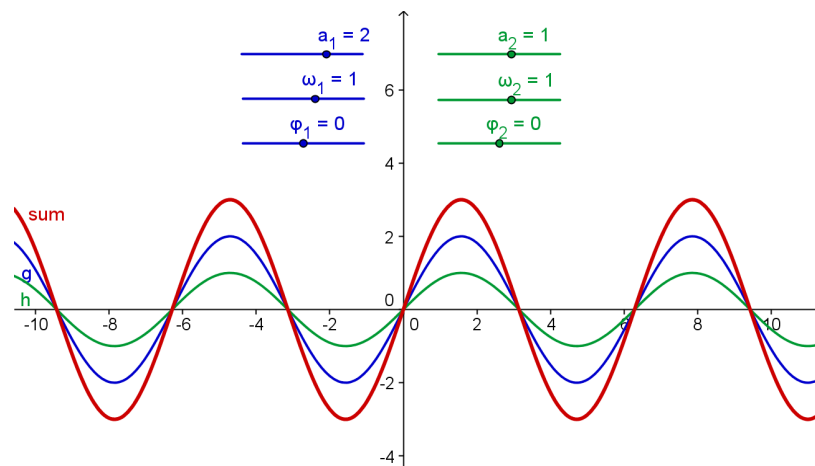
1	$\underline{a=2}$	Create three sliders a_1 , ω_1 , and φ_1 <u>Hints:</u> a_1 produces an index. You can select the Greek letters from the menu next to the text field <i>name</i> in the <i>Slider</i> dialog window.
2	$g(x) = a_1 \sin(\omega_1 x + \varphi_1)$	Enter the sine function g <u>Hint:</u> Again, you can select the Greek letters from a menu next to the input bar.



- (a) Examine the impact of the parameters on the graph of the sine functions by changing the values of the sliders.

3		Create three sliders a_2 , ω_2 , and φ_2 <u>Hint:</u> Sliders can only be moved when the <i>Slider</i> tool is activated.
4	$h(x) = a_2 \sin(\omega_2 x + \varphi_2)$	Enter another sine function h
5	$\text{sum}(x) = g(x) + h(x)$	Create the sum of both functions

- (b) Change the color of the three functions so they are easier to identify.
- (c) Set $a_1 = 1$, $\omega_1 = 1$, and $\varphi_1 = 0$. For which values of a_2 , ω_2 , and φ_2 does the sum have maximal amplitude?
Hint: In this case the resulting tone has the maximal volume.
- (d) For which values of a_2 , ω_2 , and φ_2 do the two functions cancel each other?
Hint: In this case no tone can be heard any more.






5. Export of Pictures to the Clipboard

GeoGebra's graphics view can be exported as a picture to your computer's clipboard. Thus, they can be easily inserted into text processing or presentation documents allowing you to create appealing sketches for tests, quizzes, notes or mathematical games.

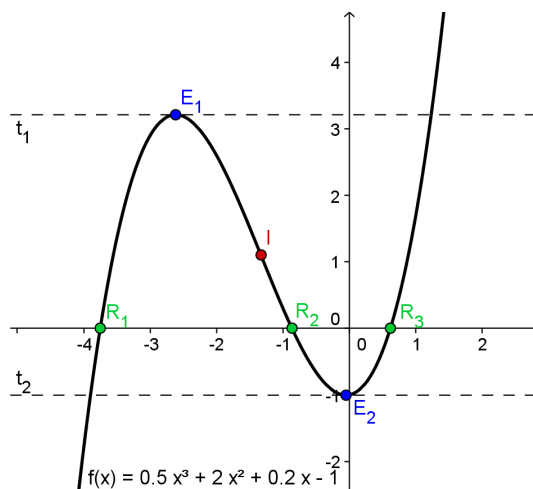
Activity 12a: Exporting Pictures to the Clipboard

Create your Sketch

Open a new GeoGebra window and make sure the algebra view, input bar and coordinate axes are shown.

1	$f(x) = 0.5x^3 + 2x^2 + 0.2x - 1$	Enter the cubic polynomial f
2	$R = \text{Root}[f]$	Create the roots of polynomial f <u>Hint:</u> If there are more than one root GeoGebra will produce indices for their names if you type in $R =$ (e.g. R_1, R_2, R_3).
3	$E = \text{Extremum}[f]$	Create the extrema of polynomial f
4		Create tangents to f in E_1 and E_2
5	$I = \text{InflectionPoint}[f]$	Create the inflection point of polynomial f

Hint: You might want to change properties of objects (e.g. color of points, style of the tangents, show name and value of the function).





Exporting a Picture to the Clipboard

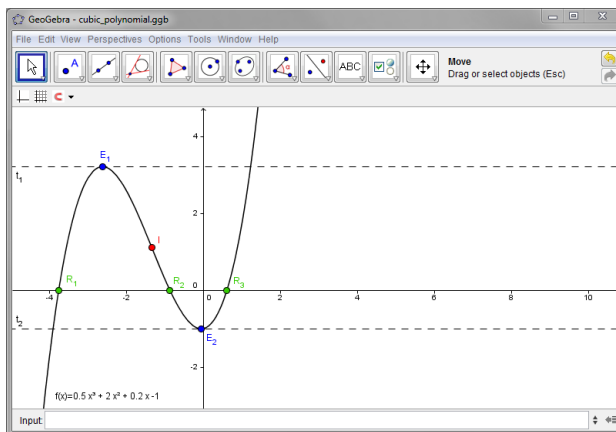
GeoGebra will export the whole graphics view into the clipboard. Thus, you need to make the GeoGebra window smaller in order to reduce unnecessary space on the graphics view:

- Move your figure (or the relevant section) to the upper left corner of the graphics view using the *Move Graphics View* tool (see left figure below).

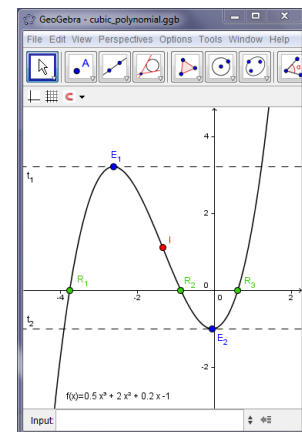
Hint: You might want to use tools *Zoom In* and *Zoom Out* in order to prepare your figure for the export process.

- Reduce the size of the GeoGebra window by dragging its lower right corner with the mouse (see right figure below).

Hint: The pointer will change its shape when hovering above an edges or corner of the GeoGebra window.



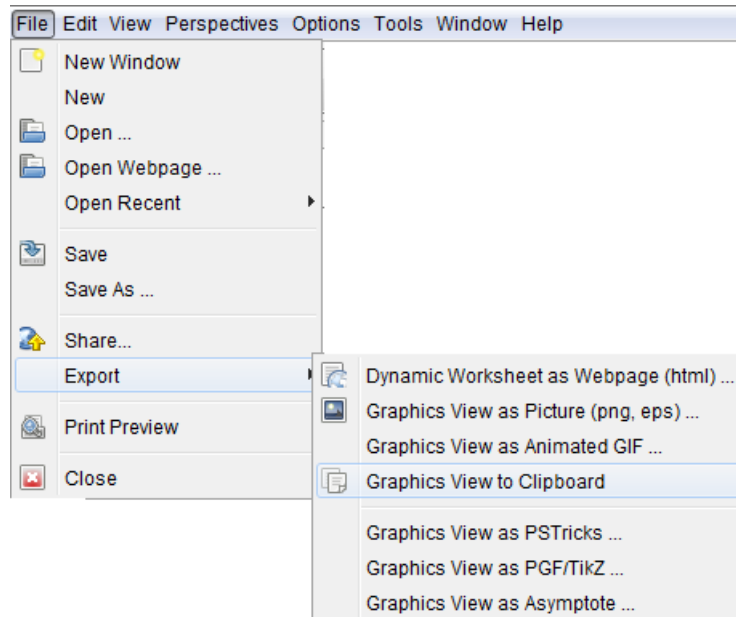
GeoGebra window before
the size reduction



GeoGebra window after
the size reduction

Use the *File* menu to export the graphics view to the clipboard:

- Export – Graphics view to Clipboard
Hint: You could also use the key combination *Ctrl – Shift – C*.
- Your figure is now stored in your computer's clipboard and can be inserted into any word processing or presentation document.



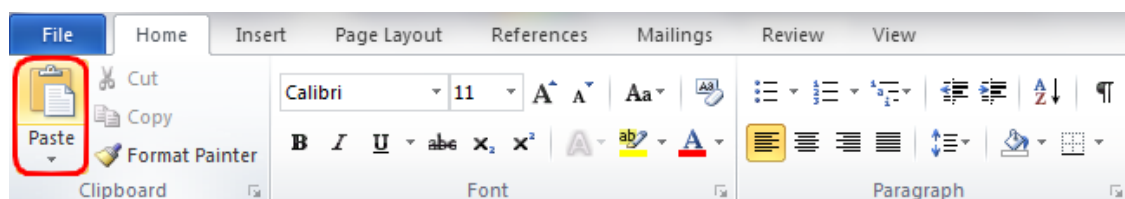
Activity 12b: Inserting Pictures into a Text Processing Document

After exporting a figure from GeoGebra into your computer's clipboard you can now paste it into a word processing document.

Inserting pictures from the clipboard to MS Word

- Open a new text processing document
- From the *Home* tab select *Paste*. The picture is inserted at the position of the cursor.

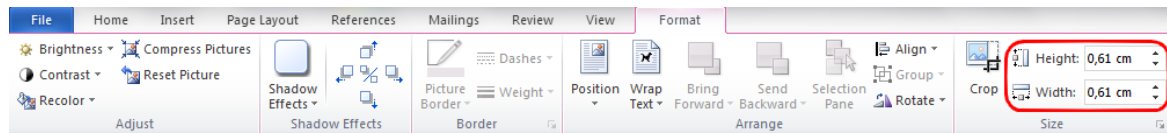
Hint: You can use the key combination *Ctrl – V* instead.



Reducing the size of pictures in MS Word

If necessary you can reduce the size of the picture in MS Word:

- Double click the inserted picture.
- Change the height/width of the picture using the *Size* group on the right.

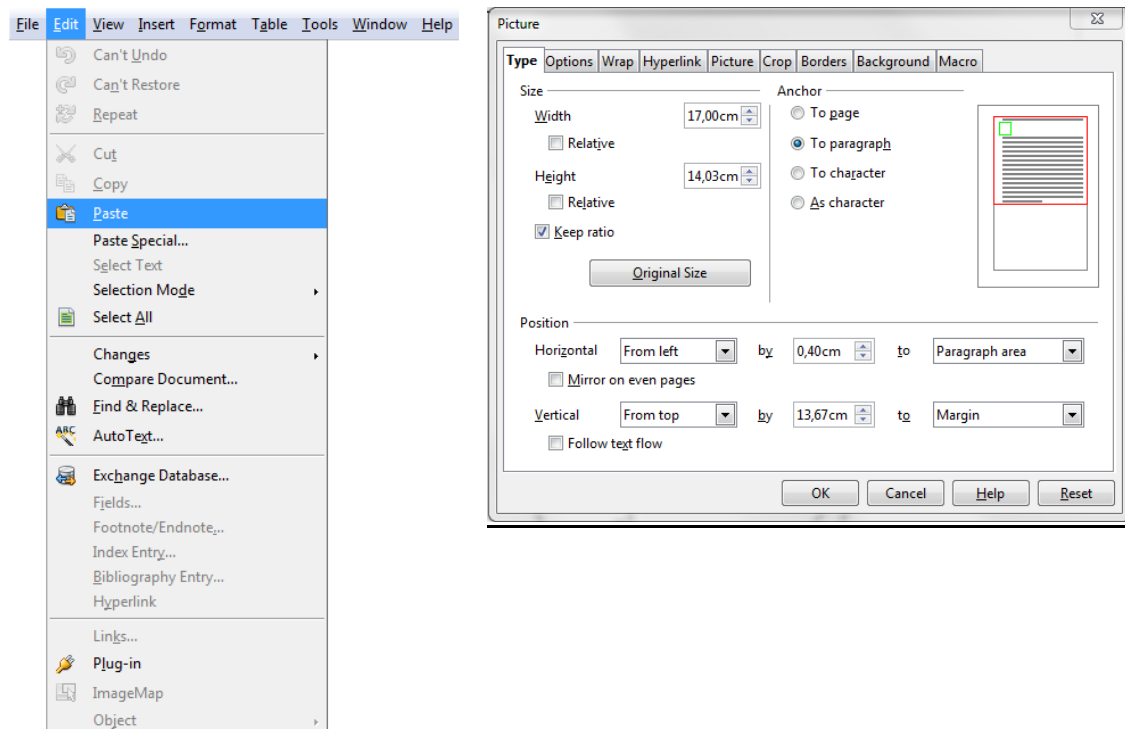


Note: If you change the size of a picture, the scale is modified. If you want to maintain the scale (e.g. for your students to measure lengths) make sure the size of the picture is 100%.

Note: If a picture is too big to fit on one page MS Word will reduce its size automatically and thus, change its scale.

Inserting pictures from the clipboard to OO Writer

- Open a new text processing document
- From the *Edit* menu select *Paste* or use the key combination *Ctrl – V*.



Reducing the size of pictures in OO Writer

- Double click the inserted picture.
- Select the *Type* tab in the appearing *Picture* window.
- Change width/height of the picture.
- Click *OK*.



6. Practice Block II

This practice block consists of a pool of activities to practice using algebraic input, commands, and functions in GeoGebra. Activities of two different difficulty levels are available: *Basic tasks* and *Advanced tasks*. You can pick tasks of your interest and work on them either on your own or together with a colleague.

Tips and Tricks







- For each of these activities, open a new GeoGebra file. Check prior to beginning the activity if you need to show or hide the algebra view, input bar and coordinate axes.
- You might want to save your files before you start a new activity.
- Don't forget about the *Undo* and *Redo* buttons in case you make a mistake.
- Frequently use the *Move* tool in order to check your construction (e.g. are objects really connected, did you create any unnecessary objects).
- Make sure you know the syntax to enter algebraic expressions and functions. Read through the *Tips and Tricks* section in the beginning of topic *Basic Algebraic Input, Commands, and Functions* if you have troubles or ask colleagues for help.
- Carefully check algebraic input before hitting the *Enter* key. If an error message appears, read it! It might help you to fix the problem.
- Make sure, you know how to use corresponding geometry tools before you begin the construction. If you don't know how to operate a certain tool, activate it and read the toolbar help.
- If you have questions, please ask a colleague before you address the presenter or assistant(s).



Activity II.a: Parameters of a Linear Equation

Classification: Basic task

In this activity you are going to use the following tools, algebraic input, and commands. Make sure you know how to use them before you begin with the actual construction.

 Slider $\text{line: } y = m x + b$	 Intersect Two Objects
 Segment Between Two Points $\text{Intersect}[\text{line}, \text{yAxis}]$	 Slope New!
	 Show / Hide Object
	 Move

Hint: You might want to have a look at the file [A_2a_parameters_line.html](#) first.

Construction Steps

1. Enter: $\text{line: } y = 0.8 x + 3.2$

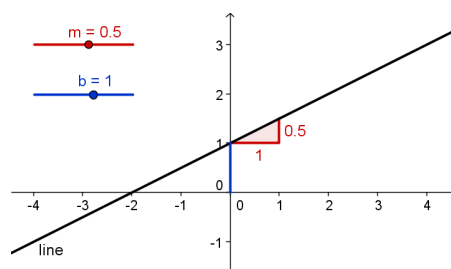
Task 1: Move the line in the algebra view using the arrow keys. Which parameter are you able to change in this way?

Task 2: Move line in the graphics view with the mouse. Which transformation can you apply to the line in this way?



2. Delete the line. Create sliders m and b using the default settings of sliders.
3. Enter $\text{line: } y = m x + b$.

Hint: Don't forget to use an asterisk or space to indicate multiplication!

4. Task 3: Write down directions for your students that guide them through examining the influence of the equation's parameters on the line by using the sliders. These directions could be provided on paper along with the GeoGebra file.



Challenge: Enhance this construction by showing the slope and y-intercept.

5. Create the intersection point between the line and the y-axis.
Hint: Use tool  *Intersect* or command $\text{Intersect}[\text{line}, \text{yAxis}]$.
6. Create a point at the origin and draw a segment between these two points.
7. Use tool  *Slope* and create the slope (triangle) of the line.
8. Hide unnecessary objects and modify the appearance of the other ones.



Activity II.b: Introducing Derivatives – The Slope Function

Classification: Advanced task

In this activity you are going to use the following tools, algebraic input, and commands. Make sure you know how to use them before you begin with the actual construction.

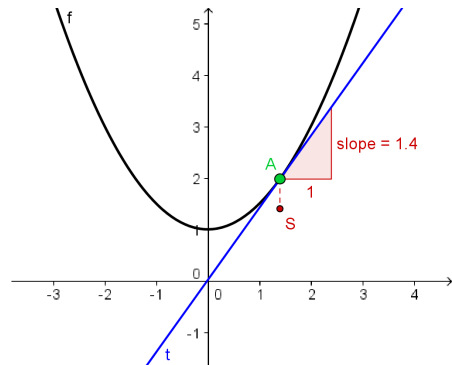
	$f(x) = x^2/2 + 1$
	New Point
	Tangent
	$\text{slope} = \text{Slope}[t]$

	$S = (x(A), \text{slope})$
	Segment Between Two Points
	Move

Hint: You might want to have a look at the file [A 2b slope function.html](#) first.

Construction Steps

1. Enter the polynomial: $f(x) = x^2/2 + 1$
2. Create new point A on function f .
Hint: Move point A to check if it is really restricted to the function graph.
3. Create tangent t to function f through point A .
4. Create the slope of tangent t using:
 $\text{slope} = \text{Slope}[t]$
5. Define point S : $S = (x(A), \text{slope})$
Hint: $x(A)$ gives you the x -coordinate of point A .
6. Connect points A and S using a segment.
7. Task: Move point A along the function graph and make a conjecture about the shape of its path, which corresponds to the slope function.



Challenge: Find the equation of the slope function.

8. Turn on the trace of point S . Move point A to check your conjecture.
Hint: Right click point S (MacOS: *Ctrl*-click) and select Trace on.
9. Find the equation of the resulting slope function. Enter the function and move point A . If it is correct the trace of point S will match the graph.
10. Change the equation of the initial polynomial f to produce a new problem.



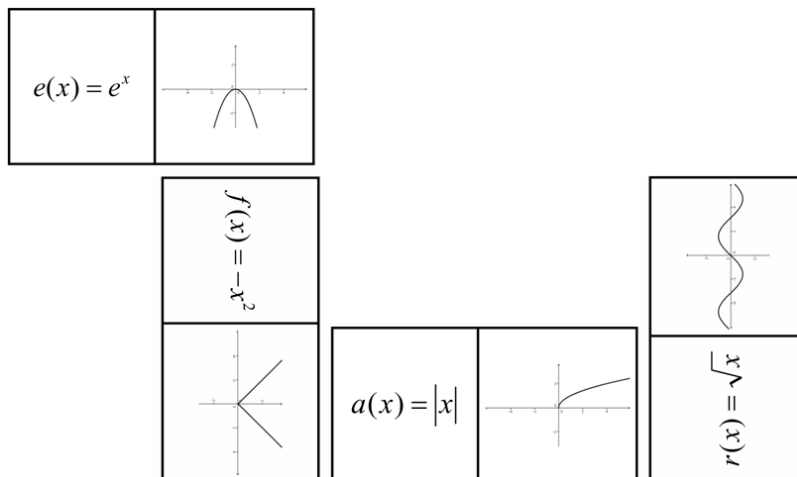
Activity II.c: Creating a 'Function Domino' Game

Classification: Basic task

In this activity you are going to practice exporting function graphs to the clipboard and inserting them into a word processing document in order to create cards for a 'Function Domino' game. Make sure you know how to enter different types of functions before you begin with this activity.

Construction Steps

1. Enter an arbitrary function. Example: $e(x) = \exp(x)$
2. Move the function graph into the upper left corner of the graphics view and adjust the size of the GeoGebra window.
3. Export the graphics view to the clipboard (menu *File – Export – Graphics View to Clipboard*).
4. Open a new word processing document.
5. Create a table (menu *Insert – Table...*) with two columns and several rows.
6. Place the cursor in one of the table cells. Insert the function graph from the clipboard (menu *Home – Paste* or key combination *Ctrl – V*).
7. Adjust the size of the picture if necessary (double click the picture to open the *Format* tab and click on *Size*).
8. Enter the equation of a different function into the cell next to the picture.
Hint: You might want to use an equation editor.
9. Repeat steps 1 through 8 with a different function (e.g. trigonometric, logarithmic).
Hint: Make sure to put the equation and graph of each function on different domino cards.





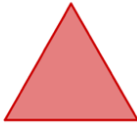
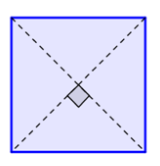
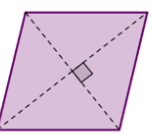

Activity II.d: Creating a ‘Geometric Figures Memory’ Game

Classification: Advanced task

In this activity you are going to practice exporting function graphs to the clipboard and inserting them into a word processing document in order to create cards for a memory game with geometric figures. Make sure you know how to construct different geometric figures (e.g. quadrilaterals, triangles) before you begin with this activity.

Construction Steps

1. Create a geometric figure in GeoGebra (e.g. isosceles triangle).
2. Use the *Properties dialog* to enhance your construction.
3. Move the figure into the upper left corner of the graphics view and adjust the size of the GeoGebra window.
4. Export the graphics view to the clipboard (menu *File – Export – Graphics View to Clipboard*).
5. Open a new word processing document.
6. Create a table (*Insert – Table...*) with three columns and several rows.
7. Set the height of the rows and the width of the columns to 5 cm (2 inches).
Hint: Place the cursor in the table and open the *Table Properties* dialog with a right click. On tab *Row* specify the row height. On tab *Column* enter the preferred width. On tab *Cell* set the vertical alignment to *Center*. Click *OK* when you are done.
8. Place the cursor in one of the table cells. Insert the picture from the clipboard (menu *File – Paste* or key combination *Ctrl – V*).
9. Adjust the size of the picture if necessary (double click the picture to open the *Format Picture* tab, click on *Size* and specify the size).
10. Enter the name of the geometric shape into another cell of the table.
11. Repeat steps 1 through 10 with different geometric figures (e.g. circle, quadrilaterals, triangles).

EQUILATERAL TRIANGLE		SQUARE	
PARALLELOGRAM		CIRCLE	



7. Inserting Pictures into the Graphics View

Activity 13: Drawing Tool for Symmetric Figures


Back to School...

Open the dynamic worksheet [A13 drawing tool symmetry.html](#). Follow the directions on the worksheet and experience how your students could explore the axes of symmetry of a flower.

Hint: You will learn how to create such dynamic worksheets later in this workshop.

Axes of Symmetry

Below you can see a point **A** who was reflected at the line in order to create its image **A'**.



1. Drag point **A** with the mouse along the outline of the flower. What do you notice? Write down your **observations**.
2. How many **axes of symmetry** does this flower have?
Hint: Drag the **green points** in order to **change the position of the line** of reflection. Then, repeat step (1) for every position of the line.
Hint: Press the keys **Ctrl + F** at the same time in order to **delete the traces**.
3. Make a **sketch** of this worksheet including the flower and all lines of symmetry you were able to find.





Discussion

- How could your students benefit from this prepared construction?
- Which tools were used in order to create the dynamic figure?



Preparations

- Open a new GeoGebra window.
- Hide algebra view, input bar and coordinate axes (*View* menu).

Construction Steps

1		New point A
2	A A	Show the label of point A
3		Line of reflection through two points
4		Mirror point at line to get image A'
5		Segment between point A and its image A'



6		Turn the <i>Trace on</i> for points A and A' <u>Hint</u> : Right click (MacOS: <i>Ctrl</i> – click) the point and select <i>Trace on</i> from the menu. Whenever point A is moved it leaves a trace in the graphics view.
7		Move point A to draw a dynamic figure



Discussion

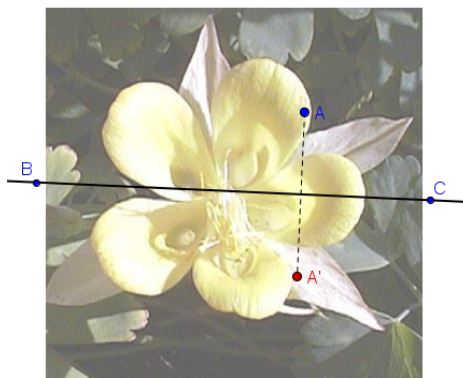
The  *Trace on* feature has some special characteristics:

- The trace is a temporary phenomenon. Whenever the graphics are refreshed, the trace disappears.
- The trace can't be saved and is not shown in the algebra view.
- To delete the trace you need to refresh the views (menu *View – Refresh Views* or key combination *Ctrl – F*. MacOS: *Open Apple-F*).

Enhancing the Construction

Note: Make sure you have the picture [A13 flower.jpg](#) saved on your computer.

8		Insert image into the graphics view <u>Hint</u> : Click in the lower left corner of the graphics view to insert the picture at this position.
9		Adjust the position of the inserted image.
10		Set the image as <i>background image</i> (<i>Properties dialog</i> , tab <i>Basic</i>).
11		Reduce the <i>filling</i> of the image (<i>Properties dialog</i> , tab <i>Style</i>). <u>Hint</u> : After specifying the picture as a background image you need to open the <i>Properties dialog</i> using the <i>Edit</i> menu. You can't select a background image in the graphics view any more.








Activity 14a: Resizing and Reflecting a Picture

In this activity you will learn how to resize an inserted picture to a certain size and how to apply transformations to the picture in GeoGebra.

Preparations

- Make sure you have the picture [A14 Sunset Palmtrees.jpg](#) saved on your computer.
- Open a new GeoGebra window.
- Close the algebra view and hide the coordinate axes.

Construction Steps

1		Insert picture A14 Sunset Palmtrees.jpg on the left part of the graphics view
2		New point A at the lower left corner of the picture
3		Set point A as the first corner point of your picture. <u>Hint:</u> Open the <i>Properties dialog</i> and select the picture in the list of objects. Click on tab 'Position' and select point A from the drop-down list next to <i>Corner 1</i> .
4		$B = A + (3, 0)$
5		Set point B as the second corner point of the picture. <u>Hint:</u> You just changed the width of the picture to 3 cm.
6		Vertical line through two points in the middle of the graphics view
7		Mirror the picture at the line <u>Hint:</u> You might want to reduce the filling of the image in order to be able to better distinguish it from the original.

Tasks

- Move point A with the mouse. How does this affect the picture?
- Move the picture with the mouse and observe how this affects its image.
- Move the line of reflection by dragging the two points with the mouse. How does this affect the image?








Activity 14b: Distorting a Picture

In this activity you will learn how to resize an inserted picture to an arbitrary size and how to distort a picture in GeoGebra.

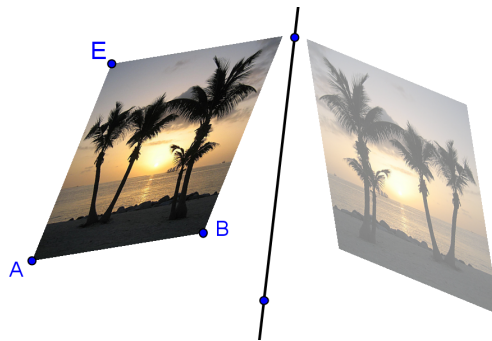
You will now modify the construction created in activity 14a. If you want to keep the original as well, you need to save your file.

Construction Steps

1		Start with the figure you created in activity 14a.
2		Delete the point B to restore the picture's original size
3		Create a new point B at the lower right corner of the original picture
4		Set the new point B as the second corner point of your picture. <u>Hint</u> : You can now resize the image by moving point B .
5		Create a new point E at the upper left corner of the original picture
6		Set the new point E as the fourth corner point of your picture

Tasks

- How does moving point E affect the picture and its image?
- Which geometric shape do the picture and the image form at any time?





Activity 14c: Exploring Properties of Reflection

In this activity you will create a dynamic figure that allows your students to explore the properties of reflection.

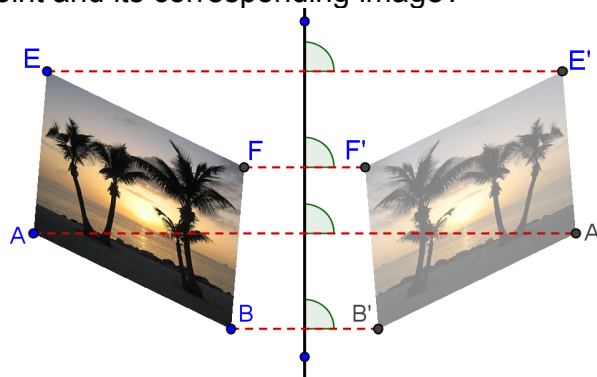
You will now modify the construction created in activity 14b. If you want to keep the original as well you need to save your file.

Construction Steps

1		Start with the figure you created in activity 14b.
2		Segment between points A and B
3		Segment between points A and E
4		Parallel line to segment AB through point E
5		Parallel line to segment AE through point B
6		Intersect the two lines to get intersection point F
7		Hide auxiliary objects
8		Reflect all four corner points at the line to get their images
9		Connect corresponding points with segments (e.g. points A and A')
10		Create angles between the line of reflection and the segments

Task

Move the corner points of the original picture, as well as the line of reflection. What do you notice about the angles between the segments and the line of reflection? What can we call the line of reflection in relation to the segments formed by each point and its corresponding image?





8. Inserting Text into the Graphics View

Activity 15: Coordinates of Reflected Points

Preparations

- Open a new GeoGebra window.
- Show the algebra view, input bar, coordinate axes and the grid (*View* menu).
- In the *Options* menu set the *Point capturing* to *Fixed to Grid*.

1		Create point $A = (3, 1)$
2		Create line $a: y = 0$
3		Mirror point A at line a to get point A'
4		Change the color of line a and point A' .
5		Create line $b: x = 0$
6		Mirror point A at line b to get point A_1'
7		Change the color of line b and point A_1' .

Inserting static text

Insert a heading into the graphics view of GeoGebra so your students know what this dynamic figure is about:

- Activate the ^{ABC} *Text* tool and click on the upper part of the graphics view.
- Type the following text into the appearing window:
Reflecting a point at the coordinate axes
- Click *OK*.
- Adjust the position of the text using the *Move* tool.

Hints: You can change the properties of the text in the *Properties dialog* (e.g. wording, font style, font size, formatting). On tab *Basic* you can fix the position of the text so it can't be moved accidentally any more.

Inserting dynamic text

Dynamic text refers to existing objects and adapts automatically to modifications, for example $A = (3, 1)$.

- Activate the ^{ABC} *Text* tool and click on the graphics view.

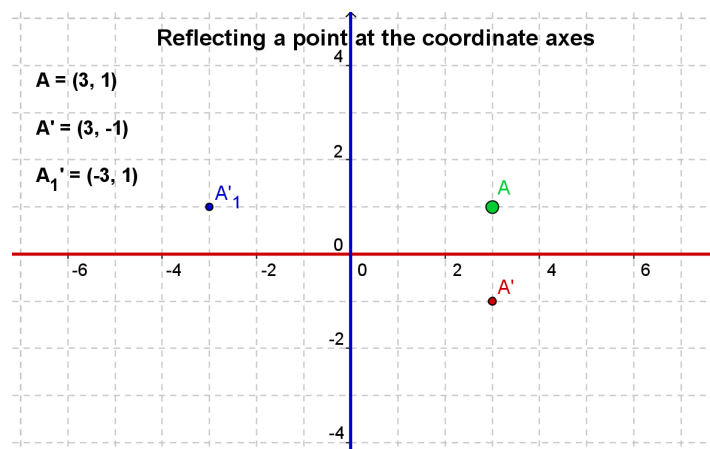


- Type $A =$ into the appearing window:
Hint: This will be the static part of the text and won't change if point A is moved.
- Insert the dynamic part of this text by selecting point A from the *Objects* drop-down list.
- Click *OK*.

Note: The text shows the coordinates of point A and adapts automatically to modifications of its position.

Enhancing the dynamic figure

- Insert dynamic text that shows the coordinates of the reflected points A' and A_1' .
- Zoom out in order to show a larger part of the coordinate plane.
Hint: You might want to adjust the distance of the grid lines.
 - Open the *Properties dialog for the Graphics View* (right click / MacOS: Ctrl-click the graphics view and select *Graphics*)
 - Select tab *Grid*
 - Check the box next to *Distance* and change the values in both text fields to 1
- Close the algebra view and fix all text so it can't be moved accidentally.



Task

Come up with instructions to guide your students towards discovering the relation between the coordinates of the original and the reflected points which could be provided along with the dynamic figure.



Activity 16: Rotation of a Polygon

Preparations

- Open a new GeoGebra window.
- Hide algebra view and input bar if necessary.
- Show the coordinate axes and the grid.
- Open the *Properties dialog for the graphics view*:
 - On tab *xAxis* change the *Distance* to 1
 - On tab *yAxis* change the *Distance* to 1


Construction Steps

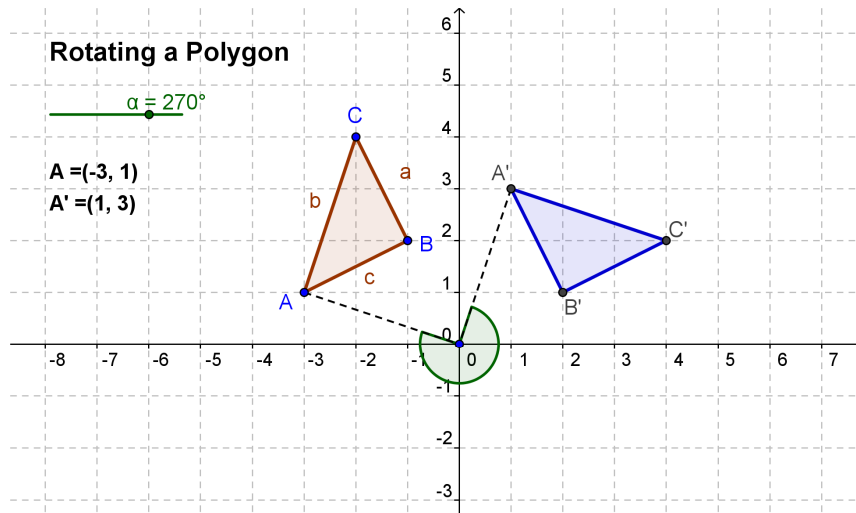
1		Create an arbitrary triangle <i>ABC</i>
2		New point <i>D</i> at the origin of the coordinate system
3		Rename the new point to <i>O</i> <u>Hint</u> : GeoGebra offers a ‘fast-renaming’ option. Activate <i>Move</i> mode and select the object. When you start typing the new name GeoGebra opens the <i>Rename</i> dialog window.
4		Create a slider for angle α <u>Hint</u> : In the slider dialog window check <i>Angle</i> and set the increment to 90° .
5		Rotate triangle <i>ABC</i> around point <i>O</i> by angle α <u>Hint</u> : Check <i>counter clockwise</i> rotation.
6		Segments <i>AO</i> and <i>A'O</i>
7		Angle <i>AOA'</i> <u>Hint</u> : Select the points in counter clockwise order. Hide the label of this angle.
8		Move slider and check the image of the triangle

Enhancing the Construction

1	ABC	Insert static text: <i>Rotation of a Polygon</i>
2	ABC	Insert dynamic text: $A =$ and select Point <i>A</i> from <i>Objects</i>
3	ABC	Insert dynamic text: $A' =$ and select Point <i>A'</i> from <i>Objects</i>



4		Move slider and text to desired positions
5		Fix the position of the slider (<i>Properties dialog – tab Slider</i>)
6		Fix the position of the text (<i>Properties dialog – tab Basic</i>)



Discussion

How could you use this file to introduce the concept of rotating an object around the origin of a coordinate system to your students?



9. Practice Block III

This practice block consists of a pool of activities to practice inserting pictures and text into the graphics view of GeoGebra. Activities of two different difficulty levels are available: *Basic tasks* and *Advanced tasks*. You can pick tasks of your interest and work on them either on your own or together with a colleague.

Tips and Tricks

- For each of these activities, open a new GeoGebra file. Check prior to beginning the activity if you need to show or hide the algebra view, input bar and coordinate axes.
- If you choose to work on an activity that involves inserting a picture into GeoGebra, make sure you have these picture files saved on your computer before you begin with the activity.
- You might want to save your files before you start a new activity.
- Don't forget about the *Undo* and *Redo* buttons in case you make a mistake.
- Frequently use the *Move* tool in order to check your construction (e.g. are objects really connected, did you create any unnecessary objects).
- Make sure you know the syntax to enter algebraic expressions and functions. Read through the *Tips and Tricks* section in the beginning of topic *Basic Algebraic Input, Commands, and Functions* if you have troubles or ask colleagues for help.
- Carefully check algebraic input before hitting the *Enter* key. If an error message appears, read it! It might help you to fix the problem.
- Make sure, you know how to use corresponding geometry tools before you begin the construction. If you don't know how to operate a certain tool, activate it and read the toolbar help.
- If you have questions, please ask a colleague before you address the presenter or assistant(s).




Activity III.a: Visualizing a System of Equations

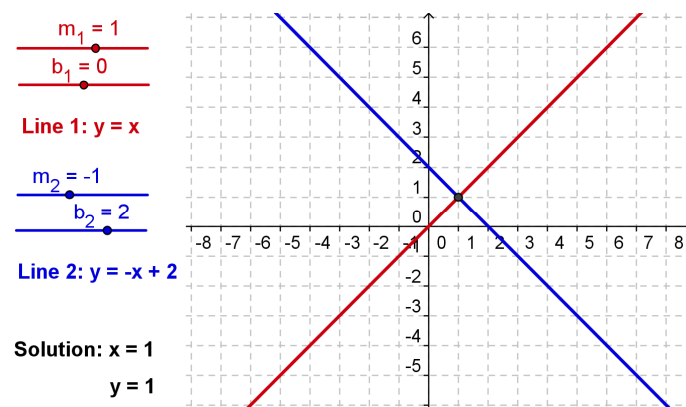
Classification: Basic task

In this activity you are going to use algebraic input and commands. Make sure you know the syntax for algebraic input and commands before you begin.

You might want look at the dynamic worksheet called [A 3a system equations.html](#) in order to find out how your students could use this construction to graphically solve a system of linear equations.

Construction Steps

1. Create sliders m_1 and b_1 using the default settings for sliders.
2. Create the linear equation l_1 : $y = m_1 x + b_1$.
3. Create sliders m_2 and b_2 using the default settings for sliders.
4. Create the linear equation l_2 : $y = m_2 x + b_2$.
5. Create dynamic *text1*: Line 1: and select l_1 from *Objects*.
6. Create dynamic *text2*: Line 2: and select l_2 from *Objects*.
7. Construct the intersection point A of both lines either using tool  *Intersect two objects* or command $A = \text{Intersect}[l_1, l_2]$.
8. Define $xcoordinate = x(A)$.
Hint: $x(A)$ gives you the x -coordinate of point A .
9. Define $ycoordinate = y(A)$.
Hint: $y(A)$ gives you the y -coordinate of point A .
10. Create dynamic *text3*: Solution: $x =$ and select $xcoordinate$ from *Objects*. Type in $y =$ and select $ycoordinate$ from *Objects*.



Challenge: Create a similar construction that allows for visualizing the graphical solution of a system of quadratic polynomials.

Hint: Functions need to be entered using the syntax $f(x) = \dots$






Note: Such a dynamic figure can also be used to visualize an equation in one variable by entering each side of the equation as one of the two functions.





Activity III.b: Translating Pictures

Classification: Basic task

In this activity you are going to use the following tools and commands. Make sure you know how to use each tool and command before you begin.

	Insert Image $A = (1, 1)$		Vector Between Two Points	New!
	Polygon $\text{Vector}[O, P]$		Translate Object By Vector	New!
			Move	
		ABC	Text	

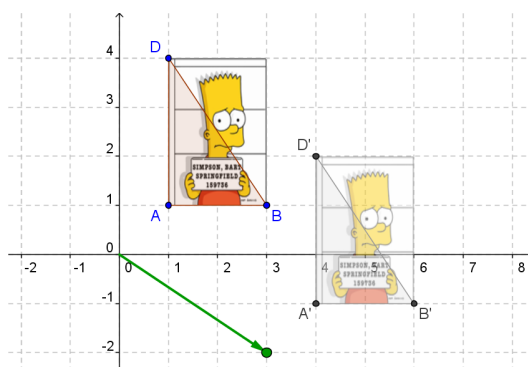
Construction Steps

1. Open a new GeoGebra window. Show the algebra view, input bar, coordinate axes, and grid. In the *Options* menu set the point capturing to *Fixed to Grid*.
2. Insert picture [A 3b Bart.png](#) into the first quadrant.
3. Create points $A = (1, 1)$, $B = (3, 1)$, and $D = (1, 4)$.
4. Set point A as the first, B as the second, and D as the fourth corner point of the picture (*Properties dialog*, tab *Position*).
5. Create triangle ABD .
6. Create point $O = (0, 0)$ and point $P = (3, -2)$.
7. Create vector $u = \text{Vector}[O, P]$.
Hint: You could also use tool  *Vector Between Two Points*.
8. Translate the picture by vector u using tool  *Translate Object By Vector*.
Hint: You might want to reduce the filling of the image.
9. Translate the three corner points A , B , and D by vector u .
10. Create triangle $A'B'D'$.
11. Hide point O so it can't be moved accidentally. Change the color and size of objects to enhance your construction.

Challenge

Insert dynamic text that shows

- the coordinates of points A , B , C , A' , B' , and D' .
- the coordinates of vector u .





Activity III.c: Constructing a Slope Triangle

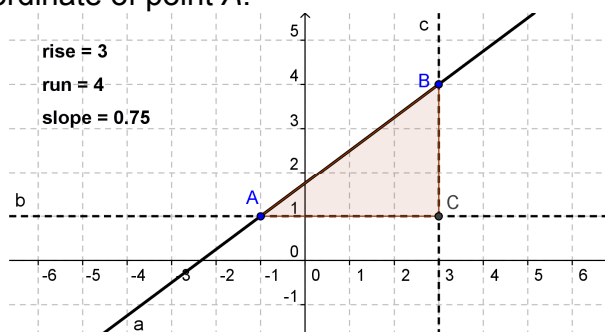
Classification: Advanced task

In this activity you are going to use the following tools and algebraic input. Make sure you know how to use each tool and the syntax for algebraic input before you begin.

	Line Through Two Points	$run = x(B) - x(A)$
	Perpendicular Line	$slope = rise / run$
	Intersect Two Objects	ABC Text
	Polygon	Midpoint or Center
	$rise = y(B) - y(A)$	Move

Construction Steps

1. Show the algebra view, coordinate axes and the grid. Set point capturing to *Fixed to Grid* and the labeling to *All new objects*.
2. Create line *a* through two points *A* and *B*.
3. Construct a perpendicular line *b* to the *y*-axis through point *A*.
4. Construct a perpendicular line *c* to the *x*-axis through point *B*.
5. Intersect perpendicular lines *b* and *c* to get intersection point *C*.
Hint: You might want to hide the perpendicular lines.
6. Create polygon *ACB* and hide the labels of the sides.
7. Calculate the rise: $rise = y(B) - y(A)$
Hint: $y(A)$ gives you the *y*-coordinate of point *A*.
8. Calculate the run:
 $run = x(B) - x(A)$
Hint: $x(B)$ gives you the *x*-coordinate of point *B*.
9. Enter the following equation into the input bar to calculate the slope of line *a*:
 $slope = rise / run$
10. Insert dynamic text:
 $rise =$ and select *rise* from *Objects*
 $run =$ and select *run* from *Objects*
 $slope =$ and select *slope* from *Objects*
11. Change properties of objects in order to enhance your construction.





Challenge 1: Insert a dynamic text that contains a fraction

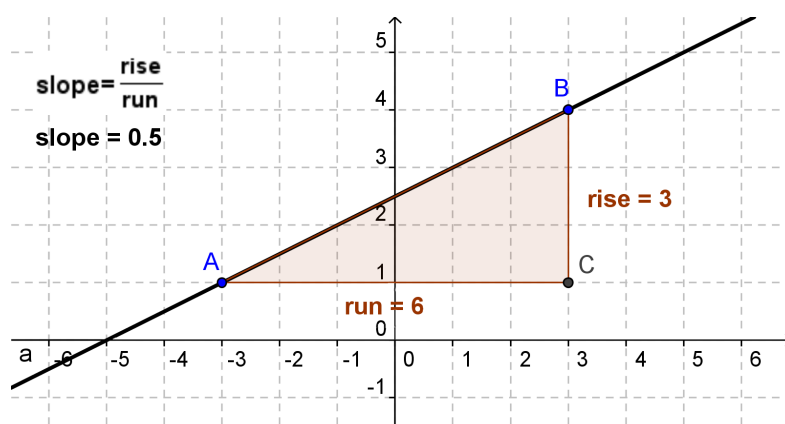
Using LaTeX formulas, text can be enhanced to display fractions, square roots, or other mathematical symbols.

1. Activate tool ^{ABC} *Insert text* and click on the graphics view.
2. Type $\text{slope} =$ into the *Insert text* window's input bar.
3. Check *LaTeX formula* and select *Roots and Fractions a/b* from the drop-down list.
4. Place the cursor within the first set of curly braces and replace *a* by number *rise* from the *Objects* drop-down list.
5. Place the cursor within the second set of curly braces and replace *b* by number *run* from the *Objects* drop-down list.
6. Click *OK*.

Challenge 2: Attach text to an object

Whenever an object changes its position, attached text adapts to the movement and follows along.

1. Create midpoint *D* of the vertical segment using tool \cdot *Midpoint or center*.
2. Create midpoint *E* of the horizontal segment.
3. Open the *Properties dialog* and select *text1 (rise = ...)*. Click on tab *Position* and select point *D* from the drop-down list next to *Starting point*.
4. Select *text2 (run = ...)* in the *Properties dialog* and set point *E* as starting point.
5. Hide the midpoints *D* and *E*.

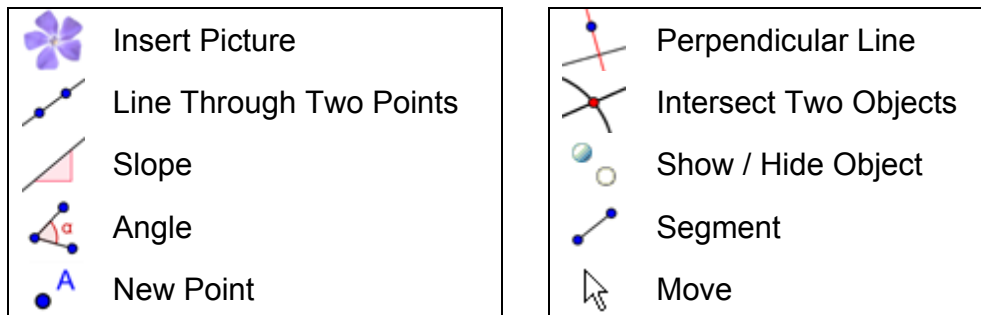




Activity III.d: Exploring the Louvre Pyramid

Classification: Advanced task

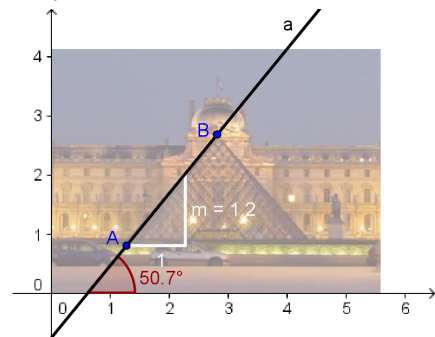
In this activity you are going to use the following tools and some algebraic input. Make sure you know how to use every single tool and the syntax for algebraic input before you begin. Also, check if you have the picture [A 3d Louvre.jpg](#) saved on your computer.



The *Louvre* in Paris is one of the most visited and famous art museums in the world. The building holds some of the world's most famous works of art, such as Leonardo da Vinci's *Mona Lisa*. In 1989 the main entrance of the museum was renovated and a glass pyramid was built (from <http://en.wikipedia.org/wiki/Louvre>, February 20, 2008).

Determine the slope of the pyramid's faces

1. Set point capturing *off*. Set the decimal places to 1. Change the labeling setting to *All new objects* (menu *Options*).
2. Insert the picture [A 3d Louvre.jpg](#) into the first quadrant of the coordinate system. The left lower corner should match the origin.
3. Reduce the filling of the picture (about 50%) and set it as background image (*Properties dialog*).
4. Create a line through two points with the first point at the base and the second point at the vertex of the pyramid.
Hint: Change the properties of line to improve its visibility.
5. Use the *Slope* tool to get slope triangle of line.
Hint: Change the properties of slope triangle to improve its visibility.
Hint: The slope triangle is attached to the point created first.
6. Task: Determine the slope of the pyramid's faces in percent.
7. Create the angle between the x-axis and the line.
Task: Determine the inclination angle for the pyramid's face.





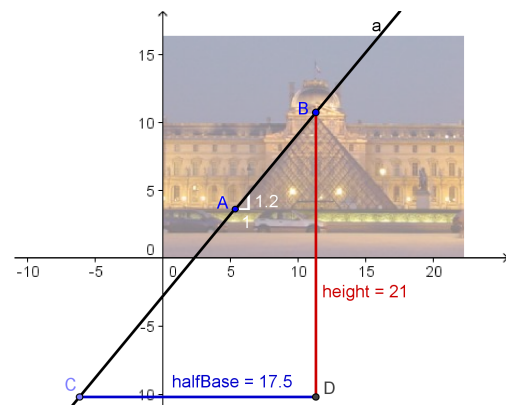
Challenge

The pyramid's base is a square with a side length of 35 meters. Determine the height of the pyramid using similar triangles.

1. Create a new point C on the line.
2. Construct the slope triangle of the line using points C and B at the pyramid's vertex.
Hint: Create a line perpendicular to the y -axis through point C and a line perpendicular to the x -axis through point B at vertex of pyramid.
 Create the intersection point D of the two lines.
Hint: Hide the auxiliary lines.
3. Use segments to connect point D with points B and C .
Hint: Change the properties of the segments to increase their visibility.
Hint: You might want to rename the vertical segment to *height* and the horizontal segment to *halfBase*.
4. Move point C along the line until the horizontal segment of the triangle matches the level of the road in front of the pyramid.
5. Task: Calculate the height of the pyramid using similar triangles.
Hint: Use the slope triangle and your new triangle. Remember that the base side length is 35 m.

Check your answer with GeoGebra

6. Show name and values of segments *height* and *halfBase*.
7. Drag point C until the horizontal segment has length $35/2 = 17.5$.
Hint: You might need to zoom out of the construction and / or move the graphics view in order to be able to do this.
8. Check if the height of the pyramid matches your answer.



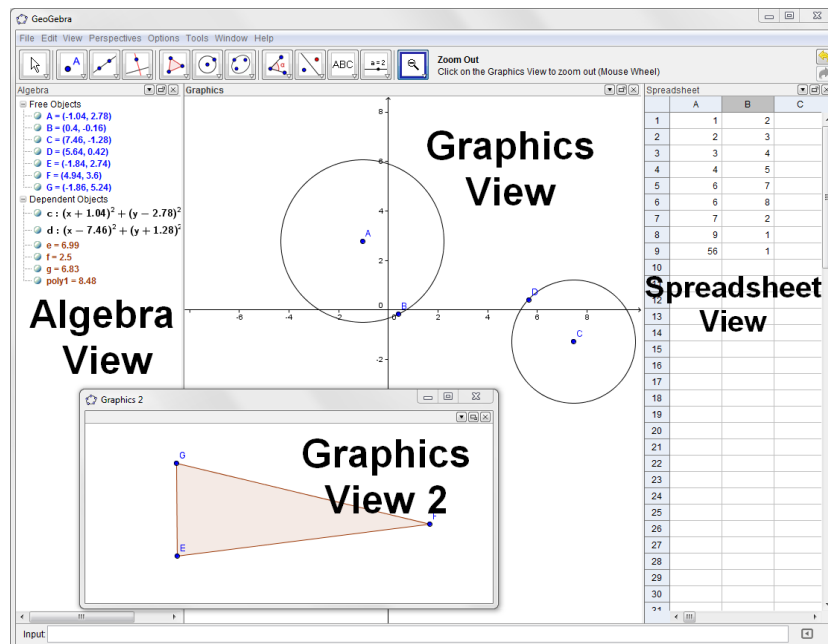
Comment

By implementing the instructions above you were able to graphically determine the approximate value for the pyramid's height. In reality, the *Louvre* pyramid has a base length of 35 m and a height of 21.65 m. Its faces have a slope of 118% and an inclination angle of about 52° (from http://de.wikipedia.org/wiki/Glaspyramide_im_Innenhof_des_Louvre#Daten, February 22, 2008).



10. Combining Spreadsheet View & Graphics View

GeoGebra provides different views of mathematical objects: a *graphics view*, a numeric *algebra view* and a *spreadsheet view*. Thereby, all representations of the same object are linked dynamically and adapt automatically to changes made to any of the representations, no matter how they were initially created.



Spreadsheet Cells Input

In GeoGebra's *spreadsheet view* every cell has a specific name that allows you to directly address each cell. For example, the cell in column A and row 1 is named *A1*.

Note: These cell names can be used in expressions and commands in order to address the content of the corresponding cell.

You can not only use numbers in spreadsheet cells, but all types of mathematical objects that are supported by GeoGebra (e.g., coordinates of points, functions, lines). If possible, GeoGebra immediately displays the graphical representation of the object you enter into a spreadsheet cell in the graphics view as well. Thereby, the name of the object matches the name of the spreadsheet cell used to initially create it (e.g., *A5*, *C1*).

Note: By default, spreadsheet objects are classified as auxiliary objects in the algebra view. You can show or hide these auxiliary objects by selecting 'Auxiliary Objects' from the styling bar at the top of the algebra view.



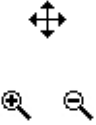



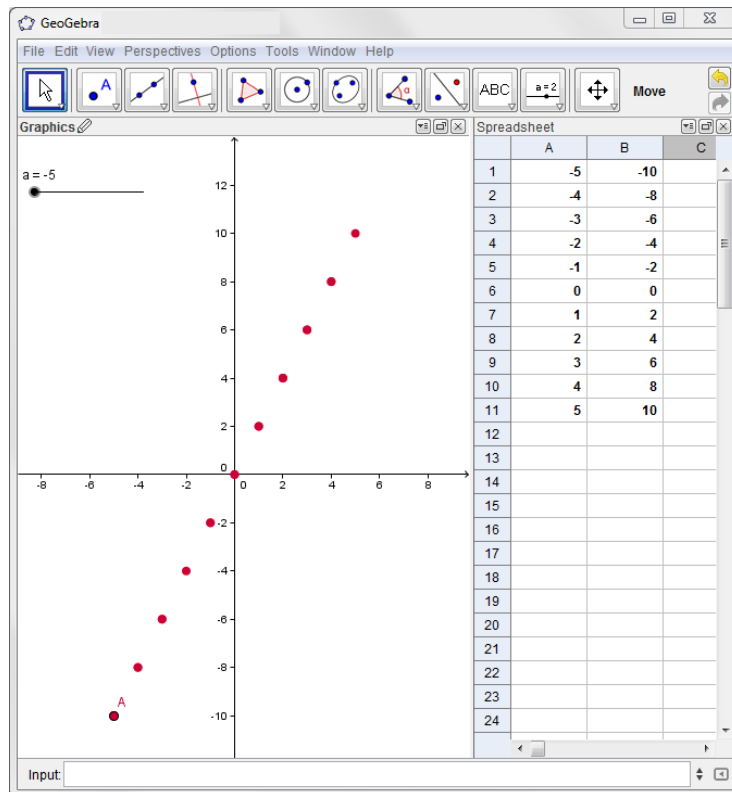
Activity 17a: Record to Spreadsheet

Preparations

- Open a new GeoGebra window.
- Go to *Perspectives – Spreadsheet & Graphics*.

Construction Steps

1		Create a slider a with default interval and increment 1. <u>Hint:</u> Select tool <i>Slider</i> and click in the <i>graphics view</i> to set the position for the slider. In the appearing dialog window change the increment to 1 and click the <i>Apply</i> button.
2	$A = (a, 2a)$	Create point A by entering $A = (a, 2a)$ into the Input Bar. <u>Hint:</u> The value of slider a determines the x-coordinate of point A while the y-coordinate is a multiple of this value.
3	AA	Show the label of point A in the graphics view.
4		Change the value of slider a to examine different positions of point A .
5		Use tools <i>Move Graphics View</i> , as well as <i>Zoom In</i> and <i>Zoom Out</i> to adjust the visible part of the graphics view and make point A visible in all positions.
6		Record the coordinates for different positions of point A to the spreadsheet: (1) Select tool <i>Record to Spreadsheet</i> . Then, click on point A in order to highlight it. <u>Note:</u> The coordinates for the actual position of point A are immediately entered into cells $A1$ (x-coordinate) and $B1$ (y-coordinate) of the spreadsheet. (2) Now, change the value of slider a in order to record the coordinates of all other possible positions of point A to the spreadsheet as well. <u>Note:</u> Do not switch to another tool before moving the slider.



Activity 17b: Relative Copy and Linear Equations



Preparations

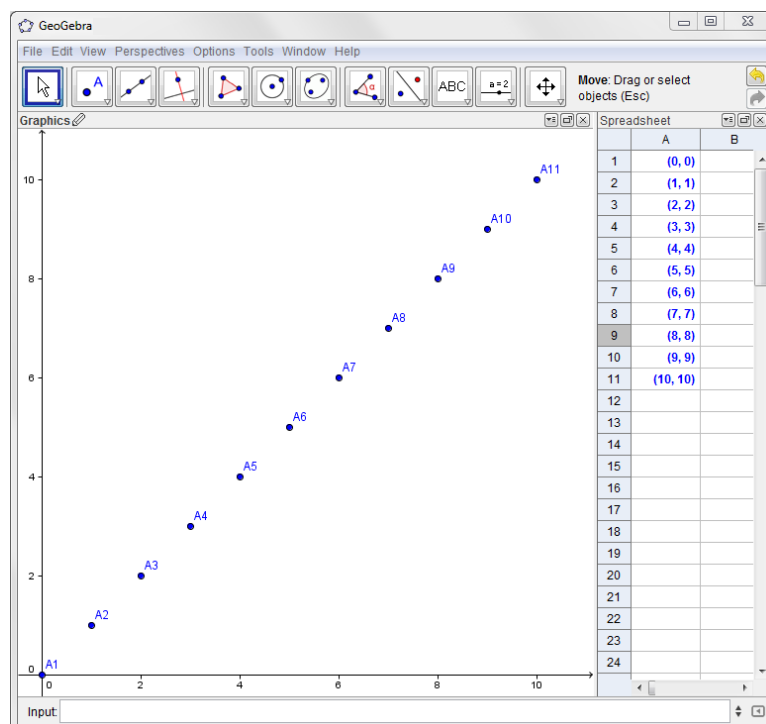
- Open a new GeoGebra window.
- Go to *Perspectives – Spreadsheet & Graphics*.

Construction Steps

1		Activate tool <i>Move Graphics View</i> and drag the origin of the coordinate system close to the lower left corner of the graphics view.
2	$(0, 0)$	In the spreadsheet view, click on cell <i>A1</i> enter the point coordinates $(0, 0)$.
3	$(1, 1)$	In the spreadsheet view, click on cell <i>A2</i> enter the point coordinates $(1, 1)$.
4	AA	Show the labels of both points in the graphics view.



5		<p>Relative copy the inserted point coordinates to other cells in column A:</p> <ol style="list-style-type: none"> (1) Highlight both cells $A1$ and $A2$ by using the mouse. (2) Click on the little square at the lower right corner of the highlighted cell range. (3) Hold the mouse button down and drag the pointer down to cell $A11$.
6		<p>Use tools <i>Move Graphics View</i>, as well as <i>Zoom In</i> and <i>Zoom Out</i> to adjust the visible part of the graphics view and make all points visible.</p>



Task 1: Examine the coordinates of the point sequence

What sequence of numbers is created if you apply the 'relative copy' feature of the GeoGebra spreadsheet the way it is described above?

Hint: Examine the x-coordinates of all created points and come up with a conjecture about how they are related. Then, check your conjecture using the y-coordinates of the points.

Task 2: Find the matching equation

Make a prediction about an equation that would create a graph going through all points of this sequence. Enter this equation into the Input bar in order to check your prediction.




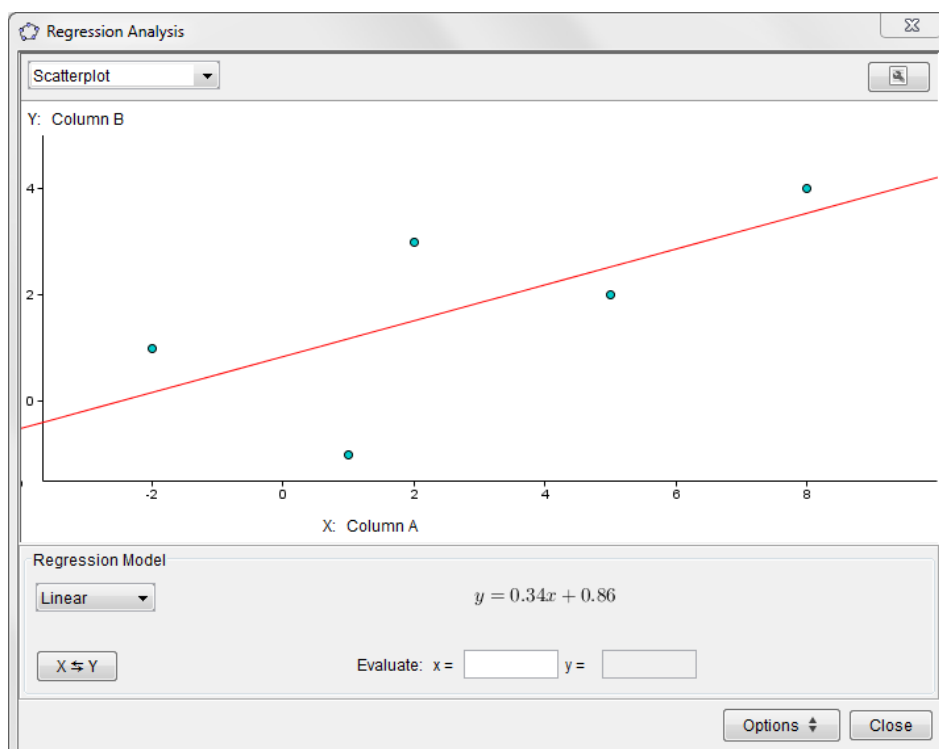
Activity 17c: Best Fit Line

Preparations

- Open a new GeoGebra window.
- Go to *Perspectives – Spreadsheet & Graphics*.
- In the *Options* menu set the *Labeling* to *New Points Only*.

Construction Steps

1		Enter the following numbers into the spreadsheet cells of column A: A1: 1 A2: 5 A3: 2 A4: 8 A5: -2
2		Enter the following numbers into the spreadsheet cells of column B: B1: -1 B2: 2 B3: 3 B4: 4 B5: 1
3		Use tool <i>Two Variable regression Analysis</i> in order to create the function that best fits your data points. Highlight the cells and then click the tool.
4		Try to find the function that best fits your points by selecting different <i>Regression Models</i> .





Task 1: Examine the regression models


Why do some models not work with the points you entered? Enter different points and try the *Two Variable Regression Analysis Tool* again.

Task 2: Polynomial Regression

Select the *Polynomial Regression Model* and observe what happens to the function when you change the order of the polynomial function.

Importing Data from other Spreadsheets

Note: GeoGebra allows you to copy and paste data from other spreadsheet software into the GeoGebra spreadsheet:

- Select and copy the data you want to import (e.g. use the keyboard shortcut *Ctrl-C* in order to copy the data to your computer's clipboard).
Note: You need to press both keys (e.g., *Ctrl*-key and *C*-key) at the same time in order to apply a keyboard shortcut.
- Open a GeoGebra window and show the spreadsheet view.
- Click on the spreadsheet cell that should contain the first data value.
- Paste the data from your computer's clipboard into GeoGebra's spreadsheet view. In order to do so, you can either use the keyboard shortcut *Ctrl-V* or right click (MacOS: *Ctrl*-click) on the highlighted cell and select  *Paste* from the appearing context menu.

Activity 17d: Exploring Basic Statistics

Yesterday, you gave a mathematics quiz to the 25 students of your 1st period math class. After the quiz, you asked your students to rate the difficulty of the quiz on a scale from 1 ('very easy') to 5 ('very difficult').

- 4 of your students rated the quiz 'very easy' (1)
- 6 students rated the quiz 'easy' (2)
- 6 other students rated the quiz 'difficult' (4)
- 1 student rated the quiz 'very difficult' (5)
- The rest of the students thought the difficulty of the quiz was 'ok' (3).

Task 1: Create a histogram

Enter the data into GeoGebra's spreadsheet and create a histogram that visualizes this data.

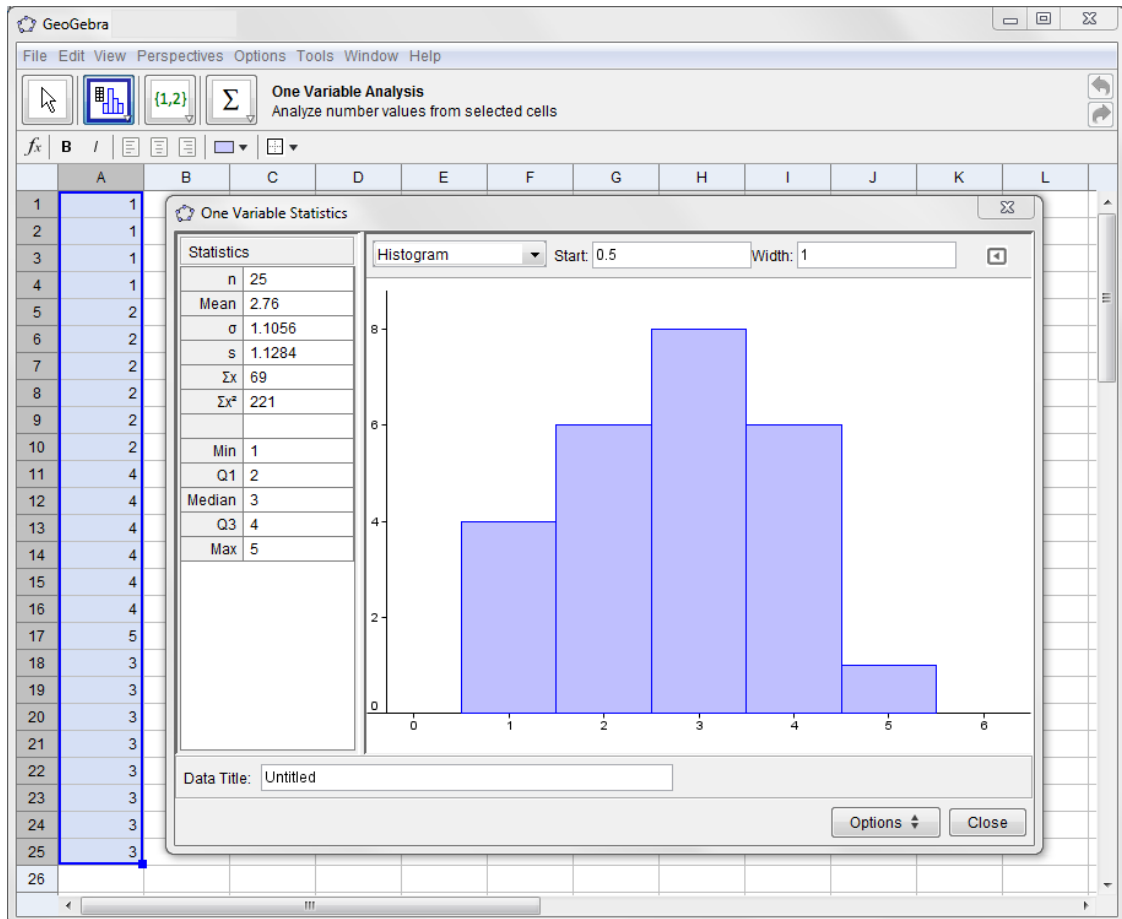
Hints:

- Use the *One Variable Analysis Tool* in order to create a histogram.
- Note: Change the slider *Classes* in the appearing window to control the number of bars that are shown in your histogram.
- Enhance the histogram by setting the classes manually and changing the start and width parameter.



Task 2: Determine mean and median

1. Make a prediction for mean and median of the data you collected.
2. Compare your solution by checking the left table of the *One Variable Statistics* window.





11. Creating Static Instructional Materials

Activity 18a: Saving Pictures as Files

In this activity you will learn how to export a picture from GeoGebra into a file. Compared to exporting it to the clipboard (see activity 12a) this export possibility has several advantages:


- You can save your picture and reuse it later on. This isn't possible with the clipboard since data is just stored temporarily there and can't be reused in a later session.
- You can determine the scale of your picture which comes in handy if you want your students to use the sketch for measurements.
- You can change the resolution of your picture and thus, change the size of the picture file. The resolution determines the quality of the picture: For a good quality printout we recommend a rather high resolution (about 300 dpi). In order to upload the picture to a website you want to have a smaller file and to use the usual resolution of 72 dpi.

Hint: *dpi* means "dots per inch".

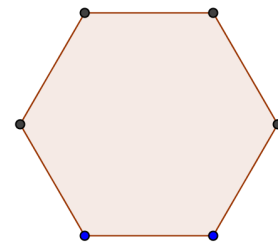
- You can determine the format of your picture. GeoGebra offers several different picture formats. The default setting is *Portable Network Graphics (png)* which is fine for inserting the picture into word processing and presentation software.

Hint: If you don't know the other formats don't worry about them.


Create your figure

Create a simple construction in GeoGebra. For example use the  *Regular polygon* tool in order to create a square, pentagon, or hexagon in the graphics view.

- Activate the tool and click on the graphics view twice in order to determine the side length of the regular polygon.
- Type the number of vertices (e.g. 6 for a hexagon) into the appearing dialog window and click *OK*.



Again, GeoGebra will export the whole graphics view into the picture file. Thus, you need to make the GeoGebra window smaller in order to reduce unnecessary space in the graphics view:

- Move your figure (or the relevant section) to the upper left corner of the graphics view using the  *Move Graphics View* tool (see left figure below).

Hint: You might want to use tools  *Zoom in* and  *Zoom out* in order to prepare your figure for the export process.

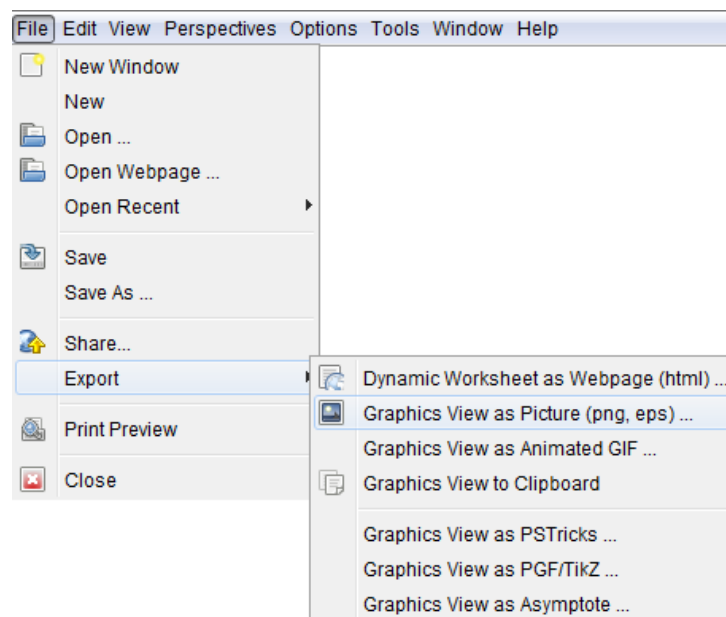


- Reduce the size of the GeoGebra window by dragging its lower right corner with the mouse.
Hint: The pointer will change its shape when hovering above an edge or corner of the GeoGebra window.

Exporting a picture as a file

Use the *File* menu to export the graphics view as a picture:

- *Export – Graphics View as Picture*
Hint: You could also use the key combination *Ctrl – Shift – P*.
- If you want to you can change the settings in the appearing window (picture format, scale, resolution).
Hint: Always check the size of your picture (in cm or dpi). If your picture doesn't fit on a piece of paper you might need to modify your figure prior to the export. This is especially important if you want to keep the scale of the picture for the word processing or presentation document.
- Click *Save* and save your picture in your *GeoGebra_Introduction* folder. Your figure is now ready to be inserted into any word processing or presentation document.



Activity 18b: Inserting Pictures into a Text Processing Document

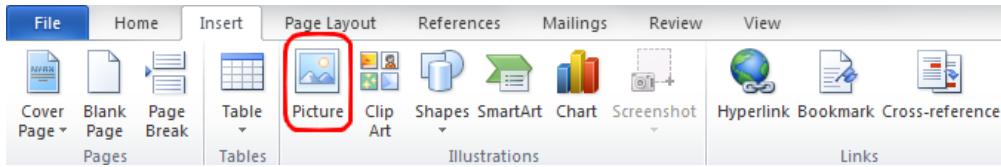
After exporting a figure from GeoGebra into a picture file you can now insert it into a word processing document.

Inserting a picture file into MS Word

- Open a new text processing document
- From the *Insert* tab, in the *Illustrations* group select *Picture*.



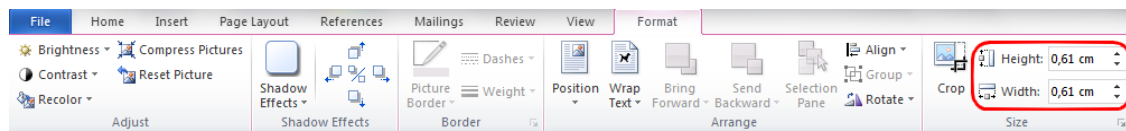
- Select your picture in the appearing dialog window.
Hint: You might have to navigate through the folders in order to find it.
- Clicking the *Picture* button inserts the picture at the position of the cursor.



Reducing the size of pictures

If necessary you can reduce the size of the picture in MS Word:

- Double click the inserted picture.
- Change the height/width of the picture using the *Size* group on the right.

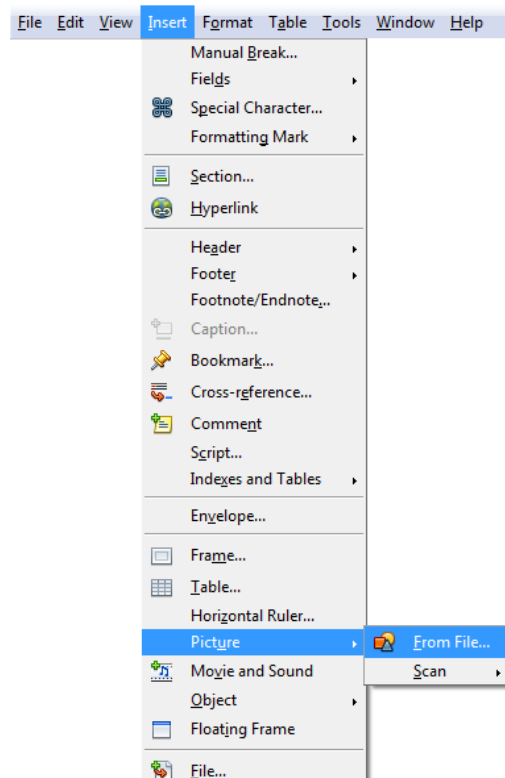


Note: If you change the size of a picture, the scale is modified. If you want to maintain the scale (e.g. for your students to measure lengths) make sure the size of the picture is 100%.

Note: If a picture is too big to fit on one page MS Word will reduce its size automatically and thus, change its scale.

Inserting a picture file into OO Writer

- Open a new text processing document
- From the *Insert* menu select *Picture – From File*.
- Select your picture in the appearing dialog window.
Hint: You might have to navigate through the folders in order to find it.
- Clicking the *Insert* button inserts the picture at the position of the cursor.





12. Creating Dynamic Worksheets

Introduction: GeoGebraTube and User Forum

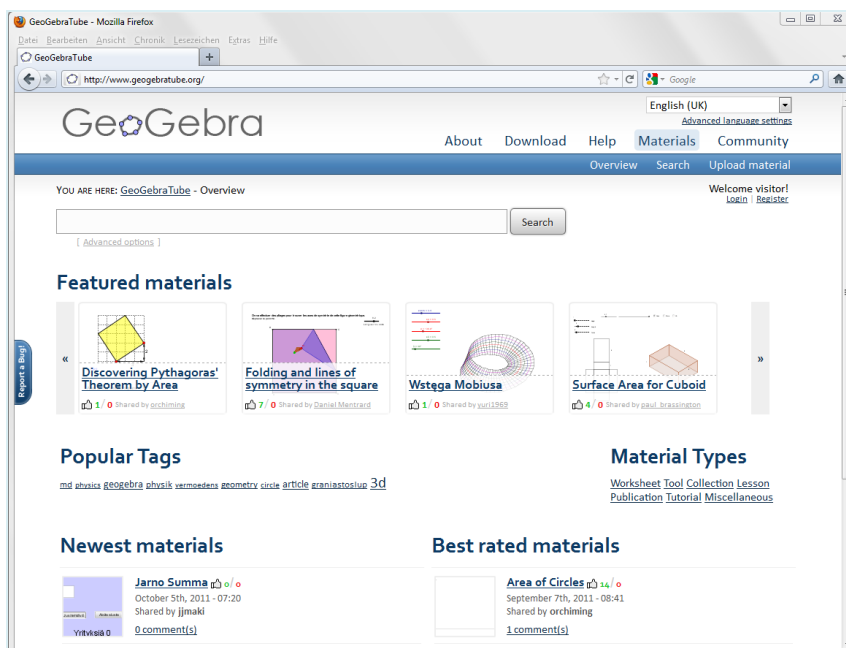
Dynamic Worksheets

GeoGebra allows you to create your own interactive instructional materials, so called *dynamic worksheets*, by exporting dynamic figures into web pages. Usually, a dynamic worksheet consists of a heading, short explanation, interactive applet, as well as tasks and directions for your students.

In order to work with dynamic worksheets your students don't need to know how to operate GeoGebra at all. The interactive web pages are independent of the software and can be provided either online or on a local storage device.

GeoGebraTube

GeoGebraTube (<http://www.geogebra.org/>) is a pool of free instructional materials that were created by people from all over the world. They are grouped in different tags in order to organize their content and make them easier to access.



All materials on GeoGebraTube are under a Creative Common Attribution-Share Alike License (www.geogebra.org/en/cc_license/cc_license.htm). This means that you are allowed to use them for free and that you can create derivative work if you give credit to the original author.



The GeoGebra User Forum

The GeoGebra User Forum (www.geogebra.org/forum) was created to offer additional support for the community of GeoGebra users. Created for teachers and maintained by teachers, it is a platform to pose and answer questions related to GeoGebra.

The screenshot shows the GeoGebra User Forum interface. At the top, there is a navigation menu with links for 'About', 'Download', 'Help', 'Materials', and 'Community'. Below this, there are links for 'Login', 'Register', 'FAQ', and 'Search'. The main content area displays a table of forum topics, organized into sections for different languages: English speaking users, German speaking users, and French speaking users/Utilisateurs francophones. Each section contains a list of topics with columns for 'Forum', 'Topics', 'Posts', and 'Last post'.

Forum	Topics	Posts	Last post
English speaking users			
Using GeoGebra Questions concerning the use of GeoGebra as a stand-alone application	1972	10458	Wed Oct 05, 2011 7:03 am Birgit Lachner ↗
Technological Questions Installation, dynamic worksheets, GeoGebraWiki, JavaScript, etc.	945	4428	Wed Oct 05, 2011 8:03 am rami ↗
German speaking users			
Bedienung von GeoGebra Fragen rund um die Bedienung von GeoGebra als Einzelanwendung	965	3836	Tue Oct 04, 2011 7:44 pm Birgit Lachner ↗
Technische Fragen Installation, dynamische Arbeitsblätter, GeoGebraWiki, JavaScript usw.	422	1622	Sat Oct 01, 2011 7:28 pm Birgit Lachner ↗
French speaking users/Utilisateurs francophones			
Français Version 3.2 Forum pour les utilisateurs de GeoGebra qui parlent français Moderators: Noel Lambert , miir	1399	9184	Tue Oct 04, 2011 3:30 pm Daniel Mentrad ↗
Français Version 4.0 Messages relatifs à la version 4.0 Moderators: Noel Lambert , miir	188	1104	Mon Oct 03, 2011 6:39 pm miir ↗
Français Version beta 4.2 (avec CAS) Moderator: miir	13	43	Tue Oct 04, 2011 6:01 pm Noel Lambert ↗

The GeoGebra User Forum consists of several discussion boards in different languages allowing users to post and answer their GeoGebra related questions in their first language.

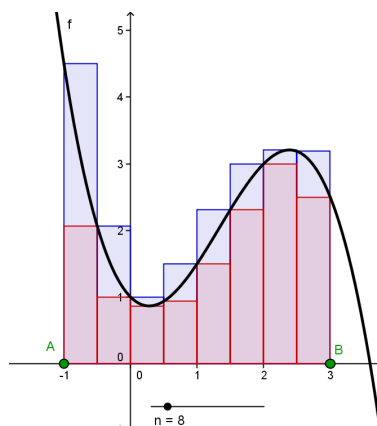


Activity 19a: Creating Dynamic Worksheets

In this activity you will learn how to create a dynamic worksheet that illustrates how lower and upper sums can be used to approximate the area between a function and the x -axis.

Preparations

- Open a new GeoGebra window.
- Show the algebra view, input bar and coordinate axes (*View menu*).



Construction Steps


1		Enter the cubic polynomial $f(x) = -0.5x^3 + 2x^2 - x + 1$
2		Create two points A and B on the x -axis <u>Hint</u> : These points will determine the interval.
3		Create a slider for the number n (interval 1 to 50; increment 1)
4		Create <code>uppersum = uppersum[f, x(A), x(B), n]</code> <u>Hint</u> : <code>x(A)</code> gives you the x -coordinate of point A .
5		Create <code>lowersum = lowersum[f, x(A), x(B), n]</code>
6	ABC	Insert dynamic text <code>Upper Sum =</code> and select <code>uppersum</code> from <i>Objects</i>
7	ABC	Insert dynamic text <code>Lower Sum =</code> and select <code>lowersum</code> from <i>Objects</i>
8		Calculate the difference <code>diff = uppersum - lowersum</code>
9	ABC	Insert dynamic text <code>Difference =</code> and select <code>diff</code> from <i>Objects</i> <u>Hint</u> : Fix the slider and the text using the <i>Properties dialog</i> .

Task: Use slider n in order to modify the number of rectangles used to calculate the lower and upper sum. What happens to the difference of the upper and lower sum (a) if n is small (b) if n is big?



Reducing the Size of the GeoGebra Window

GeoGebra will export the algebra and graphics view into the dynamic figure of the worksheet. In order to save space for explanations and tasks on the dynamic worksheet you need to make the GeoGebra window smaller prior to the export.

- If you don't want to include the algebra view you need to hide it prior to the export.
- Move your figure (or the relevant section) to the upper left corner of the graphics view using the  *Move Graphics View* tool (see left figure below).

Hint: You might want to use tools  *Zoom in* and  *Zoom out* in order to prepare your figure for the export process.

- Reduce the size of the GeoGebra window by dragging its lower right corner with the mouse (see right figure below).

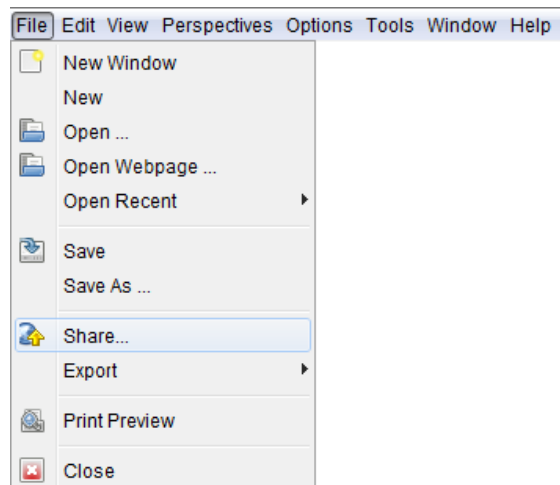
Hint: The pointer will change its shape when hovering above an edge or corner of the GeoGebra window.

Note: Although the interactive applet should fit on one screen and even leave some space for text on the worksheet you need to make sure that it is big enough to allow students manipulations and experiments.

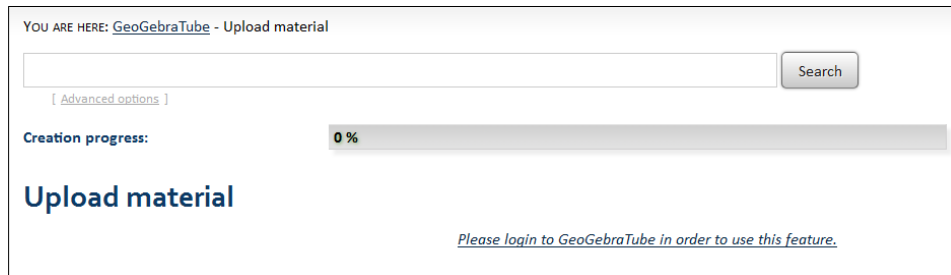
Upload to GeoGebraTube

After adjusting the size of the GeoGebra window, you are now ready to export the figure as a dynamic worksheet using the *File* menu.

- *File – Share...*



- The *GeoGebraTube* website opens automatically where you have to login (or register if you do not have an account yet) before you are able to continue your upload.



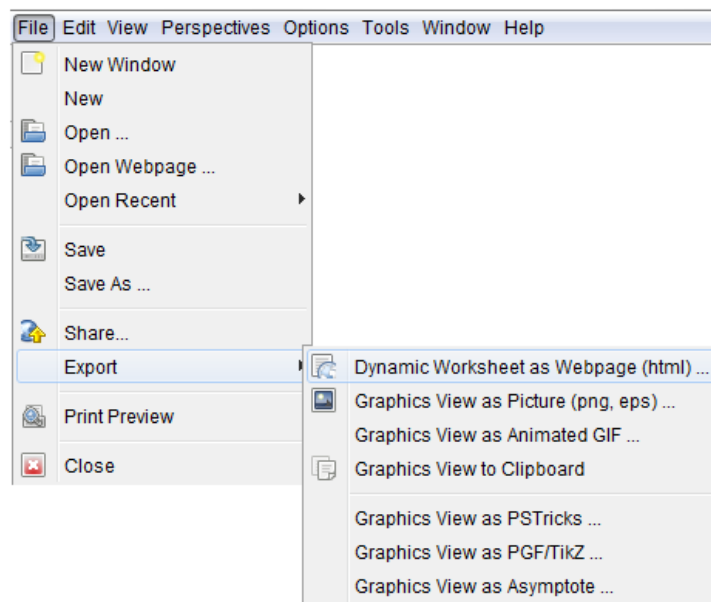
- Fill in the information for your students. If you want, you can also select to show the toolbar, the inputbar or the menubar. Click *Continue*.
- Type a short explanation for other teachers, so that they are able to use your materials, too. This information is not shown on the student worksheet. Choose a target group and select tags that describe your material to help others with searching.
- Finish your Upload with the Save button.

Your worksheet is now saved on GeoGebraTube where people are able to like/dislike the material or write comments.

Exporting a Dynamic Worksheet to a Webpage (for Advanced Users)

Instead of uploading to GeoGebraTube it is possible to export your dynamic worksheet to a webpage.

- *Export – Dynamic Worksheet as Webpage*
Hint: You could also use the key combination *Ctrl – Shift – W*.





- Fill in the text fields in the appearing window in the *Export as Webpage* Tab (title of the worksheet, name of the author, and date).
- Type a short explanation of the dynamic figure into the text field *Text above the construction*.
- Enter tasks and directions for students into the text field *Text below the construction*.
- Click *Export* and save your dynamic worksheet.
Hint: GeoGebra will create several files which always need to stay together in order to maintain the functionality of the dynamic worksheet. We recommend creation of a new folder (e.g. *Dynamic_Worksheets*) within the *GeoGebra_Introduction* folder prior to saving your dynamic worksheet.

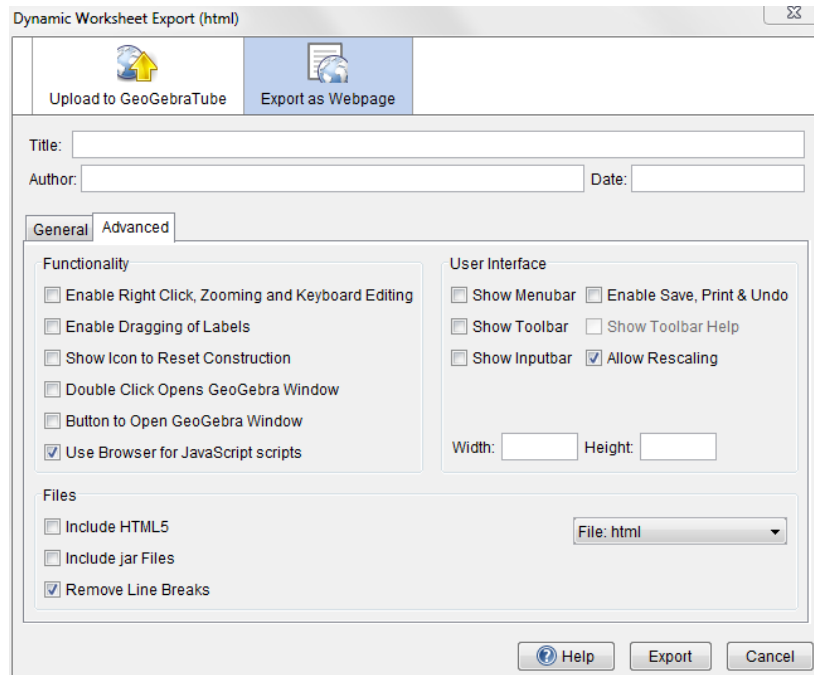
Tips and Tricks for Creating Dynamic Worksheets

- After saving the dynamic worksheet it will be automatically opened up in your web browser. Check the text you inserted as well as the functionality of the interactive applet. If you want to change your dynamic worksheet go back to the GeoGebra file and make your changes to the figure. Export the figure again (you can use the same file name to overwrite the old worksheet) in order to apply your changes.
Hint: You can change the text of the dynamic worksheet in the same way.
- GeoGebra automatically saves your entries in the export window for dynamic worksheets. If you want to make changes to your figure while filling in the export dialog you can just close it and continue later on.
- Make sure your applet is not too big. Your students shouldn't have to scroll between the tasks and the figure because this makes learning more difficult.
- Your dynamic worksheet should fit on one screen. If you want to include more than 3 tasks you should consider creation of another worksheet that includes the same dynamic figure but different tasks.



Activity 19b: Enhancing Dynamic Worksheets

The export dialog window for *Export as Webpage* consists of two tabs: *General* and *Advanced*. In the last activity you used tab *General* in order to add explanations, tasks and directions to the dynamic figure prior to the export. You will now learn how to enhance your dynamic worksheet by including different features in the interactive figure using the tab *Advanced*.



Functionality

- Enable right click, zooming and keyboard editing features: Your students will be able to right click objects or the graphics view in order to access the features of the context menu (e.g. show / hide object or label, trace on / off, *Properties dialog*). It is also possible to use the common keyboard shortcuts.
- Enable dragging of labels: By selecting this feature students are able to drag labels of points or objects.
- Show icon to reset construction: A reset icon is displayed in the upper right corner of the interactive applet allowing your students to reset the interactive figure to its initial state.
- Double click opens application window: Your students will be able to open a full GeoGebra window by double clicking the interactive applet.
- Button to open application window: A button is displayed to open the GeoGebra window directly.
- Use Browser for JavaScript Scripts: JavaScript Scripts are enabled. This allows your browser to show your construction properly.



User Interface

- Show menubar: The menubar is displayed within the interactive applet.
- Enable save, print & undo: It is possible to provide your students features for saving, printing and undoing the construction.
- Show toolbar: The toolbar is displayed within the interactive applet allowing your students to use the geometry tools.
- Show toolbar help: In combination with the toolbar, you can also display the toolbar help within the interactive applet. If you want your students to use geometry tools they can use the toolbar help in order to find out how to operate the different tools on their own.
- Show inputbar: The input bar is displayed at the bottom of the interactive applet allowing your students to use algebraic input and commands for their explorations.
- Allow rescaling: Affects the behavior of zooming within the browser, in order that the construction is rescaled, too.
- Width and height of the interactive applet: You can modify the width and height of the interactive applet.
Note: If you reduce the size of the applet important parts of the dynamic worksheets might be invisible for your students.
Hint: If you include the menubar, toolbar, or input bar you might want to adjust the height of the interactive applet.

Files

- Include HTML 5: Includes HTML 5 into your exported webpage.
- Include *.jar files: Creates not only the html file, but also the *.jar files.
- Remove Line Breaks: removes line breaks if there are any
- File-Dropdown: You can choose if the export-file is html, MediaWiki, GoogleGadget or Moodle.
- Single File: A dynamic webpage will be created using a single file.
- Single File (Tabs): More than one worksheet can be displayed in the browser. Your students are able to navigate between the different tasks by using tabs.
- Linked Files: Creates linked dynamic worksheets and provides *Next* and *Back* buttons to work on more than one exercise.
Hint: For creating several interactive applets it is necessary to create more than one construction using *File - New Window* or *Ctrl-N*.

Task

Use the dynamic figure you created in activity 18 and export it as an enhanced dynamic worksheet. Use the *Advanced* tab to try out different options and check how the applet of your dynamic worksheet is changed accordingly.



Activity 19c: Providing Dynamic Worksheets to Students

You can provide your dynamic worksheets in several ways to your students. However, in all cases it is very important to keep all the files together which were created during the export process.

Note: The files created have different file name extensions (*.ggb*, *.html*, *.jar*). If one of these files is missing your dynamic worksheet won't function any more.

Local Storage Device

Copy all files into the same folder before saving this folder on a local storage device (e.g. flash drive, CD). Have your students copy the whole folder on their computers. Your students will have to open the file with the name extension *.html* in their Internet browser.



13. Practice Block IV

This practice block consists of a pool of activities to practice exporting GeoGebra constructions as picture files and creating dynamic worksheets. Activities of two different difficulty levels are available: *Basic tasks* and *Advanced tasks*. You can pick tasks of your interest and work on them either on your own or together with a colleague.

Tips and Tricks




- For each of these activities, open a new GeoGebra file. Check prior to beginning the activity if you need to show or hide the algebra view, input bar and coordinate axes.
- You might want to save your files before you start a new activity.
- Don't forget about the *Undo* and *Redo* buttons in case you make a mistake.
- Frequently use the *Move* tool in order to check your dynamic figures (e.g. did you create any unnecessary objects, are sliders and text fixed so they can't be moved accidentally).
- Make sure, you know how to use corresponding geometry tools before you begin the construction. If you don't know how to operate a certain tool, activate it and read the toolbar help.
- Make sure you know the syntax to enter algebraic expressions and functions. Read through the *Tips and Tricks* section in the beginning of topic *Basic Algebraic Input, Commands, and Functions*. If you have trouble, ask colleagues for help.
- Carefully check algebraic input before hitting the *Enter* key. If an error message appears, read it! It might help you to fix the problem.
- Make sure to try out your dynamic worksheets in order to check for mistakes and fix them if necessary.
- Always keep the files for a dynamic worksheet together in a folder (files with the extensions *.ggb*, *.html*, *.jar*).
- You can save several dynamic worksheets into the same folder.
Note: The files with the extension *.jar* are just created once in this folder. If you want to provide one of the dynamic worksheets to your students you need to copy the *jar*-files in addition to the corresponding *ggb*- and *html*-files.
- If you have questions, please ask a colleague before you address the presenter or assistant(s).



Activity IV.a: Area Relations of Similar Geometric Figures

Classification: Basic task

In this activity you are going to use the following tools and algebraic input. Make sure you know how to use each tool before you begin.

$a = 2$  Segment With Given Length New!	 Regular Polygon  Move
--	---

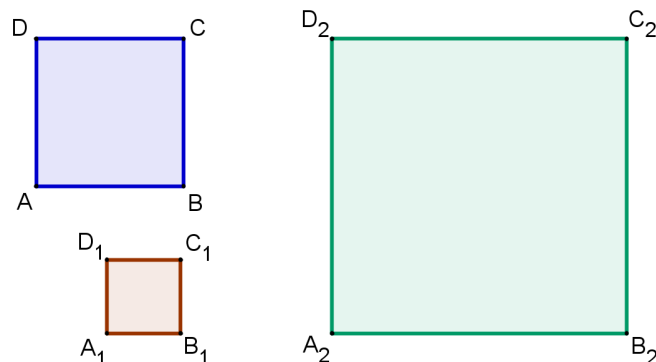
Task

In this activity you will recreate the following worksheet for your students. It allows them to discover the special relationship between the area of squares whose side lengths are a , $a/2$, and $2a$.

Area Relations

1. Measure the side length of the three squares below. Compare the side length of the blue square to the side lengths of the red and green squares. Which relation can you find?
2. Calculate the areas of the three squares. Compare the area of the blue square to the areas of the red and green squares. Which relation can you find?
3. Formulate a conjecture that compares the side length and area of the blue square to these of the red and green squares.
4. Try to prove your conjecture.

Hints: Assume that the side length of the blue square is a and calculate the areas of the corresponding squares.





Construction Steps

1. Start your construction in GeoGebra by creating the number $a = 2$.
2. Construct the blue square starting with *Segment with given length a*. Then, use the two endpoints of this segment to create a regular polygon with 4 vertices.
3. In the same way construct the red square with side length $a/2$ and the green square with side length $2a$.
4. Rename the vertices and change the properties of the squares (e.g. color, line thickness).
5. Prepare the GeoGebra window for the export of the graphics view as a picture (e.g. rearrange the squares, reduce the size of the GeoGebra window).
6. Export the graphics view as a picture and save your picture file.
Hint: Since your students are supposed to measure the side length of the squares you should not change the scale of the picture!
7. Open a word processing document and type in the heading and tasks of the worksheet.
8. Insert the picture of the squares into the worksheet.
Hint: Make a printout of the worksheet and try it out by measuring the side lengths of the squares.

Challenge 1

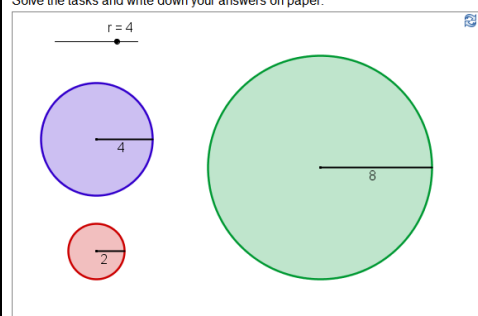
Create similar examples for different geometric shapes (e.g. circle with given radius, equilateral triangle, rectangle). For which of these shapes do the same relations between the length of the given side (radius) and the resulting area apply? Try to find an explanation for this relationship between the given length and the area of the figure.

Challenge 2

Create a dynamic worksheet based on your construction that helps your students to generalize their conjecture about the relationship between the side length and the area of such geometric shapes (for example, see [A 4a Area Circles.html](#)).

Relationship Circle Areas

Below you can see three circles with the radii r , $r/2$, and $2r$.
Solve the tasks and write down your answers on paper.



Task 1
Calculate the areas of the circles for $r = 1$. Come up with a conjecture about the relationship between the length of the radius and the area of the three circles.

Task 2
Use the slider to change the radius r . Again, calculate the areas of the circles and check your conjecture about the relationship.

Task 3
Proof that your conjecture is true for all circles whose radii are r , $r/2$, and $2r$.
Hint: Express the areas of these circles and compare them.

Task 1
Calculate the areas of the circles for $r = 1$. Come up with a conjecture about the relationship between the length of the radius and the area of the three circles.

Task 2
Use the slider to change the radius r . Again, calculate the areas of the circles and check your conjecture about the relationship.

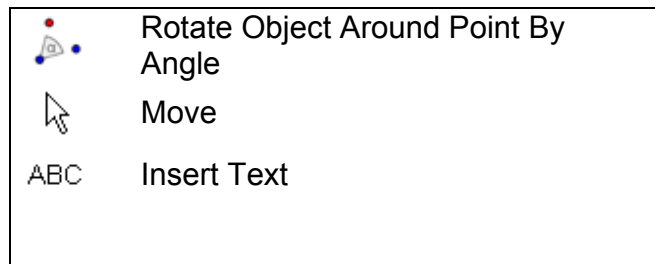
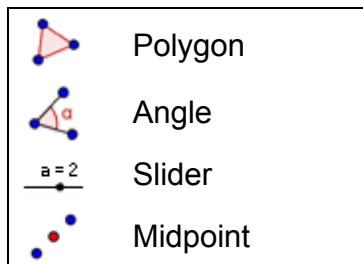
Task 3
Proof that your conjecture is true for all circles whose radii are r , $r/2$, and $2r$.
Hint: Express the areas of these circles and compare them.



Activity IV.b: Visualizing the Angle Sum in a Triangle

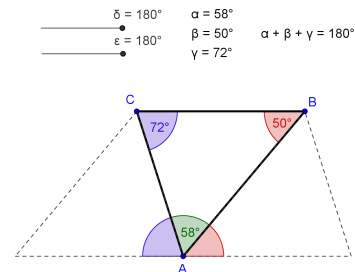
Classification: Basic task

In this activity you are going to use the following tools. Make sure you know how to use each tool before you begin.



Construction Steps

1. Create a triangle ABC .
Hint: Use counterclockwise orientation.
2. Create the angles α , β , and γ of triangle ABC .
3. Set the number of decimal places to 0 (*Options* menu).
4. Create sliders δ and ϵ with the settings *angle* (type); 0° to 180° (interval); 10° (increment).
5. Create midpoint D of segment AC and midpoint E of segment AB .
6. Rotate the triangle around point D by angle δ (setting *clockwise*).
7. Rotate the triangle around point E by angle ϵ (setting *counterclockwise*).
8. Move both sliders to show 180° before you create the angles ζ ($A'C'B'$) and η ($C'B'A'_1$).
9. Enhance your construction using the properties dialog.
Hint: Congruent angles should have the same color.



Challenge 1

Insert dynamic text showing that the interior angles add up to 180° .

Hint: Create dynamic text for the interior angles (e.g. $\alpha =$ and select α from *Objects*), calculate the angle sum using $\text{sum} = \alpha + \beta + \gamma$ and insert the sum as a dynamic text.

Match colors of corresponding angles and text. Fix the text in the graphics view.

Challenge 2

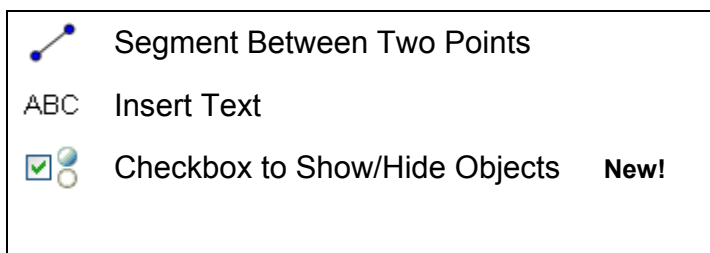
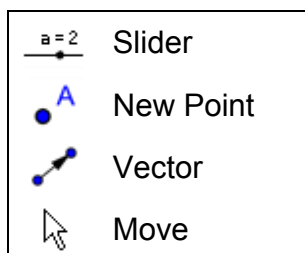
Export the figure to a dynamic worksheet. Come up with instructions that guide your students towards discovering the angle sum in a triangle. Have them check their conjecture using the provided worksheet.



Activity IV.c: Visualizing Integer Addition on the Number Line

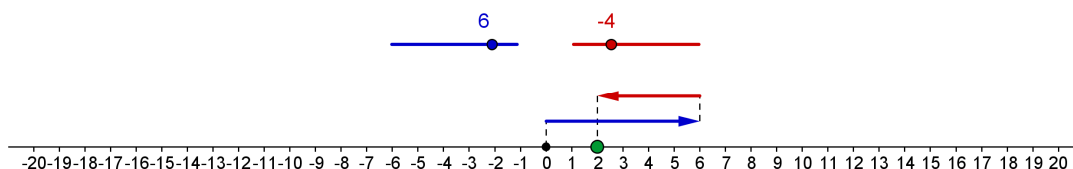
Classification: Advanced task

In this activity you can either use the following tools or corresponding commands. Make sure you know how to use them before you begin.



Construction Steps

1. Open a new GeoGebra window and hide the algebra view. Set the labeling option to *All new objects* (*Options* menu).
2. Open the *Properties dialog for the Graphics View*. On tab *yAxis*, uncheck *Show yAxis*. On tab *xAxis*, set the distance of tick marks to 1 by checking the box *Distance* and entering 1 into the text field. On tab *Basic* set the *minimum* of the *x-Axis* to -21 and the *maximum* to 21.
3. Create sliders *a* and *b* (*interval* -10 to 10; *increment* 1). Show the value of the sliders instead of their names (*Properties dialog*).
4. Create points $A = (0, 1)$ and $B = A + (a, 0)$.
5. Create vector $u = \text{Vector}[A, B]$ which has the length *a*.
6. Create points $C = B + (0, 1)$ and $D = C + (b, 0)$ as well as vector $v = \text{Vector}[C, D]$ which has the length *b*.
7. Create point $R = (x(D), 0)$.
Hint: $x(D)$ gives you the *x*-coordinate of point *D*. Thus, point *R* shows the result of the addition.
8. Create point $Z = (0, 0)$ as well as the following segments:
 $g = \text{Segment}[Z, A]$, $h = \text{Segment}[B, C]$, $i = \text{Segment}[D, R]$.
9. Use the *Properties dialog* to enhance your construction (e.g. change color, line style, fix sliders, hide labels).

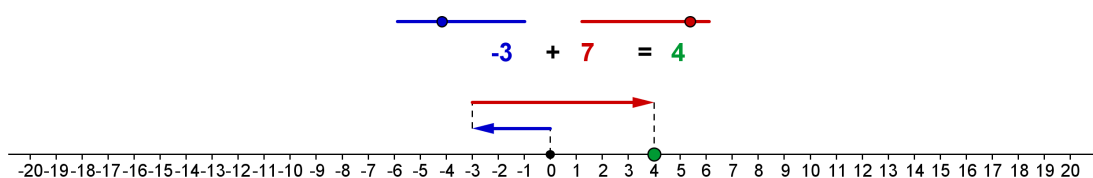




Challenge 1

Enhance your interactive figure by inserting dynamic text that displays the corresponding addition problem.

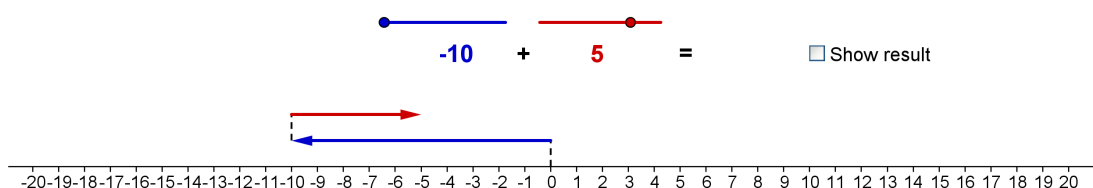
10. Calculate the result of the addition problem: $r = a + b$
11. In order to display the parts of the addition problem in different colors you need to insert the dynamic text step by step.
 - a. Insert *text1*: Select *a* from Objects
 - b. Insert *text2*: +
 - c. Insert *text3*: Select *b* from Objects
 - d. Insert *text4*: =
 - e. Insert *text5*: Select *r* from Objects
12. Match the color of *text1*, *text3*, and *text5* with the color of the corresponding sliders and point *R*. Hide the labels of the sliders and fix the text (*Properties dialog*).
13. Export the interactive figure as a dynamic worksheet.



Challenge 2

Insert a checkbox into the graphics view that allows you to show or hide the result of the addition problem (*text5*, point *R*, and segment *i*).

14. Activate tool *Checkbox* to show and hide objects.
15. Click on the graphics view next to the result of the addition problem.
16. Enter `Show result` into the *Caption* text field.
17. From the drop down menu successively select all objects whose visibility should be controlled by the checkbox (*text5*, point *R*, and segment *i*).
18. Click *Apply* to create the checkbox.
19. In *Move* mode check and uncheck the checkbox to try out if all three objects can be hidden / shown.
20. Fix the checkbox so it can't be moved accidentally any more (*Properties dialog*).
21. Export this new interactive figure as a dynamic worksheet.
Hint: You might want to use a different name for this worksheet.



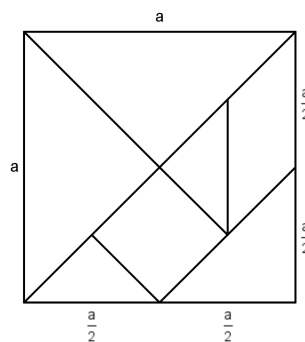


Activity IV.d: Creating a 'Tangram' Puzzle

Classification: Advanced task

In this activity you will create the 'Tangram' puzzle shown at the right hand side. It consists of 7 geometric shapes which can all be constructed using the side length a (see [A 4d tangram puzzle.html](#)).

For these constructions you will need a selection of geometry tools. Please read through the following hints before you actually start to create the geometric shapes.



1. Enter the number $a = 6$. It will provide a basis for the construction of all triangles and quadrilaterals necessary for a 'Tangram' puzzle.
2. Try to figure out the side lengths of the geometric shapes.
Hint: In some cases you might want to look at the diagonals or height. Their lengths can be expressed more easily using the variable a than the lengths of the corresponding sides.
3. Begin each of the geometric figures using a segment with given length. This will allow you to drag and rotate the figure later on.
4. Construction hints:
 - a. If the height of a right triangle is half the length of the hypotenuse you might want to use the theorem of Thales for the construction (see practice block 1).
 - b. If you know the legs of a right triangle you might want to construct it similar to the square construction presented earlier.
 - c. For constructing a square using its diagonals, it is helpful to know that they are perpendicular and bisect each other.
 - d. For constructing the parallelogram it is helpful to know the size of the acute angle.
5. Check your construction by trying out if you can manage to create a square with side length a using all figures.

Challenge 1: Arrange the geometric shapes arbitrarily around the edges of the interactive applet. Export the figure to a dynamic worksheet and add an explanation for your students (see dynamic worksheet [A 4d tangram puzzle.html](#)).

Challenge 2: With these geometric shapes other figures than a square can be created as well (see [A 4d tangram puzzle cat.html](#)). Search the Internet for a 'Tangram' figure other than a square (e.g. [A 4d tangram cat.png](#)) and import this figure into the graphics view. Export the GeoGebra construction again using a different name and different instructions.

